0775/3/2022
F. MATHS A/L

SOUTH WEST REGIONAL MOCK EXAMINATION
GENERAL EDUCATION

THE TEACHERS' RESOURCE UNIT (TRU)
IN COLLABORATION WITH

THE REGIONAL INSPECTORATE OF PEDAGOGY FOR SCIENCE EDUCTION

AND

THE SOUTH-WEST ASSOCIATION OF MATHEMATICS TEACHERS (SWAMT)

TUESDAY, 05/04/2022 ADVANCED LEVEL

Subject Title	**Further Mathematics**
Paper Number	**Paper 3**
Subject Code Number	**0775**

TWO AND A HALF HOURS

INSTRUCTIONS TO CANDIDATES:

Answer ALL questions.

For your guidance, the approximate mark allocation for parts of each question is indicated in brackets.

You are reminded of the necessity for good English and orderly presentation in your answers.

Mathematical formulae and tables published by the CGCE Board and noiseless, non-programmable calculators are allowed.

In calculations, you are advised to show all the steps in your working, giving your answer at each stage.

1. a) A force $\vec{F}$, acting parallel to the vector $(-5i + 2j + 5k)$, displaces a particle from the point with position

1. a) A force $\vec{F}$, acting parallel to the vector $(-5i+2j+5k)$, displaces a particle from the point with position vector $(3i-3j+4k)m$ to the point with position vector $(i-4j+6k)m$. Given that the work done on the particle is 36 joules, find $\vec{F}$. (3 marks)

b) Three forces $\vec{F}_1$, $\vec{F}_2$ and $\vec{F}_3$ act respectively at the points with position vectors $\vec{r}_1$, $\vec{r}_2$ and $\vec{r}_3$ such that

$\vec{F}_1 = (i+2j+3k)N$, $\vec{r}_1 = (j+k)m$

$\vec{F}_2 = (-i+2k)N$, $\vec{r}_2 = (4i+j-k)m$

$\vec{F}_3 = (pi+qj+rk)N$, $\vec{r}_3 = (\alpha i+\beta j+5k)m$. Given that $\vec{F}_1+\vec{F}_2+\vec{F}_3 = 0$ and that the sum of the moments of the three forces about the origin O is $(-7i+9j-6k)Nm$,

(i) state the system of forces that the three forces form, (1 mark)

(ii) hence find the values of p, q and r, (2 marks)

(iii) find the moments of $\vec{F}_1$ and $\vec{F}_2$ about the origin. (3 marks)

(iv) Hence, determine the values of α and β . (4 marks)

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2. a) Evaluate $\int_1^7 \sqrt{(x^2-x)}\,dx$, correct to two decimal places, using Simpson's rule with 7 ordinates. (4 marks)

b) Find in ascending powers of x, up to and including the term in x^4, the solution of the differential equation $\frac{d^2y}{dx^2} + y\frac{dy}{dx} + x = 0$, given that $y = 1$ and $\frac{dy}{dx} = 1$ when $x = 0$. Using your solution, find to four decimal places, the value of y when $x = 0.1$ (4 marks)

c) The variables x and y satisfy the differential equation $\frac{d^2y}{dx^2} = xe^{cosy}$. Using the approximation $h^2\left(\frac{d^2y}{dx^2}\right)_n \approx y_{n+1} - 2y_n + y_{n-1}$ and a step length of h = 0.1, show that when $x = 1.3, y \approx 1.6456$, given that $y = 1$ when $x = 1$ and $y = 1.2$ when $x = 1.1$ (4 marks)

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3. An unstable compound Z decomposes into two substances X and Y such that a mass 2m of Z breaks down into a mass m of X and a mass m of Y. The substances X and Y also amalgamate in equal masses to form the compound Z.

At any time, the mass Z is being reduced due to decomposition at a rate k multiplied by the mass of Z present at that instant, where k is a positive constant. At the same time, the mass of Z is being increased at a rate $3k$ multiplied by the product of the masses of X and Y present at that time.

Initially, only Z is present. Given that at time t, the masses in kg, of X, Y and Z are respectively $10x$, $10x$ and $20(2-x)$,

a) show that $\frac{dx}{dt} = k(2 - x - 15x^2)$. (4 marks)

b) By solving this differential equation, show that $In\left(\frac{2+5x}{2-6x}\right) = 11kt$. (4 marks)

Given that at a particular time t, the mass of Z present is $\frac{9}{10}$ of the initial mass of Z,

c) show that $In\left(\frac{15}{4}\right) = 11kt$ (4 marks)

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4. The radial and transverse components of the velocity of a particle at the point (r, θ) are $2r^2$ and $3\theta^2$, respectively. Initially $r = 1$ and $\theta = 1$.

a) Show that the polar equation of the path of the particle at time t is $r^2 = \frac{3\theta}{4-\theta}$ (4 marks)

b) Show that the radial component of the acceleration of the particle in terms of r and θ is $\frac{8r^4 - 9\theta^4}{r}$ and find, in terms of r and θ, the transverse component of this acceleration. (8 marks)

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5. a) A smooth sphere moving along a smooth horizontal plane with velocity $(4i - 5j)$ ms^{-1} collides with a smooth vertical wall which is parallel to j. Given that the coefficient of restitution between the sphere and the wall is $\frac{1}{2}$, find the velocity of the sphere after impact. (3 marks)

b) Two smooth spheres A and B of different masses, moving on a smooth horizontal plane collide as shown in fig.1 below. Before impact, A is moving with speed 21ms^{-1} at an angle of 60^0 to the line of centres, and B is moving with speed 7ms^{-1} perpendicular to the line of centres. Immediately after impact, the directions of motion of A and B make angles of 90^0 and 45^0 respectively with the line of centres.

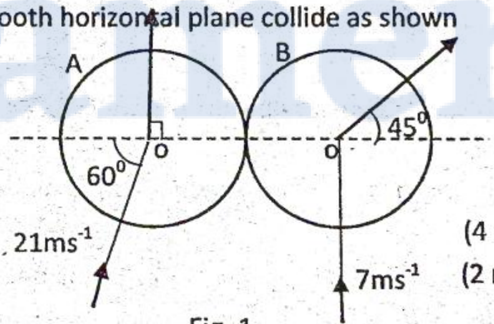

Fig. 1

(i) Find the speeds of A and B after impact (4 marks)

(ii) Show that the coefficient of restitution between A and B is $\frac{2}{3}$. (2 marks)

Given that the mass of B is 3kg,

(iii) find the mass of A, (2 marks)

(iv) find the magnitude of the impulse experienced by A, (2 marks)

(v) find the loss in kinetic energy during the impact. (2 marks)

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6. a) The random variable X is such that $X \sim Po(\lambda)$.

(i) Express $E(X^2)$ in terms of λ. (2 marks)

Hence, given also that $(X^2) = 6$, find

(ii) the value of λ (2 marks)

(iii) $P(X > E(X))$, correct to four decimal places. (2 marks)

b) Given that $\sim N(\mu, 6)$, $Y \sim N(8, \sigma^2)$ and $(2X - 3Y) \sim N(-12, 42)$, find

(i) the values of μ and σ (4 marks)

(ii) $P(X > Y)$, correct to four decimal places. (3 marks)

7.

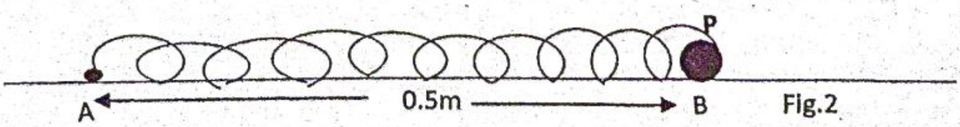

Fig.2

Figure 2 shows a spring of natural length 0.5m and modulus of elasticity 20N lying in equilibrium on a smooth horizontal table, with one end fixed to a point A on the table and the other end attached to a particle P of mass 0.4 kg, lying at the point B on the table.

The particle is displaced a distance of 0.1m from B in the sense AB, and then released from rest. In the ensuing motion, P is subject to a variable resistance of magnitude $4.8v\ N$, where v is its speed.

Given that $x\ m$ is the displacement of P from B in the sense AB, at time t seconds after P is released,

a) show that the equation of motion of P is $\frac{d^2x}{dt^2} + 12\frac{dx}{dt} + 100x = 0$ (3 marks)

b) By solving this differential equation, show that the displacement, x, of P from B at time t seconds can be expressed as $x = Re^{-\alpha t}\cos(\beta t + \lambda)$, stating the values of R, α, β and λ, and expressing R and λ each correct to three decimal places. (6 marks)

c) Hence, find, correct to four decimal places, the distance of P from A, 0.4 seconds after P is released from rest. (3 marks)

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8. Given that the moment of inertia of a uniform disc of mass m and radius a, about an axis which is a tangent to the disc is $\frac{5}{4}ma^2$, show that the moment of inertia of the disc about an axis through the centre of the disc and perpendicular to the plane of the disc is $\frac{1}{2}ma^2$. (3 marks)

This uniform disc is made to rotate in a vertical plane about a horizontal axis through its centre and perpendicular to its plane. A light inextensible string is wound round the rim of the disc and two particles of masses m and $2m$ are hanging at the ends of the string. Given that the particles are released from rest when the string is taut and the hanging parts of the string are vertical, find

a) the acceleration of each particle, (4 marks)
b) the tension in each of the vertical parts of the string, (3 marks)
c) the force exerted on the disc by the string. (2 marks)
d) the kinetic energy of the system, at the instant when the velocity of the particles is $\sqrt{\left(\frac{8ag}{7}\right)}$. (3 marks)
