REPUBLIQUE DU CAMEROUN
Paix – Travail – Patrie

MINISTERE DES ENSEIGNEMENTS SECONDAIRES

DELEGATION REGIONALE DE L'OUEST

INSPECTION REGIONALE DE PEDAGOGIE
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REPUBLC OF CAMEROON
Peace – Work - Fatherland

MINISTRY OF SECONDARY EDUCATION

REGIONAL DELGATION FOR THE WEST

REGIONAL INSPECTORATE OF PEDAGOGY
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F. MATHS 775 **West General Certificate of EDUCATION MOCK EXAMINATION**

March 2019 **ADVANCED LEVEL**

THE REGIONAL DELEGATION OF SECONDARY EDUCATION IN ASSOCIATION WITH THE **TEACHERS' RESOURCE UNIT**	SUBJECT CODE NUMBER **775**	PAPER NUMBER **2**
MAHTEMATICS TEACHERS' PEDAGOGIC GROUP	SUBJECT TITLE **Further mathematics**	

Two and a half hours

INSTRUCTIONS TO CANDIDATES

Answer ALL questions

For your guidance, the approximate mark allocation for parts of each question is indicated in brackets.

Mathematical formulae and tables, published by the Board and noiseless non-programmable electronic calculators are allowed.

In calculations, you are advised to show all the steps in your working giving your answer at each stage.

Calculators are allowed.

Start each question on a fresh page.

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1. The matrices A and B are such that

$$AB = \begin{pmatrix} k & 8 & 1 \\ 1 & 1 & 0 \\ 1 & 4 & 0 \end{pmatrix}, \text{ where } A = \begin{pmatrix} k & 6 & 8 \\ 0 & 1 & 2 \\ -3 & 4 & 8 \end{pmatrix} \text{ and } k \text{ is a constant,}$$

(a) show that AB is non-singular **(2 marks)**
(b) find $(AB)^{-1}$ in terms of k **(4 marks)**
(c) use your result in (b) to find B^{-1}. **(4 marks)**
(d) Find, in Cartesian form, the image of the line $\mathbf{r} = \mathbf{i} - \mathbf{j} + \lambda(2\mathbf{i} + \mathbf{j} - \mathbf{k})$ under the transformation represented by the matrix A, with $k = 2$. **(3 marks)**

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2. (a) Consider the vectors $\mathbf{a} = 2\mathbf{i} + \mathbf{j} + \mathbf{k}$, $\mathbf{b} = \mathbf{i} + 2\mathbf{j} - \mathbf{k}$, $\mathbf{c} = 2\mathbf{i} + \mathbf{j} + 2\mathbf{k}$;
(i) Show that the vectors **a, b** and **c** form a basis in 3-dimensions. **(2 marks)**
(ii) Express the vectors $\mathbf{d} = 3\mathbf{i} + 5\mathbf{j} + 3\mathbf{k}$ in terms of **a**, **b** and **c**. **(2 marks)**

(b) Show that the mapping, T, defined by

$$T: \mathbb{R}^2 \to \mathbb{R}^2$$
$$(x, y) \mapsto (2x - 3y, x + 2y)$$

is a linear transformation.
Find the kernel, ker (T), of this linear transformation. **(6 marks)**

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3. (a) Sketch on the same axes the half-lines Arg $(z) = 0$, Arg $(z) = \frac{3}{4}\pi$, and the circle $|z + 1 - i| = 1$. Hence, shade the region satisfied by z for which $|z + 1 - i| \le 1$ and $0 \le Arg(z) \le \frac{3}{4}\pi$. **(5 marks)**

(b) A complex transformation T from the z-plane to the w-plane is given by $T(z) = \dfrac{z-2}{z-i} = w$. Show that the image of the circle $|z| = 2$ is also a circle in the w-plane and sketch it. **(5 marks)**

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4. (a) Given that the differential equation $\frac{dy}{dx} = y \cot x, \ 0 < x < \pi$.
Show that $y = A \sin x$, where A is such that $\ln|A|$ is the constant of integration. **(2 mks)**
Sketch the family of curves for integral values of A from –3 to 3 and $0 \le x < \pi$. **(2mks)**

(b) A second order differential equation is given by $16\dfrac{d^2y}{dx^2} + 8\dfrac{dy}{dx} + 5y = 5x + 23$
(i) Find the complementary function, y_c. **(2 marks)**
(ii) Find the particular integral, y_p. **(3 marks)**
(iii) Hence, state the general solution of the above differential equation. **(1 mark)**
(iv) Deduce an approximate solution for y as $x \to +\infty$ **(1 mark)**

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5. (a) Express $f(x)$, where $f(x) = \dfrac{2x+3}{(x+1)^2(x+2)^2}$ in partial fractions. **(3 marks)**

Hence or otherwise, show that $\int_1^3 f(x)dx = \frac{7}{60}$. **(2 marks)**

(b) Use the theorems of Pappus to calculate the volume and surface area of the solid generated when the region $x^2 + (y - 3x)^2 \le 4\lambda^2$ is rotated completely about the x- axis.
(3 marks)

(c) A curve has equation $y=\sqrt{\left(1-x^{\frac{2}{3}}\right)^2}$, $0\le x\le 1$.

Show that its arc length is $\frac{3}{2}$ units. **(3 marks)**

6. Given that $f(x)=\dfrac{x^2+x-1}{2(x-1)}$

(a) The domain of definition of f is $D_f =]-\infty, r[\cup]r, +\infty[$. State the value of r.

(b) Express $f(x)$ in the form $ax+b+\dfrac{c}{2(x-1)}$, where a, b and c are constants to be found. Find $\lim\limits_{x\to\pm\infty}\left[f(x)-(ax+b)\right]$. **(1, 3 marks)**

(c) Find the limits at the boundaries of D_f. **(2 marks)**

(d) State the equations of the asymptotes to the curve of f. **(1 mark)**

(e) Find $f'(x)$ and determine the turning points on the curve of f. **(3 marks)**

(f) Sketch the curve of f. **(2 marks)**

7. (a) Prove that the equations of the tangent and normal to the ellipse $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$ at the point $P(a\cos\theta, b\sin\theta)$ are respectively $(b\cos\theta)x+(a\sin\theta)y=ab$ and $(a\sin\theta)x-(b\cos\theta)y=(a^2-b^2)\sin\theta\cos\theta$. **(6 marks)**

(b) The hyperbola $\dfrac{x^2}{\alpha^2}-\dfrac{y^2}{\beta^2}=1$ has asymptotes $y^2=m^2x^2$ and passes through the point $(a, 0)$, $a\neq 0$. Find α^2 and β^2 and hence state the equation of the hyperbola. **(4 mks)**

8. (a) Express $17\cosh x+15\sinh x$ in the form $R\cosh(x+\ln\alpha)$, where R and α are positive integers. **(3 marks)**

Hence or otherwise, find the maximum value of $\dfrac{56}{34\cosh x+30\sinh x+12}$ **(2 mks)**

(b) Given the polar equation $r=4\cos 3\theta$,

(i) sketch this curve,

(ii) find the area enclosed by this curve. **(6 marks)**

9. Given the sequences $(u_n)$and (v_n), $n\in\mathbb{N}$, defined as

$$\begin{cases} u_0=0 \\ u_{n+1}=\frac{1}{2}(u_n+v_n)\end{cases} \quad and \quad \begin{cases} v_0=12 \\ v_{n+1}=\frac{1}{3}(u_n+2v_n)\end{cases}$$

(a) Show that the sequence (w_n) where $w_n = v_n - u_n$, $n\ge 0$, is a geometric sequence with positive terms. Deduce the $\lim\limits_{n\to+\infty} w_n$. **(3 marks)**

(b) Show that (u_n) is increasing while (v_n) is decreasing. **(3 marks)**

(c) Prove that the sequence (t_n) where $t_n = 2u_n + 3v_n$, $n\ge 0$, is a constant sequence. **(2 marks)**

(d) Deduce the common limit of (v_n) and (u_n). **(2 marks)**

(e) Express u_n and v_n in terms of n. **(2 marks)**