Markscheme

May 2019

Mathematics

Higher level

Paper 1

20 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a valid **Method**; working must be seen.

(M) Marks awarded for **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Mark according to RM™ Assessor instructions. In particular, please note the following:

- Marks must be recorded using the annotation stamps. Please check that you are entering marks for the right question.
- If a part is **completely correct**, (and gains all the “must be seen” marks), use the ticks with numbers to stamp full marks.
- If a part is completely wrong, stamp ***A0*** by the final answer.
- If a part gains anything else, it **must** be recorded using **all** the annotations.
- All the marks will be added and recorded by RM™ Assessor.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *eg* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*eg* substitution into a formula) and ***A1*** for using the **correct** values.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc.*, do **not** split the marks.

- Once a correct answer to a question or part-question is seen, ignore further correct working. However, if further working indicates a lack of mathematical understanding do not award the final ***A1***. An exception to this may be in numerical answers, where a correct exact value is followed by an incorrect decimal. However, if the incorrect decimal is carried through to a subsequent part, and correct ***FT*** working shown, award ***FT*** marks as appropriate but do not award the final ***A1*** in that part.

  Examples

	Correct answer seen	**Further working seen**	**Action**
1.	$8\sqrt{2}$	$5.65685...$ *(incorrect decimal value)*	Award the final ***A1*** *(ignore the further working)*
2.	$\frac{1}{4}\sin 4x$	$\sin x$	Do not award the final ***A1***
3.	$\log a - \log b$	$\log(a-b)$	Do not award the final ***A1***

3 *N* marks

*Award **N** marks for **correct** answers where there is **no** working.*

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.

4 Implied marks

*Implied marks appear in **brackets eg (M1)**, and can only be awarded if **correct** work is seen or if implied in subsequent working.*

- Normally the correct work is seen or implied in the next line.
- Marks **without** brackets can only be awarded for work that is **seen**.

5 Follow through marks

*Follow through (**FT**) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s). To award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part.*

- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*eg* $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).
- Within a question part, once an error is made, no further **dependent *A*** marks can be awarded, but ***M*** marks may be awarded if appropriate.
- Exceptions to this rule will be explicitly noted on the markscheme.

6 Misread

*If a candidate incorrectly copies information from the question, this is a misread (**MR**).*
*A candidate should be penalized only once for a particular misread. Use the **MR** stamp to indicate that this has been a misread. Then deduct the first of the marks to be awarded, even if this is an **M** mark, but award all others so that the candidate only loses **[1 mark]**.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*eg* $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).

7 Discretionary marks *(d)*

*An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation DM should be used and a brief **note** written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for part-questions are indicated by **EITHER** . . . **OR**.
- Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

Example: for differentiating $f(x) = 2\sin(5x-3)$, the markscheme gives

$$f'(x) = \left(2\cos(5x-3)\right)5 \quad \left(=10\cos(5x-3)\right)$$ ***A1***

Award ***A1*** for $\left(2\cos(5x-3)\right)5$, even if $10\cos(5x-3)$ is not seen.

10 Accuracy of Answers

*Candidates should **NO LONGER** be penalized for an accuracy error **(AP)**.*

*If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures. Please check work carefully for **FT**.*

11 Crossed out work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

12 Calculators

No calculator is allowed. The use of any calculator on paper 1 is malpractice, and will result in no grade awarded. If you see work that suggests a candidate has used any calculator, please follow the procedures for malpractice. Examples: finding an angle, given a trig ratio of 0.4235.

13 More than one solution

Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise.

14. Candidate work

Candidates are meant to write their answers to Section A on the question paper (QP), and Section B on answer booklets. Sometimes, they need more room for Section A, and use the booklet (and often comment to this effect on the QP), or write outside the box. This work should be marked.

The instructions tell candidates not to write on Section B of the QP. Thus they may well have done some rough work here which they assume will be ignored. If they have solutions on the answer booklets, there is no need to look at the QP. However, if there are whole questions or whole part solutions missing on answer booklets, please check to make sure that they are not on the QP, and if they are, mark those whole questions or whole part solutions that have not been written on answer booklets.

Section A

1. $\boldsymbol{a} \bullet \boldsymbol{b} = \begin{pmatrix} 2 \\ k \\ -1 \end{pmatrix} \bullet \begin{pmatrix} -3 \\ k+2 \\ k \end{pmatrix}$

$= -6 + k(k+2) - k$ ***A1***

$\boldsymbol{a} \bullet \boldsymbol{b} = 0$ ***(M1)***

$k^2 + k - 6 = 0$

attempt at solving their quadratic equation ***(M1)***

$(k+3)(k-2) = 0$

$k = -3, 2$ ***A1***

Note: Attempt at solving using $|\boldsymbol{a}||\boldsymbol{b}| = |\boldsymbol{a} \times \boldsymbol{b}|$ will be ***M1A0A0A0*** if neither answer found ***M1(A1)A1A0*** for one correct answer and ***M1(A1)A1A1*** for two correct answers.

Total [4 marks]

2. attempt at binomial expansion ***M1***

$1 + \binom{11}{1}(-2x) + \binom{11}{2}(-2x)^2 + \ldots$ ***(A1)***

$\binom{11}{2} = 55$

$1 - 22x + 220x^2$ ***A1A1***

Note: ***A1*** for first two terms, ***A1*** for final term.

Note: Award ***M1(A1)A0A0*** for $(-2x)^{11} + \binom{11}{10}(-2x)^{10} + \binom{11}{9}(-2x)^9 + \ldots,$

Total [4 marks]

3. $A = P$

use of the correct formula for area and arc length ***(M1)***

perimeter is $r\theta + 2r$ ***(A1)***

Note: ***A1*** independent of previous ***M1***.

$\frac{1}{2}r^2(1) = r(1) + 2r$ ***A1***

$r^2 - 6r = 0$

$r = 6$ (as $r > 0$) ***A1***

Note: Do not award final ***A1*** if $r = 0$ is included.

Total [4 marks]

4. (a) **EITHER**

$\frac{5\sqrt{15}}{2} = \frac{1}{2} \times 4 \times 5\sin\theta$ ***A1***

OR

height of triangle is $\frac{5\sqrt{15}}{4}$ if using 4 as the base or $\sqrt{15}$ if using 5 as the base ***A1***

THEN

$\sin\theta = \frac{\sqrt{15}}{4}$ ***AG***

[1 mark]

(b) let the third side be x

$x^2 = 4^2 + 5^2 - 2 \times 4 \times 5 \times \cos\theta$ ***M1***

valid attempt to find $\cos\theta$ ***(M1)***

Note: Do not accept writing $\cos\left(\arcsin\left(\frac{\sqrt{15}}{4}\right)\right)$ as a valid method.

$\cos\theta = \pm\sqrt{1-\frac{15}{16}}$

$= \frac{1}{4}, -\frac{1}{4}$ ***A1A1***

$x^2 = 16 + 25 - 2 \times 4 \times 5 \times \pm\frac{1}{4}$

$x = \sqrt{31}$ or $\sqrt{51}$ ***A1A1***

[6 marks]

Total [7 marks]

5. let $\text{OX} = x$

METHOD 1

$\frac{\mathrm{d}x}{\mathrm{d}t} = 24$ (or -24) ***(A1)***

$\frac{\mathrm{d}\theta}{\mathrm{d}t} = \frac{\mathrm{d}x}{\mathrm{d}t} \times \frac{\mathrm{d}\theta}{\mathrm{d}x}$ ***(M1)***

$3\tan\theta = x$ ***A1***

EITHER

$3\sec^2\theta = \frac{\mathrm{d}x}{\mathrm{d}\theta}$ ***A1***

$\frac{\mathrm{d}\theta}{\mathrm{d}t} = \frac{24}{3\sec^2\theta}$

attempt to substitute for $\theta = 0$ into their differential equation ***M1***

OR

$\theta = \arctan\left(\frac{x}{3}\right)$

$\frac{\mathrm{d}\theta}{\mathrm{d}x} = \frac{1}{3} \times \frac{1}{1 + \frac{x^2}{9}}$ ***A1***

$\frac{\mathrm{d}\theta}{\mathrm{d}t} = 24 \times \frac{1}{3\left(1 + \frac{x^2}{9}\right)}$

attempt to substitute for $x = 0$ into their differential equation ***M1***

THEN

$\frac{\mathrm{d}\theta}{\mathrm{d}t} = \frac{24}{3} = 8$ (rad s^{-1}) ***A1***

Note: Accept $-8\,\text{rad}\,\text{s}^{-1}$.

continued…

Question 5 continued

METHOD 2

$\frac{dx}{dt} = 24$ (or -24) ***(A1)***

$3\tan\theta = x$ ***A1***

attempt to differentiate implicitly with respect to t ***M1***

$3\sec^2\theta \times \frac{d\theta}{dt} = \frac{dx}{dt}$ ***A1***

$\frac{d\theta}{dt} = \frac{24}{3\sec^2\theta}$

attempt to substitute for $\theta = 0$ into their differential equation ***M1***

$\frac{d\theta}{dt} = \frac{24}{3} = 8$ (rad s^{-1}) ***A1***

Note: Accept $-8\,\text{rad}\,\text{s}^{-1}$.

Note: Can be done by consideration of CX, use of Pythagoras.

METHOD 3

let the position of the car be at time t be $d - 24t$ from O ***(A1)***

$\tan\theta = \frac{d-24t}{3}\left(= \frac{d}{3} - 8t\right)$ ***M1***

Note: For $\tan\theta = \frac{24t}{3}$ award ***A0M1*** and follow through.

EITHER

attempt to differentiate implicitly with respect to t ***M1***

$\sec^2\theta\frac{d\theta}{dt} = -8$ ***A1***

attempt to substitute for $\theta = 0$ into their differential equation ***M1***

OR

$\theta = \arctan\left(\frac{d}{3} - 8t\right)$ ***M1***

$\frac{d\theta}{dt} = -\frac{8}{1+\left(\frac{d}{3} - 8t\right)^2}$ ***A1***

at O, $t = \frac{d}{24}$ ***A1***

continued…

Question 5 continued

THEN

$\frac{d\theta}{dt} = -8$ ***A1***

Total [6 marks]

6. (a) use of symmetry *eg* diagram ***(M1)***

$P(X > \mu + 5) = 0.2$ ***A1***

[2 marks]

(b) **EITHER**

$P(X < \mu + 5 \mid X > \mu - 5) = \frac{P(X > \mu - 5 \cap X < \mu + 5)}{P(X > \mu - 5)}$ ***(M1)***

$= \frac{P(\mu - 5 < X < \mu + 5)}{P(X > \mu - 5)}$ ***(A1)***

$= \frac{0.6}{0.8}$ ***A1A1***

Note: ***A1*** for denominator is independent of the previous ***A*** marks.

OR

use of diagram ***(M1)***

Note: Only award ***(M1)*** if the region $\mu - 5 < X < \mu + 5$ is indicated and used.

continued…

Question 6 continued

$\mathrm{P}\left(X > \mu - 5\right) = 0.8 \quad \mathrm{P}\left(\mu - 5 < X < \mu + 5\right) = 0.6$ ***(A1)***

Note: Probabilities can be shown on the diagram.

$= \frac{0.6}{0.8}$ ***M1A1***

THEN

$= \frac{3}{4} \ = (0.75)$ ***A1***

[5 marks]

Total [7 marks]

7. attempt at implicit differentiation ***M1***

$3y^2\frac{\mathrm{d}y}{\mathrm{d}x} + 3y^2 + 6xy\frac{\mathrm{d}y}{\mathrm{d}x} - 3x^2 = 0$ ***A1A1***

Note: Award ***A1*** for the second & third terms, ***A1*** for the first term, fourth term & RHS equal to zero.

substitution of $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ ***M1***

$3y^2 - 3x^2 = 0$

$\Rightarrow y = \pm x$ ***A1***

substitute either variable into original equation ***M1***

$y = x \Rightarrow x^3 = 9 \Rightarrow x = \sqrt[3]{9}$ (or $y^3 = 9 \Rightarrow y = \sqrt[3]{9}$) ***A1***

$y = -x \Rightarrow x^3 = 27 \Rightarrow x = 3$ (or $y^3 = -27 \Rightarrow y = -3$) ***A1***

$\left(\sqrt[3]{9}, \sqrt[3]{9}\right)$, $(3, -3)$ ***A1***

Total [9 marks]

8. (a) 3 ***A1***

[1 mark]

(b) attempt to use definite integral of $f'(x)$ ***(M1)***

$\int_0^1 f'(x)\mathrm{d}x = 0.5$

$f(1) - f(0) = 0.5$ ***(A1)***

$f(1) = 0.5 + 3$

$= 3.5$ ***A1***

[3 marks]

continued…

Question 8 continued

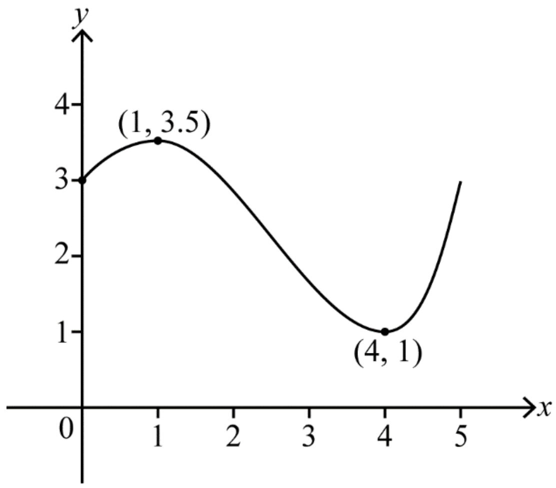
(c) $\int_1^4 f'(x)\mathrm{d}x = -2.5$ ***(A1)***

Note: ***(A1)*** is for -2.5.

$f(4) - f(1) = -2.5$
$f(4) = 3.5 - 2.5$
$= 1$ ***A1***

[2 marks]

(d)



A1A1A1

A1 for correct shape over approximately the correct domain

A1 for maximum and minimum (coordinates or horizontal lines from 3.5 and 1 are required),
A1 for y-intercept at 3

[3 marks]
Total [9 marks]

Section B

9. (a) $(\sin x + \cos x)^2 = \sin^2 x + 2\sin x\cos x + \cos^2 x$ ***M1A1***

Note: Do not award the ***M1*** for just $\sin^2 x + \cos^2 x$.

Note: Do not award ***A1*** if correct expression is followed by incorrect working.

$= 1 + \sin 2x$ ***AG***

[2 marks]

(b) $\sec 2x + \tan 2x = \frac{1}{\cos 2x} + \frac{\sin 2x}{\cos 2x}$ ***M1***

Note: ***M1*** is for an attempt to change both terms into sine and cosine forms (with the same argument) or both terms into functions of $\tan x$.

$= \frac{1 + \sin 2x}{\cos 2x}$

$= \frac{(\sin x + \cos x)^2}{\cos^2 x - \sin^2 x}$ ***A1A1***

Note: Award ***A1*** for numerator, ***A1*** for denominator.

$= \frac{(\sin x + \cos x)^2}{(\cos x - \sin x)(\cos x + \sin x)}$ ***M1***

$= \frac{\cos x + \sin x}{\cos x - \sin x}$ ***AG***

Note: Apply MS in reverse if candidates have worked from RHS to LHS.

Note: Alternative method using $\tan 2x$ and $\sec 2x$ in terms of $\tan x$.

[4 marks]

(c) **METHOD 1**

$\int_0^{\frac{\pi}{6}} \left(\frac{\cos x + \sin x}{\cos x - \sin x}\right) dx$ ***A1***

Note: Award ***A1*** for correct expression with or without limits.

EITHER

$= \left[-\ln(\cos x - \sin x)\right]_0^{\frac{\pi}{6}}$ or $\left[\ln(\cos x - \sin x)\right]_{\frac{\pi}{6}}^0$ ***(M1)A1A1***

Note: Award ***M1*** for integration by inspection or substitution, ***A1*** for $\ln(\cos x - \sin x)$, ***A1*** for completely correct expression including limits.

$= -\ln\left(\cos\frac{\pi}{6} - \sin\frac{\pi}{6}\right) + \ln(\cos 0 - \sin 0)$ ***M1***

Note: Award ***M1*** for substitution of limits into their integral and subtraction.

$= -\ln\left(\frac{\sqrt{3}}{2} - \frac{1}{2}\right)$ ***(A1)***

continued…

Question 9 continued

OR

let $u = \cos x - \sin x$ ***M1***

$\frac{du}{dx} = -\sin x - \cos x = -(\sin x + \cos x)$

$-\int_1^{\frac{\sqrt{3}}{2}-\frac{1}{2}} \left(\frac{1}{u}\right) du$ ***A1A1***

Note: Award ***A1*** for correct limits even if seen later, ***A1*** for integral.

$= \left[-\ln u\right]_1^{\frac{\sqrt{3}}{2}-\frac{1}{2}}$ or $\left[\ln u\right]_{\frac{\sqrt{3}}{2}-\frac{1}{2}}^1$ ***A1***

$= -\ln\left(\frac{\sqrt{3}}{2} - \frac{1}{2}\right)(+\ln 1)$ ***M1***

THEN

$= \ln\left(\frac{2}{\sqrt{3}-1}\right)$ ***M1***

Note: Award ***M1*** for both putting the expression over a common denominator and for correct use of law of logarithms.

$= \ln\left(1+\sqrt{3}\right)$ ***(M1)A1***

[9 marks]

METHOD 2

$\left[\frac{1}{2}\ln(\tan 2x + \sec 2x) - \frac{1}{2}\ln(\cos 2x)\right]_0^{\frac{\pi}{6}}$ ***A1A1***

$= \frac{1}{2}\ln\left(\sqrt{3}+2\right) - \frac{1}{2}\ln\left(\frac{1}{2}\right) - 0$ ***A1A1(A1)***

$= \frac{1}{2}\ln\left(4+2\sqrt{3}\right)$ ***M1***

$= \frac{1}{2}\ln\left(\left(1+\sqrt{3}\right)^2\right)$ ***M1A1***

$= \ln\left(1+\sqrt{3}\right)$ ***A1***

[9 marks]

Total [15 marks]

10. (a) (i) $p(2) = 8 - 12 + 16 - 24$ ***(M1)***

> **Note:** Award ***M1*** for a valid attempt at remainder theorem or polynomial division.

$= -12$ ***A1***
remainder $= -12$

(ii) $p(3) = 27 - 27 + 24 - 24 = 0$ ***A1***
remainder $= 0$

[3 marks]

(b) $x = 3$ (is a zero) ***A1***

> **Note:** Can be seen anywhere.

EITHER

factorise to get $(x-3)\left(x^2+8\right)$ ***(M1)A1***

$x^2 + 8 \neq 0$ (for $x \in \mathbb{R}$) (or equivalent statement) ***R1***

> **Note:** Award ***R1*** if correct two complex roots are given.

OR

$p'(x) = 3x^2 - 6x + 8$ ***A1***
attempting to show $p'(x) \neq 0$ ***M1***
eg discriminant $= 36 - 96 < 0$, completing the square
no turning points ***R1***

THEN

only one real zero (as the curve is continuous) ***AG***
[4 marks]

(c) new graph is $y = p(2x)$ ***(M1)***
stretch parallel to the x-axis (with $x = 0$ invariant), scale factor 0.5 ***A1***
[2 marks]

> **Note:** Accept “horizontal” instead of “parallel to the x-axis”.

continued…

Question 10 continued

(d) $\frac{6\lambda^3 e^{-\lambda}}{6} = \frac{3\lambda^2 e^{-\lambda}}{2} - 2\lambda e^{-\lambda} + 3e^{-\lambda}$ ***M1A1***

Note: Allow factorials in the denominator for ***A1***.

$2\lambda^3 - 3\lambda^2 + 4\lambda - 6 = 0$ ***A1***

Note: Accept any correct cubic equation without factorials and $e^{-\lambda}$.

EITHER

$4\left(2\lambda^3 - 3\lambda^2 + 4\lambda - 6\right) = 8\lambda^3 - 12\lambda^2 + 16\lambda - 24 = 0$ ***(M1)***

$2\lambda = 3$ ***(A1)***

OR

$(2\lambda - 3)\left(\lambda^2 + 2\right) = 0$ ***(M1)(A1)***

THEN

$\lambda = 1.5$ ***A1***

[6 marks]

Total [15 marks]

11. (a) (i) appreciation that two points distinct from P need to be chosen from each line ***M1***

$^4C_2 \times {}^3C_2$

$= 18$ ***A1***

(ii) **EITHER**

consider cases for triangles including P **or** triangles not including P ***M1***

$3 \times 4 + 4 \times {}^3C_2 + 3 \times {}^4C_2$ ***(A1)(A1)***

Note: Award ***A1*** for 1st term, ***A1*** for 2nd & 3rd term.

OR

consider total number of ways to select 3 points and subtract those with 3 points on the same line ***M1***

$^8C_3 - {}^5C_3 - {}^4C_3$ ***(A1)(A1)***

Note: Award ***A1*** for 1st term, ***A1*** for 2nd & 3rd term.

$56 - 10 - 4$

THEN

$= 42$ ***A1***

[6 marks]

(b) **METHOD 1**

substitution of $(4, 6, 4)$ into both equations ***(M1)***

$\lambda = 3$ and $\mu = 1$ ***A1A1***

$(4, 6, 4)$ ***AG***

METHOD 2

attempting to solve two of the three parametric equations ***M1***

$\lambda = 3$ or $\mu = 1$ ***A1***

check both of the above give $(4, 6, 4)$ ***M1AG***

Note: If they have shown the curve intersects for all three coordinates they only need to check (4,6,4) with one of "λ" or "μ".

[3 marks]

(c) $\lambda = 2$ ***A1***

[1 mark]

(d) $\vec{PA} = \begin{pmatrix} -1 \\ -2 \\ -1 \end{pmatrix}, \ \vec{PB} = \begin{pmatrix} -5 \\ -6 \\ -2 \end{pmatrix}$ ***A1A1***

Note: Award ***A1A0*** if both are given as coordinates.

[2 marks]

continued…

Question 11 continued

(e) **METHOD 1**

area triangle $\text{ABP} = \frac{1}{2}\left|\vec{\text{PB}} \times \vec{\text{PA}}\right|$ ***M1***

$\left(= \frac{1}{2}\left|\begin{pmatrix} -5 \\ -6 \\ -2 \end{pmatrix} \times \begin{pmatrix} -1 \\ -2 \\ -1 \end{pmatrix}\right| \right) = \frac{1}{2}\left|\begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix}\right|$ ***A1***

$= \frac{\sqrt{29}}{2}$ ***A1***

EITHER

$\vec{\text{PC}} = 3\vec{\text{PA}}, \ \vec{\text{PD}} = 3\vec{\text{PB}}$ ***(M1)***

area triangle $\text{PCD} = 9 \times$ area triangle ABP ***(M1)A1***

$= \frac{9\sqrt{29}}{2}$ ***A1***

OR

D has coordinates $(-11, -12, -2)$ ***A1***

area triangle $\text{PCD} = \frac{1}{2}\left|\vec{\text{PD}} \times \vec{\text{PC}}\right| = \frac{1}{2}\left|\begin{pmatrix} -15 \\ -18 \\ -6 \end{pmatrix} \times \begin{pmatrix} -3 \\ -6 \\ -3 \end{pmatrix}\right|$ ***M1A1***

Note: A1 is for the correct vectors in the correct formula.

$= \frac{9\sqrt{29}}{2}$ ***A1***

THEN

area of $\text{CDBA} = \frac{9\sqrt{29}}{2} - \frac{\sqrt{29}}{2}$

$= 4\sqrt{29}$ ***A1***

[8 marks]

continued…

Question 11 continued

METHOD 2

D has coordinates $(-11, -12, -2)$ ***A1***

area $= \frac{1}{2}\left|\vec{CB} \times \vec{CA}\right| + \frac{1}{2}\left|\vec{BC} \times \vec{BD}\right|$ ***M1***

Note: Award ***M1*** for use of correct formula on appropriate non-overlapping triangles.

Note: Different triangles or vectors could be used.

$\vec{CB} = \begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix}, \ \vec{CA} = \begin{pmatrix} 2 \\ 4 \\ 2 \end{pmatrix}$ ***A1***

$\vec{CB} \times \vec{CA} = \begin{pmatrix} -4 \\ 6 \\ -8 \end{pmatrix}$ ***A1***

$\vec{BC} = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}, \ \vec{BD} = \begin{pmatrix} -10 \\ -12 \\ -4 \end{pmatrix}$ ***A1***

$\vec{BC} \times \vec{BD} = \begin{pmatrix} -12 \\ 18 \\ -24 \end{pmatrix}$ ***A1***

Note: Other vectors which might be used are $\vec{DA} = \begin{pmatrix} 14 \\ 16 \\ 5 \end{pmatrix}, \ \vec{BA} = \begin{pmatrix} 4 \\ 4 \\ 1 \end{pmatrix}, \vec{DC} = \begin{pmatrix} 12 \\ 12 \\ 3 \end{pmatrix}$.

Note: Previous ***A1A1A1A1*** are all dependent on the first ***M1***.

valid attempt to find a value of $\frac{1}{2}\left|a \times b\right|$ ***M1***

Note: ***M1*** independent of triangle chosen.

area $= \frac{1}{2} \times 2 \times \sqrt{29} + \frac{1}{2} \times 6 \times \sqrt{29}$

$= 4\sqrt{29}$ ***A1***

Note: Accept $\frac{1}{2}\sqrt{116} + \frac{1}{2}\sqrt{1044}$ or equivalent.

[8 marks]

Total [20 marks]