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Markscheme

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Mathematical studies

Standard level

Paper 2

24 pages

International Baccalaureate
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Paper 2 Markscheme
Instructions to Examiners

Notes: If in doubt about these instructions or any other marking issues, contact your team leader for clarification.

1 Abbreviations

M Marks awarded for **Method**

A Marks awarded for an **Answer** or for **Accuracy**

R Marks awarded for clear **Reasoning**

G Marks awarded for correct solutions obtained from a **Graphic Display Calculator**, when no working shown.

AG **Answer Given** in the question and consequently, marks not awarded.

ft Marks that can be awarded as **follow through** from previous results in the question.

2 Method of Marking

(a) All marking must be done in RM Assessor using the mathematical studies annotations and in accordance with the current document for guidance in e-marking Mathematical Studies SL. It is essential that you read this document before you start marking.

(b) If a question part is completely correct use the number tick annotations to award full marks. If a part is completely wrong use the ***A0*** annotation, otherwise full annotations must be shown.

(c) Working crossed out by the candidate should not be awarded any marks.

(d) Where candidates have written two solutions to a question, only the first solution should be marked.

(e) If correct working results in a correct answer but then further working is developed, indicating a lack of mathematical understanding full marks should **not** be awarded. In most such cases it will be a single final answer mark that is lost. An exception to this may be in numerical answers, where a correct exact value is followed by an incorrect decimal.

Example:

	Correct answer seen	**Further working seen**	**Action**
1.	$8\sqrt{2}$	$5.65685...$ *(incorrect decimal value)*	Award the final ***(A1)*** *(ignore the further working)*
2.	$(x-6)\,(x+1)$	$x=6$ and -1	Do **not** award the final ***(A1)***

Example: Calculate the gradient of the line passing through the points $(5, 3)$ and $(0, 9)$.

Markscheme	**Candidates' Scripts**	**Marking**
$\frac{9-3}{0-5}$ ***(M1)*** *Award **(M1)** for correct substitution in gradient formula* $=-\frac{6}{5}$ ***(A1)***	(i) $\frac{9-3}{0-5}=-\frac{6}{5}$ Gradient is $=-\frac{6}{5}$ *(There is clear understanding of the gradient.)* $y=-\frac{6}{5}x+9$	***(M1)*** ***(A1)***
	(ii) $\frac{9-3}{0-5}=-\frac{6}{5}$ $y=-\frac{6}{5}x+9$ *(There is confusion about what is required.)*	***(M1)*** ***(A0)***

3 Follow-through (ft) Marks

Errors made at any step of a solution affect all working that follows. To limit the severity of the penalty, **follow through (ft)** marks can be awarded. Markschemes will indicate where it is appropriate to apply follow through in a question with **'(ft)'**.

(a) Follow through applies only from one part of a question to a subsequent part of the question. Follow through does not apply within the same part.

(b) If an answer resulting from follow through is extremely unrealistic (*eg*, negative distances or incorrect by large order of magnitude) then the final ***A*** mark should not be awarded.

(c) If a question is transformed by an error into a **different**, **much simpler question** then follow through may not apply.

(d) To award follow through marks for a question part, **there must be working present for that part**. An isolated follow through answer, without working is regarded as incorrect and receives no marks **even if it is approximately correct**.

(e) The exception to the above would be in a question which is testing the candidate's use of the GDC, where working will not be expected. **The markscheme will clearly indicate where this applies.**

(f) Inadvertent use of radians will be penalized the first time it occurs. The markscheme will give clear instructions to ensure that only one mark per paper can be lost for the use of radians.

Example: Finding angles and lengths using trigonometry

Markscheme	Candidates' Scripts	Marking
(a) $\frac{\sin A}{3} = \frac{\sin 30}{4}$ ***(M1)(A1)*** *Award **(M1)** for substitution in sine rule formula, **(A1)** for correct substitutions.*	(a) $\frac{\sin A}{4} = \frac{\sin 30}{3}$	***(M1)(A0)*** *(use of sine rule but with wrong values)*
$A = 22.0^{\circ}\ (22.0243\ldots)$ ***(A1)(G2)***	$A = 41.8^{\circ}$	***(A0)*** *(**Note:** the 2nd **(A1)** here was not marked* **(ft)** *and cannot be awarded because there was an earlier error in the **same** question part.)*
(b) $x = 7\tan(22.0243\ldots^{\circ})$ ***(M1)*** $= 2.83\ (2.83163\ldots)$ ***(A1)*****(ft)**	(b) case (i) $x = 7\tan 41.8^{\circ}$ $= 6.26$	***(M1)*** ***(A1)*****(ft)**
	but case (ii) 6.26	***(G0)*** *since no working shown*

4 Using the Markscheme

(a) ***A*** marks are **dependent** on the preceding ***M*** mark being awarded, it is **not** possible to award ***(M0)(A1)***. Once an ***(M0)*** has been awarded, all subsequent ***A*** marks are lost in that part of the question, even if calculations are performed correctly, until the next ***M*** mark.
The only exception to this will be for an answer where the accuracy is specified in the question – see section 5.

(b) ***A*** marks are **dependent** on the ***R*** mark being awarded, it is **not** possible to award ***(A1)(R0)***. Hence the ***(A1)*** cannot be awarded for an answer which is correct when no reason or the wrong reason is given.

(c) In paper 2 candidates are expected to demonstrate their ability to communicate mathematics using appropriate working. Answers which are correct but not supported by adequate working will **not always receive full marks**, these unsupported answers are designated ***G*** in the mark scheme as an alternative to the full marks. Example ***(M1)(A1)(A1)(G2)***.

Example: Using trigonometry to calculate an angle in a triangle.

Markscheme	**Candidates' Scripts**	**Marking**
(a) $\frac{\sin A}{3} = \frac{\sin 30}{4}$ ***(M1)(A1)*** *Award* ***(M1)*** *for substitution in sine rule formula,* ***(A1)*** *for correct substitutions.*	(i) $\frac{\sin A}{3} = \frac{\sin 30}{4}$ $A = 22.0^\circ$	***(M1)(A1)*** ***(A1)***
$A = 22.0^\circ$ $(22.0243\ldots)$ ***(A1)(G2)***	(ii) $A = 22.0^\circ$ ***Note: G*** *marks are used only if no working has been shown and the answer is correct.*	***(G2)***

(d) **Alternative methods** may not always be included. Thus, if an answer is wrong then the working must be carefully analysed in order that marks are awarded for a different method consistent with the markscheme.
Where alternative methods for complete questions are included in the markscheme, they are indicated by '**OR**' *etc*.

(e) Unless the question specifies otherwise, accept **equivalent forms**. For example: $\frac{\sin\theta}{\cos\theta}$ for $\tan\theta$.
On the markscheme, these equivalent numerical or algebraic forms will sometimes be written in brackets after the required answer.
Where numerical answers are required as the final answer to a part of a question in the markscheme, the scheme will show, in order:
the 3 significant figure answer worked through from full calculator display;
the exact value $\left(\text{for example } \frac{2}{3} \text{ if applicable}\right)$;
the full calculator display in the form $2.83163\ldots$ as in the example above.
Where answers are given to 3 significant figures and are then used in subsequent parts of the question leading to a **different** 3 significant figure answer, these solutions will also be given.

(f) As this is an international examination, all valid **alternative forms of notation** should be accepted. Some examples of these are:

Decimal points: 1.7; $1'7$; $1\cdot 7$; $1,7$.

Decimal numbers less than 1 may be written with or without a leading zero: 0.49 or $.49$.

Different descriptions of an interval: $3 < x < 5$; $(3, 5)$; $]\,3, 5\,[$.

Different forms of notation for set properties (*e.g.* complement): A'; $\overline{A}$; A^c; $U - A$;$(A$;$U \setminus A$.
Different forms of logic notation: $\neg p$; p'; $\tilde{p}$; $\overline{p}$; $\sim p$; $-p$.
$p \Rightarrow q$; $p \rightarrow q$; $q \Leftarrow p$.

Significance level may be written as α .

(g) Discretionary marks: There will be very rare occasions where the markscheme does not cover the work seen. In such cases the annotation DM should be used to indicate where an examiner has used discretion. Discretion should be used sparingly and if there is doubt an exception should be raised through RM Assessor to the team leader.

As with previous sessions there will be no whole paper penalty marks for accuracy AP, financial accuracy FP and units UP. Instead these skills will be assessed in particular questions and the marks applied according to the rules given in sections 5, 6 and 7 below.

5 Accuracy of Answers

Incorrect accuracy should be penalized once only in each question according to the rules below.

Unless otherwise stated in the question, all numerical answers should be given exactly or correct to 3 significant figures.

1. If the **candidate's answer** is seen to 4 sf or greater **and** would round to the required 3 sf answer, then award ***(A1)*** and ignore subsequent rounding.

2. If the candidate's unrounded answer is **not** seen then award ***(A1)*** if the answer given is **correctly** rounded to 2 or more significant figures, otherwise ***(A0)***.

Note: If the candidate's unrounded answer is **not** seen and the answer is given correct to 1 sf (correct or not), the answer will be considered wrong and will not count as incorrect accuracy. If this answer is used in subsequent parts, then working must be shown for further marks to be awarded.

3. If a correct 2 sf answer is used in subsequent parts, then working **must** be shown for further marks to be awarded. (This treatment is the same as for following through from an incorrect answer.)

These 3 points (see numbers in superscript) have been summarized in the table below and illustrated in the examples following.

	If candidates final answer is given …					
	Exact or to 4 or more sf (and would **round to the correct 3 sf**)	**Correct to 3 sf**	**Incorrect to 3 sf**	Correct to 2 sf [3]	Incorrect to 2 sf	Correct or incorrect to 1 sf
Unrounded answer seen[1]	Award the final ***(A1)*** irrespective of correct or incorrect rounding					
Unrounded answer not seen[2]	***(A1)***	***(A1)***	***(A0)***	***(A1)***	***(A0)***	***(A0)***
Treatment of subsequent parts	As per MS		Treat as follow through, only if working is seen.[3]			

Examples:

Markscheme	Candidates' Scripts	Marking
9.43 (9.43398…) ***(A1)***	(i) 9.43398… is seen followed by 9; 9.4; 9.43; 9.434 *etc.* (correctly rounded)	***(A1)***
	(ii) 9.43398… is seen followed by 9.433; 9.44 *etc.* (incorrectly rounded)	***(A1)***
	(iii) 9.4	***(A1)***
	(iv) 9	***(A0)*** *(correct to 1 sf)*
	(v) 9.3	***(A0)*** *(incorrectly rounded to 2 sf)*
	(vi) 9.44	***(A0)*** *(incorrectly rounded to 3 sf)*

Markscheme	Candidates' Scripts	Marking
7.44 (7.43798…) ***(A1)***	(i) 7.43798… is seen followed by 7; 7.4; 7.44; 7.438 *etc.* (correctly rounded)	***(A1)***
	(ii) 7.43798… is seen followed by 7.437; 7.43 *etc.* (incorrectly rounded)	***(A1)***
	(iii) 7.4	***(A1)***
	(iv) 7	***(A0)*** *(correct to 1 sf)*
	(v) 7.5	***(A0)*** *(incorrectly rounded to 2 sf)*
	(vi) 7.43	***(A0)*** *(incorrectly rounded to 3 sf)*

Example: ABC is a right angled triangle with angle $\mathrm{ABC} = 90^\circ$, $\mathrm{AC} = 32\text{ cm}$ and $\mathrm{AB} = 30\text{ cm}$. Find (a) the length of BC, (b) The area of triangle ABC.

Markscheme	Candidates' Scripts	Marking
(a) $\mathrm{BC} = \sqrt{32^2 - 30^2}$ ***(M1)*** *Award* ***(M1)*** *for correct substitution in Pythagoras' formula* $= 11.1\ \left(\sqrt{124}, 11.1355...\right)(\text{cm})$ ***(A1)***	(a) $\mathrm{BC} = \sqrt{32^2 - 30^2}$ $11\,(\text{cm})$ *(2 sf answer only seen, but correct)*	***(M1)*** ***(A1)***
(b) $\text{Area} = \frac{1}{2} \times 30 \times 11.1355...$ ***(M1)*** *Award* ***(M1)*** *for correct substitution in area of triangle formula* $= 167(167.032...)(\text{cm}^2)$ ***(A1)*****(ft)**	(b) case (i) $\text{Area} = \frac{1}{2} \times 30 \times 11$ *(working shown)* $= 165\ (\text{cm}^2)$ case (ii) $= 165\ (\text{cm}^2)$ *(No working shown, the answer 11 is treated as a ft, so no marks awarded here)*	***(M1)*** ***(A1)*****(ft)** ***(M0)(A0)*****(ft)**

Certain answers obtained from the GDC are worth 2 marks and working will not be seen. In these cases only one mark should be lost for accuracy.

eg, Chi-squared, correlation coefficient, mean

Markscheme	Candidates' Scripts	Marking
Chi-squared $7.68\ (7.67543\ldots)$ ***(A2)***	(a) 7.7	***(G2)***
	(b) 7.67	***(G1)***
	(c) 7.6	***(G1)***
	(d) 8	***(G0)***
	(e) 7	***(G0)***
	(e) 7.66	***(G0)***

Regression line

Markscheme	Candidates’ Scripts	Marking
$y = 0.888x + 13.5$ ***(A2)*** $(y = 0.887686\ldots x + 13.4895\ldots)$ If an answer is not in the form of an equation award at most ***(A1)(A0)***.	(a) $y = 0.89x + 13$	***(G2)*** *(both accepted)*
	(b) $y = 0.88x + 13$	***(G1)*** *(one rounding error)*
	(c) $y = 0.88x + 14$	***(G1)*** *(rounding error repeated)*
	(d) (i) $y = 0.9x + 13$ (ii) $y = 0.8x + 13$	***(G1)*** *(1 sf not accepted)*
	(e) $0.88x + 13$	***(G0)*** *(one rounding error and not an equation)*

Maximum/minimum/points of intersection

Markscheme	Candidates’ Scripts	Marking
$(2.06, 4.49)$ ***(A1)(A1)*** $(2.06020\ldots, 4.49253\ldots)$	(a) $(2.1, 4.5)$	***(A1)(A1)*** *(both accepted)*
	(b) $(2.0, 4.4)$	***(A1)*** *(same rounding error twice)*
	(c) $(2.06, 4.4)$	***(A1)*** *(one rounding error)*
	(d) $(2, 4.4)$	***(A0)*** *(1sf not accepted, one rounding error)*

Rounding of an exact answer to 3 significant figures **should be accepted if performed correctly**. Exact answers such as $\frac{1}{4}$ can be written as decimals to fewer than 3 significant figures if the result is still exact. Reduction of a fraction to its lowest terms is **not** essential, however where an answer simplifies to an integer this is expected. Fractions that include a decimal in the numerator and/or the denominator are acceptable for showing correct substitution, but not as a final answer.

Ratios of π and answers taking the form of square roots of integers or any rational power of an integer (*eg,* $\sqrt{13}, 2^{\frac{2}{3}}, \sqrt[4]{5}$,) may be accepted as exact answers. All other powers (*eg,* of non-integers) and values of transcendental functions such as sine and cosine must be evaluated.

If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy. In **all** such cases the final mark is not awarded if the rounding does not follow the instructions given in the question. A mark for specified accuracy can be regarded as a **(ft)** mark regardless of an immediately preceding ***(M0)***.

6 Level of accuracy in finance questions

The accuracy level required for answers will be specified in all questions involving money. This will usually be either whole units or two decimal places. The first answer not given to the specified level of accuracy will not be awarded the final ***A*** mark. The markscheme will give clear instructions to ensure that only one mark per paper can be lost for incorrect accuracy in a financial question.

Example: A financial question demands accuracy correct to 2 dp.

Markscheme	Candidates' Scripts	Marking
\$231.62 (231.6189) ***(A1)***	(i) 231.6	***(A0)***
	(ii) 232	***(A0)*** *(Correct rounding to incorrect level)*
	(iii) 231.61	***(A0)***
	(iv) 232.00	***(A0)*** *(Parts (iii) and (iv) are both incorrect rounding to correct level)*

7 Units in answers

There will be specific questions for which the units are required and this will be indicated clearly in the markscheme. The first correct answer with no units or incorrect units will not be awarded the final ***A*** mark. The markscheme will give clear instructions to ensure that only one or two mark per paper can be lost for lack of units or incorrect units.
The units are considered only when the numerical answer is awarded ***(A1)*** under the accuracy rules given in Section 5.

Example:

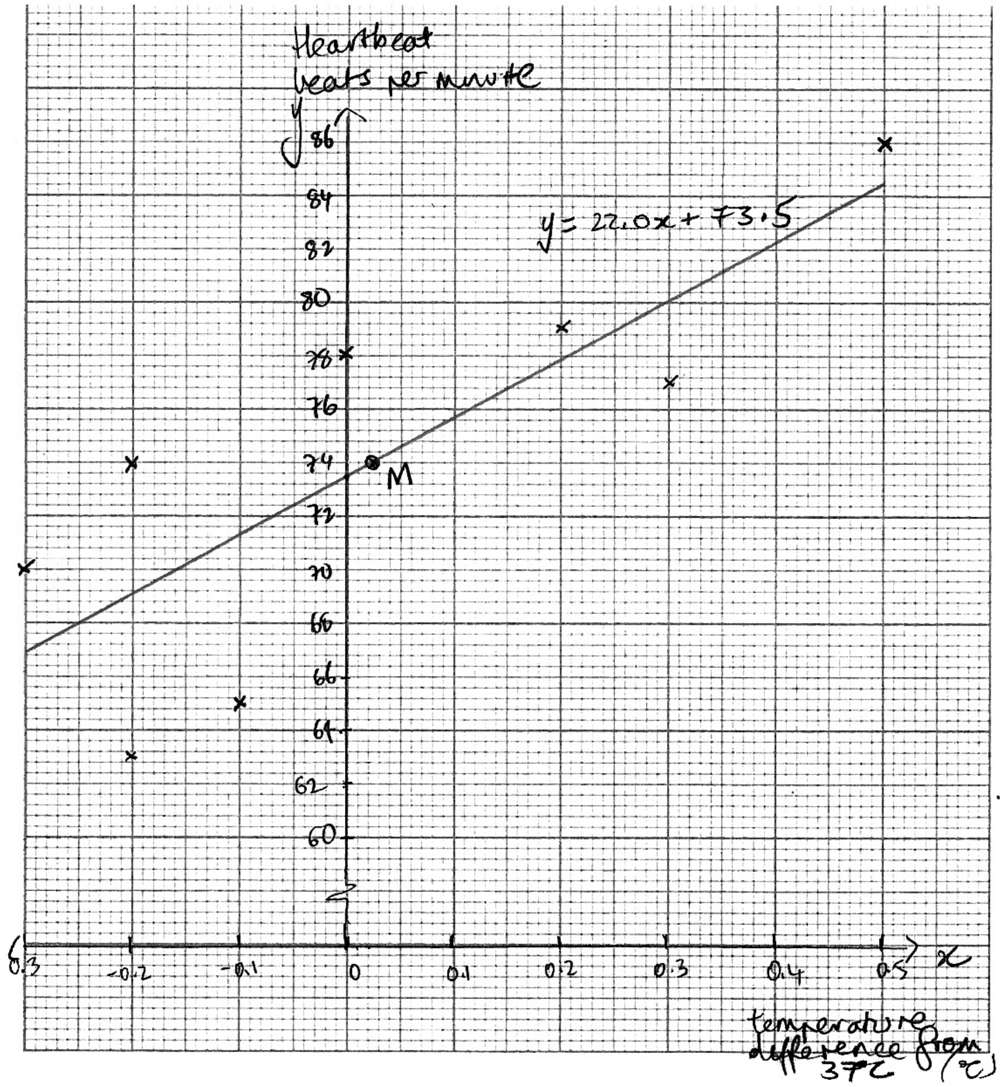
Markscheme	Candidates' Scripts	Marking
(a) 37000 m^2 ***(A1)***	(a) 36000 m^2	***(A0)*** *(Incorrect answer so units not considered)*
(b) 3200 m^3 ***(A1)***	(b) 3200 m^2	***(A0)*** *(Incorrect units)*

If no method is shown and the answer is correct but with incorrect or missing units award G marks with a one mark penalty.

8 Graphic Display Calculators

Candidates will often be obtaining solutions directly from their calculators. They must use mathematical notation, not calculator notation. No method marks can be awarded for incorrect answers supported only by calculator notation. The comment 'I used my GDC' cannot receive a method mark.

1. (a)



(A4)

Note: Award ***(A1)*** for correct scales, axis labels, minimum $x = -0.3$, and minimum $y = 60$. Award ***(A0)*** if axes are reversed and follow through for their points.
Award ***(A3)*** for all eight points correctly plotted,
(A2) for six or seven points correctly plotted.
(A1) for four or five points correctly plotted.
Allow a tolerance of half a small square.
If graph paper has not been used, award at most ***(A1)(A0)(A0)(A0)***.
If accuracy cannot be determined award ***(A0)(A0)(A0)(A0)***.

[4 marks]

continued…

Question 1 continued

(b) (i) $0.025 \left(\frac{1}{40}\right)$ ***(A1)***

(ii) 74 ***(A1)***

[2 marks]

(c) the point M labelled, correctly plotted on their diagram ***(A1)(A1)*****(ft)**

Note: Award ***(A1)*** for labelled M. Do not accept any other label. Award ***(A1)*****(ft)** for their point M correctly plotted. Follow through from part (b).

[2 marks]

(d) (i) $0.807\ (0.806797\ldots)$ ***(G2)***

(ii) (moderately) strong, positive ***(A1)*****(ft)*****(A1)*****(ft)**

Note: Award ***(A1)*** for (moderately) strong, ***(A1)*** for positive. Follow through from part (d)(i). If there is no answer to part (d)(i), award at most ***(A0)(A1).***

[4 marks]

(e) $y = 22.0x + 73.5\ (y = 21.9819\ldots x + 73.4504\ldots)$ ***(G2)***

Note: Award ***(G1)*** for $22.0x$, ***(G1)*** for 73.5.
Award a maximum of ***(G0)(G1)*** if the answer is not an equation.

[2 marks]

(f) their regression line correctly drawn on scatter diagram ***(A1)*****(ft)*****(A1)*****(ft)**

Note: Award ***(A1)*****(ft)** for a straight line, using a ruler, intercepting their mean point, and ***(A1)*****(ft)** for intercepting the y-axis at their 73.5 and the gradient of the line is positive. If graph paper is not used, award at most ***(A1)(A0).*** Follow through from part (e).

[2 marks]

Total [16 marks]

2. Units are required in part (b) of this question.

(a) $180^\circ - 27^\circ - 26^\circ$ ***(M1)***

Note: Award ***(M1)*** for correct working to find angle $\mathrm{B\hat{A}C}$ or 127 seen.

$$\frac{\mathrm{AB}}{\sin 26^\circ} = \frac{5}{\sin 127^\circ}$$ ***(M1)(A1)***

Note: Award ***(M1)*** for substitution into sine rule formula and ***(A1)*** for correct substitution.

$2.74450\,(\mathrm{m})$ ***(A1)***

$(\mathrm{AB} =)\ 2.745\,(\mathrm{m})$ ***(A1)*(ft)*(G4)***

Note: The final ***(A1)*(ft)** is for correctly rounding **their** unrounded AB to 4 sf. If 2.745 is given as the final answer, the unrounded answer need not be seen, award ***(M1)(M1)(A1)(A2)***. For all other answers, the unrounded answer must be seen to an accuracy greater than 4 sf.
Award ***(G3)*** for a final answer of $2.74450\ldots(\mathrm{m})$ with no working.
If radians are used then award at most ***(M1)(M1)(A1)(A0)(A1)*(ft)** for an answer of 3.920 (m).

[5 marks]

(b) $10 \times 2.84 + 10 \times 2.74450\ldots$ ***(M1)(M1)***

Note: Award ***(M1)*** for finding their area of each rectangle and ***(M1)*** for adding their areas.

OR

$10 \times (2.84 + 2.74450\ldots)$ ***(M1)(M1)***

Note: Award ***(M1)*** for adding AC and their AB. Award ***(M1)*** for multiplying their total area by 10.

$55.8\ (55.8450\ldots)\,\mathrm{m}^2$ ***(A1)*(ft)*(G3)***

Note: Follow through from their AB in part (a).

[3 marks]

continued…

Question 2 continued

(c) $\frac{10-2(0.3)}{1.6}$ ***(M1)***

Note: Award ***(M1)*** for correct calculation of the number of panels on the long side.

$\frac{2.745-2(0.3)}{0.95}$ **OR** $\frac{2.84-2(0.3)}{0.95}$ ***(M1)***

Note: Award ***(M1)*** for correct calculation of the number of panels on either short side with no further incorrect working.

20 ***(A1)*****(ft)*****(G2)***

Note: Follow through from part (a). Do not award ***(M0)(M1)(A1)*****(ft)**.

[3 marks]

(d) $20\times1.6\times0.95 \quad (=30.4)$ ***(M1)***

Note: Award ***(M1)*** for their $20\times1.6\times0.95$ or 30.4 seen. Follow through from their 20 in part (c). Award ***(M0)*** if their 20 is not an integer.

$\left|\frac{29-30.4}{30.4}\right|\times100\%$ ***(M1)***

Note: Award ***(M1)*** for correct substitution of their 30.4 into the percentage error formula. Their 30.4 must be **exact.** found. Accept a method in two steps where "$\times100$" is implicit from their answer. The second ***(M1)*** is contingent on the first ***(M1)*** being awarded, eg do **not** award ***(M0)(M1)(A0)***.

$4.61(\%)$ $(4.60526(\%))$ ***(A1)*****(ft)*****(G3)***

Note: Follow through from their answer to part (c). Percentage sign is not required. Award ***(G2)*** for an unsupported final answer of 4.61.

[3 marks]

continued…

Question 2 continued

(e) 1×9 (array) **OR** 18 (total panels) ***(R1)*(ft)**

Note: Award ***(R1)*** for one correct array seen (1×9) or total number of panels (18). Working is not required, but award ***(R0)*** for incorrect working seen. Correct working is as follows. $\left(\frac{10-0.6}{0.95}, \frac{2.84-0.6}{1.6}, \frac{2.745-0.6}{1.6}\right)$

Reasoning may compare both sides of the roof or just one side; accept correct comparisons with part (c) values. Follow through from their treatment of tolerances in part (c) and maximum number of panels.
Award ***(R0)*** for any approach with no clearance or for any method which includes further incorrect working.

No (new arrangement will mean fewer solar panels) ***(A1)*(ft)**

Note: Follow through from their maximum number of panels in part (c). Do not award ***(R0)(A1)*(ft)*.***

[2 marks]

Total [16 marks]

3. (a) 140 ***(A1)***

[1 mark]

(b) 1 ***(A1)***

[1 mark]

(c) (i) 2 ***(A1)***

(ii) 1 ***(A1)***

(iii) 3 ***(A1)***

[3 marks]

(d) $17:15$ **OR** $\frac{17}{15}$ ***(A1)***

Note: Award ***(A0)*** for $85:75$ or $1.13:1$.

[1 mark]

(e) (i) preferred pet is independent of “whether or not the respondent was a teenager" or "age category” ***(A1)***

Note: Accept there is no association between pet and age. Do not accept “not related” or “not correlated” or “influenced”.

(ii) preferred pet is not independent of age ***(A1)*(ft)**

Note: Follow through from part (e)(i) *i.e.* award ***(A1)*(ft)** if their alternative hypothesis is the negation of their null hypothesis. Accept “associated” or “dependent”.

[2 marks]

(f) 3 ***(A1)***

[1 mark]

(g) $\frac{85\times55}{160}$ **OR** $\frac{85}{160}\times\frac{55}{160}\times160$ ***(M1)***

$29.2\ (29.2187\ldots)$ ***(A1)(G2)***

[2 marks]

continued…

Question 3 continued

(h) $0.208\ (0.208093\ldots)$ ***(G2)***

[2 marks]

(i) $0.208 > 0.1$ ***(R1)***

accept null hypothesis **OR** fail to reject null hypothesis ***(A1)*(ft)**

[2 marks]

Note: Award ***(R1)*** for a correct comparison of their p-value to the significance level, award ***(A1)*(ft)** for the correct result from that comparison. Accept "p-value > 0.1" as part of the comparison but only if their p-value is explicitly seen in part (h). Follow through from their answer to part (h). Do not award ***(R0)(A1)***.

Total [15 marks]

4. (a) $f\left(\frac{1}{2}\right)=\left(\frac{1}{2}\right)^{3}-5\left(\frac{1}{2}\right)^{2}+6\left(\frac{1}{2}\right)-3+\frac{1}{\left(\frac{1}{2}\right)}$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into given function.

$\frac{7}{8}\ (0.875)$ ***(A1)(G2)***

[2 marks]

(b) (i) $\dfrac{0-\frac{7}{8}}{0-\frac{1}{2}}$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into gradient formula. Accept equivalent forms such as $\frac{7}{8}=\frac{1}{2}m$.

$\frac{7}{4}\quad(1.75)$ ***(A1)*****(ft)**

$P(x)=\frac{7}{4}x\ (1.75x)$ ***(A1)*****(ft)*****(G3)***

Note: Follow through from part (a).

(ii) $0<x<\frac{1}{2}$ ***(A1)(A1)***

Note: Award ***(A1)*** for both endpoints correct, ***(A1)*** for correct mathematical notation indicating an interval with two endpoints. Accept weak inequalities. Award at most ***(A1)(A0)*** for incorrect notation such as $0-0.5$ or a written description of the domain with correct endpoints. Award at most ***(A1)(A0)*** for $0<y<\frac{1}{2}$.

[5 marks]

continued…

Question 4 continued

(c) $(0.360, 1.34)$ $\left((0.359947\ldots, 1.33669)\right)$ ***(A1)(A1)***

$(3.63, 1.01)$ $\left((3.63066\ldots, 1.00926\ldots)\right)$ ***(A1)(A1)***

> **Note:** Award ***(A1)(A1)*** for each correct coordinate pair. Accept correct answers in the form of $x = 0.360,\ y = 1.34$ *etc*. Award at most ***(A0)(A1)(A1)(A1)*ft** if one or both parentheses are omitted.

[4 marks]

(d) $g(0) = 0.5(3)^0 + 1$ ***(M1)***

$1.5\,(\text{km})$ ***(A1)(G2)***

[2 marks]

(e) domain given as $x > 0$ (but equation of road is $x = 0$) ***(R1)***

OR

(equation of road is $x = 0$) the function of the river is asymptotic to $x = 0$ ***(R1)***

so it does not meet the river ***(A1)***

> **Note:** Award the ***(R1)*** for a correct mathematical statement about the equation of the river (and the equation of the road). Justification must be based on **mathematical reasoning**. Do not award ***(R0)(A1)***.

[2 marks]

Total [15 marks]

5. (a) $1.042 \times 880 \times 1.25$ **OR** $(880 + 0.042 \times 880) \times 1.25$ ***(M1)(M1)***

> **Note:** Award ***(M1)*** for multiplying 880 by 1.042 and ***(M1)*** for multiplying 880 by 1.25.

1150 (CAD) (1146.20 (CAD)) ***(A1)(G2)***

> **Note:** Accept 1146.2 (CAD).

[3 marks]

(b) $\frac{704}{880}$ **OR** $\frac{563.20}{704}$ ***(M1)***

> **Note:** Award ***(M1)*** for correctly dividing sequential terms to find the common ratio, or 0.8 seen.

$880(0.8)^{5-1}$ ***(M1)***

> **Note:** Award ***(M1)*** for correct substitution into geometric sequence formula.

360.45 (USD) ***(A1)(G3)***

> **Note:** Do not award the final ***(A1)*** if the answer is not correct to 2 decimal places. Award at most ***(M0)(M1)(A0)*** if $r = 1.25$.

[3 marks]

(c) $880(0.8)^{n-1} < 50$ ***(M1)***

> **Note:** Award ***(M1)*** for correct substitution into geometric sequence formula and (in)equating to 50. Accept weak or strict inequalities. Accept an equation. Follow through from their common ratio in part (b). Accept a sketch of their GP with $y = 50$ as a valid method.

OR

$u_{13} = 60.473$ **AND** $u_{14} = 48.379$ ***(M1)***

> **Note:** Award ***(M1)*** for their u_{13} and u_{14} **both** seen. If the student states $u_{14} = 48.379 < 50$, without $u_{13} = 60.473$ seen, this is not sufficient to award ***(M1)***.

14 or "14th year" or "after the 13th year" ***(A1)*(ft)*(G2)***

> **Note:** The context of the question requires the final answer to be an integer. Award at most ***(M1)(A0)*** for a final answer of 13.9 years. Follow through from their 0.8 in part (b).

[2 marks]

continued…

Question 5 continued

(d) $\frac{5}{2}\left((2\times 120)+\left(-3.5(5-1)\right)\right)$ ***(M1)(A1)***

Note: Award ***(M1)*** for substitution into arithmetic series formula, ***(A1)*** for correct substitution.

565 (USD) ***(A1)(G2)***

[3 marks]

(e) 2019 is the 12th year/term ***(M1)***

Note: Award ***(M1)*** for 12 seen.

75.59 (value of bicycle) **AND** 81.5 (cost of insurance policy) ***(A1)*(ft)**

Note: Award ***(A1)*** for both sequences' 12th term seen. The value of the bicycle will follow through from their common ratio in part (b). Do not award ***(M0)(A1)***.

the cost of the insurance policy is greater than the value of the bicycle ***(R1)*(ft)**

Note: Award ***(R1)*(ft)** for a reason consistent with their cost of insurance policy and their value of the bicycle. Follow through within this part. Award ***(R0)*** if the correct values are not explicitly seen. Accept the following contextualized reasons: "the insurance is not worth it", "the values are too close", "insurance is as much as the value of the bike", but only if **their** cost of insurance is greater than the value of the bicycle.

OR

$75.59 < 81.5$ ***(R1)*(ft)**

Note: Award ***(R1)*(ft)** for a correct numerical comparison showing their cost of insurance policy is greater than their value of the bicycle. Follow through within this part.

[3 marks]

Total [14 marks]

6. (a) $x^2 + x + k$ ***(A1)(A1)(A1)***

Note: Award ***(A1)*** for each correct term. Award at most ***(A1)(A1)(A0)*** if additional terms are seen or for an answer $x^2 + x - 6$. If their derivative is seen in parts (b), (c) or (d) and not in part (a), award at most ***(A1)(A1)(A0)***.

$(-3)^2 + (-3) + k = 0$ ***(M1)(M1)***

Note: Award ***(M1)*** for substituting in $x = -3$ into their derivative and ***(M1)*** for setting it equal to zero. Substituting $k = -6$ invalidates the process, award at most ***(A1)(A1)(A1)(M0)(M0)***.

$(k =)\ -6$ ***(AG)***

Note: For the final ***(M1)*** to be awarded, no incorrect working must be seen, and must lead to the conclusion $k = -6$. The final ***(AG)*** must be seen.

[5 marks]

(b) $(2, -2.33)$ **OR** $\left(2, -\frac{7}{3}\right)$ ***(A1)(A1)***

Note: Award ***(A1)*** for each correct coordinate. Award ***(A0)(A1)*** if parentheses are missing. Accept $x = 2,\ y = -2.33$. Award ***(M1)(A0)*** for their derivative, a quadratic expression with -6 substituted for k, equated to zero but leading to an incorrect answer.

[2 marks]

(c) $-3 < x < 2$ ***(A1)*(ft)*(A1)***

Note: Award ***(A1)*** for $x > -3$, ***(A1)*(ft)** for $x < 2$. Follow through for their "2" in part (b). It is possible to award ***(A0)(A1)***. For $-3 < y < 2$ award ***(A1)(A0)***. Accept equivalent notation such as $(-3, 2)$. Award ***(A0)(A1)*(ft)** for $-3 \le x \le 2$.

[2 marks]

continued…

Question 6 continued

(d) –4 ***(A1)*(ft)**

Note: Award ***(A1)*(ft)** for the gradient of the tangent seen. If an incorrect derivative was used in part (a), then working for their $f'(-2)$ must be seen. Follow through from their derivative in part (a).

gradient of normal is $\frac{1}{4}$ ***(A1)*(ft)**

Note: Award ***(A1)*(ft)** for the negative reciprocal of their gradient of tangent. Follow through within this part. Award ***(G2)*** for an unsupported gradient of the normal.

$\frac{49}{3}$ $\left(f(-2)=\frac{1}{3}(-2)^3+\frac{1}{2}(-2)^2-6(-2)+5=\frac{49}{3}\right)$ ***(A1)***

Note: Award ***(A1)*** for $\frac{49}{3}$ $(16.3333...)$ seen.

$\frac{49}{3}=\frac{1}{4}(-2)+c$ **OR** $y-\frac{49}{3}=\frac{1}{4}(x--2)$ ***(M1)***

Note: Award ***(M1)*** for substituting their normal gradient into equation of line formula.

$y=\frac{1}{4}x+\frac{101}{6}$ **OR** $y=0.25x+16.8333...$ ***(A1)*(ft)*(G4)***

Note: Award ***(G4)*** for the correct equation of line in correct form without any prior working. The final ***(A1)*(ft)** is contingent on $y=\frac{49}{3}$ and $x=-2$

[5 marks]

Total [14 marks]