0775/2/2022
F. MATHS A/L

SOUTH WEST REGIONAL MOCK EXAMINATION
GENERAL EDUCATION

THE TEACHERS' RESOURCE UNIT (TRU)
IN COLLABORATION WITH

THE REGIONAL INSPECTORATE OF PEDAGOGY FOR SCIENCE EDUCTION

AND

THE SOUTH-WEST ASSOCIATION OF MATHEMATICS TEACHERS (SWAMT)

MONDAY, 04/04/2022 ADVANCED LEVEL

Subject Title	**Further Mathematics**
Paper Number	**Paper 2**
Subject Code Number	**0775**

THREE HOURS

INSTRUCTIONS TO CANDIDATES:

*Answer **ALL TEN** questions.*

For your guidance, the approximate mark allocation for parts of each question is indicated in brackets.

You are reminded of the necessity for good English and orderly presentation in your answers.

Mathematical formulae and tables published by the CGCE Board and noiseless, non-programmable calculators are allowed.

In calculations, you are advised to show all the steps in your working, giving your answer at each stage.

1. The variables x and y are functions of t and satisfy the differential equations $\frac{dx}{dt} + 2x = y$ and $\frac{dy}{dt} + x = 0$.

   a) Show that $\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + x = 0$ (1 mark)

   b) Find the general solution for x, of the differential equation in (a) above, and deduce from $\frac{dx}{dt} + 2x = y$ the general solution for . (4 marks)

   c) Hence or otherwise, find x and y in terms of t, given that $x = 1$ and $= 0$, when $t = 0$. (2 marks)

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2. a) Show that the set M of all matrices of the form $\begin{pmatrix} 1 & n \\ 0 & 1 \end{pmatrix}$, $n \in \mathbb{Z}$, forms an Abelian group under multiplication of matrices. (Assume associativity) (4 marks)

   b) Given that $(x) = \frac{x^3}{(x^2+2)^3}$, (i) express $f(x)$ in partial fractions (3 marks)

   (ii) hence, show that $\int_0^1 f(x)dx = \frac{1}{72}$ (3 marks)

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3. a). Prove by mathematical induction that $\forall\, n \in \mathbb{Z}^+$, $\sum_{r=1}^{n}(r^2 + 1)r! = n(n + 1)!$ (4 marks)

   b) (i) Find the gcd of 54 and 21 and express it in the form d = $54x + 21y$. (2 marks)

   (ii) Hence, solve the linear congruence $54x \equiv 12(mod21)$ (3 marks)

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4. a). Solve for x and y the equations $coshx = 3sinhy$ and $2sinhx = 5 - 6coshy$, expressing your answers in logarithmic form.. (4 marks)

   b) Given that $r^2 = \frac{225}{25-16sin^2\theta}$ is the equation of an ellipse, find

   (i) the Cartesian equation of the ellipse (2 marks)

   (ii) the eccentricity of the ellipse (1 mark)

   (iii) the coordinates of the foci of the ellipse (1 mark)

   (iv) the equations of the directrices of the ellipse (1 mark)

   (v) the equations of the asymptotes of the ellipse (I mark)

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5. a) The position vectors of four non-coplanar points A, B, C and D relative to the origin O are $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ respectively, where $\vec{a} = 2i - 3j + 5k, \vec{b} = i + 6j - k, \vec{c} = 5i + k$ and $\vec{d} = 3j - 2k$.

Find (i) the Cartesian equation of the plane ABC, (3 marks)

(ii) the volume of the tetrahedron ABCD. (3 marks)

b) Find the polar coordinates of the points on the cardiod $r = a(1 + cos\theta)$ at which the tangent lines are parallel to the initial line. (5 marks)

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6. a) (i) Given that $x = 2sin^2\theta$, show that $\sqrt{\left(\frac{2a-x}{x}\right)} = cot\theta$ (1 mark)

(ii) Hence, show also that the area of the finite region bounded by the curve $xy^2 = 4a^2(2a - x)$, the x-axis and the ordinates $x = a$ and $x = 2a$ is $a^2(\pi - 2)$ (3 marks)

This area is rotated completely about the x-axis.

(iii) Find the volume of the solid of revolution obtained. (2 marks)

(iv) Hence, using a theorem of Pappus, deduce that the distance of the centroid of this area from the x-axis is $2a\frac{(2ln2-1)}{\pi-2}$ (1 mark)

b) Given that $I_n = \int_0^1 (1 + x^2)^n\, dx, n \in \mathbb{Z}^+$, show that $(2n + 1)I_n = 2^n + 2nInI_{n-1}$ (4 marks)

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7. The linear transformation $T: \mathbb{R}^3 \to \mathbb{R}^3$ has matrix $M = \begin{pmatrix} 2 & 1 & 1 \\ 3 & 1 & 3 \\ -1 & -2 & 4 \end{pmatrix}$

a) Show that every point in space is mapped onto the plane $5x - 3y + z = 0$ (4 marks)

Find the image under T of

b) the line $\frac{x-2}{2} = \frac{y+1}{-3} = \frac{z-1}{-1}$ (2 marks)

c) the plane $x + y - z = -1$ (3 marks)

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8. a) (i) Find $\sum_{r=0}^{\infty}(z^r)$ (1 mark)

(ii) Given that $z = \frac{1}{2}(cos\theta + isin\theta)$, show that $\sum_{r=0}^{\infty}(z^r) = \frac{4-2cos\theta+2isin\theta}{5-4cos\theta}$ (3 marks)

(iii) Hence, find $\sum_{r=0}^{\infty}\left(\frac{1}{2^r}sin^r\,\theta\right)$ (1 mark)

b) Two similarity transformations T and M on a complex plane are defined by $T: z \to 3iz + 2$

$M: z \to (2 - i)\bar{z} - 3 + 2i$

(i) Find the inverse, T^{-1}, of T (1 mark)

(ii) Find the composite transformation M $\circ$ T, stating its radius (3 marks)

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9. A numerical function f of real variable x is defined by $f(x) = 2x + 3 + In\left(\frac{x+1}{x-1}\right)$

a) Determine the domain of definition D_f of f. (2 marks)

b) Calculate the limits at the bounds of D_f (2 marks)

c) Evaluate $\lim_{x\to\pm\infty}\{f(x) - (2x+3)\}$, and hence, determine the equations of the asymptotes to the representative curve (C_f) of f. (2 marks)

d) Show that $f'(x) = \frac{2x^2-4}{x^2-1}$, and construct the table of variation of f. (3 marks)

e) Hence, sketch the graph of $y = f(x)$ (2 marks)

f) Hence, find the coordinates of the centre of symmetry of the curve (C_f) (2 marks)

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10. a) A sequence (U_n) is defined recursively by $(U_n): \begin{cases} U_0 = -1 \\ U_1 = 4 \\ U_{n+2} = \frac{1}{3}U_{n+1} + \frac{2}{3}U_n \end{cases}$

Given also that another sequence (V_n) is defined by $V_n = U_{n+1} - U_n$, $n \in \mathbb{N}$,

(i) show that (V_n) is a convergent geometric sequence, and express V_n in terms of n (3 marks)

(ii) by evaluating $\sum_{r=0}^{n-1}(V_n) = \sum_{r=0}^{n-1}(U_{n+1} - U_n)$, show that $U_n = 2 - 3\left(-\frac{2}{3}\right)^n$. (3 marks)

(iii) hence, find $\lim_{n\to\infty} U_n$ (1 mark)

b) (i) Show that when expanded as a series in ascending powers of $\frac{1}{x}$, up to and including the term in $\frac{1}{x^2}$, $In\left(\frac{3x}{4x+1}\right) \approx In\left(\frac{3}{4}\right) - \frac{1}{4}\left(\frac{1}{x}\right) + \frac{1}{32}\left(\frac{1}{x^2}\right)$ (3 marks)

(ii) Hence, show that $\left(\frac{4x}{4x+1}\right)^{32x^2} \approx e(1 - 8x + 32x^2)$ (3 marks)

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