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# Mathematics
# Higher level
# Paper 3 – sets, relations and groups

Wednesday 15 May 2019 (morning)

1 hour

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**Instructions to candidates**

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A graphic display calculator is required for this paper.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[50 marks]**.



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Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

**1.** [Maximum mark: 9]

The relation $R$ is defined on $\mathbb{N}$ by $xRy \Leftrightarrow x^2 + y^2 \equiv 0 \pmod 2$.

(a) Show that $R$ is an equivalence relation. [7]

(b) Determine the equivalence classes. [2]

**2.** [Maximum mark: 14]

Binary operations $\circ$ and $*$ are defined on the set of complex numbers such that $z_1 \circ z_2 = A(z_1 + z_2)$ and $z_1 * z_2 = Bz_1z_2$ where $A$ and $B$ are real non-zero constants.

(a) Determine the identity with respect to $*$. [2]

(b) Show that every element of the set of complex numbers, apart from $z = 0$, has an inverse with respect to $*$. [3]

(c) Find the value of $A$ for which the operation $\circ$ is associative. [4]

(d) Show that $*$ is distributive over $\circ$ for all values of $A$ and $B$. [5]

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**3.** [Maximum mark: 9]

A non-Abelian group, $\{G, \circ\}$, has eight elements $e, a, a^2, a^3, b, ab, a^2b, a^3b$, where $e$ is the identity. The binary operation is $\circ$ and, for example, $a \circ b$ is denoted by $ab$.
The element $a$ has order 4, the element $b$ has order 2 and $ba = a^3b$.

(a) Prove that

(i) $ba^2 = a^2b$;

(ii) $ba^3 = ab$. [4]

(b) The group $\{G, \circ\}$ has the following Cayley table. Find the elements $\boldsymbol{P, Q, R, S, T, U, V, W}$ of the group.

| $\circ$ | $e$ | $a$ | $a^2$ | $a^3$ | $b$ | $ab$ | $a^2b$ | $a^3b$ |
|---|---|---|---|---|---|---|---|---|
| $e$ | $e$ | $a$ | $a^2$ | $a^3$ | $b$ | $ab$ | $a^2b$ | $a^3b$ |
| $a$ | $a$ | $a^2$ | $a^3$ | $e$ | $ab$ | $a^2b$ | $a^3b$ | $b$ |
| $a^2$ | $a^2$ | $a^3$ | $e$ | $a$ | $a^2b$ | $a^3b$ | $b$ | $ab$ |
| $a^3$ | $a^3$ | $e$ | $a$ | $a^2$ | $a^3b$ | $b$ | $ab$ | $a^2b$ |
| $b$ | $b$ | $a^3b$ | $a^2b$ | $ab$ | $e$ | $a^3$ | $a^2$ | $a$ |
| $ab$ | $ab$ | $b$ | $a^3b$ | $a^2b$ | $a$ | $e$ | $\boldsymbol{R}$ | $\boldsymbol{U}$ |
| $a^2b$ | $a^2b$ | $ab$ | $b$ | $\boldsymbol{P}$ | $a^2$ | $a$ | $\boldsymbol{S}$ | $\boldsymbol{V}$ |
| $a^3b$ | $a^3b$ | $a^2b$ | $ab$ | $\boldsymbol{Q}$ | $a^3$ | $a^2$ | $\boldsymbol{T}$ | $\boldsymbol{W}$ |

[4]

(c) State why $\{G, \circ\}$ is not isomorphic to the group $\{\{0, 1, 2, 3, 4, 5, 6, 7\}, +_8\}$, where $+_8$ denotes addition modulo 8. [1]

**4.** [Maximum mark: 7]

Let $S$ be the set of real numbers of the form $a + b\sqrt{5}$ where $a, b \in \mathbb{Q}$, $a^2$ cannot equal $5b^2$, and $a, b$ are not simultaneously zero. Show that $S$ is a group under the operation of multiplication. Associativity may be assumed. [7]

**5.** [Maximum mark: 11]

Consider the function $f: \mathbb{R} \times \mathbb{R} \to \mathbb{R} \times \mathbb{R}$ defined by $f(x, y) = (x + y, x - y)$.

(a) Prove that the function $f$ is a bijection. [9]

Consider the function $g: \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z} \times \mathbb{Z}$ defined by $g(x, y) = (x + y, x - y)$.

(b) Explain why the function $g$ is not a bijection. [2]

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