Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Higher level
Paper 3 – calculus

Thursday 21 November 2019 (afternoon)

1 hour

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A graphic display calculator is required for this paper.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[50 marks]**.

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Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 7]

The function f is defined by $f(x)=\begin{cases} \dfrac{x-3}{x-5}, & x<3 \\ \ln(x-2), & x\geq 3 \end{cases}$.

(a) Show that f is continuous at $x=3$. [3]

(b) Show that f is not differentiable at $x=3$. [4]

2. [Maximum mark: 10]

Determine whether each of the following infinite series converges or diverges.

(a) $\sum_{n=1}^{\infty}\dfrac{3n}{2n^2+5}$ [4]

(b) $\sum_{n=1}^{\infty}\dfrac{(2n)!}{3^n(n!)^2}$ [6]

3. [Maximum mark: 11]

The function f is defined by $f(x)=\arcsin(2x)$, where $-\dfrac{1}{2}\leq x\leq\dfrac{1}{2}$.

(a) By finding a suitable number of derivatives of f, find the first two non-zero terms in the Maclaurin series for f. [8]

(b) Hence or otherwise, find $\lim\limits_{x\to 0}\dfrac{\arcsin(2x)-2x}{(2x)^3}$. [3]

4. [Maximum mark: 22]

Consider the differential equation $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{4x^2 + y^2 - xy}{x^2}$, with $y = 2$ when $x = 1$.

(a) Use Euler's method, with step length $h = 0.1$, to find an approximate value of y when $x = 1.4$. [5]

(b) Sketch the isoclines for $\frac{\mathrm{d}y}{\mathrm{d}x} = 4$. [3]

(c) (i) Express $m^2 - 2m + 4$ in the form $(m - a)^2 + b$, where $a, b \in \mathbb{Z}$.

(ii) Solve the differential equation, for $x > 0$, giving your answer in the form $y = f(x)$.

(iii) Sketch the graph of $y = f(x)$ for $1 \leq x \leq 1.4$.

(iv) With reference to the curvature of your sketch in part (c)(iii), and without further calculation, explain whether you conjecture $f(1.4)$ will be less than, equal to, or greater than your answer in part (a). [14]