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Mathematics
Higher level
Paper 3 – calculus

Wednesday 15 May 2019 (morning)

1 hour

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A graphic display calculator is required for this paper.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[50 marks]**.

Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 7]

A simple model to predict the population of the world is set up as follows. At time t years the population of the world is x , which can be assumed to be a continuous variable. The rate of increase of x due to births is $0.056x$ and the rate of decrease of x due to deaths is $0.035x$.

(a) Show that $\frac{dx}{dt} = 0.021x$. [1]

(b) Find a prediction for the number of years it will take for the population of the world to double. [6]

2. [Maximum mark: 9]

(a) Show that $1 - x^2 + x^4 - x^6 + \dots = \frac{1}{1+x^2}$, where $|x| < 1$. [1]

(b) Hence write down the first four non-zero terms of the power series for $f(x) = \frac{1}{1+4x^2}$. [2]

(c) Using the result in (b), find the first four non-zero terms of the power series for $g(x) = \arctan 2x$. [6]

3. [Maximum mark: 9]

Consider the series $\sum_{n=1}^{\infty} \frac{A \times 8^n}{3^{2n+1}}$.

(a) Given that $A = \frac{1}{n}$, use the comparison test to show that the series converges. [4]

(b) Given that $A = n$, determine whether the series diverges or converges. [5]

4. [Maximum mark: 9]

Using L'Hôpital's rule, find $\lim_{x \rightarrow 0} \left(\frac{\tan 3x - 3 \tan x}{\sin 3x - 3 \sin x} \right)$. [9]

5. [Maximum mark: 16]

Consider the differential equation $2xy \frac{dy}{dx} = y^2 - x^2$, where $x > 0$.

(a) Solve the differential equation and show that a general solution is $x^2 + y^2 = cx$ where c is a positive constant. [11]

(b) Prove that there are two horizontal tangents to the general solution curve and state their equations, in terms of c . [5]
