SOUTH WEST REGIONAL MOCK EXAMINATION
GENERAL EDUCATION

THE TEACHERS' RESOURCE UNIT (TRU)
IN COLLABORATION WITH

THE REGIONAL INSPECTORATE OF PEDAGOGY FOR THE SCIENCES

AND

THE SOUTH WEST ASSOCIATION OF MATHEMARTICS TEACHERS (SWAMT)

TUESDAY, 06/04/2022 ADVANCE LEVEL

Subject Title	**Mathematics with Mechanics**
Paper Number	**Paper 3**
Subject Code Number	**0765**

THREE HOURS

INSTRUCTIONS TO CANDIDATES:

Full marks may be obtained for answers to ALL questions.

All questions carry equal marks.

You are reminded of the necessity for good English and orderly presentation in your answers.

Mathematical formulae booklet published by Board are allowed.

In calculations, you are advised to show all the steps in your working, giving your answer at each stage.

Calculators are allowed.

Start each question on a fresh page.

1. A Particle of mass 8 kg moves such that its displacement $\mathbf{r}$ at time t seconds is given by

$$\mathbf{r} = [(t + cos\, t)\mathbf{i} + (1 + sin\, t)\mathbf{j} + 2t\mathbf{k}]\ \text{m}.$$

(a) Determine the magnitude of the momentum and kinetic energy of this particle when $t = \frac{\pi}{6}$ s. **(6 marks)**

(b) Prove that the force acting on this particle when $t = \frac{\pi}{6}\ s$ is of magnitude 8 N. **(2 marks)**

(c) Calculate the work done on this particle from the instance when $t = \frac{\pi}{6}\ s$ to $\pi\ s$. **(3 marks)**

(d) Find, to two decimal places, the angle between the acceleration and velocity of the particle when $t = \frac{\pi}{6}\ s$. **(2 marks)**

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2. (i) Two spheres S_1 and S_2 of masses $3m$ kg and m kg respectively are placed on a smooth horinzontal floor. Sphere S_1 is projected with speed $2u$ m/s and impinges directly on S_2 which is at rest. Given that S_2 receives an impulse of magnitude $2mu$ kgm/s.

Calculate,

(a) the speed of S_1 and S_2 after impact, **(4 marks)**

(b) the coefficient of restitution between S_1 and S_2. **(2 marks)**

(ii) Two forces $\mathbf{F_1}$ and $\mathbf{F_2}$ act at points with position vectors $\mathbf{a_1}$ and $\mathbf{a_2}$ respectively, where

$\mathbf{F_1} = (3\mathbf{i} + 2\mathbf{j})\text{N}$, $\mathbf{a_1} = (2\mathbf{i} + 5\mathbf{j})\ \text{m}$

$\mathbf{F_2} = (-4\mathbf{i} + 5\mathbf{j})\text{N}$, $\mathbf{a_2} = (3\mathbf{i} - 2\mathbf{j})\ \text{m}.$

Find

(c) the position vector of the point of intersection of the lines of action of the two forces **(3 marks)**

(d) the magnitude of the resultant of the two forces **(2 marks)**

(e) the magnitude of the resulatant moment of the two forces **(2 marks)**

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3. (i) Two particles P_1 and P_2 at time $t = 0$ have position vectors $(\mathbf{i} - \mathbf{j} + 2\mathbf{k})$ m and $(\mathbf{i}+5\mathbf{j} +4\mathbf{k})$ m respectively. P_1 moves with a speed of 6 m/s in the direction of the vector $\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$ and P_2 moves with a speed of 7 m/s in the direction of vector $2\mathbf{i} + 3\mathbf{j} - 6\mathbf{k}$.

(a) Determine the equation of path of each particle and show that the particles do not collide. **(7 marks)**

(ii) A uniform ladder of length 2a, $a > 0$, rests in equilibrium with its lower end on a rough horizontal ground and its upper end against a smooth vertical wall. The plane containing this ladder is perpendicular to the wall and makes an angle α with the ground where $\tan\alpha = \sqrt{3}$. Determine the coefficient of friction between the ladder and the ground. **(6 marks)**

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4. (i) A particle moves along a straight line with speed $\left(\frac{2}{7-x}\right)$ m/s where x m is the distance covered by the particle from a fixed point O on the line.

Calculate,

(a) the time taken to cover a distance of 4 m from O, **(4 marks)**

(b) the acceleration when the particle has covered a distance of 4 m from O. **(3 marks)**

(ii) Two points E and F are such that F is 3a metres vertically below E. A particle G of mass m kg is connected to E by a spring of natural length a metres. Each of the two springs has modulus of elasticity $2mg$. Determine the height above F that the particle G rests in equilibrium. **(6 marks)**

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5. (i) A ball is projected from the top of a wall which is $3m$ high, with an initial speed of 20 m/s, this ball just clears a pole 13 m high on its path which is a horizontal distance of 20 m from the foot of the wall.

(a) Determine the tangent of the two possible angles of projection. **(5 marks)**

Using the smaller value of these tangents,

(b) Find the time for which the velocity is at right angles to the initial and the horizontal distance covered by the ball in this time. **(8 marks)**

(ii) A particle Q of mass 2 kg, is attached to two points M and N by two inelastic strings of lengh 3m and 5m respectively, where QN = 5m and M is vertically beneath N. Q describes a horizontal circle with speed 9 m/s when both strigs are taut. Find the tension in each string. **(4 marks)**

(Take g as $10\ m\ s^{-2}$)

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6. A car of mass 5 tonnes is moving along a horizontal straight road at a steady speed of 10 m/s. The magnitude of the force resisting the motion of the car is proportional to its speed. Given that the engine of the car is working at 40 Kw , determine,

(a) the magnitude of the force resisting the motion. **(4 marks)**

The car now climbs a hill whose inclination to the horizontal is θ, where $20\sin\theta = 1$ at a steady speed of 5 m/s . Calculate,

(b) the rate at which the engine is working, **(4 marks)**

(c) the immediate acceleration of the car if the rate at which the engine is working suddenly increases by 10 KW, when it is climbing the hill with the speed of 5 m/s. **(5 marks)**

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7. A region of the $x - y$ plane is bounded by an arc of the curve $y = 1 + x^2$, the $x -$ axis and the lines $x = 0$ and $x = 2$.

(a) Show that the centroid of the area of this region is at the point whose coordinates are approximately (1.29, 1.47). **(4 marks)**

The coordinates of the points V , R , S and T are (0,1) , (2,0) (7,0) and (7,5) respectively and RSTU is a uniform square.

(b) Find the coordinates of the centre of gravity of the uniform lamina bouneded by VO, OS, ST, TU and the arc UV of the curve $y = 1 + x^2$ where O is the origin. **(7 marks)**

The uniform lamina VOSTU is freely suspeneded at the vertex T.

(c) Find the tangent of the angle the side ST makes with the vertical wall. **(2 marks)**

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8. (i) Two events A and B are such that $P(A/B) = \frac{7}{10}$, $P\left(\frac{B}{A}\right) = \frac{7}{15}$ and $P(AUB) = \frac{3}{15}$. Find

(a) $P(AnB)$ **(3 marks)**

(b) $P(A)$ and $P(B)$ **(2 marks)**

(c) $P(A/B^c)$ **(2 marks)**

(ii) A man has three cell phones A, B and C, each having the probability of containing only one SIM card X , Y and Z mobile networks respectively. He receives calls from these cell phones in the ration 5: 3:2. Calls which enter each of these phones are either accepted or rejected. 20% of the calls from X are rejected, 10% of calls from Y are rejected and 15% of calls from Z are rejected. **(2 marks)**

(d) Draw a tree diagram showing all the distribution of the calls. **(2 marks)**

Hence

(e) find the probability that a call is accepted. **(2 marks)**

Given that an incoming call is rejected find the probability that this call comes from Y.
