Grade 8 MTAP 2015 Elimination Questions with Solutions – Part 1

Below is the first part of the Grade 8 MTAP 2015Elimination Questions with Solutions and answers. If you find any errors, please comment on the box below.
1.) Find the average of the numbers -1, 3/2, and 1/2.

Solution
$(-1 + 3/2 + 1/2)/3 = 1/3$
Answer: 1/3
2.) How much larger is 2/3 than 1/6?

Solution
$2/3 - 1/6 = 4/6 - 1/6 = 3/6 = 1/2$
Answer: 1/2
3.) If one ream contains 500 sheets of paper and a sheet of paper is 0.3 mm thick, how thick is one ream in meters?

Solution
$500 \times 0.3\text{mm} = 150\text{ mm} = 0.15\text{m}$
Answer: 0.15m
4.) What is the second largest number among numbers $\sqrt{2}$, 3/2 , 1.4, $\sqrt{3}$ and 1.6?

Solution
$\sqrt{2}$ is around 1.41 and $\sqrt{3}$ is around 1.7.
Answer: 1.6

5.) If an inch is about 2.54 cm, what is 1 cm to the nearest hundredth of an inch?

Solution
$1 : 2.54 + = + x : + 1, + x + = + 0.39$

Answer: 0.39
6.) If $U = \{1, a, 2, b, 3, c, 4, d\}$ and $A = \{1, 2, c, d\}$, what is A^c?
Solution
A^c is the complement of A, or the elements of the set that is not in A but in U. So,
$A^c = \{a, b, 3, 4\}$
Answer: $A^c = \{a, b, 3, 4\}$
7.) Using the same sets in Item 6 and $B = \{1, 2, 3, 4\}$, how many subsets does $A \cap B^c$ have?
Solution
$A = \{1, 2, c, d\}$ and $B = \{1, 2, 3, 4\}$. The complement of B denoted by B^c are the elements of U not in B. So, $B^c = \{a, b, c, d\}$. Now, $A \cap B^c$ are the elements that are common to A and B^c. Therefore, $A \cap B^c = \{c, d\}$. Now, the number of subsets of a set with n elements is 2^n (this includes the empty set), so there are $2^2 = 4$ subsets.
Answer: 4
8.) If $|P| = 10$, $|Q| = 12$, and $|P\ Q| = 15$, what is $|P \cap Q|$?

Solution 1
We know that the cardinality of the union of two sets is equal to the sum of the cardinality of these sets less the cardinality of their intersection. That is, if we have sets P and Q, $|P \cup Q| = |P| + |Q| - |P \cap Q|$.

Substituting, we have
$15 = 10 + 22 - |P \cap Q|$
$|P \cap Q| = 7$

Answer: 7

Solution 2

x + y + z = 15 ()

x + y = 12 ()

y + z = 10 ()

Adding the (**) and (***), we have x + 2y + z = 22 (#)

Subtracting (*) from (#),

x + 2y + z – (x + y + z) = 22 – 15

y = 7

Answer: 7

9.) If |M ∩ N| = 24 and |M ∪ N| = 26, what is |M| + |N|?

Solution

From number 8, we know that |M ∪ N| = |M| + |N| – |M ∩ N|. Substituting, we have,

26 = |M| + |N| – 24

|M| + |N| = 50.

Answer: 50

10.) There were 59 participants during the recent math camp. Among them, 37 liked doing projects, 30 liked solving problems, and 13 liked both. How many of the participants did not like at least one of these two activities?

Solution

Solution will be discussed in a separate post.

Answer: 5

11.) If $x+=+4$ and $y+=+-3$, what is $x^2y+++xy^2$?

Solution

$x^2y+++xy^2+=+(4^2)+(-3)+++(4)+(-3)^2+=+(16)+(-3)+++4(9)$

$=+(-48)+++36+=+-12$

Answer: -12

12.) Simplify $x(1+++y)+-+2y(x+-+2)+++xy$.

Solution

$x(1+++y)+-+2y+(x+-+2)+++xy$

$=+x+++xy+-+2xy+++4y+++xy$

$=+x+++2xy+-+2xy+++4y$

$=+x+++4y$

Answer: $x+++4y$

13.) If a and b are positive constants, simplify $\dfrac{ab+\sqrt{ab}}{\sqrt[3]{a^4}+\sqrt[4]{b^3}}$.

Solution

Note that $\sqrt{+}ab+=+(ab)^{1/2}$, $\sqrt{+}[3]a^4+=+a^{4/3}$ and $\sqrt{+}[4]+b^3+=+b^{3/4}$

Now $\frac{+}{+(ab)+(ab)^{1/2}}\perp+a^{4/3}+b^{3/4}$

$=+\frac{+(ab)+(a^{1/2})+(b^{1/2})+}{+}a^{4/3}+b^{3/4}$

$=\frac{+}{a^{3/2}+b^{3/2}}+a^{4/3}+b^{3/4}$

$=+a^{(3/2+-+4/3)}+b^{(3/2+-+3/4)}$

$=+a^{(9/6+-+8/6)}+b^{(6/4+-3/4)}$

$=+a^{1/6}b^{3/4}$

This is already correct, but if you want your answer in radical form, the previous expression can be converted to

$$a^{2/12}b^{9/12}+=+\sqrt{+}[12]+a^2b^9$$

Answer: $a^{1/6}b^{3/4}$ or $\sqrt{+}[12]+a^2b^9$

14.) What is the quotient when $6x^4+++x^3+++4x^2+++x+++2$ is divided by $3x^2+-+x+++1$?

Solution

$$\begin{array}{r} 2x^2 + x + 1 \\ 3x^2 - x + 1 \overline{)\,6x^4 + x^3 + 4x^2 + x + 2} \\ \underline{6x^4 - 2x^3 + 2x^2} \\ 3x^3 + 2x^2 + x \\ \underline{3x^3 - x^2 + x} \\ 3x^2 + 0x + 2 \\ \underline{3x^2 - x + 1} \\ x + 1 \end{array}$$

Answer: $2x^2+++x+++1$ remainder $x+++1$.

15.) In Item 14, what is the remainder?

Answer: $x+++1$

16.) If $A+++B+=+x+-+2y$, what is $A^2+++2AB+++B^2+++4xy$?

Solution

$(A+++B)^2+=+(x-2y)^2$

$A^2+++2AB+++B^2+=+x^2+-+4xy+++4y^2$

$A^2+++2AB+++B^2+++4xy+=+x^2+-+4xy+++4xy+++4y^2$

$A^2+++2AB+++B^2+++4xy+=+x^2+++4y^2$

Answer: x^2+++4y^2

17.) If $x+++y+=+7$ and $xy+=+5$, what is x^3+++y^3?

Solution

$x+++y+=+7$ and $xy+=+5$

$(x+++y)^3+=+7^3$

$$x^3+++3x^2y+++3xy^2+++y^3+=+343$$
$$x^3+++3xy+(x+++y)+++y^3+=+343$$

Substituting the given values above,

$$x^3+++3+(5)(7)+++y^3+=+343$$
$$x^3+++105+++y^3+=+343$$
$$x^3+++y^3+=+238.$$

Answer: 238

18.) If the length, width, and height of an open-top rectangular box are $(x+++3)$ cm, x cm, and $(x+-+3)$ cm, what is its surface area?

Solution

The formula for finding the surface area S of a rectangular prism with length l, width w and height h is $S+=+12lh+++2lw+++2wh$. Since the box is open, we subtract lw, which is the top face. So, the surface area of the open box is $S+=+2lh+++lw+++2wh$.
Substituting, we have

$$S+=+2(x+++3)+(x+-+3)+++x(x+++3)+++2x(x+-+3)$$
$$=+2(x^2+-+9)+++x^2+++3x+++2x^2+-+6x$$
$$=+2x^2+-+18+++x^2+++3x+++2x^2+-+6x$$
$$=15x^2+-+3x+-+18$$

Answer: $15x^2+-+3x+-+18$

19.) A man walked x km for 2.5 hrs, then jogged $(2x+++3)$ km for 3.5 hrs, and finally walked again $(5x+-+3)$ km for 4 hrs. If his average speed for the entire exercise was 4 kph, what is x?

Solution

$$\frac{2.5x+++3.5(2x+++3)+++4(5x+-3)}{x+++2x+++5x}+=+4$$
$$\frac{29.5x+-+1.5}{8x}+=+4$$
$$29.5x+-+1.5+=+32x$$
$$2.5x+=+1.5$$
$$x+=+3/5+=+0.6$$

Answer: 3/5 or 0.6

20.) Simplify $(a+-+3)+(a+++3)+(a^2+++3a+++9)+(a^2+-+3a+++9)$.

Solution

We can group the expressions as sum and difference of two cubes.

$$[(a+-+3)+(a^2+++3a+++9)]+[(a+++3)+(a^2+-+3a+++9)]$$
$$=+(a^3+-+3^3)+(a^3+++3^3)$$
$$=+(a^3+-+27)+(a^3+++27)$$

Now, this is in the form of the difference of two squares $(x+++y)+(x+-+y)+=+x^2+-+y^2$.
So, $(a^3+-+27)+(a^3+++27)+=+(a^3)^2+-+27^2+=+a^6+-+729$

Answer: $a^6 + - + 729$

21.) When $P + (x)$ is divided by $x^2 + + + 2$, the quotient and remainder are both x. What is $P(-1)$?
Solution
If we divide 5 by 3, then we get a quotient of 1 and a remainder of 2. If we generalize latex a the dividend, $b$as the divisor, q as the quotient and r as the remainder, we can form the equation

$$a + = + bq + + + r \ (*).$$

In the given, we can see that $a + = + P(x)$, $b + = + x^2 + + + 2$, $q + = + x$ and $r + = + x$.

Substituting in *, $P(x) + = + (x^2 + + + 2) + (x) + + + x$
$= + x^3 + + + 2x + + + x$
$P(x) + = + x^3 + + + 3x$

Now $P(-1) + = + (-1)^3 + + + 3(-1) + = + - 4$.

Answer: -4
22.) If two more than twice *p* is four less than twice *q*, express *q* in terms of *p*.
Solution
$2 + + + 2p + = + 2q + - + 4$
$2q + = + 2p + + + 2 + + + 4$
$2q + = + 2p + + + 6$
$q + = + \frac{+}{2p + + + 6} 2$
$q + = + p + + + 3$
Answer: $q + = + p + + + 3$
23.) If x is nonnegative and $3x - 4 \leq x$, what is the least value of x?
Solution
$3x - 4 \leq x$
$3x - x \leq 4$
$2x \leq 4$
$x \leq 2$
So the lowest non-negative value is $x = 0$.
Answer: $x = 0$
24.) In Item 23, what is the maximum value of x?
Answer: $x = 2$ (Obvious!)
25.) Solve for x in the equation $x + + + 2(+ x + + 1) + + + 3 + (x + + + 1) + + + \cdots + + + 10 + (x + + + 1) + = + 110$.
Solution
$x + + + 2(x + + + 1) + + + 3(x + + + 1) + + + \cdots + + + 10(x + + + 1) + = + 110$
$x + + + (x + + + 1) + (2 + + + 3 + + + \cdots + + + 10) + = + 110$
$x + + + (x + + + 1) + (54) + = + 110$
$x + + + 54x + + + 54 + = + 110$
$55x + + + 54 + = + 110$
$55x + = + 110 + - + 54 + = + 56$
$x + = + \frac{+}{56} 55$

Answer: 56/55
26.) Solve for x in the equation $|2x + - + 3| + = + 5$.
There are two values for x:

$2x + - + 3 + = +5$
$2x + = +8$
$x + = +4$

Also,

$-(2x + - + 3) + = +5$
$-2x + + + 3 + = +5$
$-2x + = +5 + - + 3$
$-2x + = +2$
$x + = + \frac{+}{2} - 2$
$= + - 1.$

Answer: -1 and 4
27.) If ∠B is the complement of ∠A, and the supplement of ∠A is 138°, what is ∠B – ∠A?
Solution
Complementary angles add up to 90° and supplementary angles add up to 180°. So,
∠A + 138 = 180
∠A = 42.
Now since ∠A and ∠B are complementary,
∠A + ∠B = 90
42 + ∠B = 90
∠B = 90 – 42
∠B = 48.

Now, ∠B – ∠A = 48 – 42 = 6

Answer: 6°
28.) If the diagonals of rectangle ABCD meet E and ∠AEB = 140°, what is ∠EBC?
Solution
∠A + ∠B + ∠E = 180°
∠A + ∠B + 140 = 180°
∠A + ∠B = 40°

Since ABE is an isosceles triangle, ∠A = ∠B. This means that ∠A = ∠B = 20°.
Clearly, ∠ABC = 90°. So,
∠ABE + ∠EBC = 90°
20° + ∠EBC = 90°
∠EBC = 90 – 20°
∠EBC = 70°

Answer: 70°
29.) If two exterior angles of a triangle measures 80° and 130°, what is its smallest interior angle?
Solution
The adjacent interior and exterior angles of a triangle are supplementary. Therefore, the largest exterior angle has the smallest adjacent interior angle.

The sum of the exterior angle of any polygon is 360°so,
$80 + + + 130 + + + x + = +360$
$210 + + + x + = +360$
$x + = +360 + - + 210$
$x + = +150$

Among the three angles, 150° is the largest and its adjacent interior angle is 30°.

Answer: 30°

30.) In a grouped data, if 100 – 200 forms one class, what is the class interval of the data?

Answer:

Solution

The class interval is the difference between the upper class limit and the lower class limit of a class. Here, the upper class limit is 200 and the lower class limit is 100. So, the class interval is 200 – 100 = 100.

Answer: 100

31.) Factor completely: $a^2c^2 + + + b^2d^2 + - + a^2d^2 + - + b^2c^2$.

Solution

Rearranging the terms, we obtain

$a^2c^2 + - + a^2d^2 + - + b^2c^2 + + + b^2d^2$.

Factoring by grouping, we have

$a^2(c^2 + - + d^2) + - + b^2(c^2 + - + d^2)$

$= +(c^2 + - + d^2)(a^2 + - + b^2)$

$= +(c + - + d)(c + + + d)(a + - + b)(a + + + b)$.

Answer: (a + b)(a – b)(c + d)(c – d)

32.) Simplify $\dfrac{x^3 - +4x}{x^3 - x^2 - 6x}$.

Solution

$$\frac{x^3 - +4x}{x^3 - x^2 - 6x} + = +\frac{x(x + + + 2)(x + - + 2)}{x(x + - + 3)(x + + + 2)} + = +\frac{x - 2}{x - 3}.$$

Answer: $\frac{x+-+2}{x+-+3}$

33.) Perform the indicated operations: $\dfrac{1}{x} + - + \dfrac{1}{x+1} + - + \dfrac{1}{x + - + 1}$

Solution

$$\frac{1}{x} + - + \frac{1}{x+1} + - + \frac{1}{x + - + 1} + = +\frac{(x + + + 1)(x - 1) + - + x(x - 1) - x(x + 1)}{x(x + 1)(x - 1)}$$

$$= +\frac{x^2 + - + 1 + - + x^2 + + + x + - + x^2 + - x +}{x(x + + + 1)(x + - + 1)}$$

$$= +\frac{x^2 + - + 1}{x(x + 1)(x - 1)} + = +\frac{(x + + + 1)(x + - + 1)}{x(x + + + 1)(x + - + 1)} + = +\frac{1}{x}$$

Answer: 1/x

34.) Perform the indicated operations.

Formula does not parse.

Solution

We can factor the given completely into the following expressions and change division into multiplication by multiplying the first two expressions with the reciprocal of the third expression. The result of these operations is shown below.

.

After cancelling similar terms, we are left with $2(\frac{3}{4})+=+\frac{3}{2}$

Answer: 3/2

35.) Simplify $1+-+\frac{x}{1+-+\frac{x}{1+-+x}}$.

Solution

Simplifying the rational expressions, we have

$$1+-+\frac{x}{1+-+\frac{x}{1+-+x}}+=+1+-+\frac{x}{\frac{1-2x}{1+-+x}}+=+1+-+\frac{1+-+x^2}{1+-+2x}.$$

This simplifies further to

$$\frac{1+-+3x+++x^2}{1+-+2x}.$$

Answer: $\frac{1+-+3x+++x^2}{1+-+2x}$

36.) Solve for x in the equation $\frac{x+1}{(x-2)}+=+\frac{x-1}{x-2}$

Solution

Getting the least common denominator of the left hand side and combining the terms, we have

$$\frac{x(x+-+2)+1}{(x-2)}+=+\frac{1}{x+-+2}+=+\frac{x-1}{x-2}$$

$$\frac{x^2+-+2x+++1}{x+-+2}+=+\frac{x-1}{x-2}.$$

Now, we can only equation the numerator.

$x^2+-+2x+++1+=+x+-+1$ which is equivalent to $x^2 – 3x + 2 = 0$

Getting the solution, we have $(x+-+1)(x+-+2+)+=+0$

Therefore, $x+=+1$ or $x+=+2$. But x cannot be equal to 2, because the it will make the denominator of the original expressions undefined. So, the solution is $x+=+1$.

Answer: x = 1

37.) The points (0, 0), (2, 3), and (4, 0) form a triangle. What is its perimeter?

Solution

To find the perimeter of the triangle, we need to find the distance between these points and add them. Let's name the points A, B, and C respectively.

Using the distance formula, we can subtract the corresponding coordinates, square them, and get the square root.

Distance between A and B is $\sqrt{2^2+++3^2}+=+\sqrt{13}$
Distance between A and C is $\sqrt{4^2+++0^2}+=+4$
Distance between B and C is $2^2+++(-3)^2+=+\sqrt{13}$

So, the perimeter of ABC is $AB+++AC+++BC+=+\sqrt{13}+++4+++\sqrt{13}+=+4+++2\sqrt{13}$

Answer: $4+++2+\sqrt{13}$.
38.) Find the equation of the perpendicular bisector of the segment joining the points (-3, 2) and (5, 2)?
Solution
Let A be the point with coordinates (-3,2) and B be the point with coordinates (5,2). Notice that AB have the same y-coordinates which mean that it is a horizontal segment. This means that the perpendicular bisector is a vertical line.

To get the perpendicular bisector, we get the midpoint M of AB and find the equation of the vertical line passing through M.

$M+=+(+\frac{-3+++5}{2},+\frac{2+++2}{2})+=+(1,+2)$.

So, the equation of the perpendicular bisector of AB is $x+=+1$

Answer: x = 1
39.) A man agrees to invest part of his 1-million-peso inheritance at an annual interest rate of 5%, while the rest at 6% interest. If, at the end of the year, he needs a total interest of Php 56, 200, how much should he invest at 5%?
Solution

Let x = amount invested at 5% and 1000 000 – x be invested at 6%.

$0.05x+++0.06(1000000+-+x)+=+56200$
$0.05x+++60000+-+0.06x+=+56200$
$-0.01x+=+3800$
$x+=+380000$

Answer: Php380,000
40.) If 2x + 5y = 10 and x = 3y + 1, what is 11x + 11y?
Solution
Substituting the expression on the right hand side of the second equation to x in the left hand side of the first equation, we have

$2(3y+++1)+++5y+=+10$
$6y+++2+++5y+=+10$
$11y+=+8$ (*)

Multiplying $2x+++5y+=+10$ by 3, we get $6x+++15y+=+30$ (#).

Transposing the second equation and multiplying it by 5, we have

$5x+-+15y+=+5$ (##)

Adding # and ##, we have

$11x+=+35$ (**)

By * and **

We have 11x + 11y = 35 + 8 = 43$.

Answer: 43

41.) If $f+(2x)+=+2+-+3x$, what is $f(10)$?
Solution
$2x+=+10, x+=+5.$
$f(10)+=+3+-+3(5)+=+-13$
Answer: – 13
42.) What is the equation of the line that is parallel to $2x+++5y+++6+=+0$ and passes through $(1,+1)$?
Solution
Parallel lines have the same slope, so we get the slope of the given line. That is,

$2x+++5y+=+-6$
$5y+=+-2x+-+6$
$y+=+\frac{-2}{5}x+-+\frac{6}{5}$

So, the slope of the given line is $-\frac{2}{5}$.
By the slope-intercept form, we get the equation of the line parallel to it and passing through $(1,1)$.

$y+-+1+=+-+\frac{2}{5}(x+-+1)$

Multiplying both sides by 5, we obtain

$5(y+-+1)+=+-+2(x+-+1)$

Simplifying, we have

$5y+-+5+=+-2x+++2$
$2x+++5y+-+7+=+0$

Answer: $2x+++5y+-+7+=+0$
43.) What is the domain of the function $f(x)+=+\sqrt{x+-+1}+-+2$?
Solution
We know that we cannot have a negative square root, so $\sqrt{x+-+1}+\geq+0$. By squaring both sides and simplifying, this means that $x+\geq+1$.Since we can substitute any value for x except the mentioned restriction, the domain is therefore the set of real numbers greater than or equal to 1.
Answer: the set of real numbers greater than or equal to 1
44.) Find the range of the function in Item 43.
Solution
The minimum value for $\sqrt{x+-+1}+=+0$ and $0+-+2+=+-2$, therefore, the range of f is the set of real numbers greater than or equal to -2.

Answer: set of real numbers greater than or equal to -2.

45.) If $\triangle ABC \cong \triangle DEF$, $AB = x - 2$ cm, and $DE = 5y + 3$ cm, what is x when $y = 10$?
Solution
$DE = 5y + 3 = 5(10) + 3 = 53$

Since corresponding sides of congruent triangles are congruent, $AB = DE$. So,
$2x - 2 = 53$
$2x = 55$
$x = \frac{55}{2}$.

Answer: $55/2$ or 27.5.
46.) Let ABC be an isosceles right triangle with AC as its hypotenuse, and let D and E be midpoints on $AB$and AC, respectively, such that $DE||BC$. If $AB = BC = x$ and $DE = y$, what is the area of the trapezoid $DECB$ in terms of x and y?
Solution

The area A of a trapezoid is $A = \frac{1}{2}h(b_1 + b_2)$ where b_1 and b_2 are the bases, and h is the height. From the given we can see that $b_1 = \frac{x}{2}$, $b_2 = x$ and $h = \frac{x}{2}$.

Substituting we have $A = \frac{1}{2}(\frac{x}{2})(x + y) = \frac{x}{2}(\frac{x}{2} + \frac{y}{2})$.

This simplifies to $A = \frac{x^2}{4} + \frac{xy}{4}$

Answer: $A = \frac{x^2}{4} + \frac{xy}{4}$
47.) If $|x| + x + y = 8$ and $x + |y| - y = 14$, what is $x + y$?
Solution
Assuming that $y > 0$. From the second equation $x = 14$. This is impossible because $14 + 14 + y \geq 8$. So, y is negative.

This means that the second equation becomes $x - 2y = 14$. Now, suppose x is negative, then the first equation becomes $y = 8$ which is impossible because we have already shown that y is negative. So, we are left with the systems of linear equations.

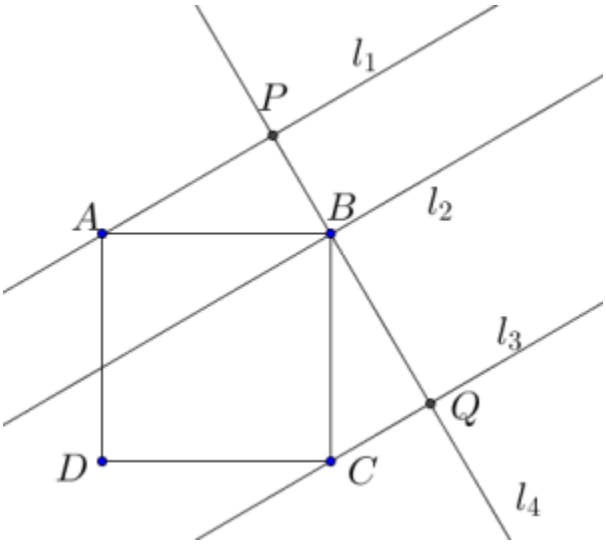
$2x + y = 8$
$x - 2y = 14$

This gives us $x = 6$ and $y = -4$. Therefore, $x + y = 2$.

Answer: 2
48.) If $90^a = 2$ and $90^b = 5$, what is $45^{\frac{1-a-b}{2-2a}}$?

Solution: To be posted later.
Answer: 3
49.) Let $ABCD$ be a square. Three parallel lines l_1, l_2, and l_3 pass through A, B, C, respectively. The distance between l1 and l2 is 4cm, and the distance between l2 and l3 is 5cm. Find the area of the square.

Solution
Draw line l_4 perpendicular to l_2 and passing through B.



Let P be the intersection of l_1 and l_4 and Q be the intersection of l_3 and l_4. Then, $\triangle BAP \cong \triangle CBQ$ (Why?). If we let s be the side of the square,

$$\frac{s}{\sqrt{s^2 + 4^2}} = \frac{s}{5}$$

which means that

$$s^2 + 4^2 = 5^2.$$

So, $s^2 = 4^2 + 5^2 = 41$

Answer: 41

50.) At least how many numbers should be selected from the set {1, 5, 9, 13, … , 125} to be assured that two of the numbers selected have a sum of 146?

Solution: To be posted later.

Answer: 20