Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Higher level
Paper 3 – discrete mathematics

Wednesday 15 May 2019 (morning)

1 hour

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A graphic display calculator is required for this paper.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[50 marks]**.

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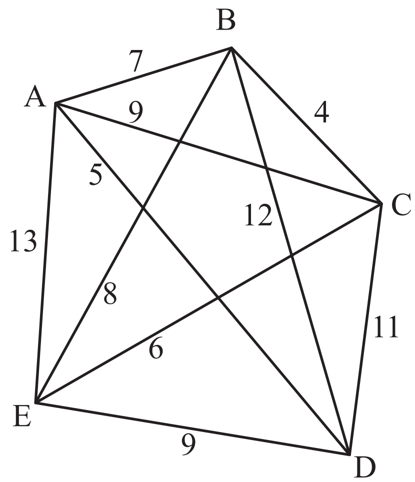
Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 14]

In a housing complex a TV company is installing cables to five houses A, B, C, D and E. The possible routes for the cables are given by the following graph G where each vertex represents a house and each edge represents a possible route for the cables. The weights of the edges represent the cost, in thousands of dollars, of installing the cables between the houses.



The company wants to find the lowest installation cost that includes each house and returns to the start.

(a) Starting at A, use the nearest-neighbour algorithm to find an upper bound for the lowest installation cost. [3]

(b) By removing A, use the deleted vertex algorithm to find a lower bound for the lowest installation cost. [5]

(c) (i) State a route which produces the lower bound.

(ii) State why this is a solution to finding the lowest installation cost. [2]

(This question continues on the following page)

(Question 1 continued)

In a second housing complex the design for installing cables between five houses P, Q, R, S and T is given in the following adjacency table.

	P	Q	R	S	T
P	–	1	0	0	1
Q	1	–	1	1	0
R	0	1	–	0	1
S	0	1	0	–	1
T	1	0	1	1	–

In the adjacency table 0 denotes no cable connecting the two houses and 1 denotes a cable connecting the two houses.

(d) Draw a graph H to represent the second housing complex. [2]

(e) Explain why the graph H is bipartite. [2]

2. [Maximum mark: 16]

(a) Use the Euclidean algorithm to find $\gcd(564, 254)$. [5]

(b) Find a general solution to the linear Diophantine equation $564x + 254y = 94$. [8]

(c) Find the two solutions such that $x, y \in [-300, 300]$. [3]

3. [Maximum mark: 12]

On the 1st March in a country there are 5000 environmentally contaminated sites requiring clean-up. By the 1st April 80 % of these 5000 contaminated sites are cleaned up but 200 new sites requiring clean-up are identified. This situation is assumed to recur every month. Jim sets up a first-degree recurrence relation that represents this information.

(a) (i) State Jim's first-degree recurrence relation for the number of sites, u_n, requiring clean-up after n months in the form $u_n = Au_{n-1} + B$, where A and B are non-zero constants.

(ii) State the value of u_0. [2]

(b) Solve Jim's first-degree recurrence relation. [5]

Jim now sets up a second-degree recurrence relation that gives information regarding environmental clean-up in a different country.
The second model is $d_n = 0.6d_{n-1} - 0.09d_{n-2}$ with initial conditions $d_0 = d_1 = 4000$.

(c) Solve Jim's second-degree recurrence relation. [5]

4. [Maximum mark: 8]

(a) Using Fermat's little theorem, show that the congruence $x^{22} + x^{11} \equiv 2 \pmod{11}$ can be expressed in the form $(x+6)^2 - 36 \equiv 2 \pmod{11}$. [4]

(b) Hence solve $x^{22} + x^{11} \equiv 2 \pmod{11}$. [4]