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Mathematical studies
Standard level
Paper 1

Monday 18 November 2019 (afternoon)

Candidate session number

1 hour 30 minutes

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Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- A clean copy of the **mathematical studies SL formula booklet** is required for this paper.
- Answer all questions.
- Answers must be written within the answer boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- The maximum mark for this examination paper is **[90 marks]**.



Maximum marks will be given for correct answers. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. Answers must be written within the answer boxes provided. Solutions found from a graphic display calculator should be supported by suitable working, for example, if graphs are used to find a solution, you should sketch these as part of your answer.

1. Complete the following table by placing ticks (\checkmark) to show which of the number sets \mathbb{N} , \mathbb{Z} , \mathbb{Q} , and \mathbb{R} these numbers belong to. The first row has been completed as an example. [6]

	\mathbb{N}	\mathbb{Z}	\mathbb{Q}	\mathbb{R}
$-\frac{3}{11}$			\checkmark	\checkmark
42				
3.14				
$\sqrt{23}$				
-113				
$\frac{24}{6}$				
$-5\frac{1}{5}$				

Working:



2. M-Line is a company that prints and sells custom designs on T-shirts. For each order, they charge an initial design fee and then an additional fee for each printed T-shirt.

M-Line charges M euros per order. This charge is modelled by the linear function $M(x) = 5x + 40$, where x is the number of T-shirts in the order.

- (a) Write down the initial design fee charged for each order. [1]
- (b) Find the total amount charged for an order of 94 T-shirts. [2]

EnYear is another company that prints and sells T-shirts. The price, N euros, that they charge for an order can be modelled by the linear function $N(x) = 9x$, where x is the number of T-shirts in the order.

- (c) Write down the number of T-shirts in an order for which EnYear charged 63 euros. [1]

An order of p T-shirts will be charged the same price by both M-Line and EnYear.

- (d) Find the value of p . [2]

Working:

Answers:

- (a)
- (b)
- (c)
- (d)



3. Popularized on the internet, fun names are often given for different types of dogs. Three of these names are given in the following propositions.

p : it is not a doggo

q : it is a floofer

r : it is a woofer

- (a) Write down in words the compound proposition $p \Rightarrow \neg r$. [2]
- (b) In symbolic form, write down the converse of $p \Rightarrow (\neg q \wedge r)$. [1]
- (c) Complete the following truth table. [2]

p	q	r	$p \Rightarrow q$	$q \Rightarrow r$	$(p \Rightarrow q) \wedge (q \Rightarrow r)$	$p \Rightarrow r$	$((p \Rightarrow q) \wedge (q \Rightarrow r)) \vee (p \Rightarrow r)$
T	T	T	T	T		T	
T	T	F	T	F		F	
T	F	T	F	T		T	
T	F	F	F	T		F	
F	T	T	T	T		T	
F	T	F	T	F		T	
F	F	T	T	T		T	
F	F	F	T	T		T	

- (d) Hence, justify why $((p \Rightarrow q) \wedge (q \Rightarrow r)) \vee (p \Rightarrow r)$ is not a logical contradiction. [1]

Working:

Answers:

- (a)
- (b)
- (d)
-
-



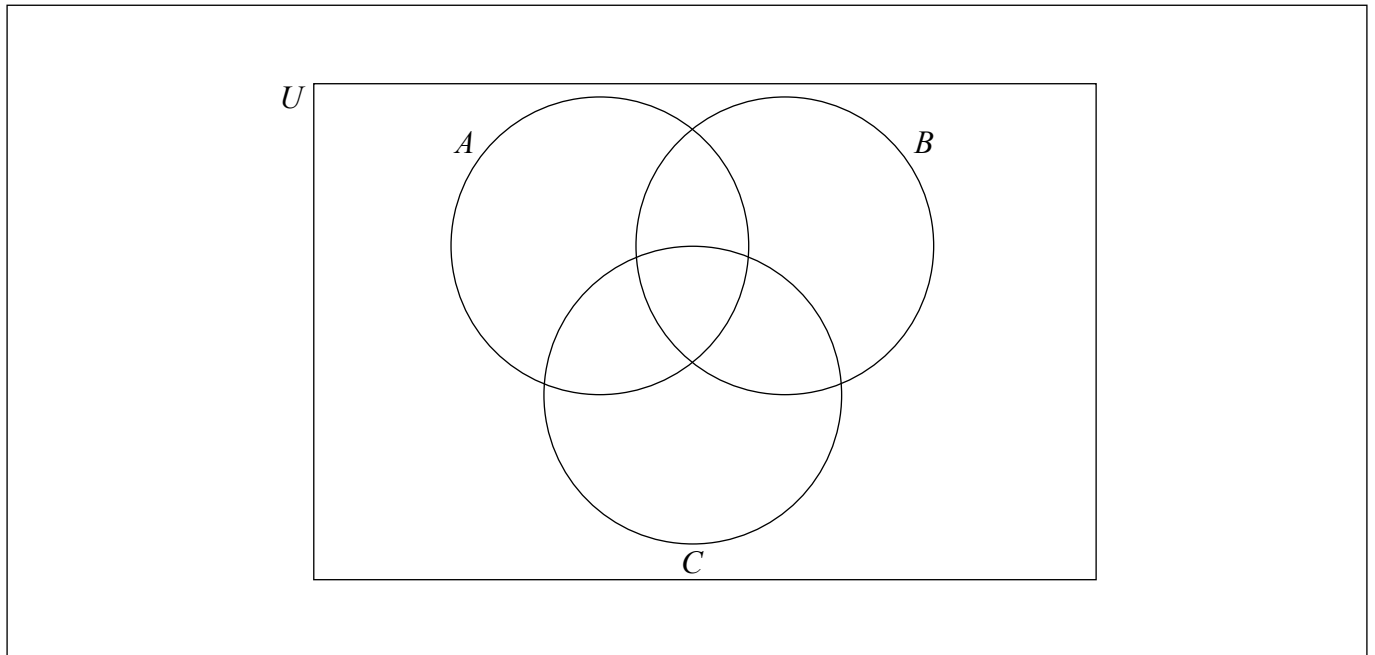
4. Let the universal set, U , be the set of all integers x such that $1 \leq x < 11$.
 A , B and C are subsets of U .

$$A = \{1, 2, 3, 4, 6, 8\}$$

$$B = \{2, 3, 5, 7\}$$

$$C = \{1, 3, 5, 7, 9\}$$

- (a) Write down $n(B)$. [1]
(b) Complete the following Venn diagram using **all** elements of U . [4]



- (c) Write down an element that belongs to $(A \cup B)' \cap C$. [1]

Working:

Answers:

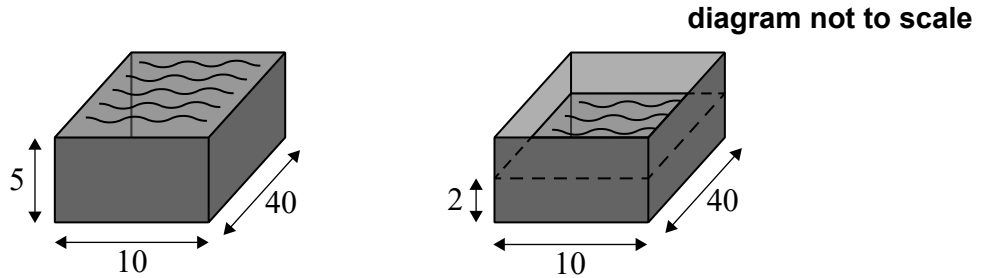
(a)

(c)



5. Yao drains the oil from his motorbike into two identical cuboids with rectangular bases of width 10 cm and length 40 cm. The height of each cuboid is 5 cm.

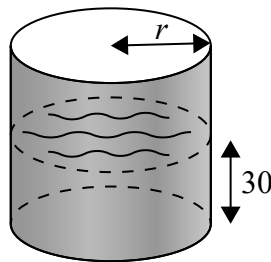
The oil from the motorbike fills the first cuboid completely and the second cuboid to a height of 2 cm. The information is shown in the following diagram.



- (a) Calculate the volume of oil drained from Yao's motorbike. [3]

Yao then pours all the oil from the cuboids into an empty cylindrical container. The height of the oil in the container is 30 cm.

diagram not to scale



- (b) Find the internal radius, r , of the container. [3]

(This question continues on the following page)



(Question 5 continued)

Working:

Answers:

(a)

(b)



20EP07

Turn over

6. Galois Airways has flights from Hong Kong International Airport to different destinations. The following table shows the distance, x kilometres, between Hong Kong and the different destinations and the corresponding airfare, y , in Hong Kong dollars (HKD).

Destination	Bali, Indonesia	Sydney, Australia	Bengaluru, India	Singapore	Auckland, New Zealand	Bangkok, Thailand
Distance x , (km)	3400	7400	4000	2600	9200	1700
Airfare y , (HKD)	1550	3600	2800	1300	4000	1400

The Pearson's product-moment correlation coefficient for this data is 0.948, correct to three significant figures.

- (a) Use your graphic display calculator to find the equation of the regression line y on x . [2]

The distance from Hong Kong to Tokyo is 2900 km.

- (b) Use your regression equation to estimate the cost of a flight from Hong Kong to Tokyo with Galois Airways. [2]
- (c) Explain why it is valid to use the regression equation to estimate the airfare between Hong Kong and Tokyo. [2]

Working:

Answers:

- (a)
- (b)
- (c)



7. A geometric sequence has a first term of $\frac{8}{3}$ and a fourth term of 9.

(a) Find the common ratio. [2]

(b) Write down the second term of this sequence. [1]

The sum of the first k terms is greater than 2500.

(c) Find the smallest possible value of k . [3]

Working:

Answers:

(a)

(b)

(c)



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Answers written on this page
will not be marked.



8. Siân invests 50 000 Australian dollars (AUD) into a savings account which pays a nominal annual interest rate of 5.6% **compounded monthly**.

(a) Calculate the value of Siân's investment after four years. Give your answer correct to two decimal places. [3]

After the four-year period, Siân withdraws 40 000 AUD from her savings account and uses this money to buy a car. It is known that the car will depreciate at a rate of 18% per year.

The value of the car will be 2500 AUD after t years.

(b) Find the value of t . [3]

Working:

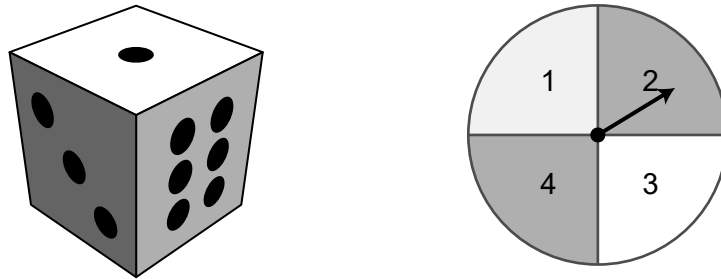
Answers:

(a)

(b)



9. Sungwon plays a game where she rolls a fair 6-sided die and spins a fair spinner with 4 equal sectors. During each turn in the game, the die is rolled once and the spinner is spun once. The **score** for each turn is the sum of the two results. For example, 1 on the die and 2 on the spinner would receive a score of 3.



The following diagram represents the sample space.

		Die					
		1	2	3	4	5	6
Spinner	1	●	●	●	●	●	●
	2	●	●	●	●	●	●
	3	●	●	●	●	●	●
	4	●	●	●	●	●	●

- (a) Find the probability that Sungwon's score on her first turn is greater than 4. [2]
Sungwon takes a second turn.
- (b) Find the probability that Sungwon scores greater than 4 on both of her first two turns. [2]
Sungwon will play the game for 11 turns.
- (c) Find the expected number of times the score on a turn is greater than 4. [2]

(This question continues on the following page)



(Question 9 continued)

Working:

Answers:

(a)

(b)

(c)



20EP13

Turn over

10. In this question, give your answers to the nearest whole number.

Criselda travelled to Kota Kinabalu in Malaysia. At the airport, she saw the following information at the Currency Exchange counter.

Currency Exchange – Malaysian Ringgit (MYR)		
Currency	Sell	Buy
1 Singapore Dollar (SGD)	3.22	3.07
1 US Dollar (USD)	4.45	4.25

This means the Currency Exchange counter would **buy** USD from a traveller and in exchange return MYR at a rate of 1 USD = 4.25 MYR. There is no commission charged.

Criselda changed 460 SGD to MYR.

- (a) Calculate the amount of MYR that Criselda received. [3]

While in Kota Kinabalu, Criselda spent 440 MYR. She returned to the Currency Exchange counter and changed the remainder of her MYR into USD.

- (b) Calculate the amount of USD she received. [3]

Working:

Answers:

(a)

(b)



11. Elvis Presley is an extremely popular singer. Although he passed away in 1977, many of his fans continue to pay tribute by dressing like Elvis and singing his songs.

The number of Elvis impersonators, $N(t)$, can be modelled by the function

$$N(t) = 170 \times 1.31^t,$$

where t , is the number of years since 1977.

- (a) Write down the number of Elvis impersonators in 1977. [1]
- (b) Calculate the time taken for the number of Elvis impersonators to reach 130 000. [2]
- (c) Calculate the number of Elvis impersonators when $t = 70$. [2]

The world population in 2047 is projected to be 9 500 000 000 people.

- (d) Use this information to explain why the model for the number of Elvis impersonators is unrealistic. [1]

Working:

Answers:

- (a)
- (b)
- (c)
- (d)
.....
.....



12. The Malthouse Charity Run is a 5 kilometre race. The time taken for each runner to complete the race was recorded. The data was found to be normally distributed with a mean time of 28 minutes and a standard deviation of 5 minutes.

A runner who completed the race is chosen at random.

- (a) Write down the probability that the runner completed the race in more than 28 minutes. [1]
- (b) Calculate the probability that the runner completed the race in less than 26 minutes. [2]

It is known that 20% of the runners took more than 28 minutes and less than k minutes to complete the race.

- (c) Find the value of k . [3]

Working:

Answers:

- (a)
- (b)
- (c)



13. Chicken eggs are classified by grade (4, 5, 6, 7 or 8), based on weight. A mixed carton contains 12 eggs and could include eggs from any grade. As part of the science project, Rocky buys 9 mixed cartons and sorts the eggs according to their weight.

Grade	Weight, w (grams)	Frequency
4	$40 \leq w < 50$	3
5	$50 \leq w < 60$	30
6	$60 \leq w < 70$	45
7	$70 \leq w < 80$	25
8	$80 \leq w < 90$	5

- (a) State whether the weight of the eggs is a continuous or discrete variable. [1]
- (b) Write down the modal grade of the eggs. [1]
- (c) Use your graphic display calculator to find an estimate for the standard deviation of the weight of the eggs. [2]

The mean weight of these eggs is 64.9 grams, correct to three significant figures.

- (d) Use the table and your answer to part (c) to find the **smallest possible** number of eggs that could be within one standard deviation of the mean. [2]

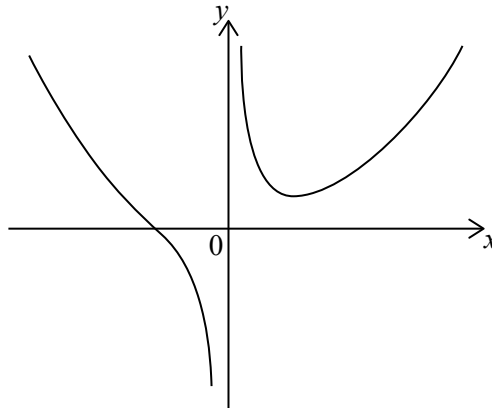
Working:

Answers:

- (a)
- (b)
- (c)
- (d)



14. The diagram shows the curve $y = \frac{x^2}{2} + \frac{2a}{x}$, $x \neq 0$.



The equation of the vertical asymptote of the curve is $x = k$.

(a) Write down the value of k . [1]

(b) Find $\frac{dy}{dx}$. [3]

At the point where $x = 2$, the gradient of the tangent to the curve is 0.5.

(c) Find the value of a . [2]

Working:

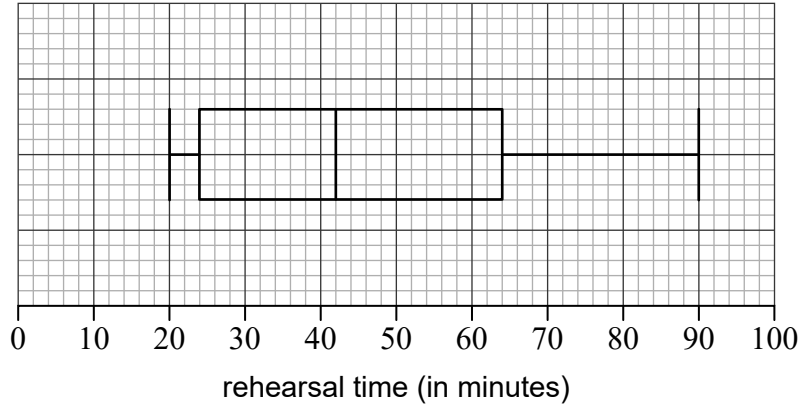
Answers:

- (a)
- (b)
- (c)



15. Stephen was invited to perform a piano recital. In preparation for the event, Stephen recorded the amount of time, in minutes, that he rehearsed each day for the piano recital.

Stephen rehearsed for 32 days and data for all these days is displayed in the following box-and-whisker diagram.



- (a) Write down the median rehearsal time. [1]

Stephen states that he rehearsed on each of the 32 days.

- (b) State whether Stephen is correct. Give a reason for your answer. [2]

On k days, Stephen practiced exactly 24 minutes.

- (c) Find the possible values of k . [3]

Working:

Answers:

- (a)
(b)
(c)



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20EP20