Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematical studies
Standard level
Paper 2

Tuesday 14 May 2019 (morning)

1 hour 30 minutes

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- A clean copy of the **mathematical studies SL formula booklet** is required for this paper.
- Answer all the questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- The maximum mark for this examination paper is **[90 marks]**.

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Answer **all** questions in the answer booklet provided. Please start each question on a new page. You are advised to show all working, where possible. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. Solutions found from a graphic display calculator should be supported by suitable working, for example, if graphs are used to find a solution, you should sketch these as part of your answer.

1. [Maximum mark: 16]

A healthy human body temperature is $37.0\,^{\circ}\mathrm{C}$. Eight people were medically examined and the difference in their body temperature (°C), from $37.0\,^{\circ}\mathrm{C}$, was recorded. Their heartbeat (beats per minute) was also recorded.

Temperature difference from 37°C (x)	–0.2	0.3	–0.3	–0.2	–0.1	0	0.2	0.5
Heartbeat (y)	63	77	70	74	65	78	79	86

(a) Draw a scatter diagram for temperature difference from $37\,^{\circ}\mathrm{C}$ (x) against heartbeat (y). Use a scale of $2\,\mathrm{cm}$ for $0.1\,^{\circ}\mathrm{C}$ on the horizontal axis, starting with $-0.3\,^{\circ}\mathrm{C}$. Use a scale of $1\,\mathrm{cm}$ for 2 heartbeats per minute on the vertical axis, starting with 60 beats per minute. [4]

(b) Write down, for this set of data

(i) the mean temperature difference from $37\,^{\circ}\mathrm{C}$, $\overline{x}$;

(ii) the mean number of heartbeats per minute, $\overline{y}$. [2]

(c) Plot and label the point $\mathrm{M}(\overline{x}, \overline{y})$ on the scatter diagram. [2]

(d) (i) Use your graphic display calculator to find the Pearson's product–moment correlation coefficient, r.

(ii) Hence describe the correlation between temperature difference from $37\,^{\circ}\mathrm{C}$ and heartbeat. [4]

(e) Use your graphic display calculator to find the equation of the regression line y on x. [2]

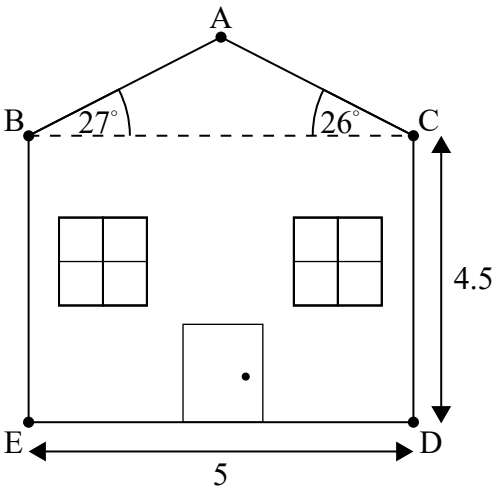
(f) Draw the regression line y on x on the scatter diagram. [2]

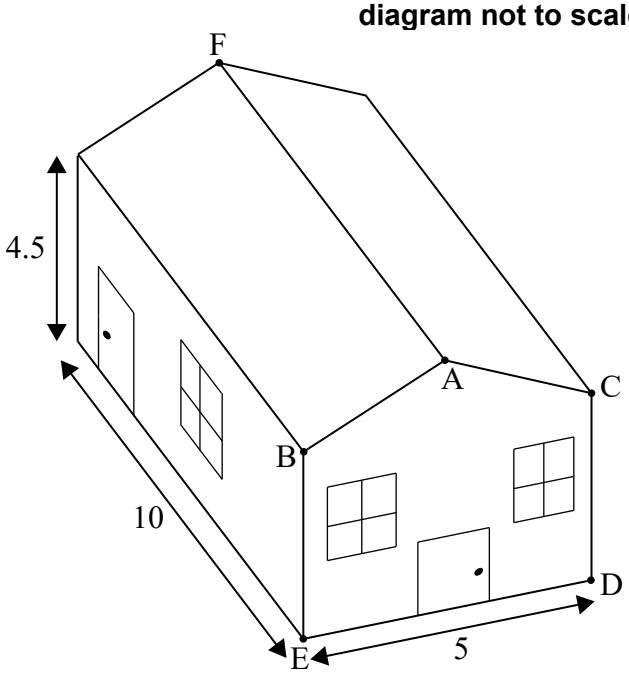
2. [Maximum mark: 16]

Olivia's house consists of four vertical walls and a sloping roof made from two rectangles. The height, CD, from the ground to the base of the roof is $4.5\,\mathrm{m}$.

The base angles of the roof are $\mathrm{A\hat{B}C} = 27^\circ$ and $\mathrm{A\hat{C}B} = 26^\circ$.

diagram not to scale





The house is $10\,\mathrm{m}$ long and $5\,\mathrm{m}$ wide.

(a) Find the length AB, giving your answer to **four significant figures**. [5]

The length AC is approximately $2.84\,\mathrm{m}$.

(b) Find the total area of the two rectangles that make up the roof. [3]

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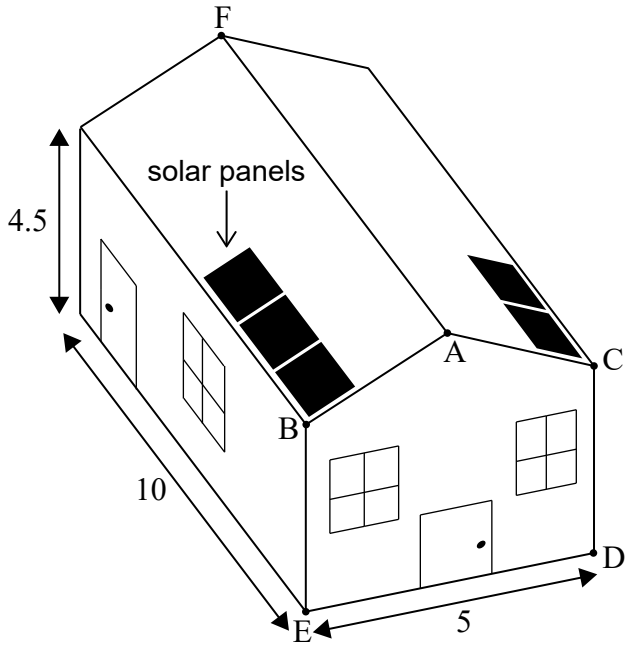
(Question 2 continued)

Olivia decides to put solar panels on the roof. The solar panels are fitted to both sides of the roof.

diagram not to scale



Each panel is $1.6\,\text{m}$ long and $0.95\,\text{m}$ wide. All the panels must be arranged in uniform rows, with **the shorter edge** of each panel parallel to AB or AC. Each panel must be at least $0.3\,\text{m}$ from the edge of the roof and the top of the roof, AF.

(c) Find the maximum number of complete panels that can be fitted to the whole roof. [3]

Olivia estimates that the solar panels will cover an area of $29\,\text{m}^2$.

(d) Find the percentage error in her estimate. [3]

Olivia investigates arranging the panels, such that **the longer edge** of each panel is parallel to AB or AC.

(e) State whether this new arrangement will allow Olivia to fit more solar panels to the roof. Justify your answer. [2]

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3. [Maximum mark: 15]

A survey was conducted on a group of people. The first question asked how many pets they each own. The results are summarized in the following table.

Number of pets owned	0	1	2	3	4	5
Number of people	20	45	40	30	20	5

(a) Write down the total number of people, from this group, who are **pet owners**. [1]

(b) Write down the modal number of pets. [1]

(c) For these data, write down

(i) the median number of pets;

(ii) the lower quartile;

(iii) the upper quartile. [3]

The second question asked each member of the group to state their age and preferred pet. The data obtained is organized in the following table.

	Age	
Preferred pet	**Teenager**	**Non-teenager**
cat	23	32
dog	35	23
bird	16	13
other	11	7

(d) Write down the ratio of teenagers to non-teenagers in its simplest form. [1]

A χ^2 test is carried out at the 10% significance level.

(e) State

(i) the null hypothesis;

(ii) the alternative hypothesis. [2]

(f) Write down the number of degrees of freedom for this test. [1]

(g) Calculate the expected number of teenagers that prefer cats. [2]

(h) Use your graphic display calculator to find the p-value for this test. [2]

(i) State the conclusion for this test. Give a reason for your answer. [2]

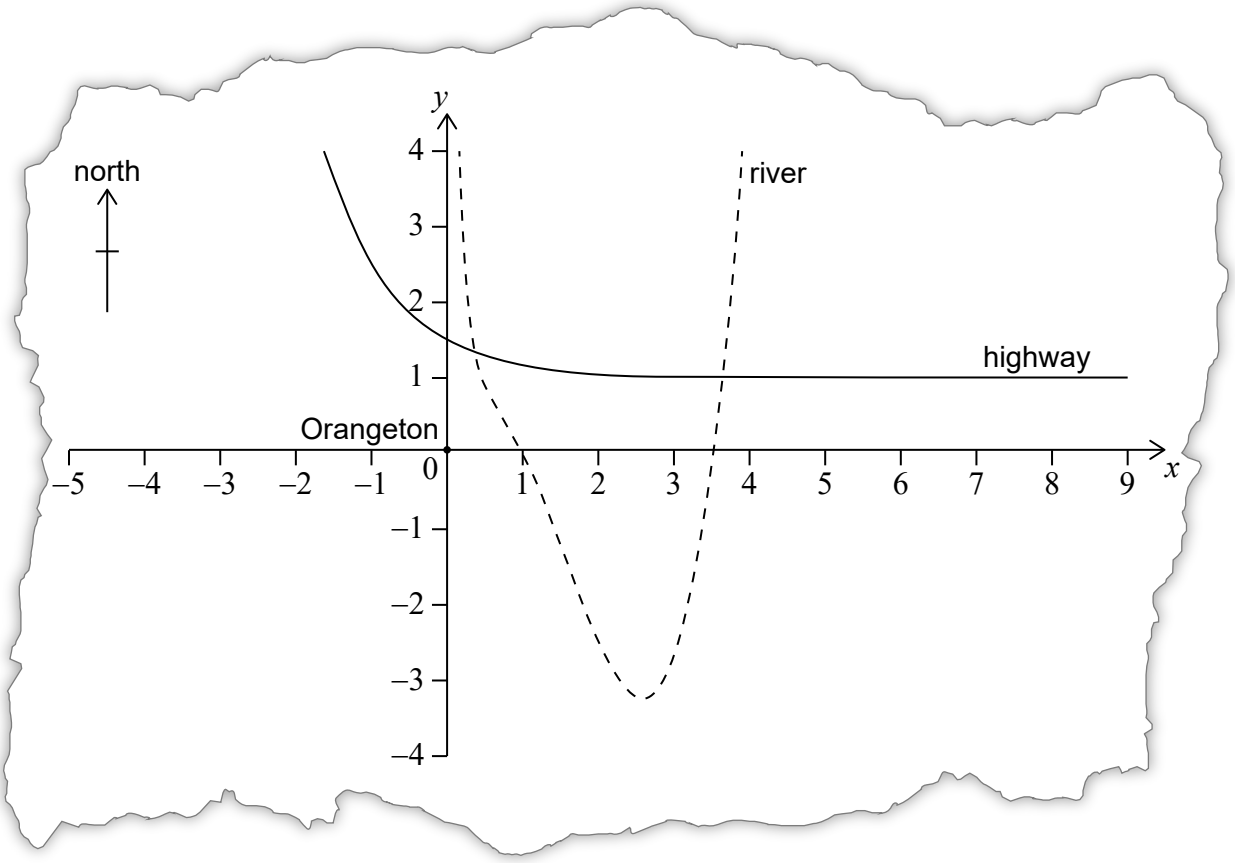
Turn over

4. [Maximum mark: 15]

Consider the function $f(x) = x^3 - 5x^2 + 6x - 3 + \frac{1}{x}$, $x > 0$.

(a) Find the value of $f(x)$ when $x = \frac{1}{2}$. [2]

The function $f(x) = x^3 - 5x^2 + 6x - 3 + \frac{1}{x}$, $x > 0$, models the path of a river, as shown on the following map, where both axes represent distance and are measured in kilometres. On the same map, the location of a highway is defined by the function $g(x) = 0.5(3)^{-x} + 1$.



The origin, $O(0, 0)$, is the location of the centre of a town called Orangeton.

A straight footpath, P, is built to connect the centre of Orangeton to the river at the point where $x = \frac{1}{2}$.

(b) (i) Find the function, $P(x)$, that would define this footpath on the map.

(ii) State the domain of P. [5]

(This question continues on the following page)

(Question 4 continued)

Bridges are located where the highway crosses the river.

(c) Find the coordinates of the bridges relative to the centre of Orangeton. [4]

A straight road is built from the centre of Orangeton, due north, to connect the town to the highway.

(d) Find the distance from the centre of Orangeton to the point at which the road meets the highway. [2]

This straight road crosses the highway and then carries on due north.

(e) State whether the straight road will ever cross the river. Justify your answer. [2]

5. [Maximum mark: 14]

John purchases a new bicycle for 880 US dollars (USD) and pays for it with a Canadian credit card. There is a transaction fee of 4.2% charged to John by the credit card company to convert this purchase into Canadian dollars (CAD).

The exchange rate is $1 \text{ USD} = 1.25 \text{ CAD}$.

(a) Calculate, in CAD, the total amount John pays for the bicycle. [3]

John insures his bicycle with a US company. The insurance company produces the following table for the bicycle's value during each year.

Year	Value of the bicycle (USD)
1st	880
2nd	704
3rd	563.20
…	…

The values of the bicycle form a geometric sequence.

(b) Find the value of the bicycle during the 5th year. **Give your answer to two decimal places**. [3]

(c) Calculate, in years, when the bicycle value will be less than 50 USD. [2]

During the 1st year John pays 120 USD to insure his bicycle. Each year the amount he pays to insure his bicycle is reduced by 3.50 USD.

(d) Find the total amount John has paid to insure his bicycle for the first 5 years. [3]

John purchased the bicycle in 2008.

(e) Justify why John should not insure his bicycle in 2019. [3]

6. [Maximum mark: 14]

The function $f(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 + kx + 5$ has a local maximum and a local minimum. The local maximum is at $x = -3$.

(a) Show that $k = -6$. [5]

(b) Find the coordinates of the local **minimum**. [2]

(c) Write down the interval where the gradient of the graph of $f(x)$ is negative. [2]

(d) Determine the equation of the normal at $x = -2$ in the form of $y = mx + c$. [5]