HIGH STANDARDS IN MATHEMATICS

A QUICK LOOK AT MATHEMATICS PAPER 2 SYLLABUS D (4024/2)

FINAL EXAMINATION QUESTIONS (2016– 2018) WITH ANSWERS

MATHEMATICAL FORMULAE

$$\int ax^n dx = \frac{ax^{n+1}}{n+1} + c$$

Conical Frustum $\mathbf{V} = \frac{1}{3}\pi(R^2H - r^2h)$

Pyramid frustum with a square base and top $\mathbf{V} = \frac{1}{3}h(L^2 + l^2 + Ll)$

$$\text{Mean} = \bar{x} = \frac{\sum fx}{\sum f}, \quad SD = \sqrt{\left\{\frac{\sum f(x-\bar{x})^2}{\sum f}\right\}} = \sqrt{\left\{\frac{\sum fx^2}{\sum f} - (\bar{x})^2\right\}}$$

n^{th} $term$ for **AP**: $T_n = a + (n-1)d$, **GP**: $T_n = ar^{n-1}$

AP: $\boldsymbol{S_n} = \frac{n}{2}[2a + (n-1)d]$, **GP**: $\boldsymbol{S_n} = \frac{(1-r^n)}{1-r}$, r < 1

Cosine Rule for ΔABC: $\boldsymbol{a^2 = b^2 + c^2 - 2bcCosA}$

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This pamphlet consists of final examination questions from 2016 - 2018 for both grade twelve (12) and G.C.E ordinary levels. Answers with all necessary working methods are shown at the end of questions. All the Questions are copied directly from the examination past papers for both July/August and October/ November exams.

"We believe, this pamphlet will be of great help to you even as you prepare for your final examinations:"

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Topic **page**

TOPIC 1: ALGEBRA

1. 2018 Oct/Nov Exams

(a) Simplify $\frac{b-a}{a^2-b^2}$

(b) Simplify $\frac{12dn^3}{15cd^3} \div \frac{9c^3n}{10c^2d^2}$

(c) Express $\frac{3}{x+1} - \frac{4}{x-1}$ as a single fraction in its lowest terms.

2. 2018 July/Aug Exams

(a) Simplify $\frac{7st^3}{15u^3v^2} \times \frac{5u^3v}{28s^3t^2}$

(b) Express $\frac{3}{2x-5} - \frac{4}{x-3}$ as a single fraction in its lowest terms.

3. 2017 Oct/Nov Exams

(a) Simplify $\frac{14x^3}{9y^2} \div \frac{7x^4}{18y^3}$

(b) Simplify $\frac{2x^2-8}{x+2}$

(c) Express $\frac{1}{x-4} - \frac{2}{5x-1}$ as a single fraction in its lowest terms.

4. 2017 July/ Aug Exams

(a) Simplify $\frac{m^2-1}{m^2-m}$

(b) Simplify $\frac{p^2q^3}{4} \times \frac{8}{pq} \div 2p^2q$

(c) Express $\frac{3}{5x-2} - \frac{2}{x+3}$ as a single fraction in its lowest terms.

5. 2016 October Exams

(a) Simplify $\frac{x-1}{x^2-1}$

(b) Simplify $\frac{17k^2}{20a^2} \div \frac{51k^2}{5a}$

(c) Express $\frac{2}{2x-1} - \frac{1}{3x+1}$ as a single fraction in its lowest terms.

TOPIC 2: MATRICES

1. 2018 Oct/Nov Exams, Q1(a)

Given that $A = \begin{pmatrix} 4 & -5 \\ 1 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 8 & y \\ 3 & 5 \end{pmatrix}$,

(a) find the value of y, for which the determinants of A and B are equal,

(b) hence find the inverse of B.

2. 2018 Jul/August Exams: Q1(a)

Given that $\mathrm{A} = \begin{pmatrix} 2x & 2 \\ 3 & x \end{pmatrix}$,

(a) find the positive value of x for the determinant of A is 12,

(b) hence or otherwise find A^{-1}.

3. 2017 Oct/Nov Exams: Q1(a)

Given that matrix $\mathrm{M} = \begin{pmatrix} 3 & -2 \\ 5 & x \end{pmatrix}$

(a) find the value of x for which the determinant of M is 22,

(b) hence find the inverse of M.

4. 2017 July/ Aug Exams: Q1(a)

Given that $\mathrm{K} = \begin{pmatrix} 10 & -2 \\ 11 & -2 \end{pmatrix}$, find

(a) the determinant of K,

(b) the inverse of K.

5. 2016 Oct/Nov Exams: Q1(a)

Given that $Q = \begin{pmatrix} 3 & -2 \\ x & 4 \end{pmatrix}$, find

(a) the value of x, given that the determinant of Q is 2,

(b) the inverse of Q.

TOPIC3: SETS

1. 2018 Oct/Nov Exams: Q2(b)

   At Sambilileni College, 20 students study at least one of the three subjects; Mathematics (M), Chemistry (C) and Physics (P). All those who study chemistry also study mathematics. 3 students study all the three subjects. 4 students study mathematics only, 8 students study chemistry and 14 students study mathematics.

   (i) Draw a Venn diagram to illustrate the information

   (ii) How many students study

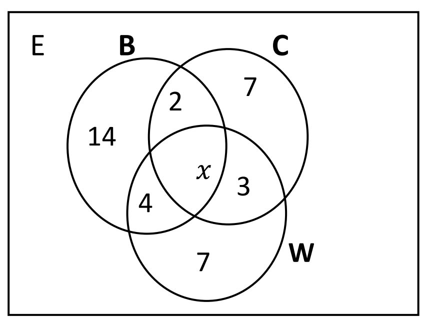
   (a) Physics only

   (b) two types of subjects only,

   (c) Mathematics and physics but not chemistry.

2. 2018 Jul/Aug Exams: Q3(a)

   The diagram below shows how learners at Twatenda School travel to school. The learners use either buses (B), cars (C) or walk (W) to school.



   (i) If 22 learners walk to school, find the value of x.

   (ii) How many learners use

   (a) only one mode of transport,

   (b) two different mode of transport.

3. 2017 Oct/Nov Exams: Q1 (b).

A survey carried out at Kamulima Farming Block showed that 44 farmers planted maize, 32 planted sweet potatoes, 37 planted cassava, 14 planted both maize and sweet potatoes, 24 planted both sweet potatoes and cassava, 20 planted both maize and cassava, 9 planted all the three crops and 6 did not plant any of these crops.

(i) Illustrate this information on a Venn diagram.

(ii) How many farmers

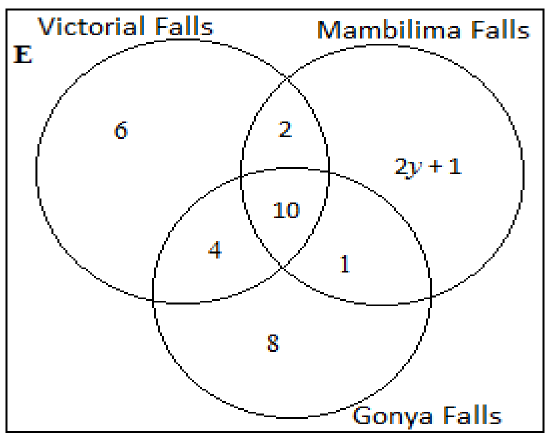
(a) Where at this farming block,

(b) Planted maize only

(c) Planted two different crops

4. 2017 July/Aug Exams: Q3 (b)

The Venn diagram below shows tourist attractions visited by certain students in a certain week.



(i) Find the value of y if 7 students visited Mambilima Falls only.

(ii) How many students visited

(a) Victoria falls but not Gonya Falls,

(b) Two tourist attractions only,

(c) One tourist attraction only?

5. 2016 October Exams,Q2 (a)

Of the 50 villagers who can tune in to Kambani Radio Station,29 listen to news,25 listen to sports, 22 listen to music, 11 listen to both news and sports,9 listen to both sports and music,12 listen to both news and music,4 listen to all the three programmes and 2 do not listen to any programme.

(i) Draw a Venn diagram to illustrate this information.

(ii) How many villagers

(a) Listen to music only,

(b) Listen to one type of programme only,

(c) Listen to two types of programs only.

TOPIC 4: PROBABILITY

1. 2018 Oct/Nov Exams, Q1(b)

   A small bag contains 6 black and 9 red pens of the same type. Two pens are taken at random one after the other without replacement. Calculate the probability that both pens

   (a) One is black,

   (b) are of different colour.

2. 2018 Jul/Aug Exams, Q5 (a)

   A box contains identical buttons of different colours. There are 20 black, 12 red and 4 white buttons in the box. Two buttons are picked at random one after the other and not replaced in the box.

   (a) Draw a tree diagram to show all the possible outcomes.

   (b) What is the probability that both buttons are white?

3. 2017 Oct/Nov Exams, Q2 (a)

   A box of chalk contains 5 white, 4 blue and 3 yellow pieces of chalk. A piece of chalk is selected at random from the box and not replaced. A second piece of chalk is then selected.

   (a) Draw a tree diagram to show all the possible outcomes.

   (b) Find the probability of selecting pieces of chalk of the same colour.

4. 2017 July Exams, Q3 (a)

   In a box of 10 bulbs, 3 are faulty. If two bulbs are drawn at random one after the other, find the probability that

   (a) Both are good.

   (b) One is faulty and the other one is good.

5. 2016 Oct/Nov Exams, Q2 (b)

A survey was carried out at certain hospital indicated that the probability that patient tested positive for malaria is 0.6. What is the probability that two patients selected at random

(a) one tested negative while the other positive,

(b) both patients tested negative.

TOPIC 5: SEQUENCES AND SERIES

1. 2018 Oct/Nov Exams, Q5(b)

The first three terms of a geometric progression are $k+4,\ k\ and\ 2k-15$ where k a positive integer.

(a) find the value of k,

(b) list the first three terms of the geometric progression,

(c) find the sum to infinity.

2. 2018 Jul/Aug Exams, Q2(b)

In a geometric progression, the third term is $\frac{2}{9}$ and the fourth term is $\frac{2}{27}$. Find

(a) The first term and the common ratio,

(b) The sum of the first 5 terms of the geometric progression,

(c) The sum to infinity.

3. 2017 Oct/Nov Exams, Q5(a)

For the geometric progression 20, 5,$1\frac{1}{4}$, . . . , find

(a) the common ratio,

(b) the n^{th} term,

(c) the sum of the first 8 terms.

4. 2017 July/ Aug Exams, Q2(b)

The first three terms of a geometric progression are $6+n, 10+n$ and $15+n$. Find

(a) the value of n,

(b) the common ratio,

(c) the sum of the first 6 terms of this sequence.

5. 2016 Oct/Nov Exams, 5(b)

The first three terms of a geometric progression are $x + 1, x - 3$ and $x - 1$.

(a) the value of x,

(b) the first term,

(c) the sum to infinity.

TOPIC 6: PSEUDO CODE AND FLOW CHARTS

1. 2018 Oct/Nov Exams, Q6(b)

The program below is given in a form of a Pseudo code

```
Start
  Enter x, y
  Let M = square root(x squared + y squared)
      IF M< 0
      THEN display error message " M must be positive"
       ELSE
       END IF
   Display M
Stop
```

Draw the corresponding flow chart for the information given above.

2. 2018 July/Aug Exams, Q5(b)

Study the pseudo code below.

Start

Enter a, r, n

R= $1 - r$

If $\mathrm{R} = 0$ THEN

Print "the value of r is not valid"

Else $\mathrm{S_n} = \frac{a(1-r^n)}{\mathrm{R}}$

End if

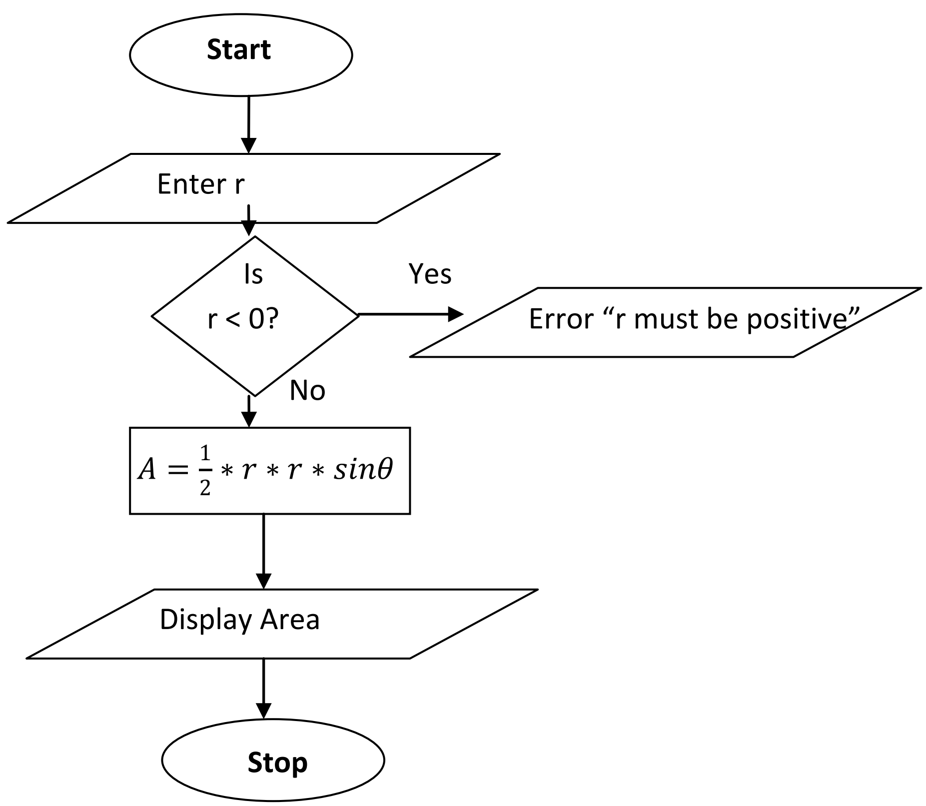
Print $\mathrm{S_n}$

Stop

Construct a flow chart corresponding to the Pseudo code above.

3. 2017 Oct/Nov Exams, Q6

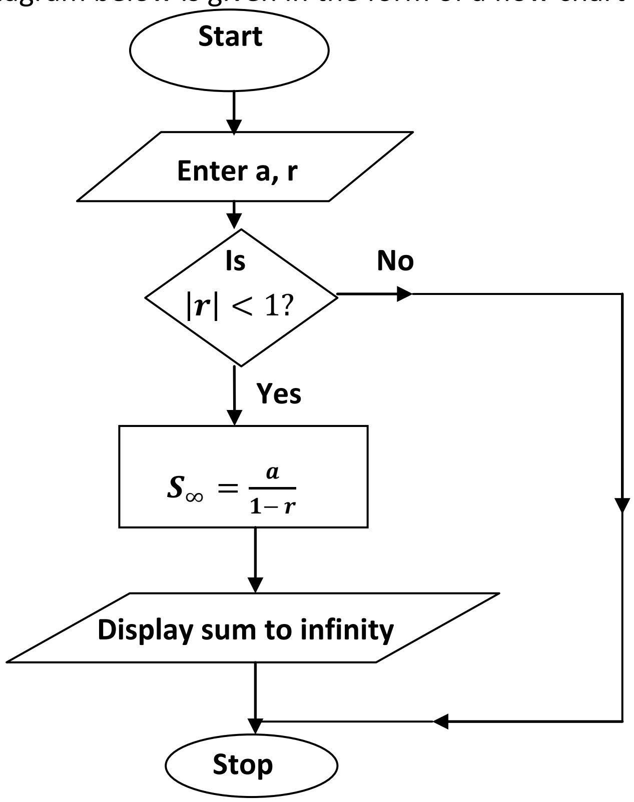
Study the flow chart below



Write a pseudo code corresponding to the flow chart program above

4. 2017 July/Aug Exams, Q6(b)

The diagram below is given in the form of a flow chart



Write a pseudo code corresponding to the flow chart program above

5. 2016 Oct/Nov Exams, Q3(b)

The program below is given in the form of a pseudo code.

Start
Enter radius
If radius < 0
The display "error message" and re-enter positive radius
Else enter height
If height < 0
The display "error message" and re-enter positive height
Else Volume = $\frac{1}{3} * \pi *$ square radius $*$ height
End if
Display volume
Stop

Draw the corresponding flowchart for the information given above.

TOPIC 7: LOCI & CONSTRUCTION

Answer the whole of these questions on a sheet of plain papers

1. 2018 Oct/Nov Exams, Q4

(a) (i) Construct triangle **XYZ** in which XY = 9cm, YZ = 7cm and angle XYZ $= 38°$

(ii) Measure and write the length of XZ

(b) On your diagram within triangle **XYZ**, construct the locus of points which are

(i) 6cm from **Y**

(ii) equidistant from **XY** and **XZ**

(c) Mark clearly with letter **P**, within triangle **XYZ**, a point which is 6cm from **Y** and equidistance from XY and XZ.

(d) A point Q within triangle **XYZ** is such that its distance from Y is less than or equal to 6cm and its nearer to **XY** than **XZ**. Indicate clearly by shading the region in which **Q** must lie.

2. 2018 Jul/Aug Exams, Q4

(a) **(i)** Construct triangle **PQR** in which **PQ** = 10cm, **QR** = 8cm and $\mathbf{P\hat{Q}R = 50°}$

(ii) Measure and write the length of **PR**

(b) On your diagram, within triangle PQR, construct the locus of points which are

(i) equidistant from **P** and **Q**

(ii) equidistant from **PR** and **PQ**

(iii) 5cm from **R**

(c) A point **T** within triangle **PQR** is such that it is 5cm from **R** and equidistant from **P** and **Q**. Label point **T**.

(d) Another point **X** is such that it is less than or equal to 5cm from **R**, nearer to **Q** than **P** and nearer to **PQ** than **PR**. Indicate by shading, the region in which **X** must lie.

3. 2017 Oct/Nov Exams, Q3

(a) Construct a quadrilateral ABCD in which AB = 10cm, and angle ABC = 120°, angle BAD = 60°, BC = 7cm and AD = 11cm

(b) Measure and write the length of CD

(c) Within the quadrilateral ABCD, draw the locus of points which are

(i) 8cm from A

(ii) Equidistant from BC and CD

(d) A point P, within the quadrilateral ABCD, is such that it 8cm from A and equidistant from BC and CD. Label point P.

(e) Another point Q, within the quadrilateral ABCD, is such that, it is nearer to CD than BC and greater than or equal to 8cm from A. indicate, by shading, the region in which Q must lie.

4. 2017 July/Aug Exams, Q4

(a) **(i)** Construct triangle PQR in which PQ is 9cm, angle PQR = 60° and QR = 10cm

(ii) Measure and write the length of PR.

(b) On your diagram, draw the locus of points with triangle PQR which are

(i) 3cm from PQ,

(ii) 7cm from R,

(iii) Equidistant from P and R.

(c) A point M, within triangle PQR, is such that it is nearer to R than P, less than or equal to 7cm from R and less than or equal to 3cm from PQ. Shade the region in which M must lie

5. 2016 Oct/Nov Exams, Q4

(a) **(i)** Construct triangle ABC where AB = BC = CA = 7cm

(ii) Measure and write the size of $\angle CAB$.

(b) Within triangle ABC construct the locus of points which are

(i) Equidistant from AB and BC

(ii) 4cm from B

(iii) 3cm from AB

(c) A point R, within triangle ABC, is such that it is nearer to BC than AB, less that 3cm from AB and less than 4cm from B. Shade the region in which R must lie.

TOPIC 8: CALCULUS

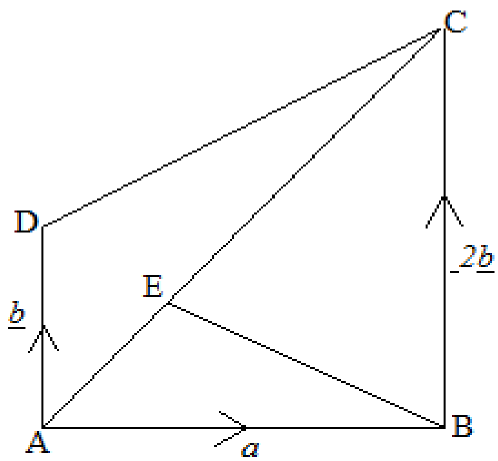
1. 2018 Oct/Nov Exams, Q3
   (a) Evaluate $\int_{-1}^{2}(2+x-x^2)dx$
   (b) Find the equation of the normal to the curve $y = x + \frac{4}{x}$ at the point where $x = 4$
2. 2018 Jul/Aug Exams, Q7(b)
   (a) Evaluate $\int_{0}^{1}(x^2-2x+3)dx$
   (b) Determine the equation of the normal to the curve $y = 2x^2 - 3x - 2$ that passes through (3, 7)
3. 2017 Oct/Nov Exams, Q9 (b & c)
   (a) Find the coordinates of the points on the curve $y = 2x^3 - 3x^2 - 36x - 3$ where the gradient is zero.
   (b) Evaluate $\int_{-1}^{3}(3x^2-2x)dx$
4. 2017 July/Aug Exams, Q4b & 9c
   (a) Evaluate $\int_{2}^{5}(3x^2+2)dx$
   (b) Find the equation of the tangent to the curve $y = x^2 - 3x - 4$ at a point where $x = 2$
5. 2016 Oct / Nov Exams, Q6
   The equation of the curve is $y = x^3 - \frac{3}{2}x^2$. Find
   (a) equation of the normal where $x = 2$,
   (b) the coordinates of the stationary points.

Continue working hard, Rome was not built in one day

TOPIC 9: VECTORS

1. 2018 Oct/Nov Exams, Q3

In the quadrilateral ABCD, $\overrightarrow{AB} = \underline{a}$ and $\overrightarrow{AD} = \underline{b}$, $\overrightarrow{BC} = 2\underline{b}$ and AE: AC = 1: 3



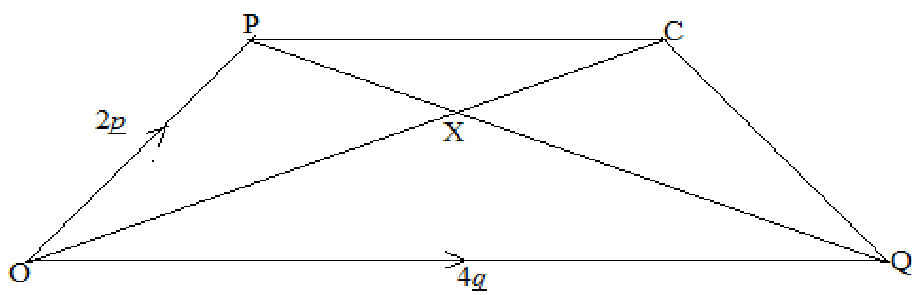
(i) Find in terms of $\underline{a}$ and/or $\underline{b}$

(a) $\overrightarrow{AE}$ **(b)** $\overrightarrow{BE}$ **(c)** $\overrightarrow{BD}$

(ii) Hence or otherwise show that the points B, D and E are collinear

2. 2017 Oct/Nov Exams, Q2(b)

In the diagram below, $\overrightarrow{OP} = 2\underline{p}$, $\overrightarrow{OQ} = 4\underline{q}$ and $PX : XQ = 1 : 2$



(i) Express in terms of p and / or q.

(a) $\overrightarrow{PQ}$

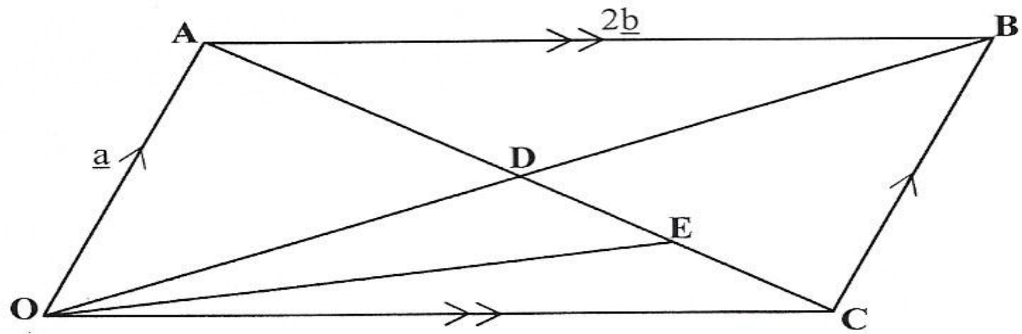
(b) $\overrightarrow{PX}$

(c) $\overrightarrow{OX}$

(ii) Given that $\overrightarrow{OC} = h\overrightarrow{OX}$, show that $\overrightarrow{CQ} = 4\left(1 - \frac{h}{3}\right)\underline{q} - \frac{4h}{3}\underline{p}$.

3. 2017 July/Aug Exams, Q6(a)

In the diagram below, OABC is a parallelogram in which $\overrightarrow{OA} = \underline{a}$ and $\overrightarrow{AB} = 2b$. OB and AC intersect at D. E is the midpoint of CD. E is the mid - point of CD.



Express in terms of a and / or b.

(a) $\overrightarrow{OB}$,

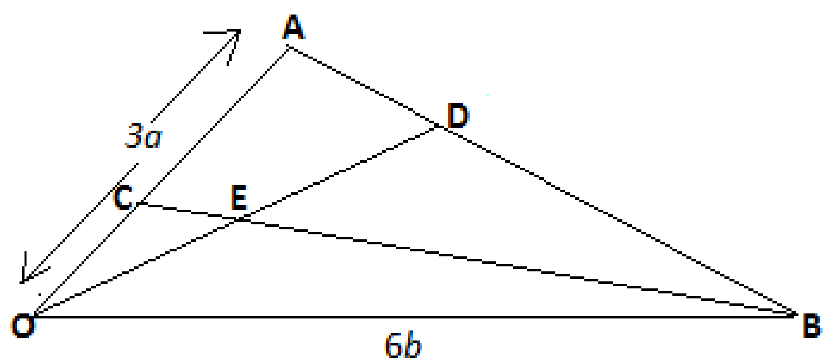
(b) $\overrightarrow{OE}$,

(c) $\overrightarrow{CD}$.

4. 2017 Oct/Nov Exams, Q6(a)

In the diagram below, OAB is a triangle in which $\overrightarrow{OA} = 3\underline{a}$ and $\overrightarrow{OB} = 6\underline{b}$.

OC : CA = 2 : 3 and AD : DB = 1 : 2. OD meets CB at E.



(i) Express each of the following in terms of $\underline{a}$ and / or $\underline{b}$

(a) $\overrightarrow{AB}$

(b) $\overrightarrow{OD}$

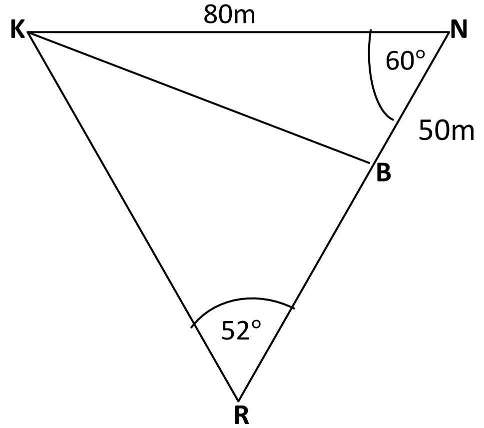
(c) $\overrightarrow{BC}$

(ii) Given that $\overrightarrow{BE} = h\overrightarrow{BC}$, express $\overrightarrow{BE}$ in terms of h, $\underline{a}$ and $\underline{b}$

TOPIC10: TRIGONOMETRY

1. 2018 Oct/Nov Exams, Q8

   In the diagram below, K, N, B and R are places on horizontal surface. KN = 80m, NB = 50m and $K\hat{R}N = 52°$



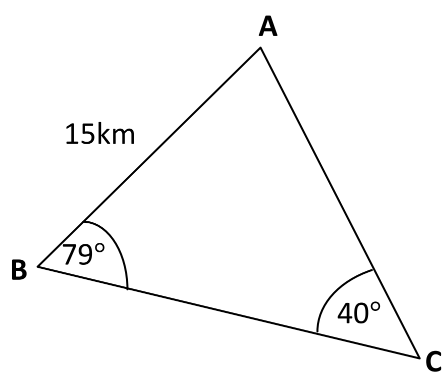
   (a) Calculate
   - **(i)** KR
   - **(ii)** the area of triangle KNB

   (b) Given that the area of triangle KNR is equal to 3 $260m^2$, calculate the shortest distance from R to KN.

   (c) Sketch the graph of $y = \cos\theta \quad for \quad 0° \leq \theta \leq 360°$

2. 2018 July/Aug Exams, Q8

   (a) Three villages A, B and C are connected by straight paths as shown in the diagram below.



   Given that AB = 15km, angle ABC = 79° and angle ACB = 40°, calculate the

   (i) Distance AC **(ii)** area of triangle ABC **(iii)** shortest distance from B to AC

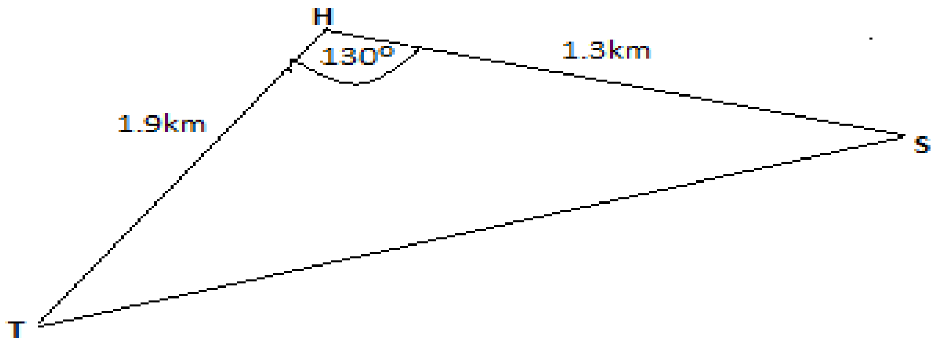
   (b) Solve the equation $\cos\theta = 0.937$ for $for \quad 0° \leq \theta \leq 360°$

   (c) Sketch the graph of $y = \sin\theta \ for \quad 0° \leq \theta \leq 360°$

3. 2017 Oct/Nov Exams, Q7

(a) The diagram below shows the Location of houses for a village Headman (H), his Secretary (S) and a Trustee (T). H is 1.3 km from S, T is 1.9 km from H and

$T\hat{H}S = 130°$



Calculate

(i) the area of triangle THS

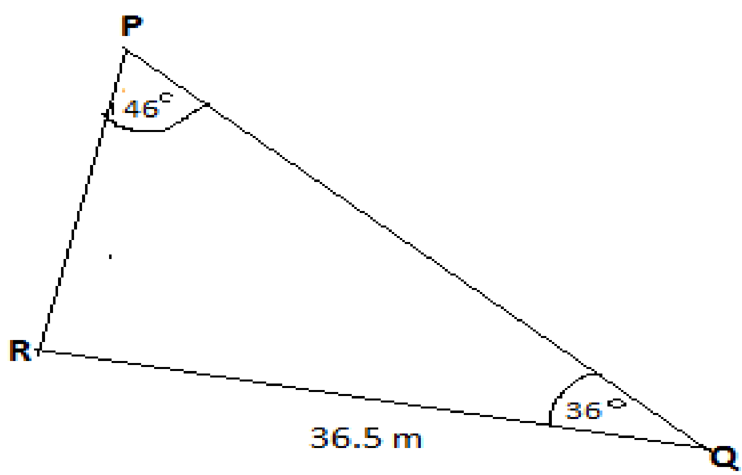
(ii) the distance TS

(iii) the shortest distance from H to TS

(b) Find the angle between $0°$ and $90°$ which satisfies the equation $\cos\theta = \frac{2}{3}$

4. 2017 July/ Aug Exams, Q10

(a) In Triangle PQR below, QR = 36.5, angle PQR = 36° and angle QPR = 46°.



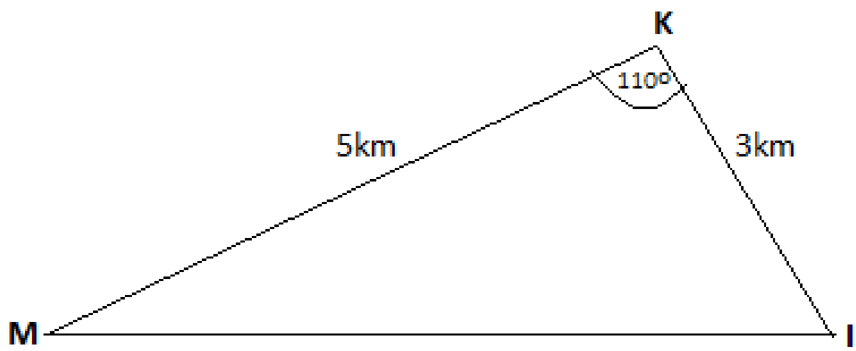
Calculate

- **(i)** PQ
- **(ii)** the area of triangle PQR
- **(iii)** the shortest distance from R to PQ

(b) Solve the equation $\sin\theta = 0.6792$ for $0° \leq \theta \leq 360°$

5. 2016 Oct/Nov Exams, Q10

(a) The diagram below shows the location of three secondary schools, namely Mufulira (M), Kantanshi (K) and Ipusukilo (I) in Mufulira district. M is 5km from K, I is 3km from K and angle MKI is 110°



Calculate

- **(i)** MI
- **(ii)** the area of triangle MKI
- **(iii)** the shortest distance from K to MI

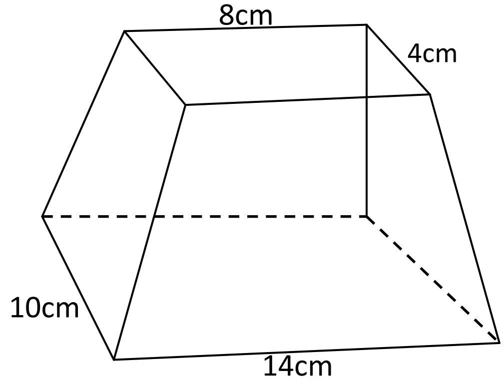
(b) Solve the equation $\tan\theta = 0.7$ for $0° \leq \theta \leq 180°$.

Don't wait for somebody to tell you that you can do it, just believe in yourself that you can do it

TOPIC 11: MENSURATION

1. 2018 Oct/ Nov Exams, Q12(a)

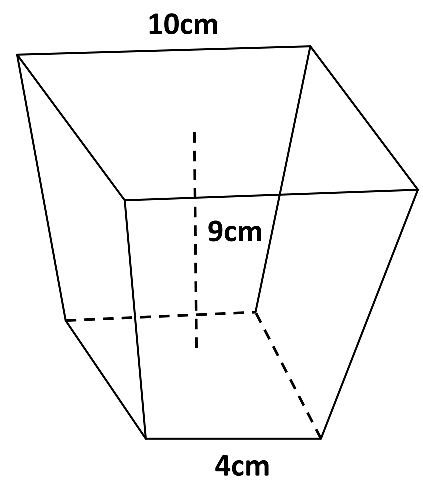
   The diagram below is a frustum of a rectangular pyramid with a base 14cm long and 10cm wide. The top of a frustum is 8cm long and 4cm wide.



   Given that the height of the frustum is 11.4cm, calculate its volume.

2. 2018 July/August Exams, Q6

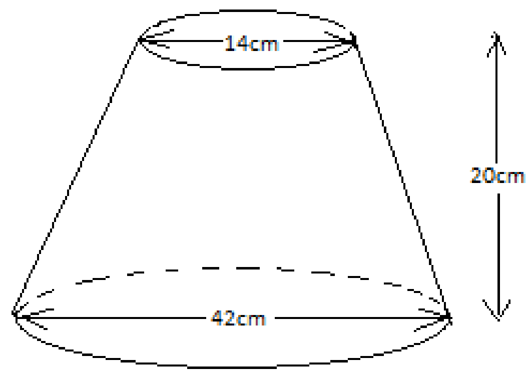
   The diagram below shows a bin in the form of a frustum with square base ends of t sides 4cm and 10cm respectively. The height of the bin is 9cm.



   Find the volume of the bin.

3. 2017 Oct/ Nov Exams, Q4(b)

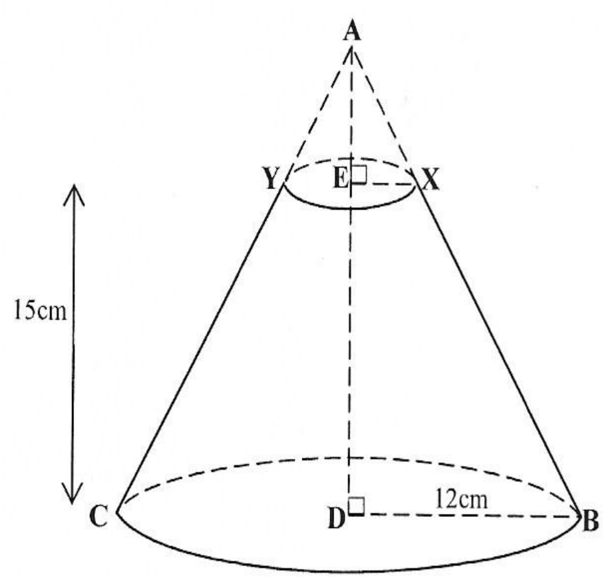
The figure below is a frustum of a cone. The base diameter and top diameter are 42cm and 14 cm respectively, while the height is 20cm. (**Take**$\boldsymbol{\pi} = \mathbf{3.142}$)



Calculate its volume

4. 2017 July/Aug Exams, Q12 (a)

The figure below is a cone ABC form which BCXY remained after the small cone AXY was cut of [Take $\pi = 3.142$]



Given that EX = 4cm, DB = 12cm and DE = 15cm, calculate

(i) the height AE, of the smaller con AXY.

(ii) the volume of XBCY, the shape that remained.

5. 2016 Oct/ Nov Exams; Q 9(b and c)

(a) The cross section of a rectangular tank measures 1.2m by 0.9. if it contains fuel to a depth of 10m, find the number of litres of fuel in the tank. ($1\text{m}^3 = 1000\text{litres}$)

(b) A cone has a perpendicular height of 12 cm and slant height of 13 cm, calculate its total surface area. (Take$\pi = 3.142$).

TOPIC 12: EARTH GEOMETRY

1. 2018 Oct/Nov Exams, Q12(b)

The points A $(15°\text{N}, 40°\text{E})$, $\text{B}(35°\text{N}, 70°\text{E})$and C $(35°\text{S}, 40°\text{E})$ are on the surface of the Earth. [**Use** $\boldsymbol{\pi} = \mathbf{3.142}$ **and** $\boldsymbol{R} = \mathbf{6370}\boldsymbol{km}$]

(a) Calculate the distance AC in kilometers.

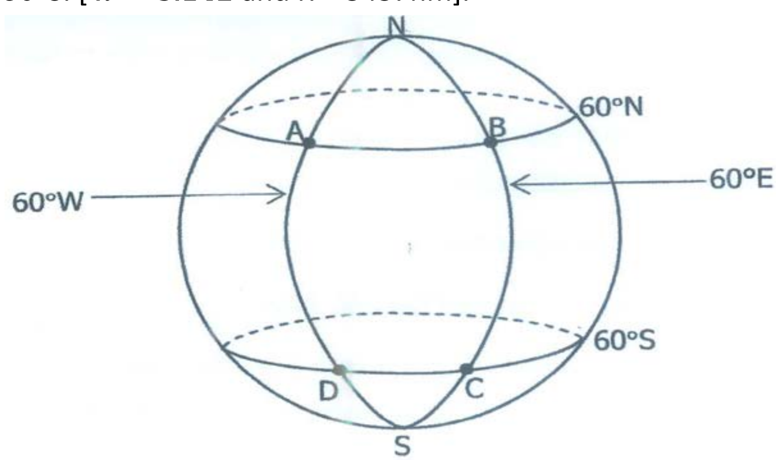
(b) An aero plane takes off from point B and flies due west on the same latitude covering a distance of 900km to point Q.

(i) Calculate the difference in longitudes between B and Q.

(ii) Find the position of Q.

2. 2018 July/ Aug Exams, Q7(a)

In the diagram below, A and B are points on latitude $60°$N while C is a point on latitude $60°$S. [$\pi = 3.142$ and R = 3437nm].

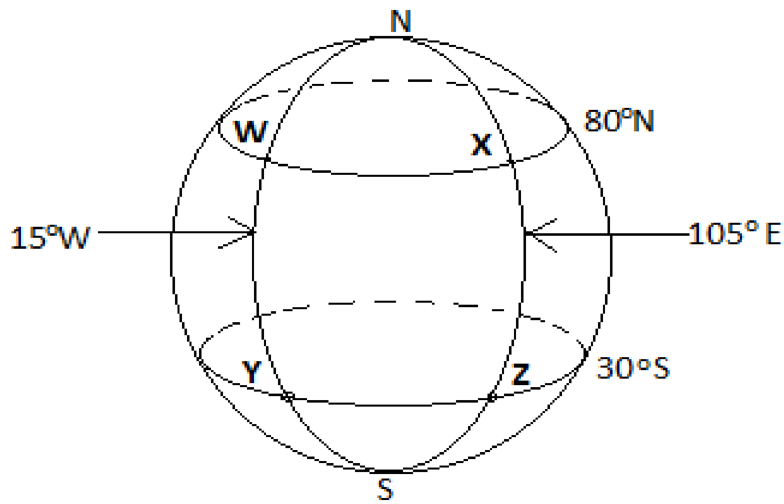


(a) Calculate the distance BC along the latitude $60°E$ in nautical miles.

(b) A ship sails from C to D in 12 hours. Find its speed in notes.

3. 2017 Oct/ Nov Exams, Q9(a)

W, X, Y and Z are four points on the surface of the earth as shown in the diagram below. **(Take $\pi = 3.142$ and $R = 3437$)**



(a) Calculate the difference in latitudes between W and Y.

(b) Calculate the distance in nautical miles between

(i) X and Z along the longitudes 105°E

(ii) Y and Z along the circle of latitude 30°S

4. 2017 July/Aug Exams, Q12 (a)

P $(80°\text{N}, 10°\text{E})$, $\mathbf{Q}(80°\text{N}, 70°\text{E})$, $\mathbf{R}(85°\text{S}, 70°\text{E})$ and $\mathbf{S}(85°\text{S}, 10°\text{E})$ are the points on the surface of the earth.

(i) Show the points on a clearly labeled sketch of the surface of the earth

(ii) Find in nautical miles

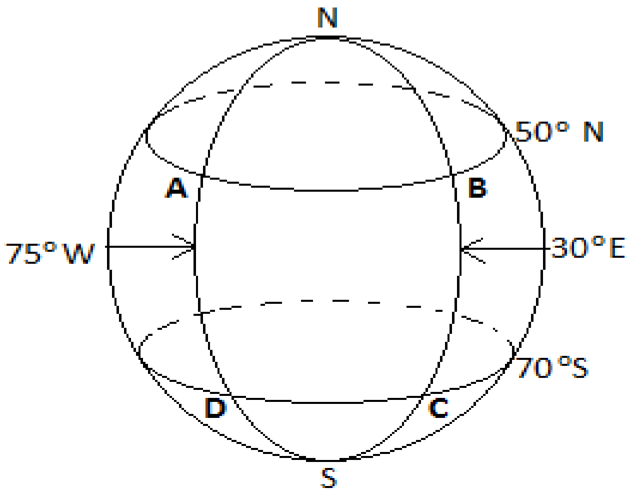
(a) The distance QR along the longitude,

(b) The circumference of latitude 85°S.

[Take $\pi = 3.142$ and $R = 3437$nm]

5. 2016 Oct/ Nov Exams, Q9(a)

The points A, B, C and D are on the surface of the earth.
(Take $\pi = 3.142$ and $R = 3437nm$)



(a) Find the difference in latitude between points C and B.

(b) Calculate the length of the circle of latitude 50°N in nautical miles.

(c) Find the distance AD in nautical miles.

TOPIC 13: QUADRATIC FUNCTIONS

1. 2018 Oct/Nov Exams, Q7(a)

The values x and y are connected by the equation $y = 2x^3 - 3x^2 + 5$. Some corresponding values of x and y are given in the table below.

x	-2	-1.5	-1	-0.5	0	0.5	1	1.5	2
y	p	-8.5	0	4	5	4.5	4	5	9

(a) Calculate the value of P

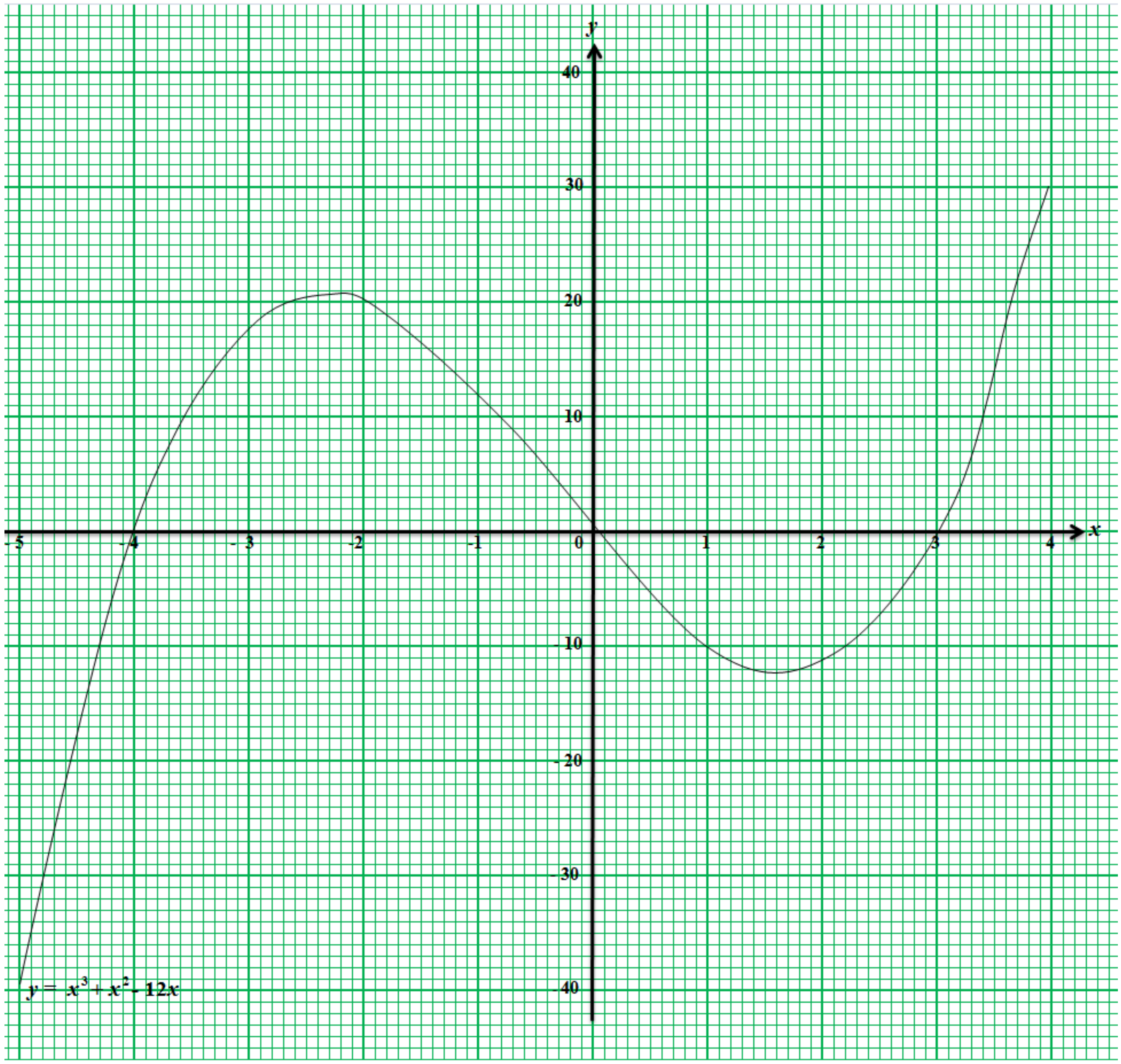
(b) Using a scale of 4cm to represent 1 unit on the x −axis for $-2 \leq x \leq 2$ and a scale of 2cm to represent 5 units on the y −axis for $-25 \leq y \leq 10$, draw the graph of $y = 2x^3 - 3x^2 + 5$.

(c) Use your graph to solve the equation $2x^3 - 3x^2 + 5 = x$

(d) Calculate an estimate of the gradient of the curve at the point where $x = 1.5$

2. 2018 July/ Aug Exams, Q12a)

The diagram below shows the graph of $y = x^3 + x^2 - 12x$



(a) Use the graph to solve the equation

(i) $x^3 + x^2 - 12x = 0$

(ii) $x^3 + x^2 - 12x = x + 10$

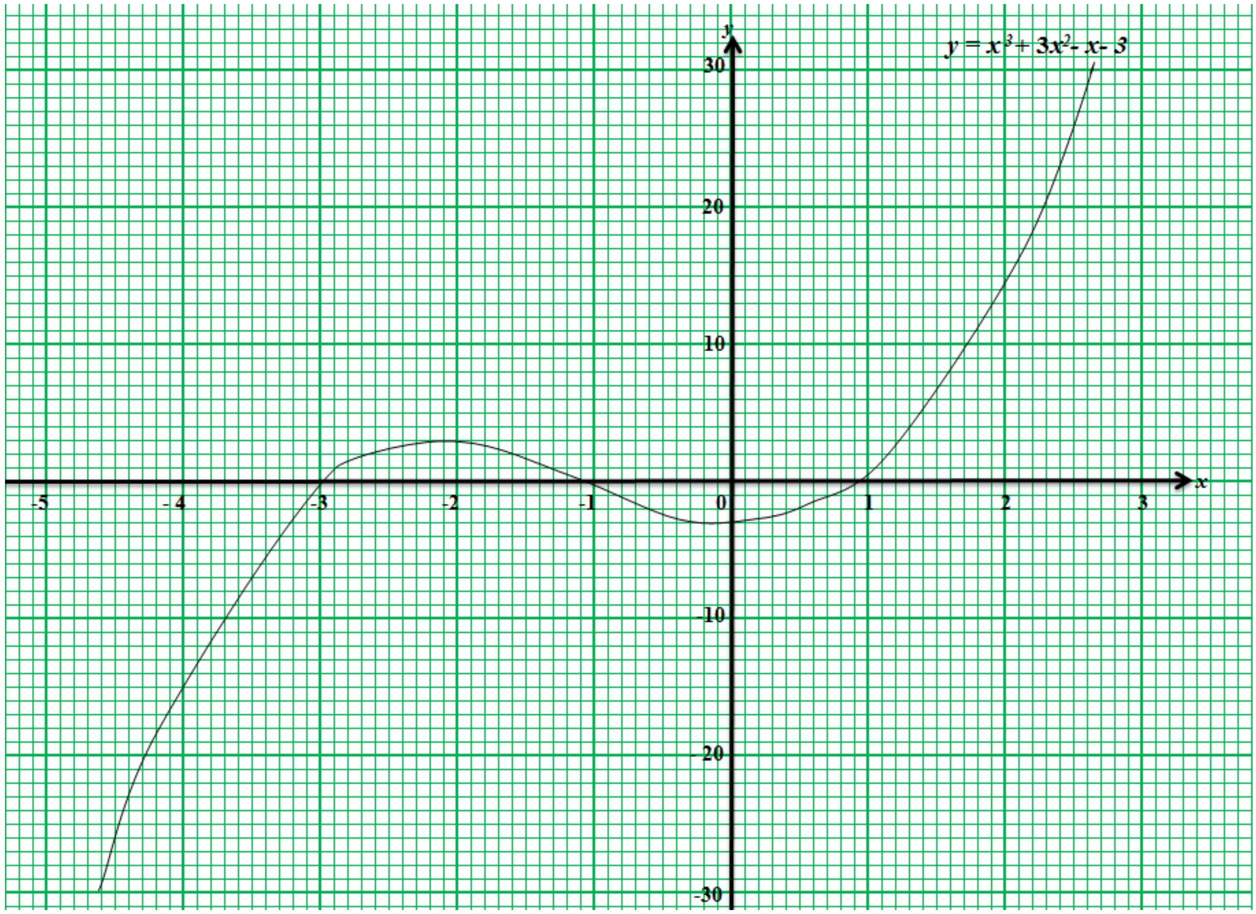
(b) Calculate an estimate of the

(i) Gradient of the curve at the point where $x = -3$

(ii) Area bounded by the curve, $x = -3, x = -1$ and $y = -10$

3. 2017 Oct / Nov Exams, Q10(a)

The diagram below shows the graph of $y = x^3 + 3x^2 - x - 3$



(a) Use the graph to find the solutions of the equations

(i) $x^3 + 3x^2 - x - 3 = 0$

(ii) $x^3 + 3x^2 - x - 3 = 5$

(b) Calculate an estimate of

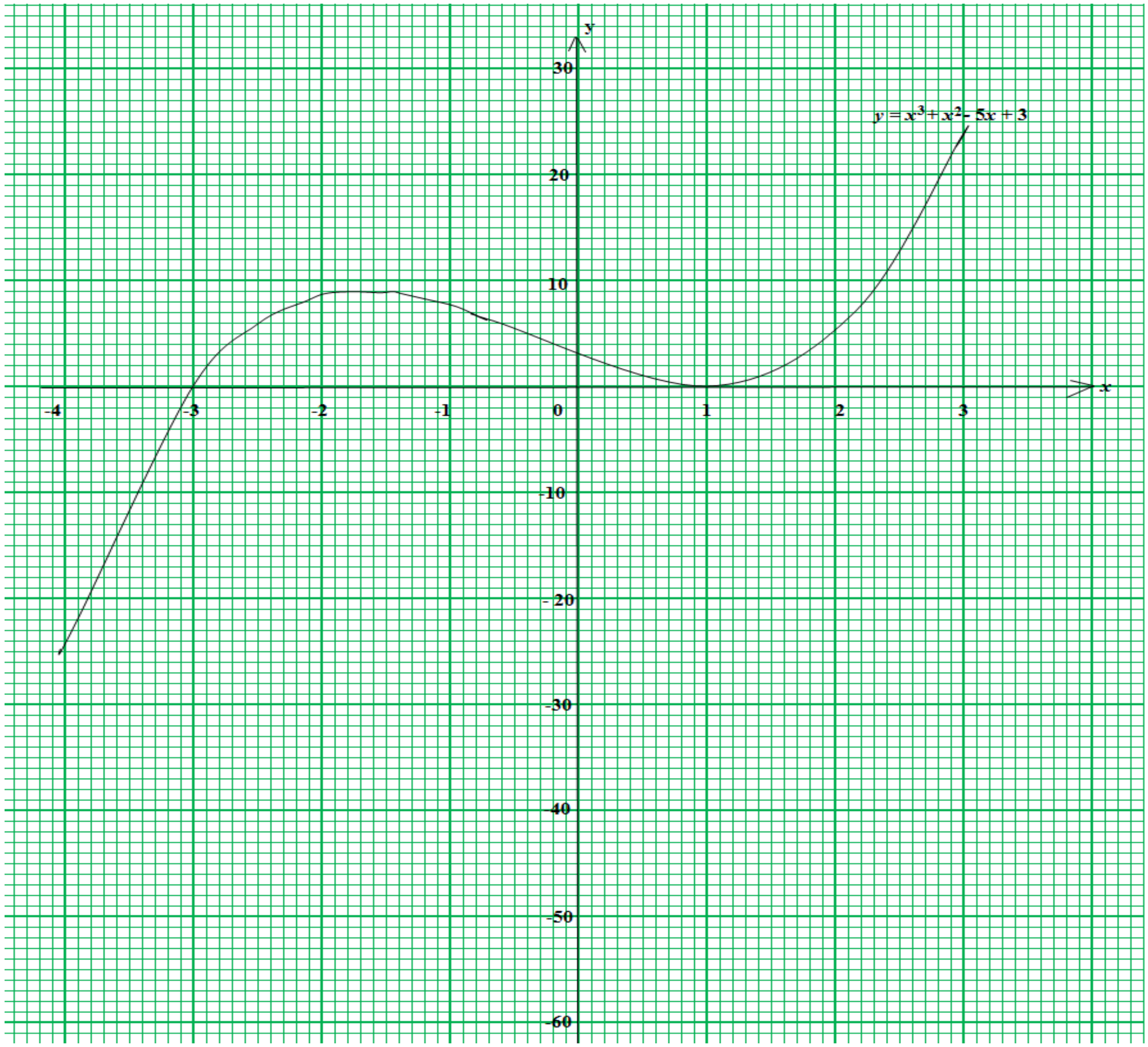
(i) the gradient of the curve at the point $(-3,0)$

(ii) the area bounded by the curve, $x = 0, y = 0$ and $y = 20$

I was not born a mathematician, but I am determined to Be a mathematician (Young Prof)

4. 2017 July/Aug Exams, Q9(a)

The diagram below shows the graph of $y = x^3 + x^2 - 5x + 3$



Use the graph

(a) to calculate an estimate of the gradient of the curve at the point (2,5).

(b) to solve the equations

(i) $x^3 + x^2 - 5x + 3 = 0$

(ii) $x^3 + x^2 - 5x + 3 = 5x$

(c) to calculate an estimate of the area bounded by the curve $x = 0, y = 0$ and $x = -2$

5. 2016 Oct/ Nov Exams, Q9(a)

The values of x and y are connected by the equation $y = x(x-2)(x+2)$. Some corresponding values of x and y are given in the table below

x	-3	-2	-1	0	1	2	3
y	-15	0	3	0	-3	0	k

(a) Calculate value of k

(b) Using a scale of 2cm to represent 1 unit on the x – axis for $-3 \leq x \leq 3$ and 2cm to represent 5 units on the y- axis for $-16 \leq y \leq 16$. Draw the graph of $y = x(x-2)(x+2)$

(c) Use your graph to solve the equations

(i) $x(x-2)(x+2) = 0$

(ii) $x(x-2)(x+2) = x+2$

TOPIC 14: LINEAR PROGRAMING

All the questions in this topic are to be answered on the sheet of graph papers.

1. 2018 Oct/Nov Exams, Q7(a)

A hired bus is used to take learners and teachers on a trip. The number of learners and teachers must be more than 60. There must be at least 35 people on the trip. There must be at least 6 teachers on the trip. The number of teachers on the trip should not be more than 14.

Let x be the number of learners and y be the number of teachers.

(a) Write four inequalities which present the information above.

(b) Using a scale of 2cm to represent 10 units on both axes, draw the x and y axes for $0 \leq x \leq 70$ and $0 \leq y \leq 70$ respectively and shade the unwanted region to indicate clearly where the solution of the inequalities lie.

(c) (i) If the group has 25 learners, what is the minimum number of teachers that must accompany them?

(ii) If 8 teachers go on the trip, what is the maximum number of learners that can be accommodated on the bus?

(d) If T is the amount in Kwacha paid by the whole group, what is the cost per learner if $\mathrm{T} = 30x + 50y$

2. 2018 July/ Aug Exams, Q12a)

A tailor at a certain market intends to make dresses and suits for sale.

(a) Let x represent the number of dresses and y the number of suits. Write the inequalities which represent each of the conditions below.

(i) The number of dresses should not exceed 50

(ii) The number of dresses should not be more than the number of suits.

(iii) The cost of making a dress is K140.00 and that o a suit is K210.00. The total should be at least K10 500.00

(b) Using a scale of 2cm to represent 10 units on both axes, draw x and y axes for $0 \leq x \leq 60$ and $0 \leq y \leq 80$. Shade the unwanted region to indicate clearly the region where (x, y) must lie.

(c) **(i)** The profit on a dress is K160.00 and on a suit is K270.00. Find the number of dresses and suits the tailor must make for maximum profit.

(ii) Calculate this maximum profit.

3. 2017 Oct / Nov Exams, Q10(a)

Himakwebo orders maize and groundnuts for sale. The order price for a bag of maize is K75.00 and that of a bag of groundnuts is K150.00. He is ready to spend up to K7 500.00 altogether. He intends to order at least 5 bags of maize and at least 10 bags of groundnuts. He does not want to order more than 70 bags altogether.

(a) If x and y are the number of bags of maize and groundnuts respectively, Write four inequalities which represent these conditions.

(b) Using a scale of 2cm to represent 10 bags on each axis, draw the x and y axes for $0 \leq x \leq 70 \; and \; 0 \leq y \leq 70$ respectively and shade the unwanted region to show clearly the region where the solution of the inequalities lie.

(c) Given that a profit on the bag of maize is K25.00 and on the bag of groundnuts is K50.00, how many bags of each type should he order to have the maximum profit?

(d) What is this estimate of the maximum profit?

4. 2017 July/Aug Exams, Q9(a)

Makwebo prepares two types of sausages, hungarian and beef, daily for sale. She prepares at least 40 hungarian and at least 10 beef sausages. She prepares not more than 160 sausages altogether. The number o beef sausages prepared are not more than the number of Hungarian sausages.

(a) Given that x represents the number of Hungarian sausages and y the number of beef sausages, write four inequalities which represent these conditions.

(b) Using a scale of 2cm to present 20cm sausages on both axes, draw the x and y axes for $0 \leq x \leq 160$ and $0 \leq y \leq 160$ respectively and shade the unwanted region to show clearly the region where the solution of the inequalities lie.

(c) The profit on the sale of each Hungarian sausage is K3.00 and on each beef sausage is K2.00. How many of each type of sausages are required to make maximum profit?

(d) Calculate this maximum profit

5. 2016 Oct/ Nov Exams, 11(a)

A Health Lobby group produced a guide to encourage healthy living among local community. The group produced the guide in two formats: a short video and a printed book. The group needs to decide the number of each format to produce for sale to maximize profit.

Let x represent the number of videos produced and y the number of printed books produced.

(a) Write the inequalities which represent each of the following conditions

(i) the total number of copies produced should not be more than 800,

(ii) the number of video copies to be at least 100

(iii) the number of printed books to be at least 100.

(b) Using a scale of 2cm to represent 100 copies on both axes, draw the x and y axes for $0 \leq x \leq 800\ and\ 0 \leq y \leq 800$ respectively and shade the unwanted region to indicate clearly the region where the solution of the inequalities lie.

(c) The profit on the sale of each video copy is K15.00 while the profit on each printed book is K8.00. How many of each type were produced to make maximum profit

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TOPIC 15: STATISTICS

1. 2018 Oct/Nov Exams, Q9

The frequency table below shows the distribution of marks obtained by 90 learners on a test.

Marks(x)	$10 < x \leq 20$	$20 < x \leq 30$	$30 < x \leq 40$	$40 < x \leq 50$	$50 < x \leq 60$	$60 < x \leq 70$
frequency	2	10	15	23	30	10

(a) Calculate the standard deviation

(b) Answer this part of this question on the sheet of graph paper

(i) copy and complete the cumulative frequency table

Marks(x)	≤ 10	≤ 20	≤ 30	≤ 40	≤ 50	≤ 60	≤ 70
Cumulative frequency	0	2	12	27	50	80	90
Relative Cumulative frequency	0	0.02	0.13	0.3			

(ii) Using a scale of 2cm to represent 10 units on the x- axis for $0 \leq x \leq 70$ and a scale of 2cm to represent 0.1 units on the y – axis for $0 \leq y \leq 1$, draw a smooth relative cumulative frequency curve.

(iii) Showing your method clearly, use your graph to estimate the 65th Percentile.

2. 2018 July/ Aug Exams, Q11

A farmer planted 60 fruit trees. In a certain month, the number of fruits per tree was recorded and the results were as shown in the table below.

Fruits per tree	2	3	4	5	6	7	8
frequency	1	5	4	6	10	16	18

(a) Calculate the standard deviation

(b) Answer this part of the question on the sheet of graph paper

(i) Using the table above, copy and complete the relative cumulative frequency table below

Fruits per tree	2	**3**	4	5	6	7	8
Cumulative frequency	1	6	10	16	26	42	60
Relative cumulative frequency	0.02	0.1	0.17	0.27			

(ii) Using a scale of 1cm to represent 1 unit on the x-axis for $0 \leq x \leq 8$ and a scale of 2cm to represent 0. 1 unit on the y – axis for $0 \leq y \leq 1$, draw a smooth relative frequency curve.

(iii) Showing your method clearly, use your graph to estimate the 70^{th} Percentile.

3. 2017 Oct / Nov Exams, Q8

The table below shows the amount of money spent by 100 learners at school on a particular day.

Amount in Kwacha	$0 < x \leq 5$	$5 < x \leq 10$	$0 < x \leq 15$	$15 < x \leq 20$	$20 < x \leq 25$	$25 \leq x \leq 30$
Frequency	13	27	35	16	7	2

(a) Calculate the standard deviation.

(b) Answer this part of the question on a sheet of graph paper

(i) Using the table above, copy and complete the cumulative frequency table below.

Amount in Kwacha	≤ 0	≤ 5	≤ 10	≤ 15	≤ 20	≤ 25	≤ 30
Cumulative frequency	0	13	40				100

(ii) Using a scale of 2cm to represent 5 units on the horizontal axis and 2cm to represent 10 units on the vertical axis, draw a smooth cumulative frequency curve.

(iii) Showing your method clearly, use your graph to estimate the semi– interquartile range.

4. 2017 July/ Aug Exams, Q8

The frequency table below shows the number of copies of newspapers allocated to 48 newspaper vendors.

Copies of Newspaper	$25 < x \leq 30$	$30 < x \leq 35$	$35 < x \leq 40$	$40 < x \leq 45$	$45 < x \leq 50$	$50 < x \leq 55$	$55 < x \leq 60$
Number of vendors	6	4	7	11	12	8	1

(a) Calculate the standard deviation.

(b) Answer this part of the question on a sheet of graph paper

(i) Using the table above, copy and complete the cumulative frequency table below.

Copies of newspaper	≤ 25	≤ 30	≤ 35	≤ 40	≤ 45	≤ 50	≤ 55	≤ 60
Number of Vendors	0	5	9	16	27			

(ii) Using a horizontal scale of 2cm to represent 10 newspapers on the x –axis for $0 \leq x \leq 60$ and a vertical scale of 4cm to represent 10 vendors on the y – axis for $0 \leq y \leq 50$, draw a smooth cumulative frequency curve.

(iii) Showing your method clearly, use your graph to estimate the 50^{th} Percentile.

5. 2016 Oct/ Nov Exams, 7

The ages of people living at Pamodzi Village are recorded in the frequency table below.

Ages	$0 < x \leq 10$	$10 < x \leq 20$	$20 < x \leq 30$	$30 < x \leq 40$	$40 < x \leq 50$	$50 < x \leq 60$
Number of People	7	22	28	23	15	5

(a) Calculate the standard deviation

(b) Answer part of this question on a sheet of a graph paper

(i) Using the table above, copy and complete the cumulative frequency table below.

Age	≤ 10	≤ 20	≤ 30	≤ 40	≤ 50	≤ 60
Number of people	7	29				100

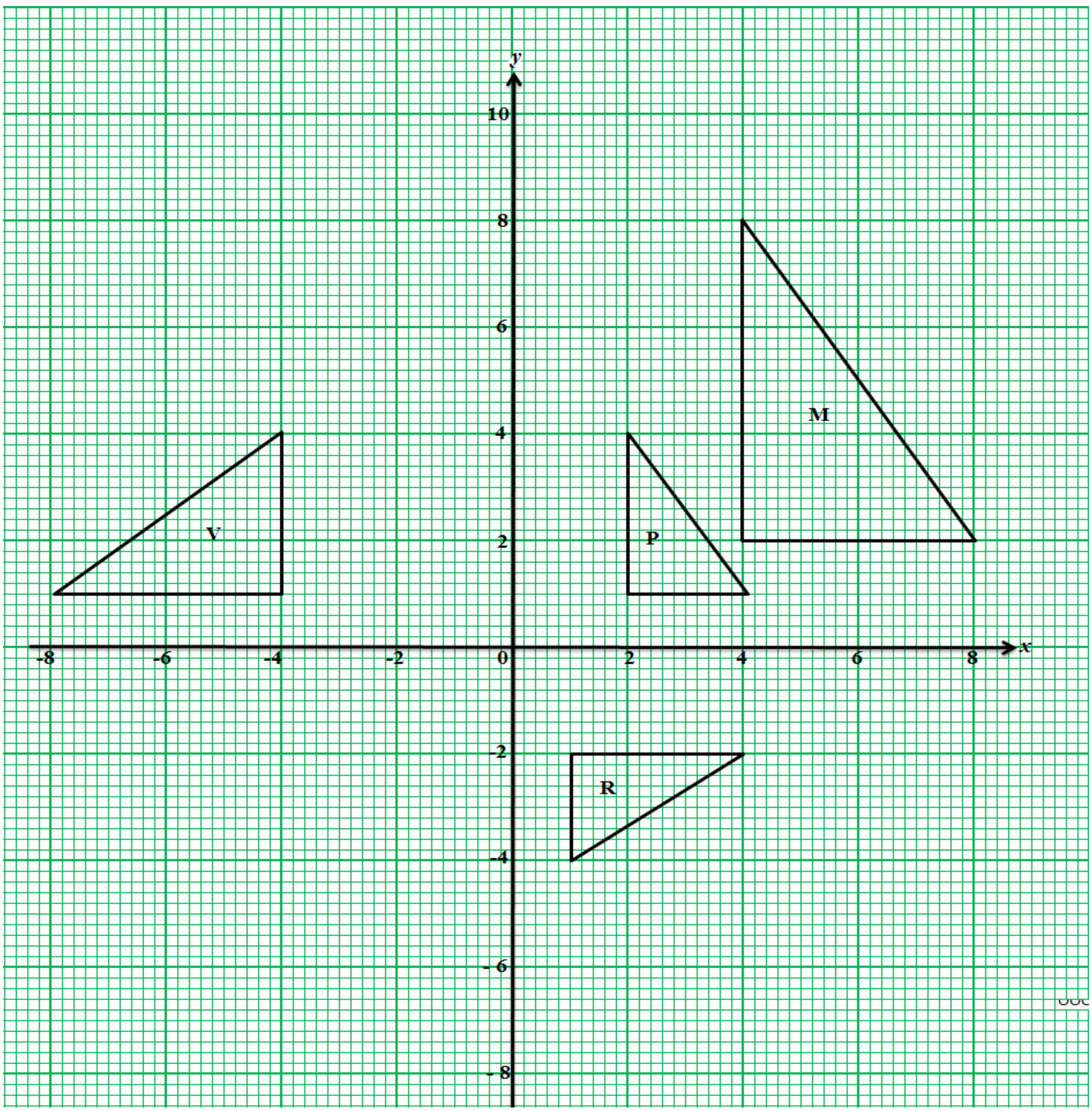
(ii) Using the scale of 2cm to represent 10 units on both axes, draw a smooth cumulative frequency curve where $0 \leq x \leq 60$ and $0 \leq y \leq 100$.

(iii) Showing your method clearly, use your graph to estimate the Semi– interquartile range.

TOPIC 16: TRANSFORMATION

1. 2018 Oct/Nov Exams, Q10

Study the diagram below and answer the questions that follow.



(a) Triangle R is the image of triangle P under a rotation. Find the coordinates of the centre, angle and the direction of the rotation.

(b) A single transformation maps triangle P onto triangle M. describe fully this transformation.

(c) Triangle P maps onto triangle V by a stretch. Find the matrix of this transformation

(d) If triangle P is mapped onto triangle S by a shear represented by the matrix $\begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix}$, find the coordinates of S.

2. 2018 July/Aug Exams, Q10

Using a scale of 1cm to represent 1 unit, on both axes, draw x and y axes for

$-8 \leq x \leq 12$ and $-6 \leq y \leq 14$.

(a) Draw and label triangle X with vertices $(2,4)$, $(4,4)$ and $(4,1)$.

(b) Triangle X is mapped onto triangle U with vertices $(6,12)$, $(12,12)$ and $(12,3)$ by a single transformation.

(i) Draw and label triangle U.

(ii) Describe fully this transformation.

(c) A 90° clockwise rotation about the origin maps triangle X onto triangle W. Draw and label triangle W.

(d) A shear with X –axis as the invariant line and shear factor -2 maps triangle X onto triangle S. Draw and label triangle S.

(e) Triangle X is mapped onto triangle M with vertices $(4,4)$, $(8,4)$ and $(8,1)$.

(i) Draw and label triangle M

(ii) Find the matrix which represents this transformation

3. 2017 Oct/Nov Exams, Q12

Using a scale of 1cm to represent 1 unit on each axis, draw x and y axes for

$-6 \leq x \leq 10$ and $-10 \leq y \leq 8$.

(a) A quadrilateral ABCD has vertices $A(-5,7)$, $B(-4,8)$, $C(-3,7)$ and $D(-4,4)$ while its imagine has vertices $A_1(-5,-3)$, $B_1(-6,-2)$, $C_1(-5,-1)$ and $D_1(-2,-2)$.

(i) Draw and label the quadrilateral ABCD and its image $A_1B_1C_1D_1$.

(ii) Describe fully the transformation which maps the quadrilateral ABCD onto quadrilateral $A_1B_1C_1D_1$.

(b) The matrix $\begin{pmatrix} -2 & 0 \\ 0 & 1 \end{pmatrix}$ maps the quadrilateral ABCD on the quadrilateral $A_2B_2C_2D_2$

(i) Find the coordinates of the vertices of the quadrilateral $A_2B_2C_2D_2$

(ii) Draw and label the quadrilateral $A_2B_2C_2D_2$.

(c) The quadrilateral ABCD is mapped onto quadrilateral $A_3B_3C_3D_3$ where A_3 is $(4, -8)$, B_3 is $(2, -10)$, is $C_3(0, -8)$and D_3 is $(2. -2)$. Describe fully this transformation.

4. 2017 July/Aug Exams, Q7

Using a scale of 1cm to represent 1 unit on each axis, draw x and y axes for $-6 \leq x \leq 10$ and $-6 \leq y \leq 12$.

(a) A quadrilateral ABCD has vertices $A(1,1), B(2,1), C(3,2)$ and $D(2,3)$ while its imagine has vertices $A_1(3,2), B_1(6,1), C_1(9,2)$ and $D_1(6,3)$.

(i) Draw and label the quadrilateral ABCD and its image $A_1B_1C_1D_1$.

(ii) Describe fully the transformation which maps the quadrilateral ABCD onto quadrilateral $A_1B_1C_1D_1$.

(b) The matrix $\begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix}$ maps the quadrilateral ABCD on the quadrilateral $A_2B_2C_2D_2$.

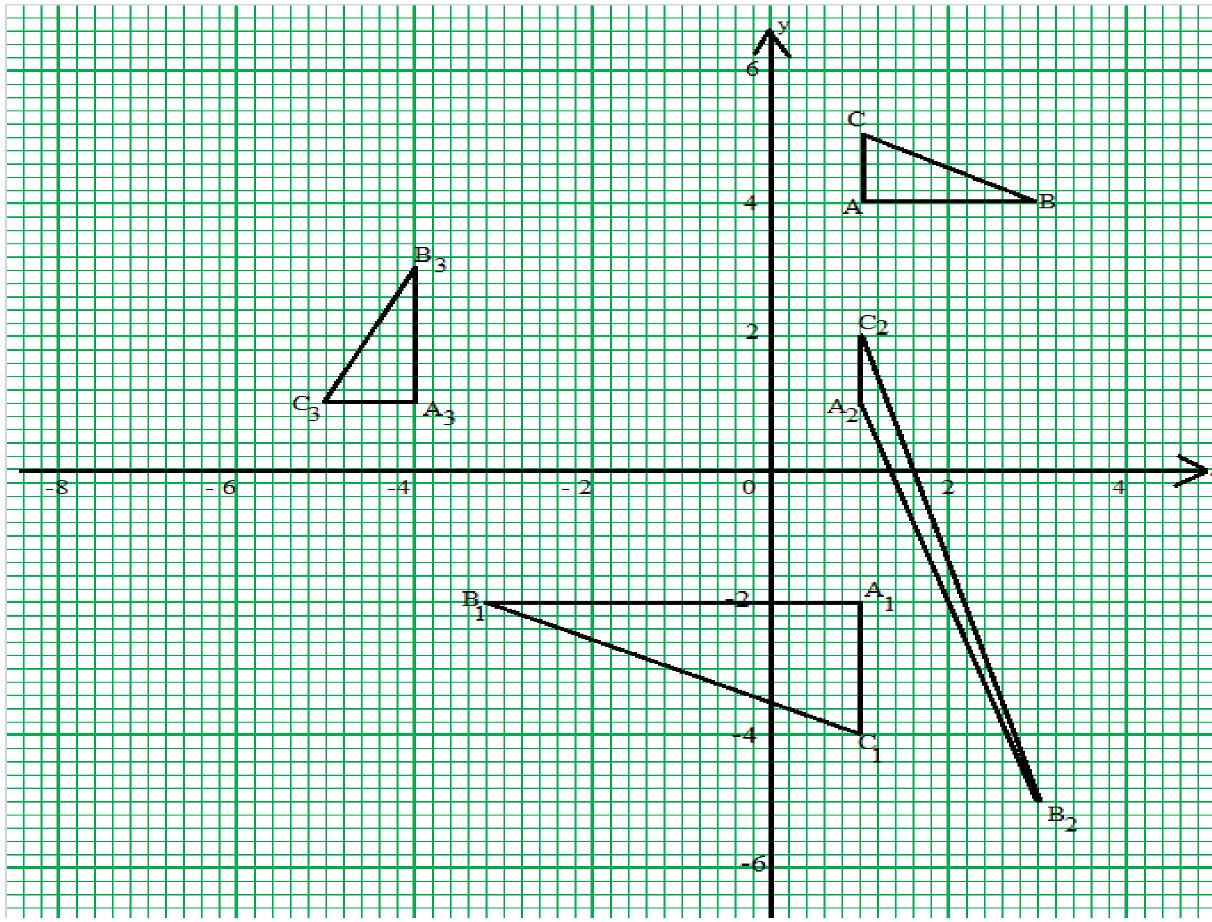
(i) Find the coordinates of quadrilateral $A_2B_2C_2D_2$.

(ii) Draw and label quadrilateral $A_2B_2C_2D_2$.

(c) Quadrilateral $A_3B_3C_3D_3$ has vertices $A_3(-2, -4), B_3(-4, -2), C_3(-6, -4)$ and $D_3(-4, -6)$. Describe fully the transformation which maps quadrilateral ABCD onto $A_3B_3C_3D_3$.

5. 2017 Oct/Nov Exams, Q12

Study the diagram below and answer questions that follow.



(a) An enlargement maps triangle ABC onto triangle $A_1B_1C_1$. Find

(i) the centre of enlargement

(ii) the scale factor

(b) Triangle ABC is mapped onto triangle $A_2B_2C_2$ by a shear. Find the matrix which presents this transformation.

(c) Triangle ABC is mapped onto triangle $A_3B_3C_3$by a single transformation. Describe this transformation fully.

(d) A transformation with matrix $\begin{pmatrix}-3 & 0\\ 0 & 1\end{pmatrix}$ maps triangle ABC onto triangle $A_4B_4C_4$ not drawn on the diagram. Find

(i) The scale factor of this transformation

(ii) The coordinates of $A_{4,}B_4$ and C_4.

Answers to All the Topic Questions

TOPIC1: ALGEBRA

1. **(a)** $\frac{b-a}{a^2-b^2}$

$= \frac{-1(a-b)}{(a+b)(a-b)}$

$= \frac{\mathbf{-1}}{\mathbf{a+b}}$ Ans

(b) $\frac{12dn^3}{15cd^3} \div \frac{9c^3n}{10c^2d^2}$

$= \frac{12dn^3}{15cd^3} \times \frac{10c^2d^2}{9c^3n}$

$= \frac{12\times d\times n\times n\times n}{15\times c\times d\times d\times d} \times \frac{10\times c\times c\times d\times d}{9\times c\times c\times c\times n}$

$= \frac{\mathbf{8n^2}}{\mathbf{9c^2}}$ **Ans**

(c) $\frac{3}{x+1} - \frac{4}{x-1}$

$= \frac{3(x-1)-4(x+1)}{(x+1)(x-1)}$

$= \frac{3x-3-4x-4}{(x+1)(x-1)}$

$= \frac{3x-4x-3-4}{(x+1)(x-1)}$

$= \frac{\mathbf{-x-7}}{\mathbf{(x+1)(x-1)}}$ **Ans**

2. **(a)** $\frac{7st^3}{15u^3v^2} \times \frac{5u^3v}{28s^3t^2}$

$= \frac{7\times s\times t\times t\times t}{15\times u\times u\times u} \times \frac{5\times u\times u\times u\times u\times v}{28\times s\times s\times s\times t\times t}$

$= \frac{\mathbf{t}}{\mathbf{12vs^2}}$ **Ans**

(b) $\frac{3}{2x-5} - \frac{4}{x-3}$

$= \frac{3(x-3)-4(2x-5)}{(2x-5)(x-3)}$

$= \frac{3x-9-8x+20}{(2x-5)(x-3)}$

$= \frac{3x-8x-9+20}{(2x-5)(x-3)} = \frac{\mathbf{-5x+11}}{\mathbf{(2x-5)(x-3)}}$ **Ans**

3. **(a)** $\frac{14x^3}{9y^2} \div \frac{7x^4}{18y^3}$

$= \frac{14x^3}{9y^2} \times \frac{18y^3}{7x^4}$

$= \frac{14\times x\times x\times x}{9\times y\times y} \times \frac{18\times y\times y\times y}{7\times x\times x\times x\times x}$

$= \frac{\mathbf{4y}}{\mathbf{x}}$ **Ans**

(b) $\frac{2x^2-8}{x+2}$

$= \frac{2(x^2-4)}{x+2}$

$= \frac{2(x^2-2^2)}{x+2}$

$= \frac{2(x+2)(x-2)}{x+2}$

$= \mathbf{2(x-2)}$ **Ans**

(c) $\frac{1}{x-4} - \frac{2}{5x-1}$

$= \frac{1(5x-1)-2(x-4)}{(x-4)(5x-1)}$

$= \frac{5x-1-2x+8}{(x-4)(5x-1)}$

$= \frac{5x-2x-1+8}{(x-4)(5x-1)}$

$= \frac{\mathbf{3x+7}}{\mathbf{(x-4)(5x-1)}}$ **Ans**

4. **(a)** $\frac{m^2-1}{m^2-m}$

$= \frac{m^2-1^2}{m(m-1)}$

$= \frac{(m+1)(m-1)}{m(m-1)}$

$= \frac{\mathbf{m+1}}{\mathbf{m}}$ **Ans**

(b) $\frac{p^2q^3}{4} \times \frac{8}{pq} \div 2p^2q$

$= \frac{p^2q^3}{4} \times \frac{8}{pq} \times \frac{1}{2p^2q}$

$= \frac{p\times p\times q\times q\times q}{4} \times \frac{8}{p\times q} \times \frac{1}{2\times p\times p\times q}$

$= \frac{\mathbf{q}}{\mathbf{p}}$ **Ans**

(c) $\frac{3}{5x-2} - \frac{2}{x+3}$

$= \frac{3(x+3)-2(5x-2)}{(5x-2)(x+3)}$

$= \frac{3x+9-10x+4}{(5x-2)(x+3)}$

$= \frac{3x-10x+9+4}{(5x-2)(x+3)}$

$= \frac{\mathbf{-7x+13}}{\mathbf{(5x-2)(x+3)}}$ **Ans**

5. **(a)** $\frac{x-1}{x^2-1}$

$= \frac{x-1}{x^2-1^2}$

$= \frac{x-1}{(x+1)(x-1)}$

$= \frac{\mathbf{1}}{\boldsymbol{x}+\mathbf{1}}$ **Ans**

(b) $\frac{17k^2}{20a^2} \div \frac{51k^2}{5a}$

$= \frac{17k^2}{20a^2} \times \frac{5a}{51k^2}$

$= \frac{17\times k\times k}{20\times a\times a} \times \frac{5\times a}{51\times k\times k}$

$= \frac{\mathbf{1}}{\mathbf{12}\boldsymbol{a}}$ **Ans**

(c) $\frac{2}{2x-1} - \frac{1}{3x+1}$

$= \frac{2(3x+1)-1(2x-1)}{(2x-1)(3x+1)}$

$= \frac{6x+2-2x+1}{(2x-1)(3x+1)}$

$= \frac{6x-2x+2+1}{(2x-1)(3x+1)}$

$= \frac{\mathbf{4}\boldsymbol{x}+\mathbf{3}}{(\mathbf{2}\boldsymbol{x}-\mathbf{1})(\mathbf{3}\boldsymbol{x}+\mathbf{1})}$ **Ans**

TOPIC 2: MATRICES

1. **(a)** deter A = $(4 \times 2) - (1 \times -5)$

= $8 - (-5)$

= $8 + 5$

$\therefore$ ***Det A*** **= 13 Ans**

deter $B = (8 \times 5) - (3 \times y)$

$13 = 40 - 3y$

$13 - 40 = -3y$

$-27 = -3y$ dividing both sides by 3 we have,

$\boldsymbol{y} = \mathbf{9}$ **Ans**

(b) $B = \begin{pmatrix} 8 & y \\ 3 & 5 \end{pmatrix}$,

B = $\begin{pmatrix} 8 & 9 \\ 3 & 5 \end{pmatrix}$

$\mathbf{B}^{-1} = \frac{\mathbf{1}}{\mathbf{13}}\begin{pmatrix} \mathbf{5} & \mathbf{-9} \\ \mathbf{-3} & \mathbf{8} \end{pmatrix}$ **Ans**

2. (a) Deter A = $(2x \times x) - (2 \times 3)$

$2x^2 - 6 = 12$

$2x^2 = 18$

$x^2 = 9$

$x = \sqrt{9}$

$x = \pm 3$

$\therefore \boldsymbol{x} = \mathbf{3}$ **Ans**

(b) A = $\begin{pmatrix} 2\times 3 & 2 \\ 3 & 3 \end{pmatrix} = \begin{pmatrix} 6 & 2 \\ 3 & 3 \end{pmatrix}$

$\boldsymbol{A}^{-1} = \frac{\mathbf{1}}{\mathbf{12}}\begin{pmatrix} \mathbf{3} & \mathbf{-2} \\ \mathbf{-3} & \mathbf{6} \end{pmatrix}$ **Ans**

3. **(a)** Deter M= $(3\times x) - (5 \times -2)$

$22 = 3x - (-10)$

$22 = 3x + 10$

$22 - 10 = 3x$

$12 = 3x$

$\boldsymbol{x} = \mathbf{4}$ **Ans**

(b) $M = \begin{pmatrix} 3 & -2 \\ 5 & 4 \end{pmatrix}$

$\boldsymbol{M}^{-1} = \frac{\mathbf{1}}{\mathbf{22}}\begin{pmatrix} \mathbf{4} & \mathbf{2} \\ \mathbf{-5} & \mathbf{3} \end{pmatrix}$ **Ans**

4. **(a)** $|K| = (10 \times -2) - (11 \times -2)$ **(b)** $\mathbf{K^{-1}} = \frac{1}{2}\begin{pmatrix} \mathbf{-2} & \mathbf{2} \\ \mathbf{-11} & \mathbf{10} \end{pmatrix}$ **Ans**

$= -20 - (-22)$

$= -20 + 22$

$\therefore deter\ K = $ **2 Ans**

5. **(a)** deter Q = $(3 \times 4) - (x \times -2)$ **(b)** $Q = \begin{pmatrix} 3 & -2 \\ -5 & 4 \end{pmatrix}$

$2 = 12 - (-2x)$ $\quad \boldsymbol{Q^{-1}} = \frac{1}{2}\begin{pmatrix} \mathbf{4} & \mathbf{-2} \\ \mathbf{5} & \mathbf{3} \end{pmatrix}$ **Ans**

$2 = 12 + 2x$

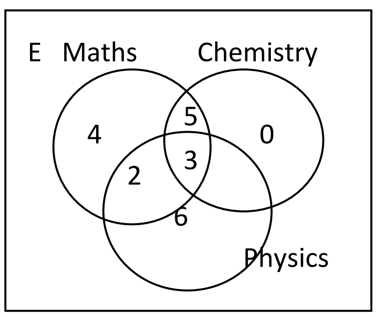
$2 - 12 = 2x$

$-10 = 2x$

$\boldsymbol{x = -5}$ **Ans**

TOPIC 3: SETS

1. **(i)**



Ans

(ii) **(a)** Physics only = **6 students Ans**

(b) two subject only = $2 + 5$

= **7 students Ans**

(c) Maths and Physics not chem = **2 Ans**

2. **(i)** $4 + x + 3 + 7 = 22$

$x + 14 = 22$

$x = 22 - 14$

$\boldsymbol{x} = \mathbf{8}$ **Ans**

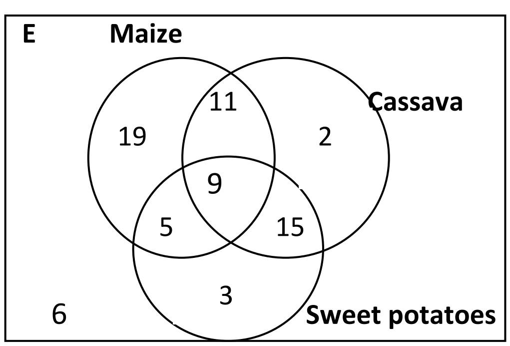
(ii) (a) only 1 mode of transport = $7 + 14 + 7 = \mathbf{28}$

(b) 2 different modes of transport

$= 4 + 2 + 3 + 8$

= 17 Ans

3. **(i)**



(ii) **(a) total =** 6 +19 + 5+ 9+ 3+ 15+ 2+ 11

= 70 farmers

(b) maize only = **19 farmers**

(c) two different crops = 11+5+15+9

= 40 farmers

4. **(i)** $2y + 1 = 7$

$2y = 7 - 1$

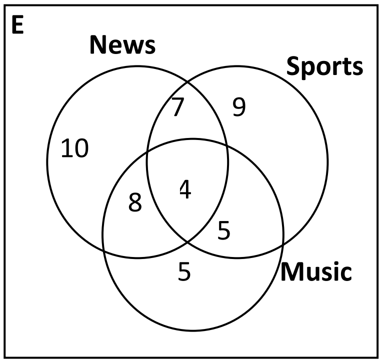
$2y = 6 \rightarrow \boldsymbol{y}$ **= 3** Ans

(ii) (a) Victoria falls but not Gonya = 6+2 = **8 students**

(b) two tourist attraction only = 4 +1 + 2 = **7 students**

(c) one tourists attraction only = 6 + 8 + 7 = **21 students**

5. **(i)**



(ii) **(a)** Music only = **5 villagers**

(b) one type only = $10 + 9 + 5$

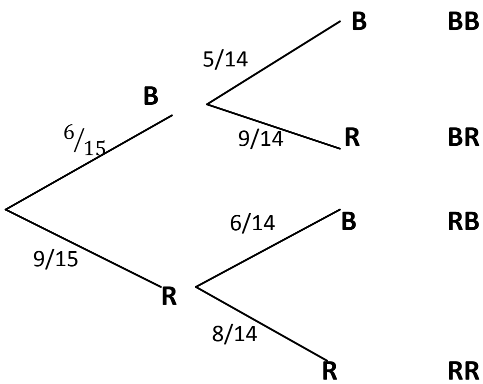
= **24 villagers**

(c) two types of programs only = $8 + 7 + 5$

= **20 Villagers**

TOPIC 4: PROBABILITY

1. Total $6 + 9 = 15$ (hint: use the tree diagram for easy calculations of probabilities)



(a) P(one is black) = P(BB) + P(BR) + P(RB)

$= \left(\frac{6}{15} \times \frac{5}{14}\right) + \left(\frac{6}{15} \times \frac{9}{14}\right) + \left(\frac{9}{15} \times \frac{6}{14}\right)$

$= \frac{30}{210} + \frac{54}{210} + \frac{54}{210}$

$= \frac{138}{210}$

$= \mathbf{\frac{23}{35}}$ **Ans**

(b) P(different colour) = P(BR)+P(RB)

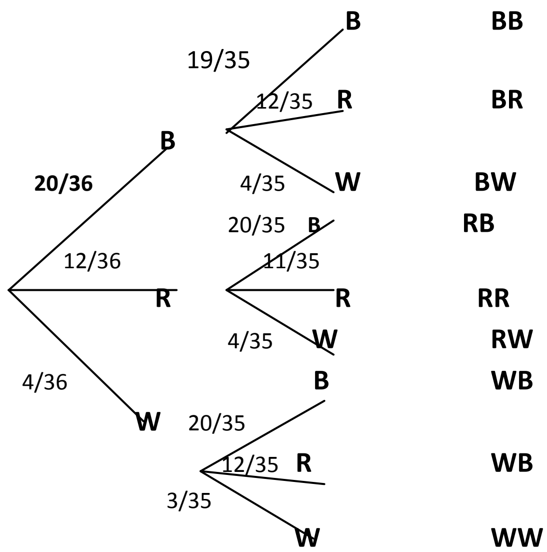
$= \left(\frac{6}{15} \times \frac{9}{14}\right) + \left(\frac{9}{15} \times \frac{6}{14}\right)$

$= \frac{54}{210} + \frac{54}{210}$

$= \frac{108}{210}$

$= \mathbf{\frac{18}{35}}$ **Ans**

2. **(a) Total $20 + 12 + 4 = 36$**



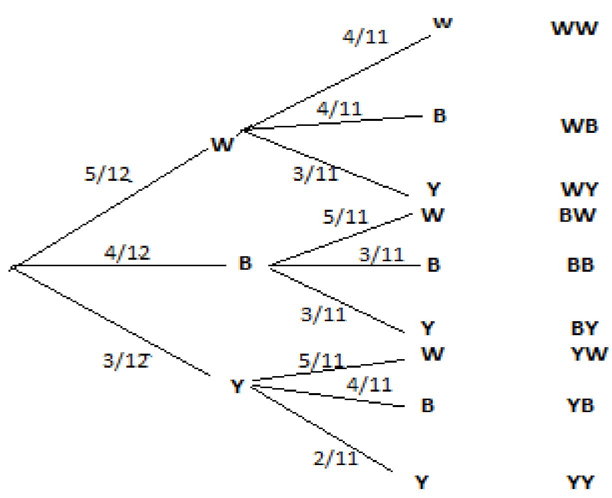
(b) P(both white) = P(WW)

$$= \frac{4}{36} \times \frac{3}{35}$$

$$= \frac{12}{1260}$$

$$= \frac{\mathbf{1}}{\mathbf{105}}$$ **Ans**

3. **(a) Total $5 + 4 + 3 = 12$**

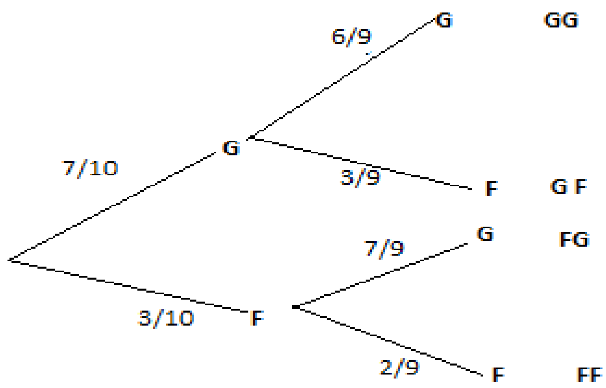


(b) P(Same colour) = P(WW) + P(BB) + (YY)

$$= \left(\frac{5}{12} \times \frac{4}{11}\right) + \left(\frac{4}{12} \times \frac{3}{11}\right) + \left(\frac{3}{12} \times \frac{2}{11}\right)$$

$$= \frac{20}{132} + \frac{12}{132} + \frac{6}{132} = \frac{38}{132} = \frac{\mathbf{19}}{\mathbf{69}}$$ **Ans**

4. **Faulty = 3 and good = 10 – 3 = 7**
   Use a tree diagram below for easy calculations



(a) **P(both good)** = P(G, G)

$= \left(\frac{7}{10} \times \frac{6}{9}\right)$

$= \frac{56}{90}$

$= \frac{\mathbf{7}}{\mathbf{15}}$ **Ans**

(b) P (one is faulty and the one is good) = $P(G, F) + P(F, G)$

$= \left(\frac{7}{10} \times \frac{3}{9}\right) + \left(\frac{3}{10} \times \frac{7}{9}\right)$

$= \frac{21}{90} + \frac{21}{90}$

$= \frac{42}{90}$

$= \frac{\mathbf{7}}{\mathbf{15}}$ **Ans**

5. P(negative) = 1- P(positive)

   = 1-0.6

   = 0.4

   (a) P (1 negative & other positive) = $(0.6\times 0.4) + (0.4 \times 0.6)$

   $= 0.24 + 0.24$

   = 0.48

   (b) P(both positive) = (0.4×0.4)

   $= \mathbf{0.16}$ **Ans**

TOPIC 5: SEQUENCES AND SERIES

1. **(a)** $k+4, k, 2k-15$

$\frac{k}{k+4} = \frac{2k-15}{k}$

$k^2 = (k+4)(2k-15)$

$k^2 = 2k^2 - 15k + 8k - 60$

$2k^2 - k^2 - 7k - 60 = 0$

$k^2 - 7k - 60 = 0$

$k^2 + 5k - 12k - 60 = 0$

$k(k+5) - 12(k+5) = 0$

$(k+5)(k-12) = 0$

$k = -5 \; or \; k = 12$

$\therefore \boldsymbol{k = 12}$ **Ans**

(b) $k+4, k, 2k-15$

$12+4, 12, 2(12)-15$

16, 12, 9,… Ans

(c) $S_\infty = \frac{a}{1-r}$, r = $\frac{12}{16} = \frac{3}{4}$

$S_\infty = \frac{16}{1-\frac{3}{4}}$

$S_\infty = \frac{16}{\frac{4-3}{4}}$

$S_\infty = \frac{16}{\frac{1}{4}}$

$\boldsymbol{S_\infty = 64}$ **Ans**

2. **(a)** $T_n = ar^{n-1}$

$T_3 = ar^{3-1}$

$\frac{2}{9} = ar^2$

$a = \frac{2}{9r^2}$ .... equation (i)

$T_4 = ar^{4-1}$

$\frac{2}{27} = ar^3$ .... Equation (ii)

Replacing a by $\frac{2}{9r^2}$ in (ii) we have

$\frac{2}{27} = \frac{2}{9r^2} \times r^3$

$\frac{2}{27} = \frac{2r}{9}$

$54r = 18$

$r = \frac{18}{54}$

$r = \frac{1}{3}$

$a = \frac{2}{9\times\left(\frac{1}{3}\right)^2} = \frac{2}{9\times\frac{1}{9}}$

$a = 2$

$\therefore$ the first term is 2 and common ratio is $\frac{1}{3}$ Ans

(b) $S_n = \frac{a(1-r^n)}{1-r}$

$S_5 = \frac{2(1-\left(\frac{1}{3}\right)^5)}{1-\frac{1}{3}}$

$S_5 = \frac{2(1-\frac{1}{3^5})}{\frac{2}{3}}$

$S_5 = 2(1-\frac{1}{243}) \div \frac{2}{3}$

$S_5 = 2(\frac{243-1}{243}) \times \frac{3}{2}$

$S_5 = 3(\frac{242}{243})$

$S_5 = \frac{242}{81} = 2\frac{80}{81} = \mathbf{2.99}$ **Ans**

(c) $S_\infty = \frac{a}{1-r}$

$S_\infty = \frac{2}{1-\frac{1}{3}}$

$S_\infty = \frac{2}{\frac{2}{3}}$

$\boldsymbol{S_\infty = 3}$ **Ans**

3. **(a)** $r = \frac{5}{20} = \frac{1}{4} = 0.25$

(b) $T_n = ar^{n-1}$

$T_n = 20\left(\frac{1}{4}\right)^{n-1}$

$T_n = 20\frac{1^{n-1}}{4^{n-1}}$

$\therefore \boldsymbol{T_n} = \frac{\mathbf{20}}{\mathbf{4^{n-1}}}$ **Ans**

(c) $S_n = \frac{a(1-r^n)}{1-r}$ for r < 1

$S_8 = \frac{20(1-(0.25)^8)}{1-0.25}$

$S_8 = \frac{20\ (1-0.00001558906\ )}{0.75}$

$S_8 = \frac{20(0.9999847412)}{0.75}$

$S_8 = 26.66625977$

$\boldsymbol{S_8} = \mathbf{27.7}$ **Ans**

4. **(a)** to find n, we use the common ratio formula

That is r= $\frac{T_2}{T_1} = \frac{T_3}{T_2} = \cdots \frac{T_n}{T_{n-1}}$

$\frac{10+n}{6+n} = \frac{15+n}{10+n}$

$(10+n)\,(10+n) = (6+n)\,(15+n)$

$100 + 20n + n^2 = 90 + 21n + n^2$

$100 - 90 = 21n - 20n$

$10 = n$

$\therefore \boldsymbol{n} = \mathbf{10}$

The GP is: 16, 20, 25 . . .

(b) $\boldsymbol{r} = \frac{20}{16} = \frac{5}{4}$ or $\mathbf{1.25}$

(c) $S_n = \frac{a(r^n-1)}{r-1}$ for r > 1

$S_6 = \frac{16((1.25)^6-1)}{1.25-1}$

$S_6 = \frac{16(3.814697266-1)}{0.25}$

$S_6 = \frac{16(2.814697266)}{0.25}$

$S_6 = \frac{45.03515625}{0.25}$

$S_6 = 180.140625$

$\therefore S_6 = \mathbf{180}$**Ans**

5. **(a)** We know that r = $\frac{T_2}{T_1} = \frac{T_3}{T_2} = \cdots \frac{T_n}{T_{n-1}}$

$\frac{x-3}{x+1} = \frac{x-1}{x-3}$

$(x-3)\,(x-3) = (x-1)\,(x+1)$

$x^2 - 6x + 9 = x^2 - 1$

$-6x = -1 - 9$

$-6x = -10$

$x = \frac{10}{6}$

$x = \frac{5}{3}$

$\boldsymbol{x} = \mathbf{1\frac{2}{3}}$ **Ans**

Hence the GP is; $\frac{5}{3} + 1. \frac{5}{3} - 3, \frac{5}{3} - 1 \ldots\ldots$

$\frac{8}{3}, \frac{-4}{3}, \frac{2}{3}, \ldots$

(b) the first term "a" $= \frac{\mathbf{8}}{\mathbf{3}}$ **Ans**

(C) $S_\infty = \frac{a}{1-r}$

$S_\infty = \frac{8/3}{1-(-\frac{1}{2})}$

$S_\infty = \frac{8/3}{\frac{3}{2}}$

$S_\infty = \frac{8}{3} \div \frac{3}{2}$

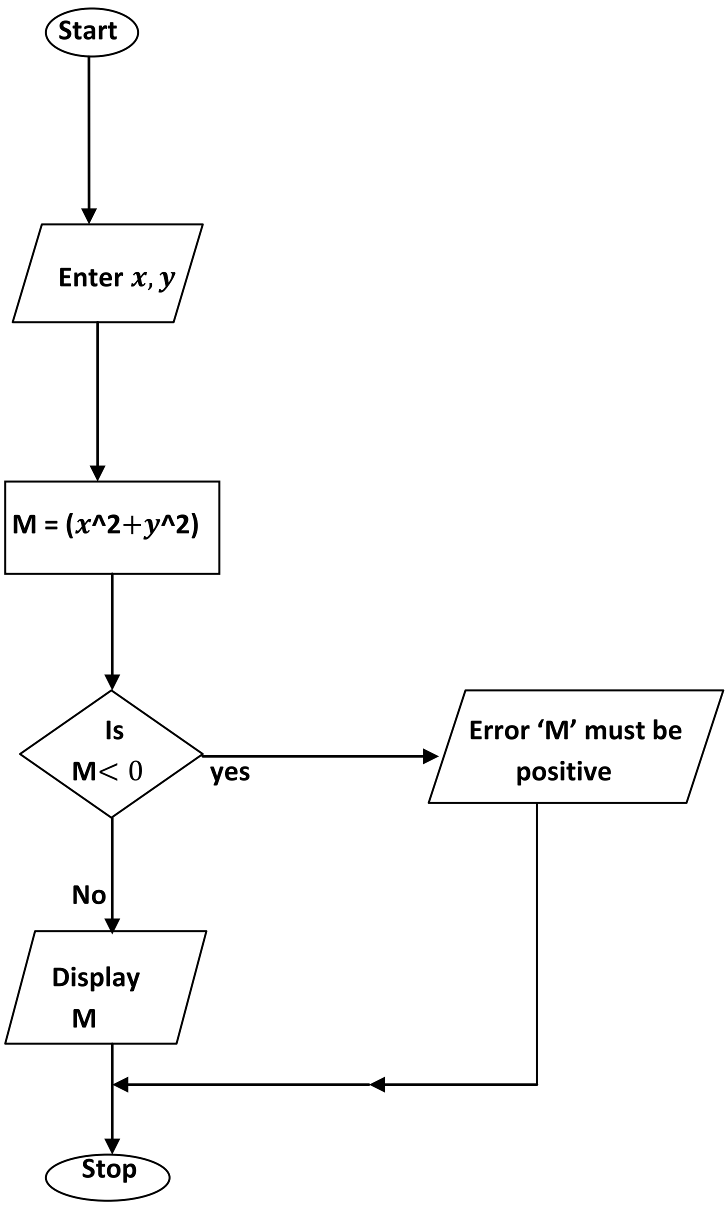
$S_\infty = \frac{8}{3} \times \frac{2}{3}$

$S_\infty = \frac{16}{9}$

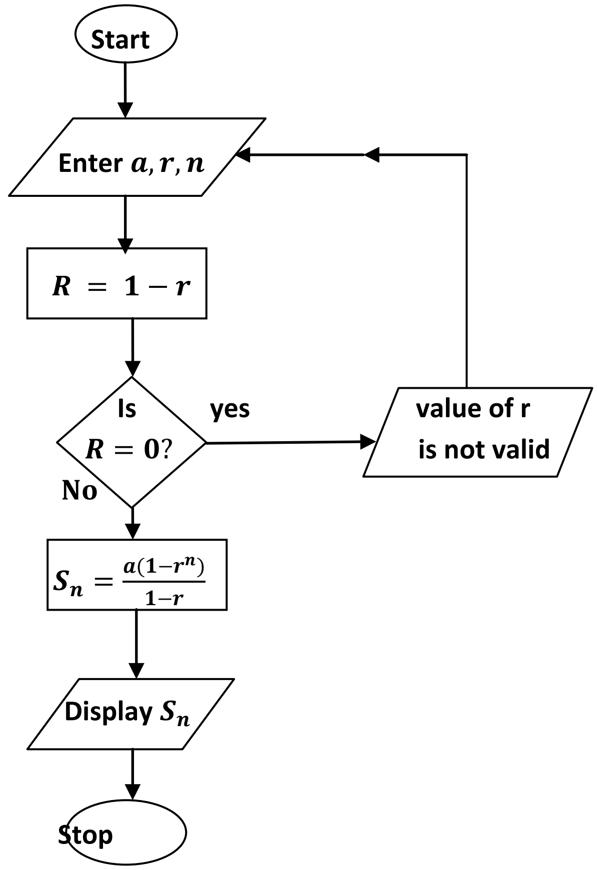
$\therefore \boldsymbol{S_\infty} = \mathbf{1\frac{7}{9}}$ **Ans**

TOPIC 6: PSEUDO CODE AND FLOWCHART

1.



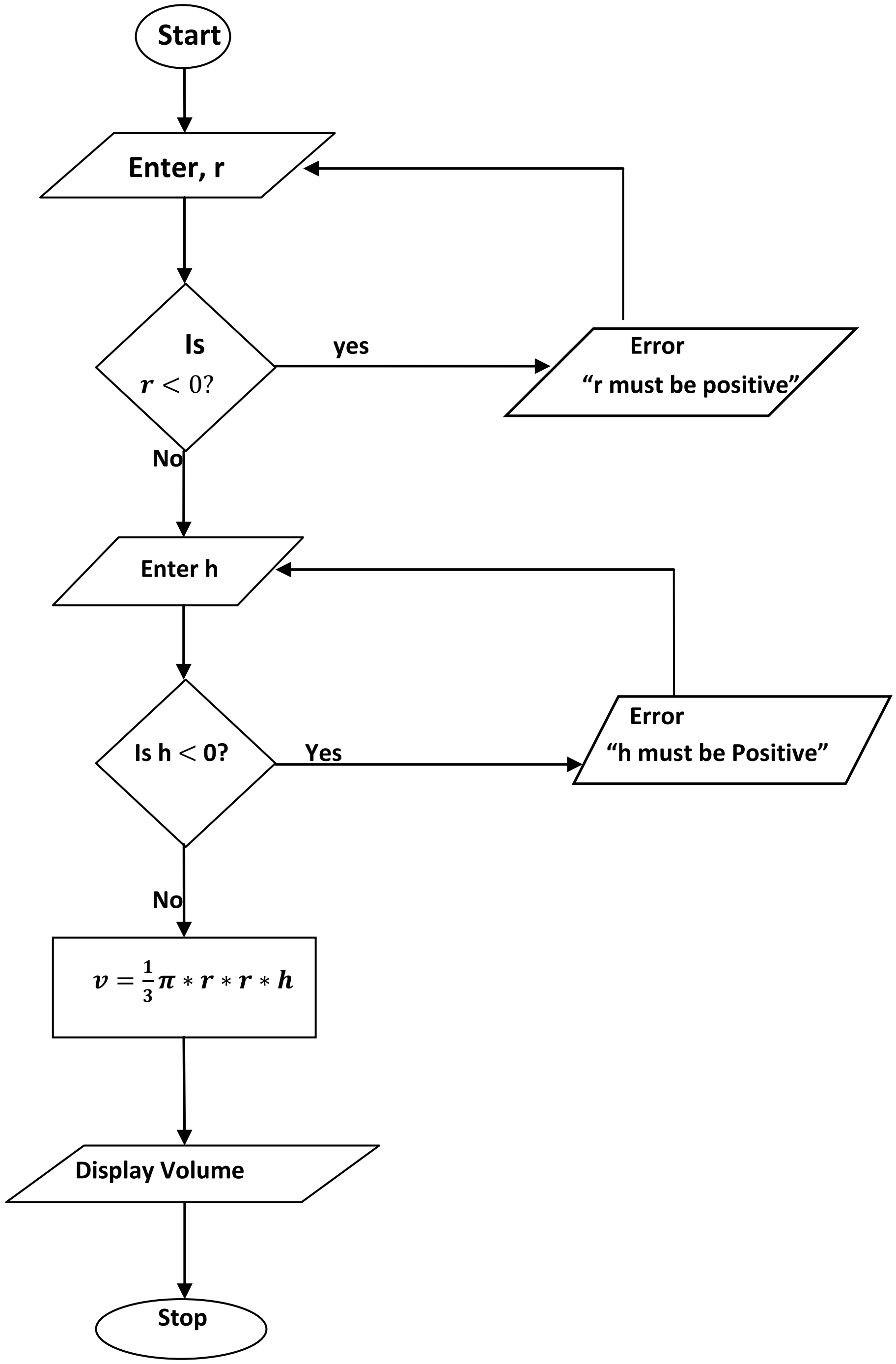
2.



3. Start
   Enter radius
   If radius < 0
   Then display "error message "
   Else Area = $\frac{1}{2} * r * r * sin\theta$
   End if
   Display Area
   Stop

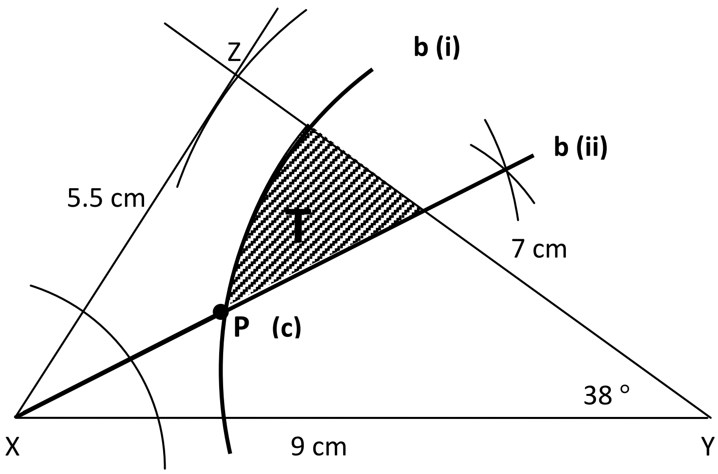
4. Start
   Enter a and r
   If $|r|$ < 1
   Then sum to infinity = $\frac{a}{1-r}$
   Else
   End if
   Display sum to infinity
   Stop

5.

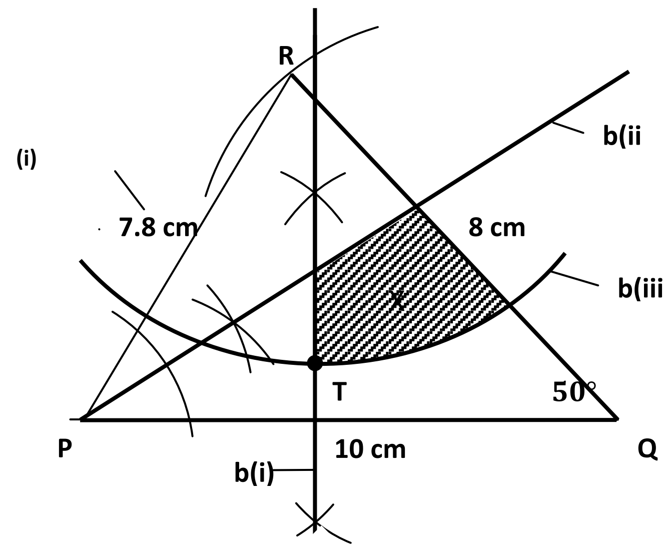

Start
Enter, r
Is
$r < 0$?
yes
Error
"r must be positive"
No
Enter h
Is h < 0?
Yes
Error
"h must be Positive"
No
$v = \frac{1}{3}\pi * r * r * h$
Display Volume
Stop

TOPIC 7: LOCI AND CONSTRUCTION

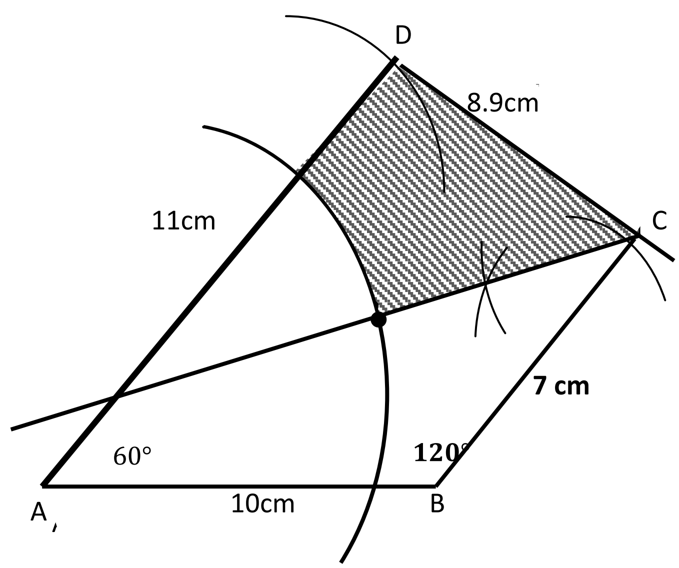
1.



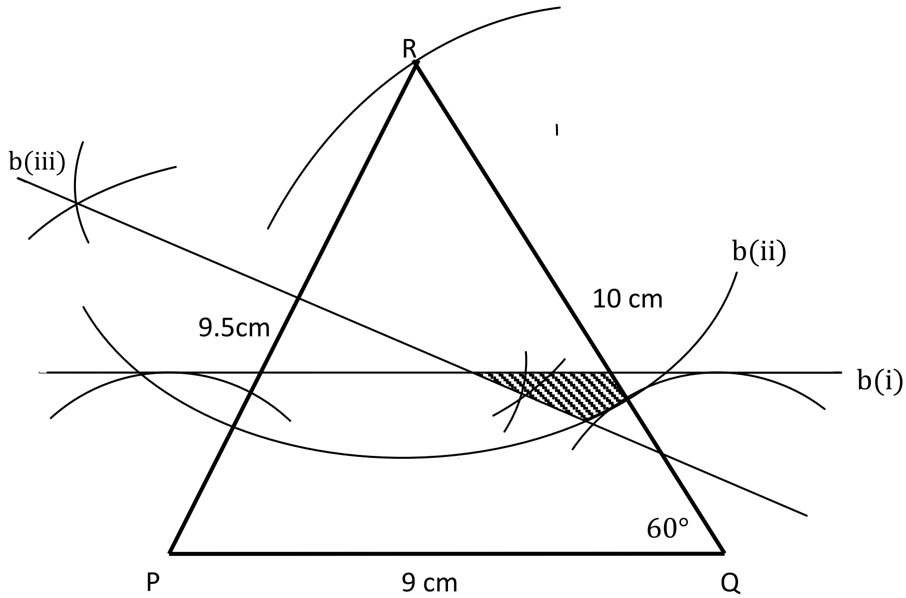
2.



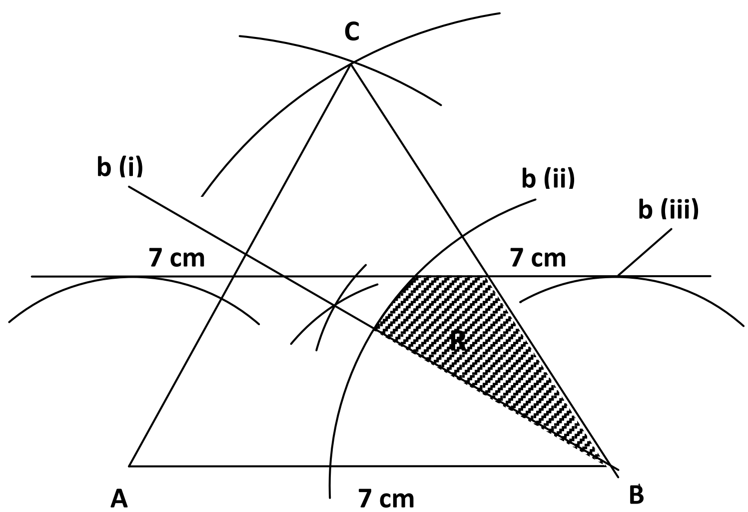
3.



4.



5.



TOPIC 8: CALCULUS

1. **(a)** $\int_{-1}^{2}(2 + x - x^2)dx$

$= \left[2x + \frac{x^2}{2} - \frac{x^3}{3}\right]_{-1}^{2}$

$= \left(\frac{2(2)}{1} + \frac{2^2}{2} - \frac{2^3}{3}\right) - \left(2(-1) - \frac{(-1)^2}{2} - \frac{(-1)^3}{3}\right)$

$= \left(4 + 2 - \frac{8}{3}\right) - \left(2 - \frac{1}{2} + \frac{1}{3}\right)$

$= \left(\frac{10}{3}\right) - \left(\frac{-7}{6}\right)$

$= \frac{10}{3} + \frac{7}{6}$

$= \frac{27}{6}$

$= 4.\mathbf{5}$ **Ans**

(b) $y = x + \frac{4}{x} \rightarrow \frac{dy}{dx} = 1 - \frac{4}{x^2}$ at $x = 4$

$m_1 = \frac{dy}{dx} = 1 - \frac{4}{4^2} = 1 - \frac{4}{16} = \frac{16-4}{16} = \frac{12}{16} = \frac{3}{4}$

$m_1 \times m_2 = -1$ (tangent ⊥ to normal)

$m_2 = -1 \times \frac{4}{3} = -\frac{4}{3}$

To find y replace $x = 4$ in the original equat

$y = 4 + \frac{4}{4} = 4 + 1 = 5$

∴ Equation of the normal is given by

$\boldsymbol{y - y_1} = m_2(x - x_1)$

$y - 5 = -\frac{4}{3}(x - 4)$

$y = -\frac{4}{3}x + \frac{16}{3} + 5$

$\boldsymbol{y = -\frac{4}{3}x + \frac{31}{3}}$ Multiplying through by

3

$\mathbf{3y = -4x + 31}$ **Ans**

2. **(a)** $\int_{0}^{1}(x^2 - 2x - 3)dx$

$= \left[\frac{x^3}{3} - \frac{2x^2}{2} - 3x\right]_{0}^{1}$

$= \left(\frac{1^3}{3} - \frac{2(1)^2}{2} - (3)\right) - \left(\frac{0}{3} - \frac{0}{2} + 3(0)\right)$

$= \left(\frac{1}{3} - 1 - 3\right) - 0$

$= \frac{1}{3} - 4 - 0$

$= \frac{1-12}{3}$

$= -\frac{11}{3}$

$= \mathbf{-3\frac{2}{3}}$ **Ans**

(b) $y = 2x^2 - 3x - 2$

$\frac{dy}{dx} = 4x - 3\ at\ (3,7) \rightarrow \frac{dy}{dx} = m_1 = 4(3) - 3 = 9$

$m_2 = -1 \times \frac{1}{9} = -\frac{1}{9}$

∴ equation of the normal to the curve is given by

$\boldsymbol{y - y_1} = m_2(x - x_1)$

$y - 7 = -\frac{1}{9}(x - 3)$

$y - 7 = -\frac{1}{9}x + \frac{3}{9}$

$y = -\frac{1}{9}x + \frac{3}{9} + \frac{7}{1}$

$y = -\frac{1}{9}x + \frac{3+63}{9}$

$y = -\frac{1}{9}x + \frac{66}{9}$

$\mathbf{9y = -x + 66}$ **Ans**

A journey of thousand miles starts with one step

3. (a) $y = 2x^3 - 3x^2 - 36x - 3$

$\frac{dy}{dx} = 6x^2 - 6x - 36$

$0 = 6x^2 - 6x - 36$ dividing through by 6

$x^2 - x - 6 = 0$

$(x + 2)(x - 3) = 0$

$x = -2$ *or* $x = 3$

When $x = -2$

$y = -16 - 12 + 72 - 3$

$y = 41$

When $x = 3$

$y = 2(-2)^3 - 3(-2)^2 - 36(-2) - 3$

$y = 2(3)^3 - 3(3)^2 - 36(3) - 3$

$y = -84$

∴ the coordinates on curve are $(-\mathbf{2}, \mathbf{41})$ and $(\mathbf{3}, \mathbf{-84})$ **Ans**

(b) $\int_{-1}^{3}(3x^2 - 2x)dx$

$= \left[\frac{3x^3}{3} - \frac{2x^2}{2}\right]_{-1}^{3}$

$= [x^3 - x^2]_{-1}^{3}$

$= [(3)^3 - (3)^2] - [(-1)^3 - (-1)^2]$

$= (27 - 9) - (-1 - 1)$

$= 18 - (-2)$

$= 18 + 2 = 20$

∴ $\int_{-1}^{3}(3x^2 - 2x)dx$ = **20 Ans**

4. (a) $\int_{2}^{5}(3x^2 + 2)dx$

$= \left[\frac{3x^{2+1}}{2+1} + \frac{2x^{0+1}}{0+1}\right]_{2}^{5}$

$= \left[\frac{3x^3}{3} + \frac{2x}{1}\right]_{2}^{5}$

$= [x^3 + 2x]_{2}^{5}$

$= ((5)^3 + 2(5)) - (2^3 + 2(2))$

$= (125 + 10) - (8 + 4)$

$= (135) - (12)$

$= \mathbf{123}$ **Ans**

(b) $y = x^2 - 3x - 4$

$m = \frac{dy}{dx} = 2x - 3$ at $x = 2, m = 2(2) - 3 = 1$

$y = (2)^2 - 3(2) - 4 = -6$

∴ equation of the tangent is given by

$y - y_1 = m(x - x_1)$

$y - (-6) = 1(x - 2)$

$y + 6 = x - 2$

$\boldsymbol{y = x + 8}$ **Ans**

5. (a) $y = x^3 - \frac{3}{2}x^2$

$\frac{dy}{dx} = 3x^2 - 3x = 3(2)^2 - 3(2) = 6$

∴ $m_1 = 6$ which is the gradient of tangent to the curve

we know that tangent is perpendicular to the normal,

$m_1 m_2 = -1$, at $x = 2, y = 2$

$m_2 = -\frac{1}{6}$ Which is the gradient of the normal?

$y - y_1 = m_2(x - x_1)$ (2,2)

$y - 2 = -\frac{1}{6}(x - 2)$

$y = -\frac{1}{6}x + \frac{2}{6} + 2$

$y = -\frac{1}{6}x + \frac{2+12}{6}$

$y = -\frac{1}{6}x + \frac{14}{6}$ → Multiplying throughout by 6, we get

$\boldsymbol{6y = -x + 14}$ **Ans**

(b) At the stationary points, $\frac{dy}{dx} = 0$

$3x^2 - 3x = 0$

$3x(x - 1) = 0$

$x = 0$ *or* $x = 1$

For $x = 0$.

$y = (0)^3 - \frac{3}{2}(0)^2 = 0$

For $x = 1$

$y = (1)^3 - \frac{3}{2}(1)^2 = -\frac{1}{2}$

∴ the stationary points are;

(0, 0) and (1, $-\frac{1}{2}$) Ans

TOPIC 9: VECTORS

1. **(i)** **(a)** $\overrightarrow{AE} = \frac{1}{3}\overrightarrow{AC}$

$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$

$\overrightarrow{AC} = \underline{a} + 2\underline{b}$

$\therefore \overrightarrow{AE} = \frac{1}{3}(\underline{a} + 2\underline{b})$ **Ans**

(b) $\overrightarrow{BE} = \overrightarrow{BA} + \overrightarrow{AE}$

$\overrightarrow{BE} = -\underline{a} + \frac{1}{3}(\underline{a} + 2\underline{b})$

$\overrightarrow{BE} = -\underline{a} + \frac{1}{3}\underline{a} + \frac{2}{3}\underline{b}$

$\overrightarrow{BE} = \frac{-3a+a}{3} + \frac{2}{3}\underline{b}$

$\overrightarrow{BE} = -\frac{2}{3}a + \frac{2}{3}\underline{b}$

$\therefore \overrightarrow{BE} = \frac{2}{3}\underline{b} - \frac{2}{3}\underline{a}$

$\overrightarrow{BE} = \frac{2}{3}(\underline{b} - \underline{a})$ **Ans**

(ii) Since $\overrightarrow{BE} = \frac{2}{3}(\underline{b} - \underline{a})$

and $\overrightarrow{BD} = \underline{b} - \underline{a}$

$\overrightarrow{BE} = \frac{2}{3}\overrightarrow{BD}$

$\therefore$ the points B, E and D are collinear

(c) $\overrightarrow{BD} = -\overrightarrow{AB} + \overrightarrow{AD}$

$\overrightarrow{BD} = -\underline{a} + \underline{b}$

$\overrightarrow{BD} = \underline{b} - \underline{a}$ **Ans**

2. (i) **(a)** $\overrightarrow{PQ} = \overrightarrow{PO} + \overrightarrow{OQ}$

$\overrightarrow{PQ} = -2\underline{p} + 4\underline{q}$

$\overrightarrow{PQ} = 4\underline{q} - 2\underline{p}$

$\therefore \overrightarrow{PQ} = 2(2\underline{q} - \underline{p})$ **Ans**

(b) $\overrightarrow{PX} = \frac{1}{3}\overrightarrow{PQ}$

$\overrightarrow{PX} = \frac{1}{3}(4\underline{q} - 2\underline{p})$ **Ans**

(c) $\overrightarrow{OX} = \overrightarrow{OP} + \overrightarrow{PX}$

$\overrightarrow{OX} = 2\underline{p} + \frac{4}{3}q - \frac{2}{3}\underline{p}$

$\overrightarrow{OX} = 2p - \frac{2}{3}p - \frac{4}{3}\underline{q}$

$\overrightarrow{OX} = \frac{6p-2p}{3} - \frac{4q}{3}$

$\overrightarrow{OX} = \frac{4}{3}p - \frac{4}{3}q$

$\overrightarrow{OC} = h\overrightarrow{OX}$

$\overrightarrow{OC} = h\left(\frac{4p}{3} + \frac{4q}{3}\right)$

$\overrightarrow{OC} = \frac{4h}{3}\underline{p} + \frac{4h}{3}\underline{q}$

$\therefore \overrightarrow{CQ} = \overrightarrow{CO} + \overrightarrow{OQ}$

$\overrightarrow{CQ} = -\left(\frac{4h}{3}\underline{p} + \frac{4h}{3}\underline{q}\right) + 4\underline{q}$

$\overrightarrow{CQ} = 4\underline{q} - \frac{4h}{3}\underline{q} - \frac{4h}{3}p$

$\overrightarrow{CQ} = 4\left(1 - \frac{h}{3}\right)\underline{q} - \frac{4h}{3}p$ hence shown

3. **(a)** $\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB}$

$\overrightarrow{\mathbf{OB}} = \underline{\boldsymbol{a}} + 2\underline{\boldsymbol{b}}$

(b) To find $\overrightarrow{OE}$, first find $\overrightarrow{CA}$ and

$\overrightarrow{CA} = \overrightarrow{CO} + \overrightarrow{OA}$

$\overrightarrow{CA} = -2\underline{b} + \underline{a}$

$\overrightarrow{AE} = -\frac{3}{4}\overrightarrow{CA}$

$\overrightarrow{AE} = -\frac{3}{4}(\underline{a} - 2\underline{b})$

$\therefore \overrightarrow{OE} = \overrightarrow{OA} + \overrightarrow{AE}$

$\overrightarrow{OE} = \underline{a} - \frac{3}{4}(\underline{a} - 2\underline{b})$

$\overrightarrow{OE} = \underline{a} - \frac{3}{4}\underline{a} + \frac{6}{4}\underline{b}$

$\overrightarrow{OE} = \frac{4a - 3a}{4} + \frac{3}{4}\underline{b}$

$\overrightarrow{OE} = \frac{1}{4}\underline{a} + \frac{3}{4}\underline{b}$

$\overrightarrow{\boldsymbol{OE}} = \frac{\mathbf{1}}{\mathbf{4}}(\boldsymbol{a} + 3\underline{\boldsymbol{b}})$ **Ans**

(b) $\overrightarrow{CD} = \frac{1}{2}\overrightarrow{CA}$

$\overrightarrow{CD} = \frac{1}{2}(-2\underline{b} + \underline{a})$

$\overrightarrow{CD} = -\underline{b} + \frac{1}{2}\underline{a}$

$\overrightarrow{\boldsymbol{CD}} = \frac{\mathbf{1}}{\mathbf{2}}\boldsymbol{a} - \underline{\boldsymbol{b}}$ **Ans**

4. **(i) (a)** $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$

$\overrightarrow{AB} = -3\underline{a} + 6\underline{b}$

$\overrightarrow{AB} = 6\underline{b} - 3\underline{a}$

$\therefore \overrightarrow{\boldsymbol{AB}} = 3(2\underline{\boldsymbol{b}} - \underline{\boldsymbol{a}})$ **Ans**

(b) $\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{AD}$

$\overrightarrow{OD} = \overrightarrow{OA} + \frac{1}{3}\overrightarrow{AB}$

$\overrightarrow{OD} = 3\underline{a} + \frac{1}{3}(6\underline{b} - 3\underline{a})$

$\overrightarrow{OD} = 3\underline{a} + 2\underline{b} - \underline{a}$

$\overrightarrow{OD} = 2\underline{a} + 2\underline{b}$

$\overrightarrow{\boldsymbol{OD}} = 2(\underline{\boldsymbol{a}} + \underline{\boldsymbol{b}})$ **Ans**

(c) $\overrightarrow{BC} = \overrightarrow{BO} + \overrightarrow{OC}$

$\overrightarrow{BC} = \overrightarrow{-OB} + \frac{2}{5}\overrightarrow{OA}$

$\overrightarrow{BC} = -6\underline{b} + \frac{2}{5}(3\underline{a})$

$\overrightarrow{BC} = -6b + \frac{6a}{5}$

$\overrightarrow{\boldsymbol{BC}} = \frac{\mathbf{6}}{\mathbf{5}}\underline{\boldsymbol{a}} - 6\underline{\boldsymbol{b}}$ **Ans**

(ii) $\overrightarrow{BE} = h\overrightarrow{BC}$

$\overrightarrow{BE} = h(\frac{6}{5}\underline{a} - 6\underline{b})$

$\overrightarrow{\boldsymbol{BE}} = 6(\frac{\boldsymbol{h}}{\mathbf{5}}\underline{\boldsymbol{a}} - h\underline{\boldsymbol{b}})$ **Ans**

TOPIC 10: TRIGONOMETRY

1. **(a)(i)** $\frac{n}{sinN} = \frac{r}{sinR}$

$\frac{n}{\sin 60} = \frac{80}{\sin 52}$

$n = \frac{80 \sin 60}{\sin 52}$

$n = 87.92016097$

$\therefore$ **KR = 87.9 m Ans**

(ii) Area of ΔKNB

$A = \frac{1}{2} kn\ sinN$

$A = \frac{1}{2}(50)(80) \sin 60°$

$A = 2000\ sin60°$

$A = 1732.050808$

$\therefore \boldsymbol{A = 1830m^2}$ **Ans**

(b) Shortest distance from R to KN

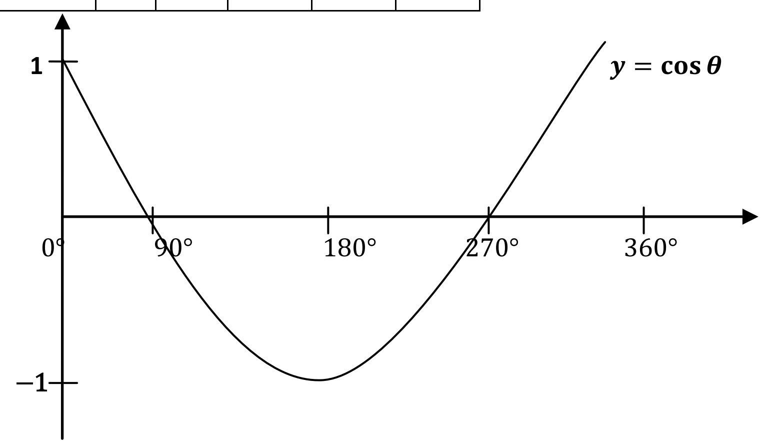
Sd = $\frac{2A}{b}$

S .d = $\frac{\mathbf{3260}}{\mathbf{80}}$

$= \mathbf{40.8m}$ **Ans**

(c) Graph of $y = \cos\theta$

θ	0°	90°	180°	270°	360°
$\cos\theta$	1	0	−1	0	1



2. (a) (i) $\frac{b}{sinB} = \frac{c}{sinC}$

$\frac{b}{\sin 79} = \frac{15}{\sin 40}$

$b = \frac{15 \sin 79}{\sin 40}$

$b = 22.9$

AC = 22.9km Ans

(ii) first find angle BAC

Angle BAC = $180 - (79 + 40) = 61$

$A = \frac{1}{2} bc\ sinA$

$A = \frac{1}{2}(15)(22.9) sin61°$

$A = 150.2159349$

$\boldsymbol{A = 150km^2}$ **Ans**

(iii) S.d = $\frac{2 \times 150}{22.9}$

= 13.100

= 13.1 Ans

(b) $cos\theta = 0.937$

$\theta = (cos^{-1}(0.937)$

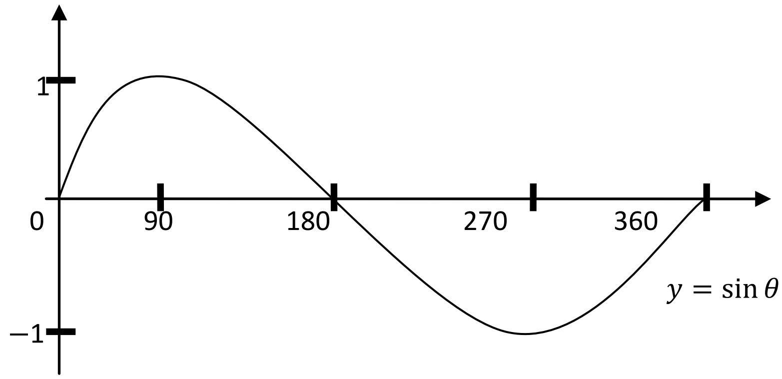
$\theta = 20.4°$

$\theta = 360° - 20.4) = 339.4$

$\therefore \boldsymbol{\theta = 20.4}$ **and** $\boldsymbol{\theta = 339.4}$ **Ans**

(c) $\boldsymbol{y = sin\ \theta}$

θ	0°	90°	180°	270°	360°
Sin θ	0	1	0	-1	0



3. **(a) (i)** A = $\frac{1}{2} ts \sin H$

$A = \frac{1}{2} \times 1.3 \times 1.9\ sin130°$

A = 1.235 sin 130°

A = 0.946

A = 0.946 km² Ans

(ii) $h^2 = t^2 + s^2 - 2tscosH$

$h^2 = (1.3)^2 + (1.9)^2 - 2(1.3)(1.9)\cos 130°$

$h^2 = 5.3 - (-3.175370792)$

$h^2 = 5.3 + 3.17537$

$h^2 = 8.475370792$

$h^2 = \sqrt{8.475370792}$

h = 2.911249009

$\therefore$ **TS = 2.91km Ans**

(iii) Shortest distance = $\frac{2A}{b}$

$= \frac{2 \times 0.95}{2.9}$

= 0.65517

S.d = **0.655 km Ans**

(b) $\cos\theta = \frac{2}{3}$

$\theta = \cos^{-1}\left(\frac{2}{3}\right)$

$\theta = 48.1896851$

$\therefore \theta =$ **48.2° Ans**

4. **(a) (i)** to find PQ, first find angle R

$\hat{R} = 180° - (46° + 36°)$

= 98°

$\therefore \frac{\sin R}{r} = \frac{\sin P}{p}$

$\frac{\sin 98°}{r} = \frac{\sin 46°}{36.5}$

rsin 46° = 36.5 sin98°

r = $\frac{36.5 \sin 98°}{\sin 46°}$

(ii) A = $\frac{1}{2} \times p \times r \times \sin Q$

$= \frac{1}{2} \times\ 36.5 \times 50.2 \times \sin 36°$

= 916.15 sin 36°

$\therefore A = 538.4994589 \approx \mathbf{538.5\ km^2}$ **Ans**

(iii) Shortest distance = $\frac{2A}{b} = \frac{2 \times 538..2}{50.2} = \mathbf{10.7\ km\ A}$

(b) $\sin\theta = 0.6792$

$\theta = \sin^{-1}(0.6792)$ → press shift / 2nd f and then sin r = 50.24716343

∴ $\mathbf{PQ = 50.2km}$ **Ans**

$\theta = 42.8°$ and $\theta = 180 - 42.8 = 137.2°$

∴ $\boldsymbol{\theta = 42.8°, 137.2°}$ **Ans**

5. **(a) (i)** $k^2 = i^2 + m^2 - 2im\cos K$

$k^2 = 5^2 + 3^2 - 2(5)(3)\cos 110°$

$k^2 = 34 - (-10.2606043)$

$k^2 = 44.26060643$

$k = \sqrt{44.26060643}$

k = 6.656864368

∴ $\mathbf{MI = 6.65km}$ **Ans**

(ii) $A = \frac{1}{2} \times i \times m\ sinK$

$A = \frac{1}{2}(5)(3)sin110°$

A = 7.5 sin110°

A = 7.047694656

$\mathbf{A = 7.05km^2}$ **Ans**

(iii) shortest $d = \frac{2A}{b}$

$= \frac{2 \times 7.05}{6.65}$

$\mathbf{= 2.12km}$ **Ans**

(b) $\tan\theta = 0.7$

$\theta = \tan^{-1}(0.7)$

$\theta = 34.9920202$

$\theta = 34.9920202$

$\boldsymbol{\theta = 35°}$ **Ans**

TOPIC 11: MENSURATION

1. $\frac{H}{H-11.4} = \frac{14}{8}$

$14(H - 11.4) = 8H$

$14H - 159.6 = 8H$

$14H - 8H = 159.6$

$6H = 159.6$

$H = 26.6$

$h = 26.6 - 11.4 = 15.2$ cm

$\therefore V = \frac{1}{3}(LBH - lbh)$

$V = \frac{1}{3}(14 \times 10 \times 26.6 - 8 \times 4 \times 15.6)$

$V = \frac{1}{3}(3724 - 486.4)$

$V = \frac{1}{3}(3237.6)$

V = 1079.2

V = 1080 $\mathbf{cm^3}$ Ans

2. $\frac{H}{H-9} = \frac{10}{4}$

$10(H - 9) = 4H$

10H−90 = 4H

10H−4H = 90

6H = 90

H = 15

$\therefore H = 15cm\ and\ h = 15 - 9 = 6cm$

∴ $V = \frac{1}{3}[L^2H - l^2h]$ (square base)

$V = \frac{1}{3}[10^2 \times 15 - 4^2 \times 6]$

$V = \frac{1}{3}(1500 - 96)$

$V = \frac{1}{3}(1404)$

V = 468 $\boldsymbol{cm^3}$ Ans

3. First find the height of the small cone that was Cutoff.

$\frac{h}{7} = \frac{20+h}{21}$

$21h$ = 7(20+h)

$21h$ = 140 + $7h$

$21h$ $-7h$ = 140

$14h$= 140

h= 10

$\boldsymbol{h = 10\ cm}$ and H = 10 + 20 = **30cm**

$\therefore V = \frac{1}{3}\pi R^2 H - \frac{1}{3}\pi r^2 h$

V = $\frac{1}{3}\pi(R^2H - r^2h)$

V = $\frac{3.142}{3}$ $(21^2 \times 30 - 7^2 \times 10)$

V = $\frac{3.142}{3}(13230 - 490)$

V = $\frac{3.142}{3}(12740)$

V = 13343.02667

V $\boldsymbol{= 13300cm^3}$ **Ans**

4. (i) Let height EA = x, then

$\frac{x}{4} = \frac{15+x}{12}$

$12x$ = 4(15 +x)

$12x$ = 60 +$4x$

$12x$ $-4x$ =60

$8x$ = 60

$x = 7.5$

$\therefore$ $\mathbf{AE = 7.5}$ And AD = 7.5 +15 = **22.5cm**

(ii) Volume that remained is the that of the frustum

V= $\frac{1}{3}\pi R^2 H - \frac{1}{3}\pi r^2 h$

V = $\frac{1}{3}\pi(R^2H - r^2h)$

V = $\frac{1}{3} \times 3.142(12^2 \times 22.5 - 4^2 \times 7.5)$

V = $\frac{3.142}{3}(3240 - 120)$

V = $\frac{3.142}{3}(3120)$

V = 3267.68

V = 3270 $\boldsymbol{cm^3}$ **(correct to 3 sig figures)**

5. **(a)** V = lbh

V= 1.2$\times$ 0.9×10

V= 10.8cm^3

1cm^3 $\rightarrow$ 1000l

10.8cm^3$\rightarrow$ x

$x = 10.8 \times 1000l$

$x = 10800l$

$\therefore$ $\boldsymbol{V = 10800l}$ **(3 sig fig)**

(b) first find the radius of a cone

r^2= $13^2 - 12^2$

r^2 = 25

r = $\sqrt{25}$

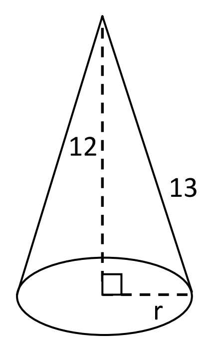
r = 5cm

$\therefore T.S.A = \pi r^2 + \pi rl$

T.S.A = $\pi r(r + l)$

T.S.A = $3.142 \times 5(5 + 13)$

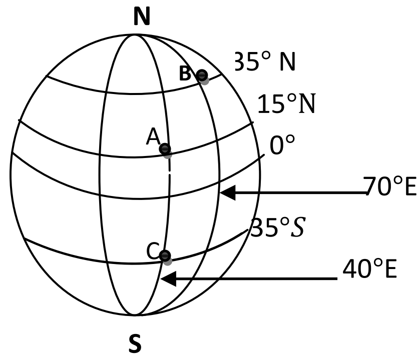
T. S. A = 282.78cm$\boldsymbol{^2}$



(Kachama Dickson.C)

TOPIC 9: EARRTH GEOMETRY

1. **(a)**



AC = $\frac{\theta}{360} \times 2\pi R$ where $\theta = 15° + 35° = 50°$

AC = $\frac{50°}{360°} \times 2 \times 3.142 \times 6370$

AC = $\frac{2001454}{360}$

AC = 5,559.594444

AC = 5,560km Ans

(b) (i) BQ = $\frac{\alpha}{360} \times 2\pi R\cos\theta$

900 = $\frac{2\pi R\alpha\cos\theta}{360}$

$2\pi R\cos\theta\alpha = 900 \times 360°$

$\alpha = \frac{32400}{2\pi R\cos\theta}$

$\alpha = \frac{32400}{(2\times3.142\times6370\times\cos 35°)}$

$\alpha = 9.88109$

∴ the difference in longitude is **9.9**°

(ii) longitude of Q = 70° − 9.9° = 60.1°

∴ position of **Q** $(\mathbf{35°S, 60.1°E})$ **Ans**

2. **(a)** D = $\frac{\theta}{360} \times 2\pi R$ where $\theta = 60° + 60° = 120°$

D = $\frac{120}{360} \times 2 \times 3.142 \times 3437$

D = $\frac{2591772.96}{360}$

D = 7199.369333

∴ **distance BC = 7200nm Ans**

or D = $\theta \times 60$

D = 120° × 60

D = 7200 nm Ans

(b) To find speed, first find distance CD

D = $\frac{\boldsymbol{\alpha}}{\mathbf{360}} \times \mathbf{2\pi R\cos\theta}$

D = $\frac{120}{360} \times 2 \times \pi \times 3437 \cos 60$

D = $\frac{\mathbf{1295886.48}}{\mathbf{360}}$

D = 3599.684667

∴ **speed =** $\frac{\boldsymbol{D}}{\boldsymbol{T}}$

speed = $\frac{3600}{12}$

speed = 300knots Ans

D = 3600nm

3. **(a)** Difference in latitudes between W and Y

$\boldsymbol{\theta} = 80° + 30° = \mathbf{110}°$

(b) (i) XZ = $\frac{\theta}{360} \times 2\pi R$

XZ $= \frac{110°}{360°} \times 2 \times 3.142 \times 3437$

XZ $= 6599.421889$

XZ = 6600nm Ans

(ii) YZ = $\frac{\alpha}{360°} \times 2\pi R cos\theta$ where $\alpha = 15° + 105 = 120°$

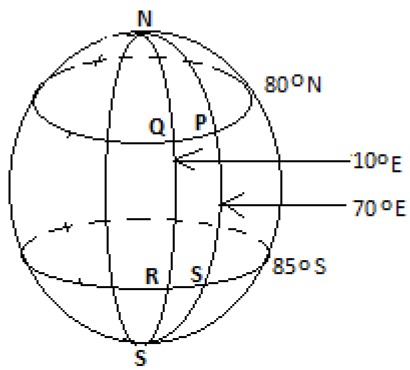
YZ $= \frac{120°}{360°} \times 2 \times 3.142 \times 3437 \times cos30°$

YZ $= \frac{2244541.224}{360}$

YZ $= 6234.836734$

YZ =6230nm Ans

4. **(i)**



(ii)(a) Distance QR = $\frac{\theta}{360°} \times 2\pi R$ $\quad \theta = 80° + 85° = 165°$

QR = $\frac{165°}{360°} \times 2 \times 3.142 \times 3437$

QR = 9899.132833

QR =9900 nm Ans

(b) C = $2\pi R cos\theta$

C = 21600 Cos θ

C = 21600$\times cos\ 85°$

C = 1882.564043

C =1900nm Ans

5. **(a)** Difference in latitudes

$\theta = 50° + 70° = \mathbf{120°}$

(b) C = $2\pi R cos\theta$

C = 21600 Cos $50°$

C=13884.21237nm

C= 13900nm

(c) AD = $\frac{\theta}{360°} \times 2\pi R$

AD = $\frac{120°}{360°} \times 2 \times 3.142 \times 3437$

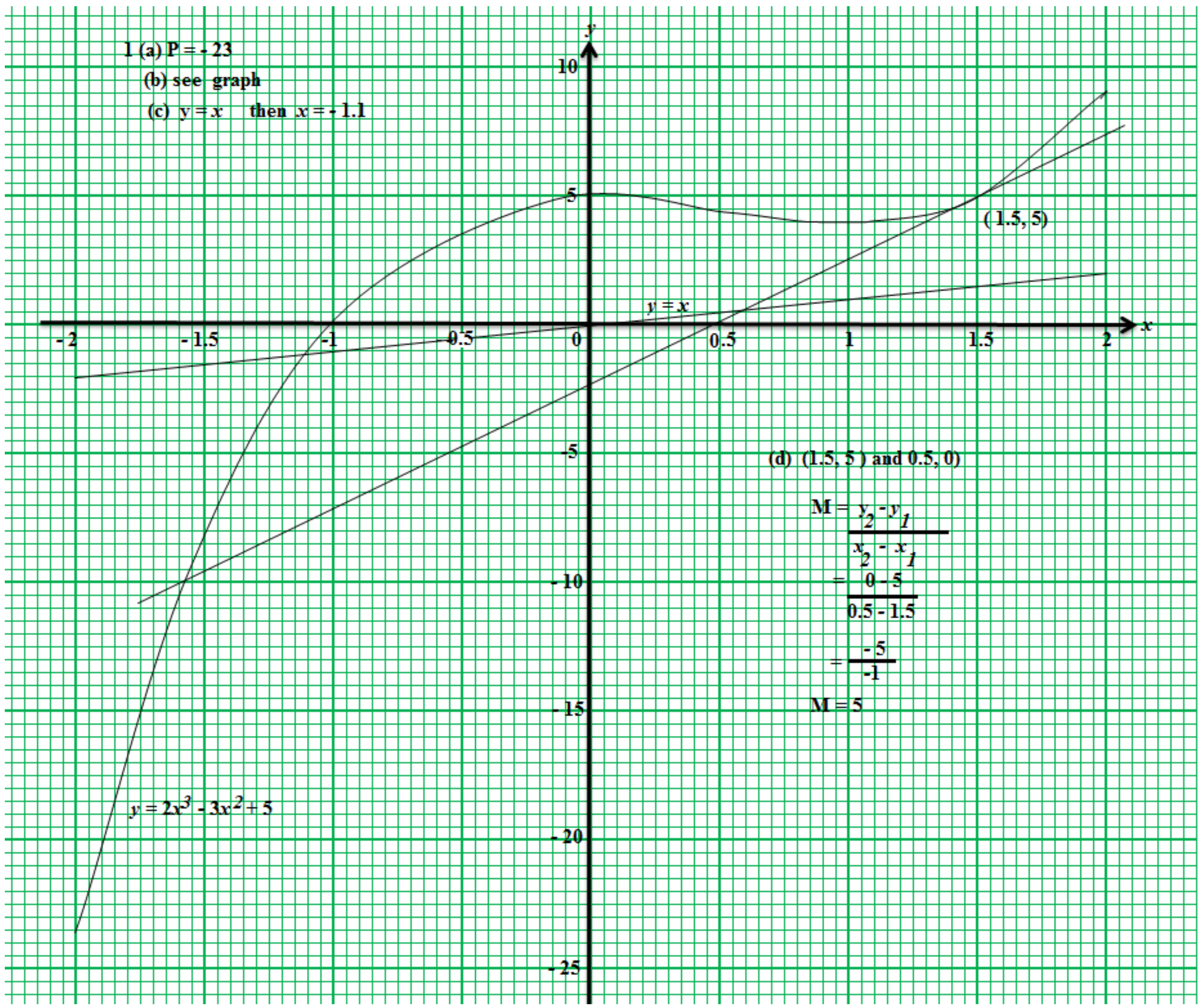
AD = 7199.369333

AD = 7200nm Ans

Produced by Sir DK & Sir JC

TOPIC 13: QUADRATIC FUNCTIONS

1.



2. **(a) (i)** $y = x^3 + x^2 - 12x - (x^3 + x^2 - 12x)$

$y = x^3 + x^2 - 12x - x^3 - x^2 + 12x$

$y = 0 \leftrightarrow x - axis$

$\therefore \boldsymbol{x = -4, x = 0}$ **and** $\boldsymbol{x = 3}$

Note that the values of x are the points where the curve $y = x^3 + x^2 - 12x$ meets the line $y = 0$

(ii) From part (i) we can see that

$y = x + 10$

when we draw this line on the same graph using any points we have

$\boldsymbol{x = -3.7 \pm 1, x = -0.7 \pm}$ **and** $\boldsymbol{x = 3.5 \pm 1}$

$\boldsymbol{x}$	-3	0	2	3

y	7	10	12	13

2. **(b)(i)** Draw a straight line touching only at (-3, 18) and pick any two points lying on the same line

e. g $(-3, 18)$ and $(-4.5,4)$

$m = \frac{y_2-y_1}{x_2-x_1}$

$m = \frac{4-18}{-4.5-(-3)}$

$m = \frac{-14}{-1.5}$

$\boldsymbol{m = 9.3 \pm 1}$

(ii) From the graph, we can see that the required area is simply the areas of the two trapeziums

A $= \frac{1}{2}(a+b)h$

A $= \frac{1}{2}(28+30)1 + \frac{1}{2}(30+22)1$

A = $\frac{1}{2}(58) + \frac{1}{2}(52)$

A $= 29 + 26$

$\mathbf{A = 55 \pm 1}$ **Square units**

3. **(a) (i)** $y = x^3 + 3x^2 - x - 3 - (x^3 + 3x^2 - x - 3)$

$y = x^3 + 3x^2 - x - 3 - x^3 - 3x^2 + x + 3$

$y = 0$

On the graph, when $y = 0, (x - axis)$

$\boldsymbol{x = -3, x = -1}$ and $\boldsymbol{x = 1}$

(ii) $y = x^3 + 3x^2 - x - 3 - (x^3 + 3x^2 - x - 5)$

$y = x^3 + 3x^2 - x - 3 - x^3 - 3x^2 + x + 5)$

$y = -3 + 5$

$y = 2$

On the graph when y=2, then

$\boldsymbol{x = -2.6, x = -1.5}$ and $\boldsymbol{x = 1.2}$ **Ans**

(b) (i) to find the gradient, draw a straight line touching the curve only at $(-3,0)$ and pick any two points lying on this line for Example $(-3,0)$ and $(-2,8)$

$m = \frac{y_2-y_1}{x_2-x_1}$

$m = \frac{8-0}{-2-(-3)}$

$\boldsymbol{m = \frac{8}{1} = 8}$ **Ans**

(ii) Area = A of rectangle + trapezium

A = l× $b + \frac{1}{2}(a+b)h$

A = $1 \times 20 + \frac{1}{2}(20+5)$

A $= 20 + 12.5$

A = 32.5 square units

4. (a) To find the gradient of the curve, simply draw a straight line touching the curve at $(2,5)$ and pick any two points lying on the line drawn, for example $(2,5)$ and $(1,-7)$ and use the gradient formula to find the gradient

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{-7-5}{1-2}$$

$$m = \frac{-12}{-1}$$

$\boldsymbol{m = 12}$ **Ans**

4. **(b) (i)** $\boldsymbol{y} = x^3 + x^2 - 5x + 3 - (x^3 + x^2 - 5x + 3)$

$\boldsymbol{y} = x^3 + x^2 - 5x + 3 - x^3 - x^2 + 5x - 3$

$y = 0$

On the graph, when $y = 0(x - axis)$

$\boldsymbol{x = -3}$ **and** $\boldsymbol{x = 1}$ **Ans**

(ii) similarly we have y=5x

Use any points to draw the line $y = 5x$ and find the values of x where it meets the curve for example use the points in this table below

x	-1	0	1	5
y	-5	0	5	10

$\therefore$ $\boldsymbol{x = -3.8}$, $\boldsymbol{x}$ **= 0.3 and** $\boldsymbol{x}$**=**$\boldsymbol{2.4}$

(c) From the graph in the given bounds, we can make two trapeziums.

Hence to find the area, we find the total areas of these two trapeziums:

A = $\frac{1}{2}(\boldsymbol{a} + \boldsymbol{b})\boldsymbol{h}$

A= $\frac{1}{2}(9 + 8)1 + \frac{1}{2}(8 + 3)$

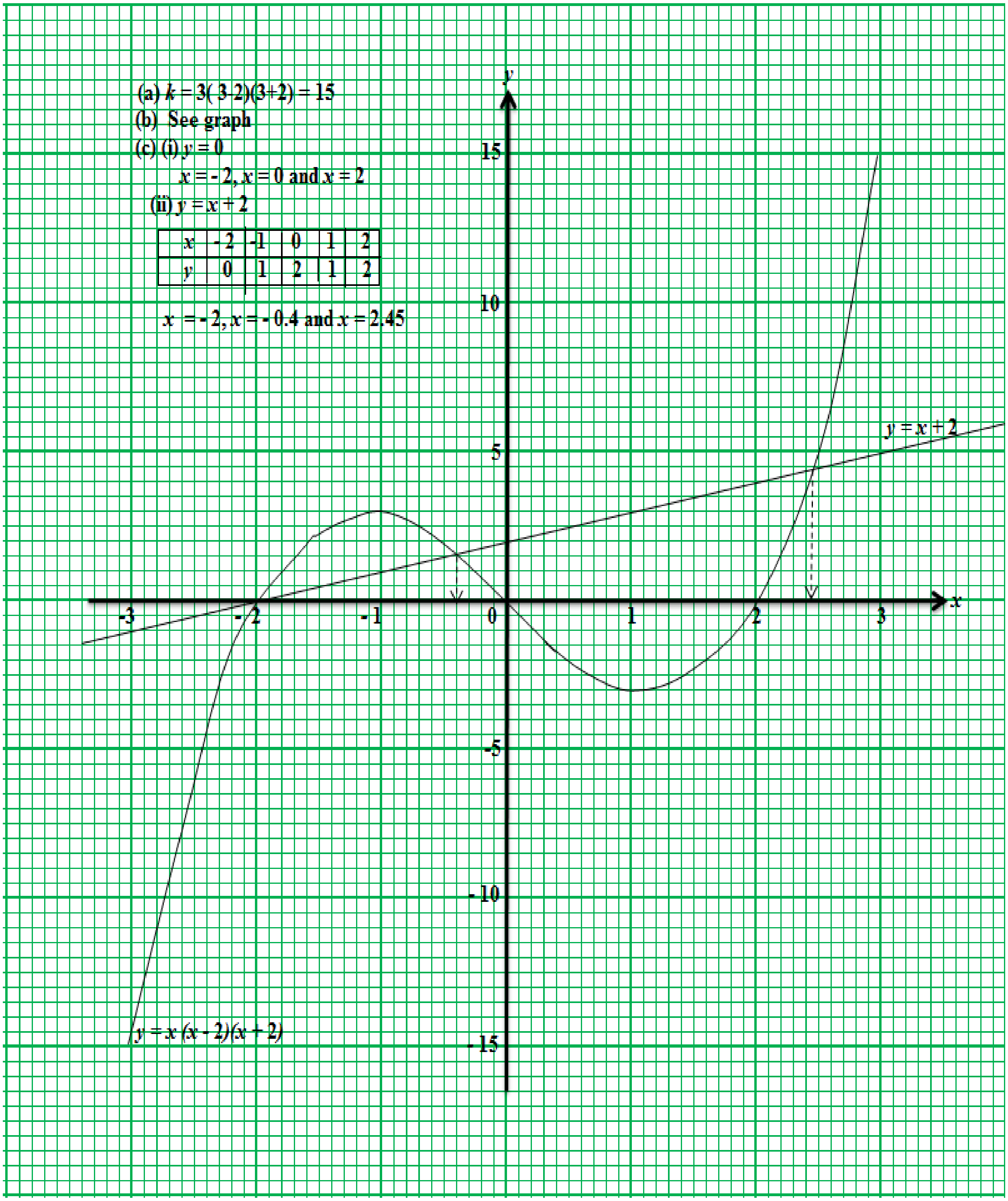
A= $\frac{1}{2}(17) + \frac{1}{2}(11)$

A= $8.5 + 5.5$

A= $\boldsymbol{14}$ **Square units**

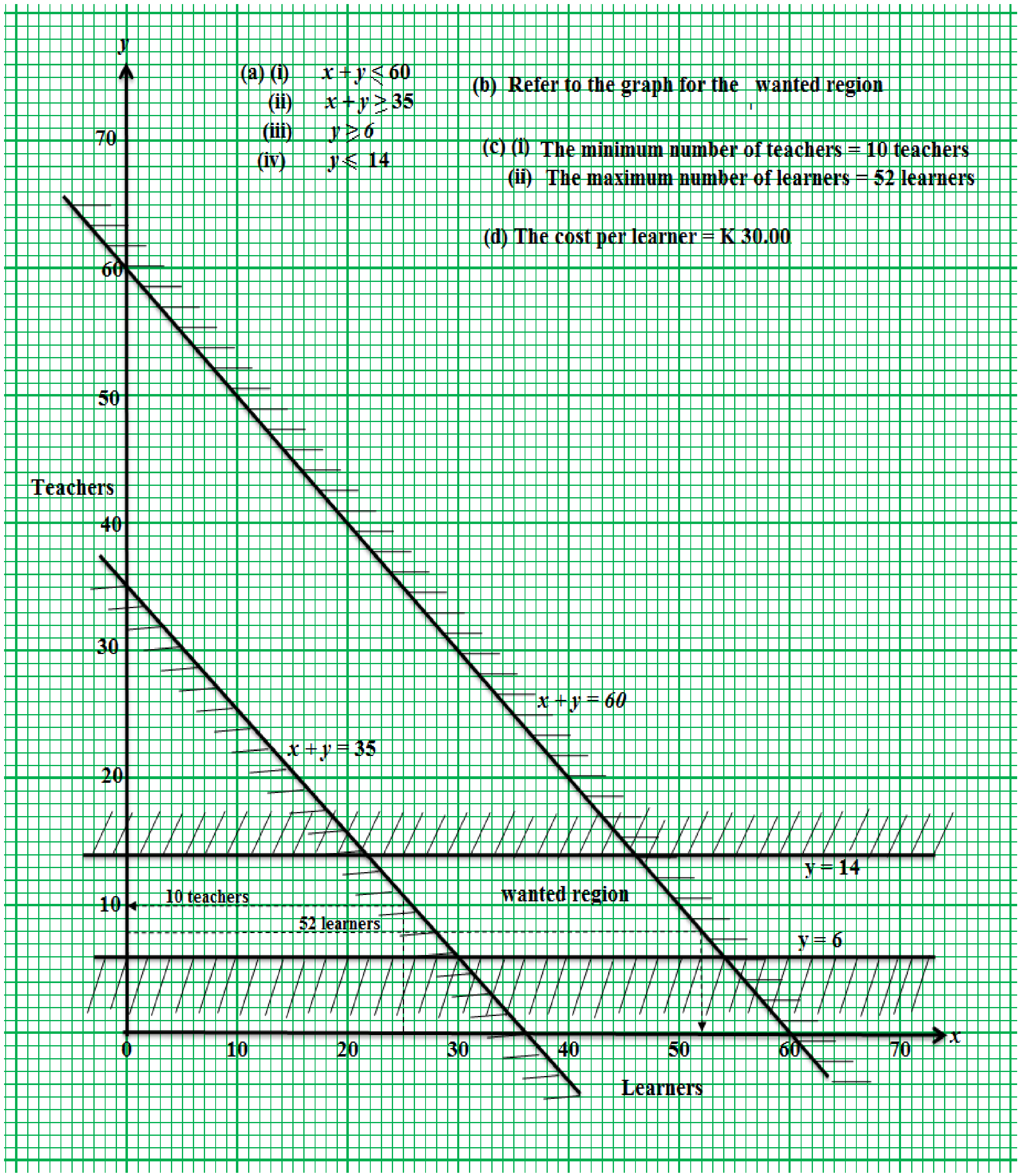
Why should you fail mathematics when success is guaranteed .

5.

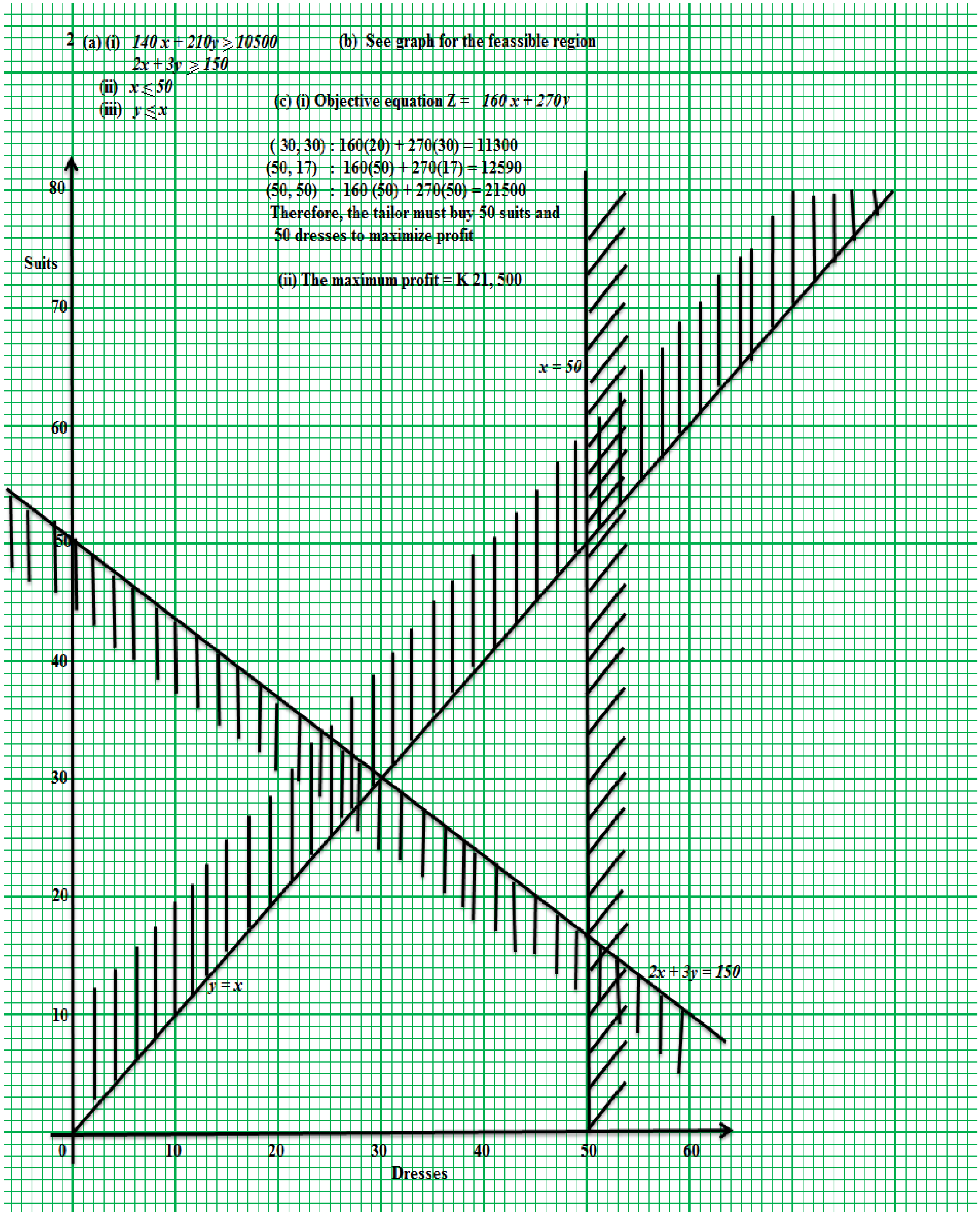

(a) k = 3(3-2)(3+2) = 15
(b) See graph
(c) (i) y = 0
x = - 2, x = 0 and x = 2
(ii) y = x + 2
x | - 2 | -1 | 0 | 1 | 2
y | 0 | 1 | 2 | 1 | 2
x = - 2, x = - 0.4 and x = 2.45
y
x
15
10
5
0
-5
- 10
- 15
-3
- 2
- 1
1
2
3
y = x + 2
y = x (x - 2)(x + 2)

TOPIC 14: LINEAR PROGRAMING

1.

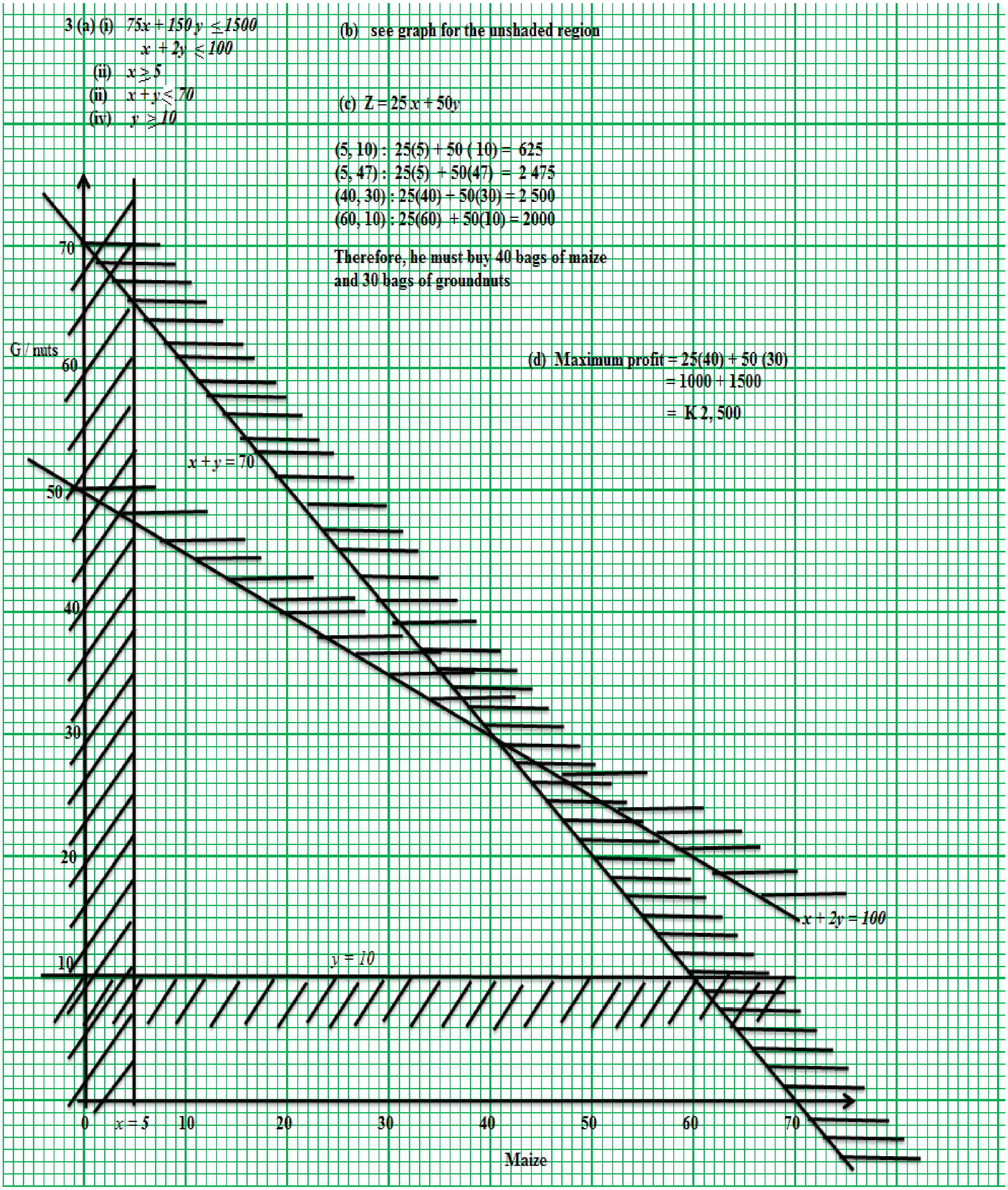


2.

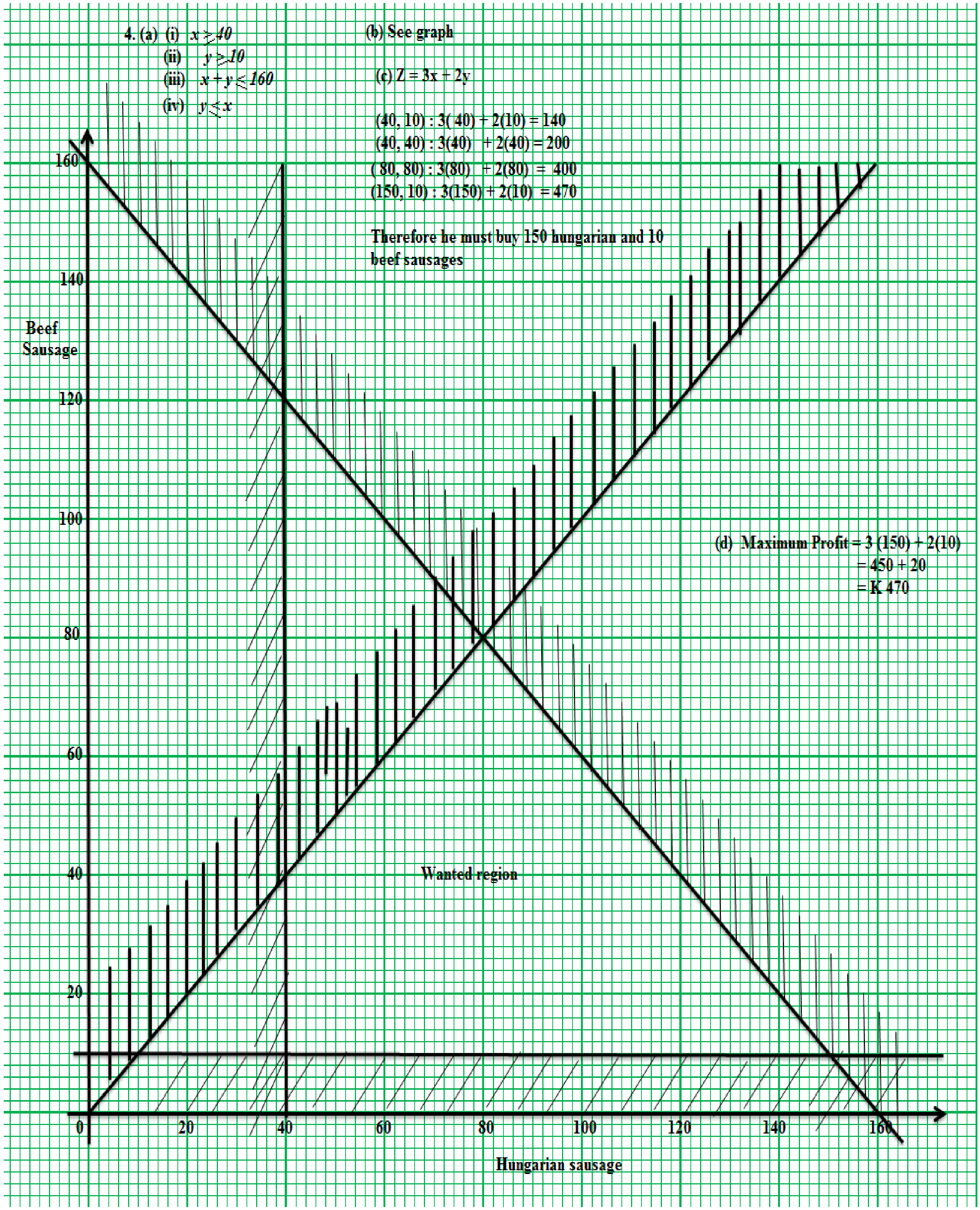


2 (a) (i) 140 x + 210y ≥ 10500
2x + 3y ≥ 150
(ii) x ≤ 50
(iii) y ≤ x
(b) See graph for the feassible region
(c) (i) Objective equation Z = 160 x + 270y
(30, 30) : 160(20) + 270(30) = 11300
(50, 17) : 160(50) + 270(17) = 12590
(50, 50) : 160 (50) + 270(50) = 21500
Therefore, the tailor must buy 50 suits and
50 dresses to maximize profit
(ii) The maximum profit = K 21, 500
Suits
80
70
60
50
40
30
20
10
0
10
20
30
40
50
60
Dresses
x = 50
y = x
2x + 3y = 150

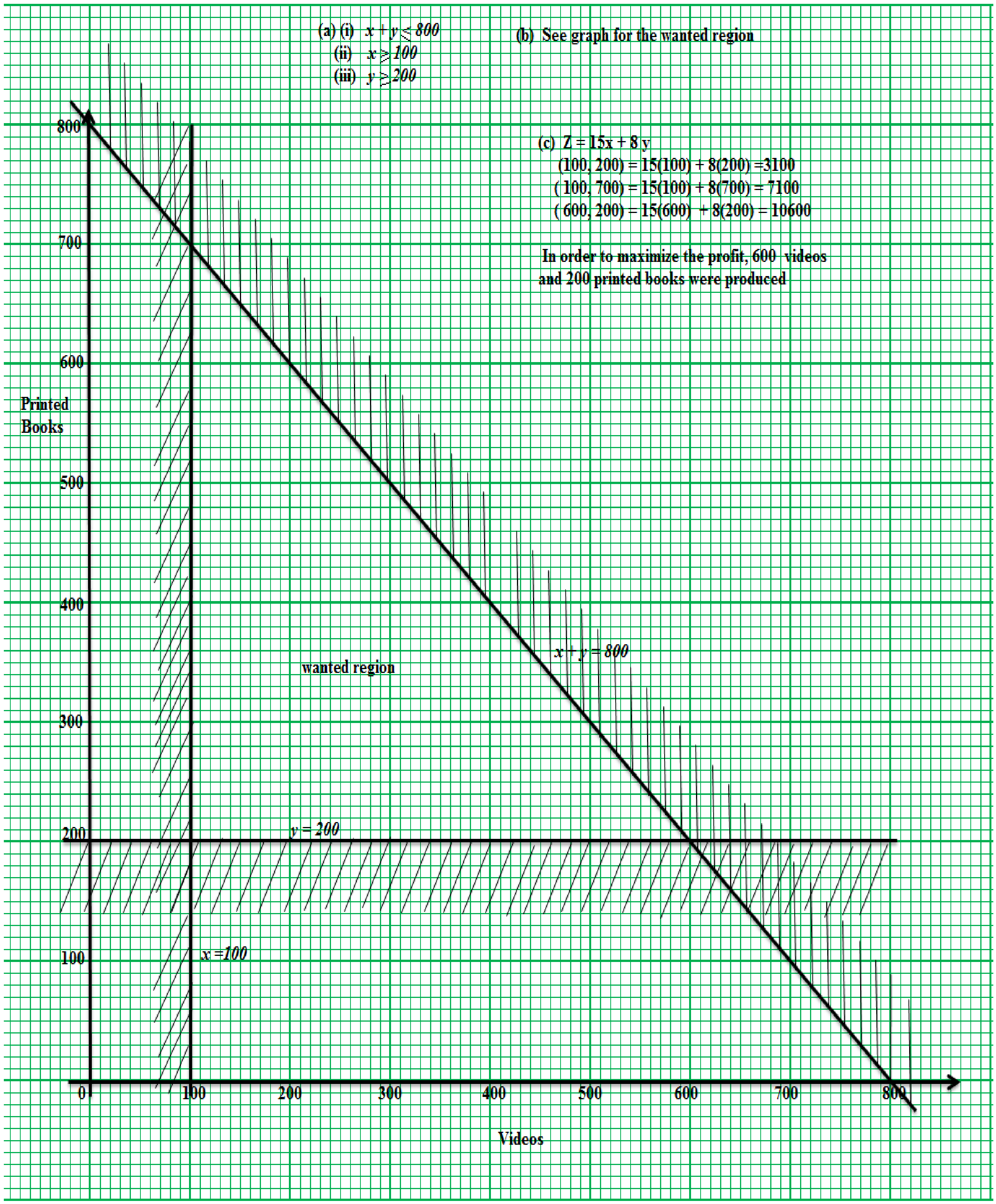
3.


3 (a) (i) 75x + 150y ≤ 1500
x + 2y ≤ 100
(ii) x ≥ 5
(ii) x + y ≤ 70
(iv) y ≥ 10
(b) see graph for the unshaded region
(c) Z = 25x + 50y
(5, 10) : 25(5) + 50 (10) = 625
(5, 47) : 25(5) + 50(47) = 2 475
(40, 30) : 25(40) + 50(30) = 2 500
(60, 10) : 25(60) + 50(10) = 2000
Therefore, he must buy 40 bags of maize
and 30 bags of groundnuts
(d) Maximum profit = 25(40) + 50 (30)
= 1000 + 1500
= K 2, 500
G / nuts
Maize
x + y = 70
x + 2y = 100
y = 10
x = 5
0
10
20
30
40
50
60
70

4.


4. (a) (i) x ≥ 40
(ii) y ≥ 10
(iii) x + y ≤ 160
(iv) y ≤ x
(b) See graph
(c) Z = 3x + 2y
(40, 10) : 3(40) + 2(10) = 140
(40, 40) : 3(40) + 2(40) = 200
(80, 80) : 3(80) + 2(80) = 400
(150, 10) : 3(150) + 2(10) = 470
Therefore he must buy 150 hungarian and 10 beef sausages
(d) Maximum Profit = 3 (150) + 2(10)
= 450 + 20
= K 470
Beef Sausage
Wanted region
Hungarian sausage
0
20
40
60
80
100
120
140
160

5.



(a) (i) x + y ≤ 800
(ii) x ≥ 100
(iii) y ≥ 200
(b) See graph for the wanted region
(c) Z = 15x + 8 y
(100, 200) = 15(100) + 8(200) =3100
(100, 700) = 15(100) + 8(700) = 7100
(600, 200) = 15(600) + 8(200) = 10600
In order to maximize the profit, 600 videos and 200 printed books were produced
Printed Books
800
700
600
500
400
300
200
100
0
100
200
300
400
500
600
700
800
Videos
wanted region
x + y = 800
y = 200
x =100

TOPIC 15: STATISTICS

1. **(a)**

class	x	x^2	f	fx	fx^2
$10< x \leq 20$	15	225	2	30	450
$20 < x \leq 30$	25	625	10	250	6250
$30 < x \leq 40$	35	1225	15	525	18375
$40< x \leq 50$	45	2025	23	1035	46575
$50 < x \leq 60$	55	3025	30	1650	90750
$60 < x \leq 70$	65	4225	10	1650	42250
Totals			$\sum f = 90$	$\sum fx = 4140$	$\sum fx^2 = 204650$

Mean $\bar{x} = \frac{\sum fx}{\sum f}$

$= \frac{4140}{90}$

$= 46$

SD = $\sqrt{\frac{\sum fx^2}{\sum fx} - (\overline{x})^2}$

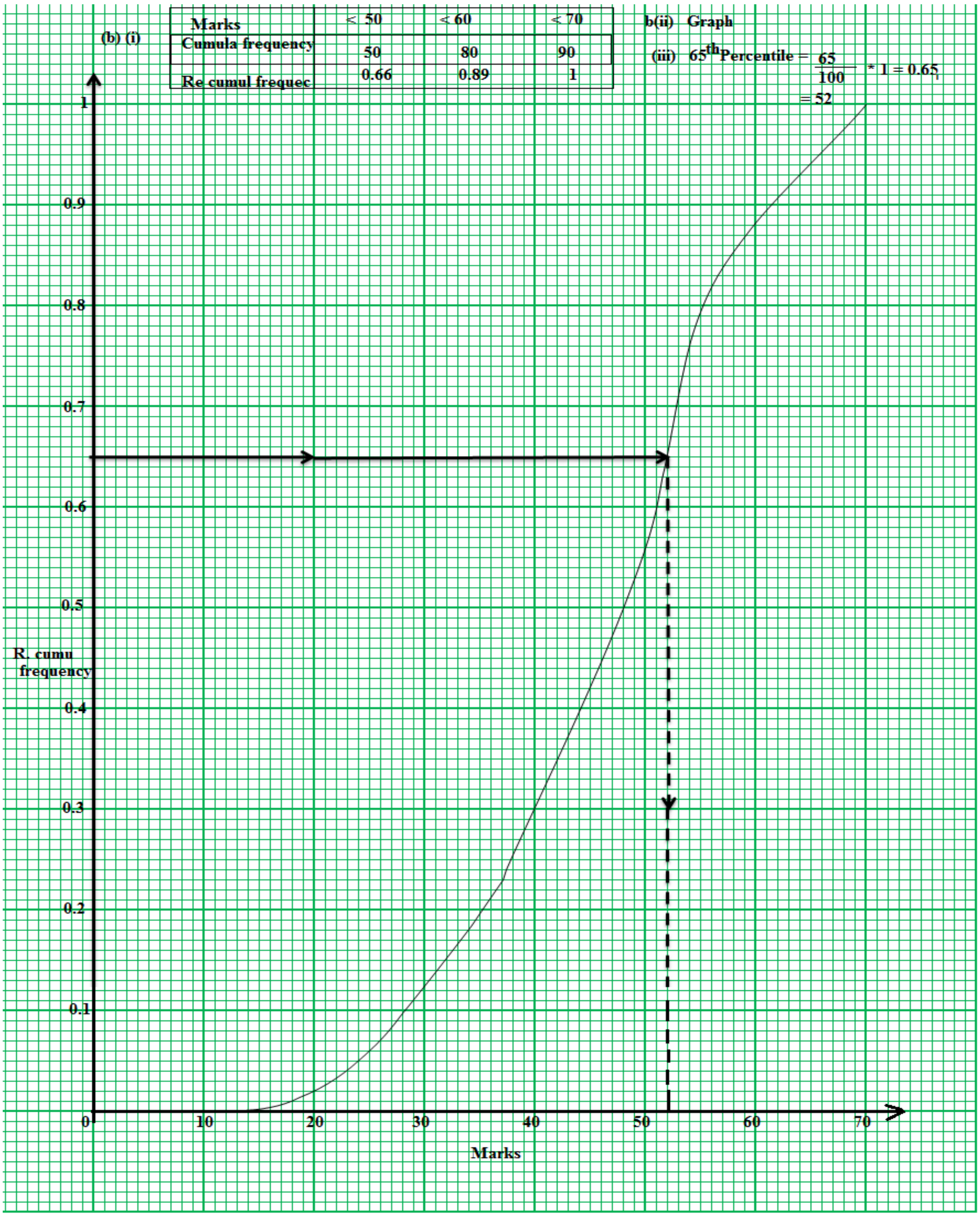
= $\sqrt{\frac{204650}{90} - (46)^2}$

= $\sqrt{2273.888889 - 2116}$

= $\sqrt{157.8888889}$

SD = 12.57 Ans

(b)


(b) (i)
Marks
< 50
< 60
< 70
Cumula frequency
50
80
90
Re cumul frequec
0.66
0.89
1
b(ii) Graph
(iii) 65th Percentile = 65/100 * 1 = 0.65
= 52
R. cumu frequency
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
10
20
30
40
50
60
70
Marks

2. **(a)**

x	x^2	f	fx	fx^2
2	4	1	2	4
3	9	5	15	45
4	16	4	16	64
5	25	6	30	150
6	36	10	60	360
7	49	16	112	784
8	64	18	144	1152
		$\sum f = 60$	$\sum fx = 379$	$\sum fx^2 = 2559$

Mean $(\bar{x}) = \frac{\sum fx}{\sum f}$

$= \frac{379}{60}$

$= \mathbf{6.32}$

SD = $\sqrt{\frac{\sum fx}{\sum f} - (\bar{x})^2}$

$= \sqrt{\frac{2559}{60} - (6.32)^2}$

$= \sqrt{42.65 - 39.9424}$

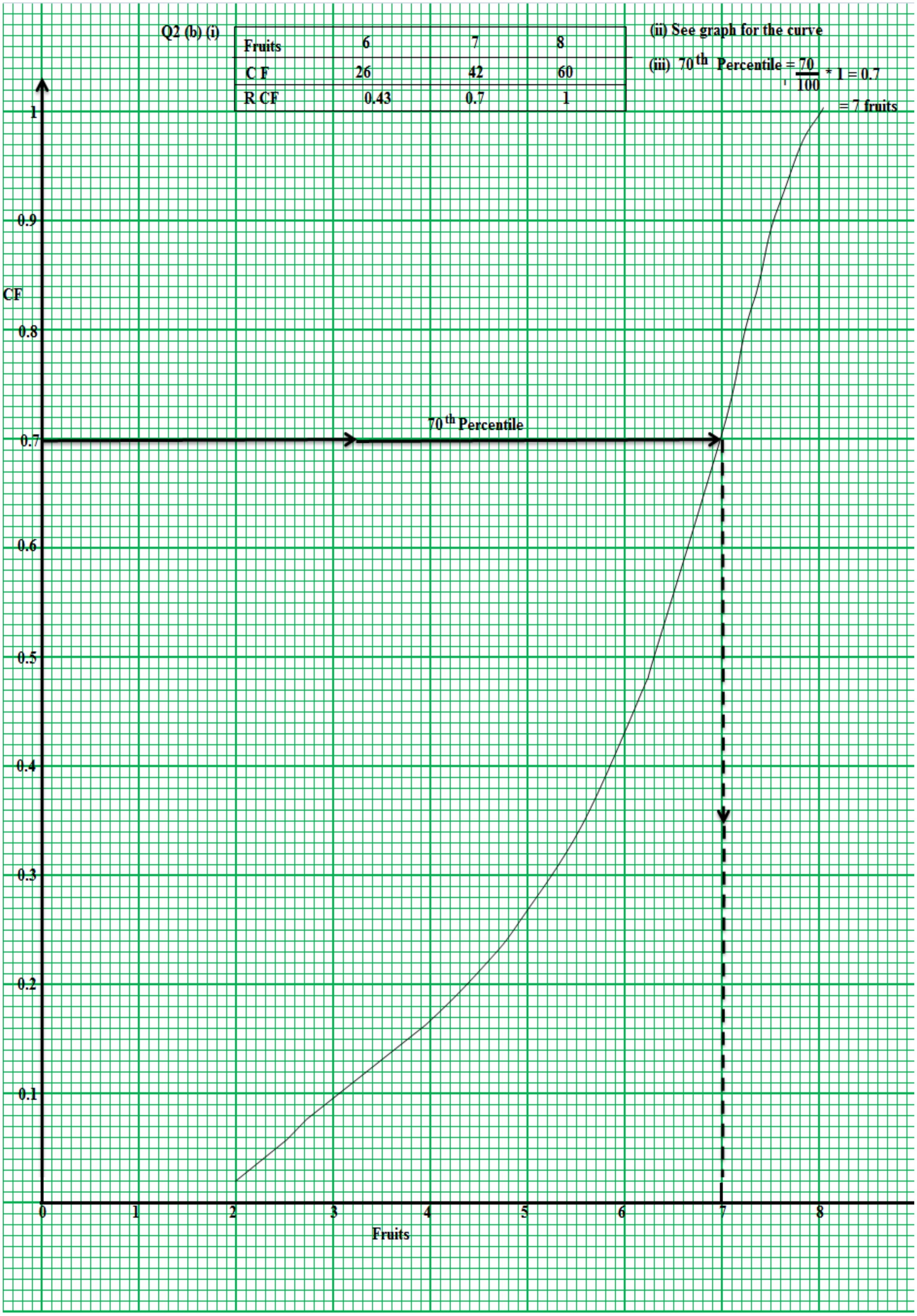
$= \sqrt{2.7076}$

$= 1.645478654$

$= \mathbf{1.65}$ **Ans**

Hard work need to be appreciated.

(b)


Q2 (b) (i)
Fruits 6 7 8
C F 26 42 60
R CF 0.43 0.7 1
(ii) See graph for the curve
(iii) 70th Percentile = 70/100 * 1 = 0.7
= 7 fruits
70th Percentile
CF
Fruits
0 1 2 3 4 5 6 7 8
0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1

3. **(a)**

Class marks	x	f	fx	$(x-\overline{x})^2$	$f(x-\overline{x})^2$
$0 < x \leq 5$	2.5	13	32.5	12.25	159.25
$5 < x \leq 10$	7.5	27	205.5	17.2225	465.0
$10 < x \leq 15$	12.5	35	437.5	0.7225	25.288
$15 < x \leq 20$	17.5	16	280	34.2225	547.58
$20 < x \leq 25$	22.5	7	157.5	117.7225	824.058
$25 < x \leq 30$	27.5	2	55	251.2225	502.445
		$\sum f = 100$	$\sum fx = 1168$		$\sum f(x-\bar{x})^2 = 2523.621$

$\overline{x} = \frac{\sum fx}{\sum f}$

$= \frac{1165}{100}$

$= \mathbf{11.65}$

$\text{SD} = \sqrt{\frac{\sum f(x-\overline{x})^2}{\sum f}}$

$\text{SD} = \sqrt{\frac{\mathbf{2523.621}}{\mathbf{100}}}$

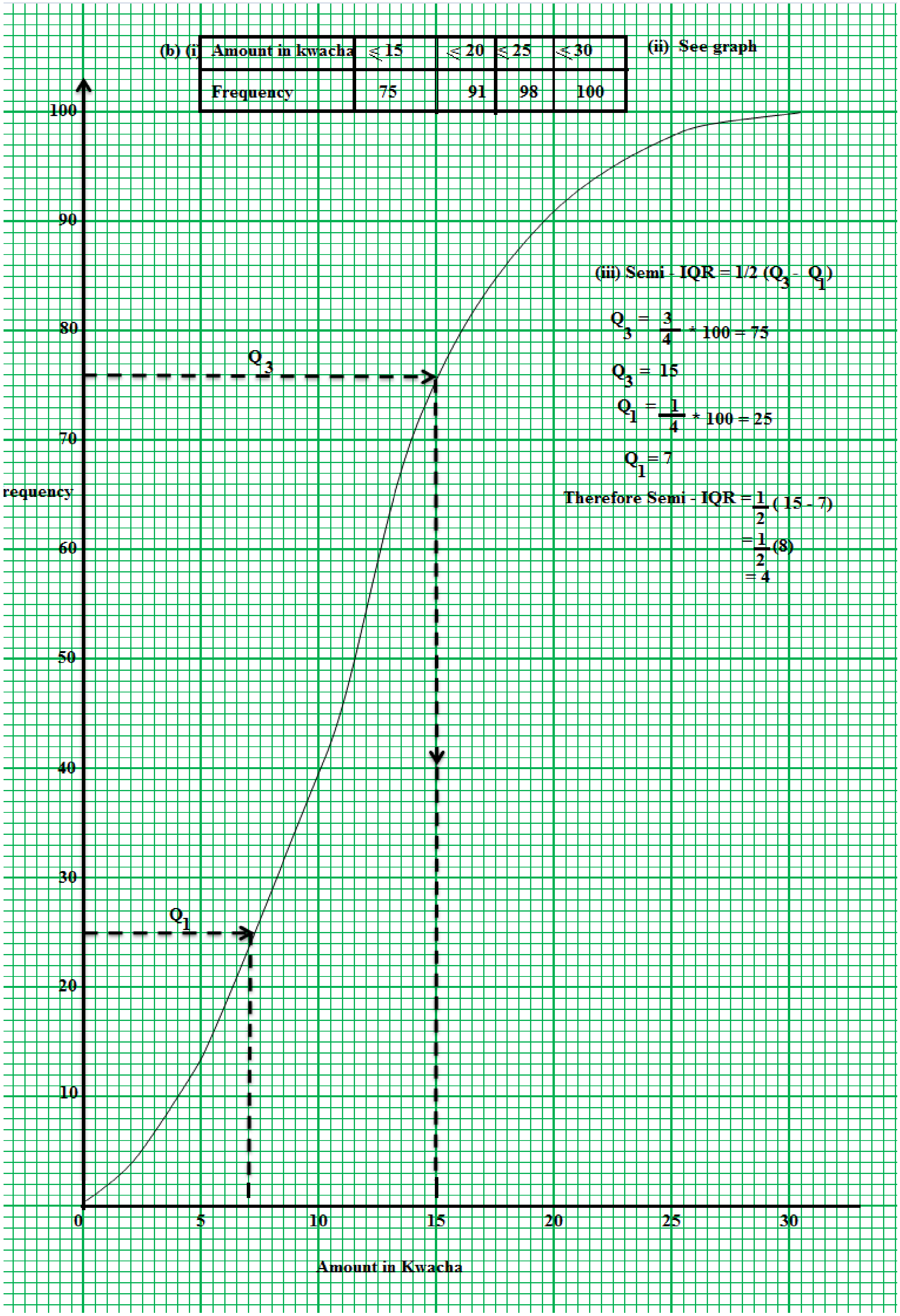
$\text{SD} = \sqrt{\mathbf{25.23621}}$

$\text{SD} = 5\ .023565467$

$\text{SD} = \mathbf{5}$ Ans

Great works are done not by hardworking, but by persistence and perseverance

(b)


(b) (i)
Amount in kwacha | ⩽ 15 | ⩽ 20 | ⩽ 25 | ⩽ 30
Frequency | 75 | 91 | 98 | 100
(ii) See graph
(iii) Semi - IQR = 1/2 (Q_3 - Q_1)
$Q_3 = \frac{3}{4} * 100 = 75$
$Q_3 = 15$
$Q_1 = \frac{1}{4} * 100 = 25$
$Q_1 = 7$
Therefore Semi - IQR = $\frac{1}{2}$ (15 - 7)
$= \frac{1}{2}$ (8)
= 4
Q_3
Q_1
requency
Amount in Kwacha
100
90
80
70
60
50
40
30
20
10
0
5
10
15
20
25
30

4. **(a)**

Class marks	Midpoints (x)	Frequency (f)	fx	x^2	fx^2
$25 < x \leq 30$	27.5	5	137.5	756.25	3781.25
$30 < x \leq 35$	32.5	4	130	1056.25	4225
$35 < x \leq 40$	37.5	7	262.5	1406.25	9843.75
$40 < x \leq 45$	42.5	11	467.5	1806.25	19868.75
$45 < x \leq 50$	47.5	12	570	2256.25	27075
$55 < x \leq 55$	52.5	8	420	2256.25	22050
$55 < x \leq 60$	57.5	1	57.5	3306.25	3306.25
Totals		$\sum f = 48$	$\sum fx = 2045$		$\sum fx^2 = 90150$

Mean $(\bar{x}) = \frac{\sum fx}{\sum f}$

$= \frac{2045}{48}$

= **42 .6**

$SD = \sqrt{\left\{\frac{\sum fx^2}{\sum f} - (\bar{x})^2\right\}}$

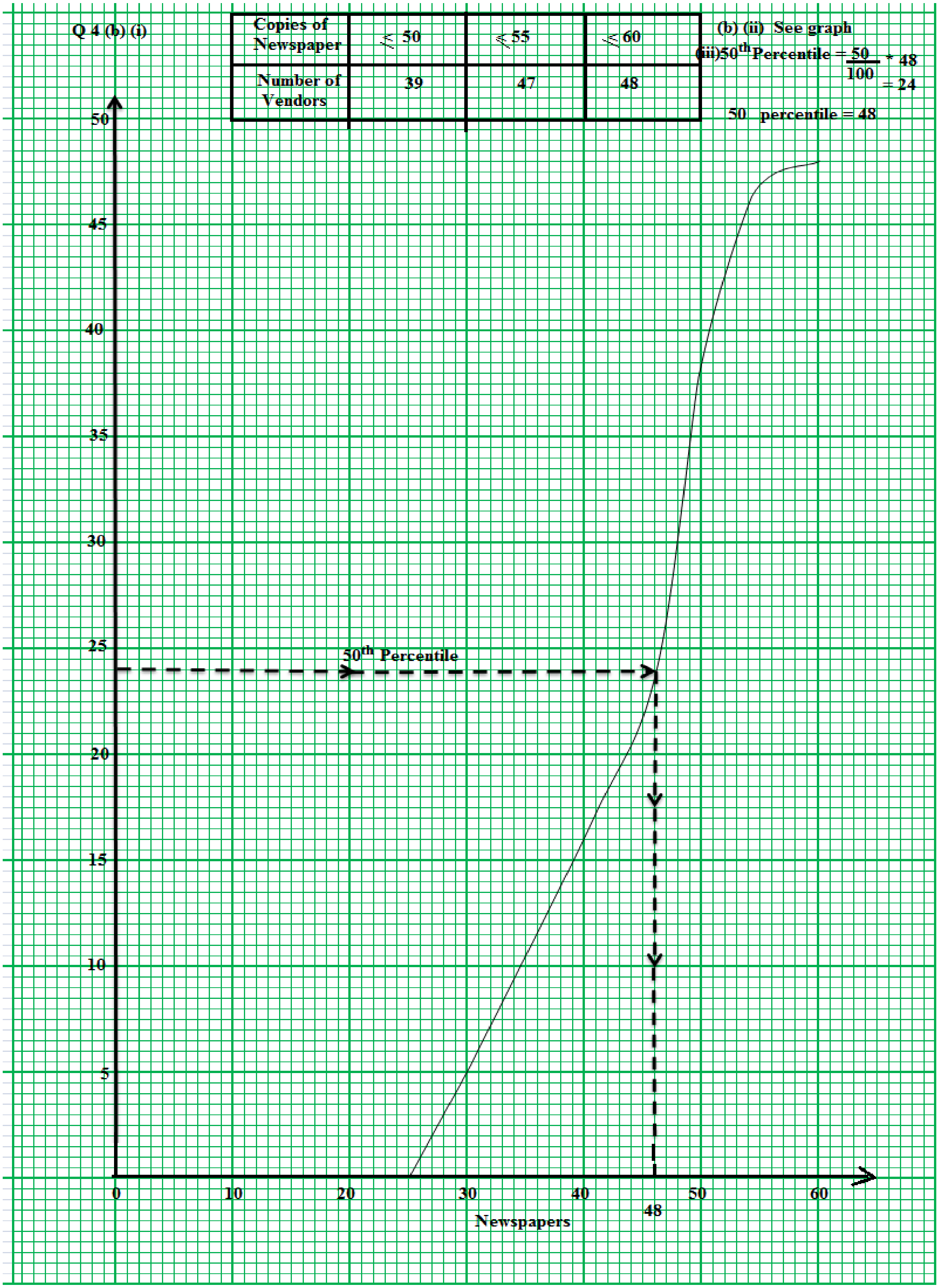
$= \sqrt{\frac{90150}{48} - (42.6)^2}$

$= \sqrt{1878.125 - 1814.76}$

$= \sqrt{63.365}$

$= \mathbf{7.96}$ **Ans**

(b)



Copies of Newspaper	≤ 50	≤ 55	≤ 60
Number of Vendors	39	47	48

5. **(a)**

Class marks	x	f	fx	x^2	fx^2
$0 < x \leq 10$	5	7	35	25	175
$10 < x \leq 20$	15	22	330	225	4950
$20 < x \leq 30$	25	28	700	625	17500
$30 < x \leq 40$	35	23	805	1225	28175
$40 < x \leq 50$	45	15	675	2025	30375
$50 < x \leq 60$	55	5	275	3025	15125
		$\sum f = 100$	$\sum fx = 2820$		$\sum f x^2 = 96300$

Mean, $\bar{x} = \frac{\sum fx}{\sum f}$

$= \frac{2820}{100}$

$= 28.2$

$\therefore SD = \sqrt{\frac{\sum f x^2}{\sum f} - (\bar{x})^2}$

$\text{SD} = \sqrt{\frac{96300}{100} - (28.2)^2}$

$\text{SD} = \sqrt{963 - 795.24}$

$\text{SD} = \sqrt{167.76}$

$\text{SD} = 12.95221989$

SD $= 13.95$ Ans

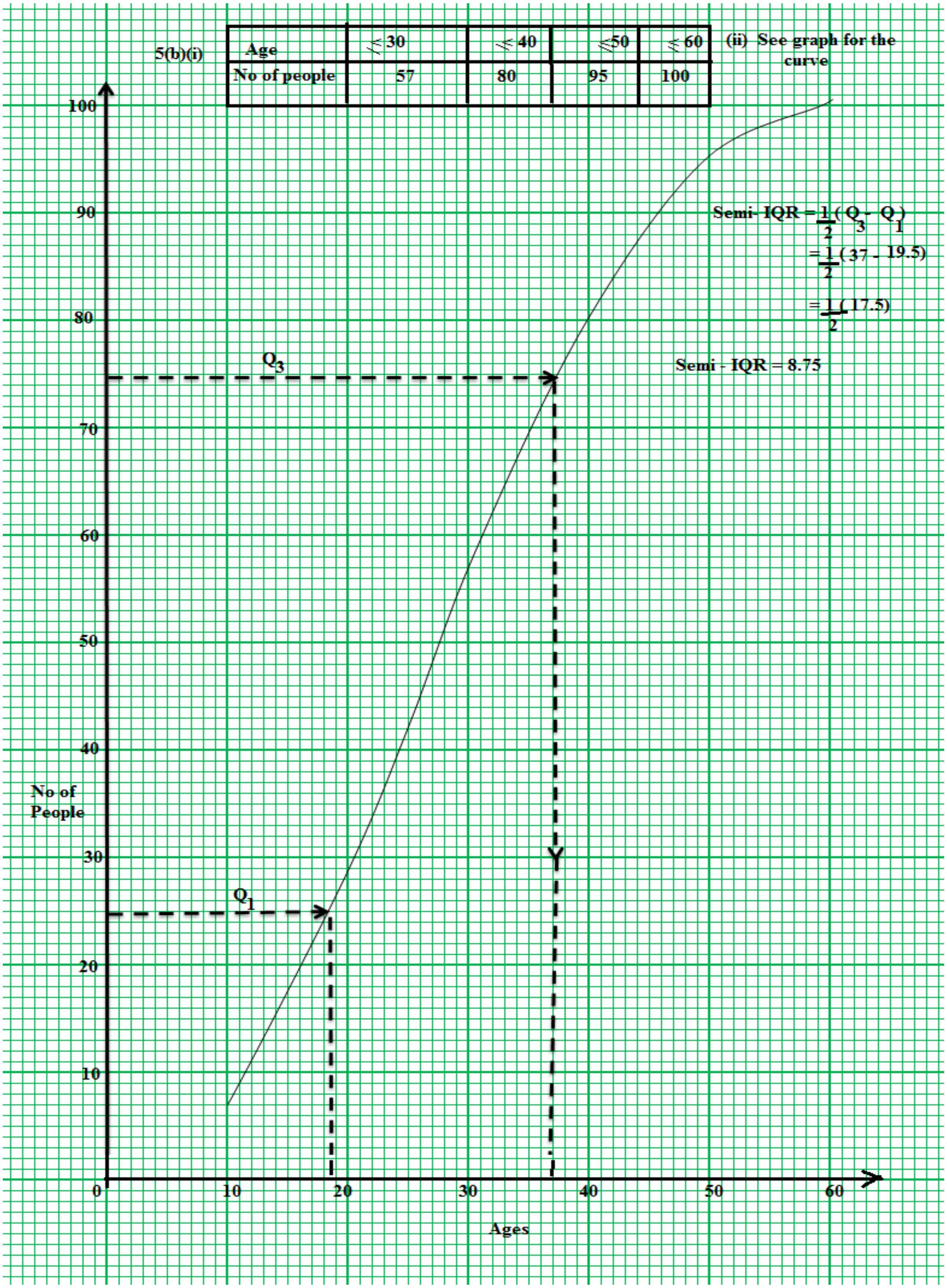
Get your own copy direct from Mr. Kachama D (Mufulira) and Mr. Chansa. J (Kitwe)

(b)

5(b)(i)

Age	≤ 30	≤ 40	≤ 50	≤ 60
No of people	57	80	95	100

(ii) See graph for the curve



Semi- IQR $= \frac{1}{2}(Q_3 - Q_1)$

$= \frac{1}{2}(37 - 19.5)$

$= \frac{1}{2}(17.5)$

Semi - IQR = 8.75

TOPIC 16: TRANSFORMATION

1. **(a)** It is a clockwise rotation of 90° about the origin.

   (b) It is an enlargement, centre (0, 0) and scale factor 2

   (c) Let the matrix be $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$, then pick any two coordinates of P which corresponds to V and form four equations as follows:

   $\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 2 & 4 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} -4 & -8 \\ 4 & 1 \end{pmatrix}$

   $a + 4b = -4$ ……………………(i) $2c + d = 1$………………(iii)

   $a + b = -8$ ……………………(ii) $4c + d = 1$………………….(iv)

   Solving these equations (i) and (ii) simultaneously, we have $a = -2$ and $b = 0$

   Similarly solving equations (iii) and (iv) we have $c = 0$ and $d = 1$

   ∴ the required matrix is

   $\begin{pmatrix} \boldsymbol{a} & \boldsymbol{b} \\ \boldsymbol{c} & \boldsymbol{d} \end{pmatrix} = \begin{pmatrix} \mathbf{-2} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}$

   (d) To find the coordinates of S, we need multiply the given matrix by the coordinates of P.

   $$\begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix}\begin{pmatrix} 2 & 2 & 4 \\ 1 & 4 & 1 \end{pmatrix}$$

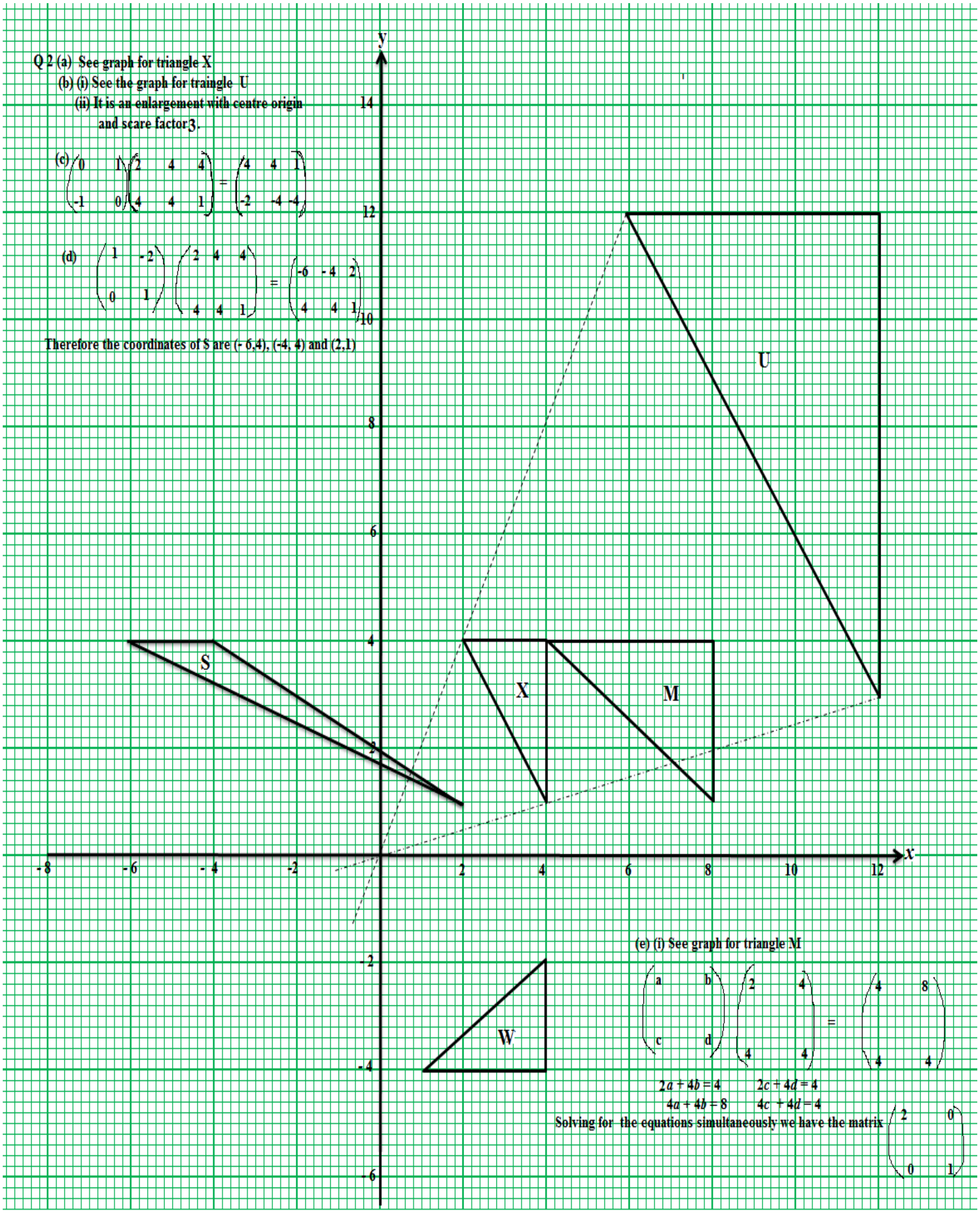
   $$\begin{pmatrix} 2+0 & 2+0 & 4+0 \\ -4+1 & -4+4 & -8+1 \end{pmatrix}$$

   $$\begin{pmatrix} \mathbf{2} & \mathbf{2} & \mathbf{4} \\ \mathbf{-3} & \mathbf{0} & \mathbf{-7} \end{pmatrix}$$

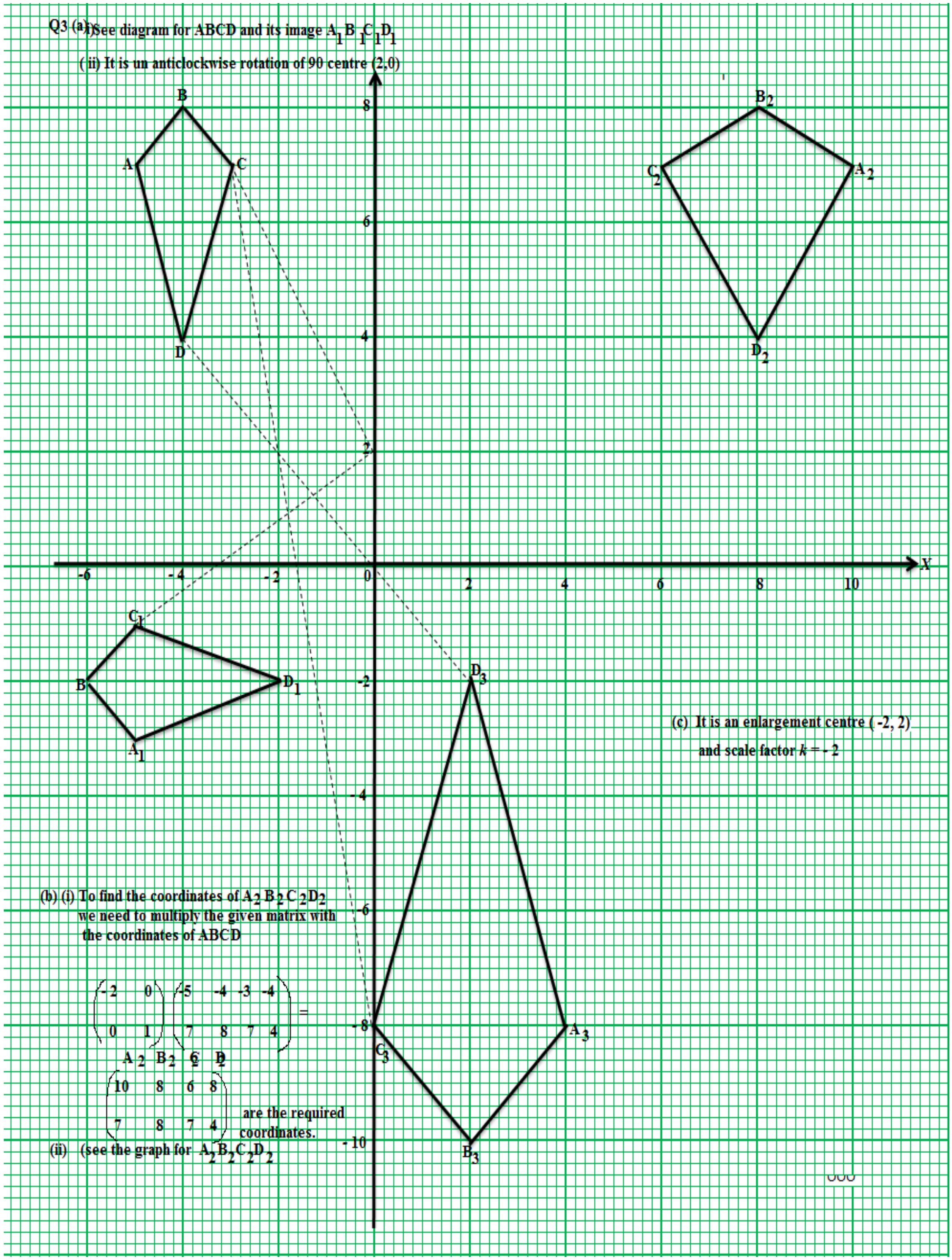
   There the coordinates of S are $(\mathbf{2}, \mathbf{-3})$, $(\mathbf{2}, \mathbf{0})$and $(\mathbf{4}, \mathbf{-7})$

Your life is in your hands, to make of it what you choose

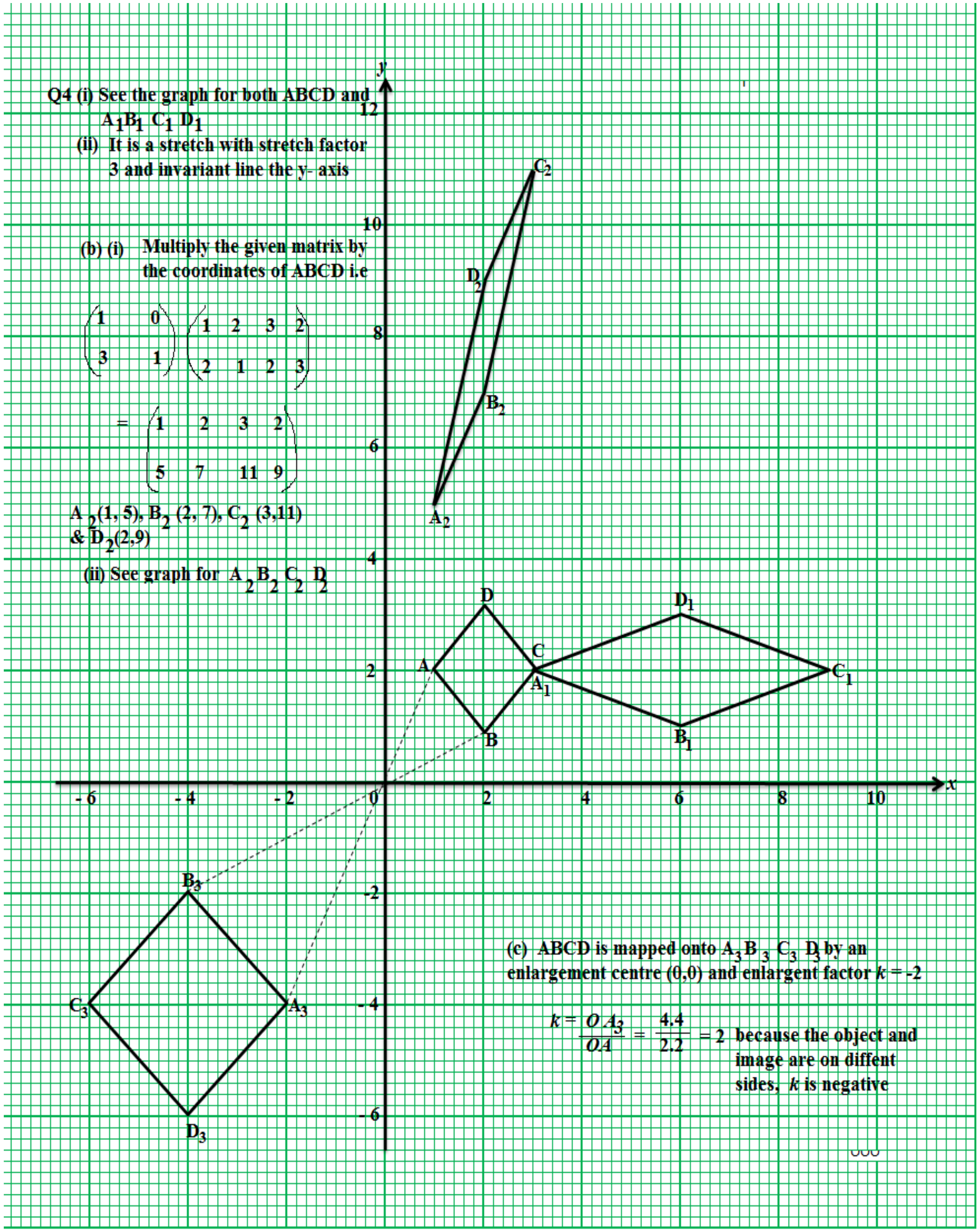
2.


y
Q 2 (a) See graph for triangle X
(b) (i) See the graph for traingle U
(ii) It is an enlargement with centre origin
and scare factor 3.
(c) $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 2 & 4 & 4 \\ 4 & 4 & 1 \end{pmatrix} = \begin{pmatrix} 4 & 4 & 1 \\ -2 & -4 & -4 \end{pmatrix}$
(d) $\begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 2 & 4 & 4 \\ 4 & 4 & 1 \end{pmatrix} = \begin{pmatrix} -6 & -4 & 2 \\ 4 & 4 & 1 \end{pmatrix}$
Therefore the coordinates of S are (- 6,4), (-4, 4) and (2,1)
14
12
10
8
6
4
2
-2
-4
-6
-8
-6
-4
-2
2
4
6
8
10
12
x
U
S
X
M
W
(e) (i) See graph for triangle M
$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 2 & 4 \\ 4 & 4 \end{pmatrix} = \begin{pmatrix} 4 & 8 \\ 4 & 4 \end{pmatrix}$
$2a + 4b = 4$ $2c + 4d = 4$
$4a + 4b = 8$ $4c + 4d = 4$
Solving for the equations simultaneously we have the matrix $\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$

3.


Q3 (a) i) See diagram for ABCD and its image A1 B1 C1 D1
(ii) It is un anticlockwise rotation of 90 centre (2,0)
B
A
C
D
B2
C2
A2
D2
8
6
4
2
0
-6
-4
-2
2
4
6
8
10
X
C1
B1
D1
A1
D3
-2
(c) It is an enlargement centre (-2, 2)
and scale factor k = - 2
-4
(b) (i) To find the coordinates of A2 B2 C2 D2
we need to multiply the given matrix with
the coordinates of ABCD
-6
-2 0 -5 -4 -3 -4
0 1 7 8 7 4
=
-8
C3
A3
A2 B2 C2 D2
10 8 6 8
7 8 7 4
are the required
coordinates.
(ii) (see the graph for A2B2C2D2
-10
B3

4.


Q4 (i) See the graph for both ABCD and
A1B1 C1 D1
(ii) It is a stretch with stretch factor
3 and invariant line the y- axis
(b) (i) Multiply the given matrix by
the coordinates of ABCD i.e
(1 0 / 3 1)(1 2 3 2 / 2 1 2 3)
= (1 2 3 2 / 5 7 11 9)
A2(1, 5), B2 (2, 7), C2 (3,11)
& D2(2,9)
(ii) See graph for A2 B2 C2 D2
(c) ABCD is mapped onto A3 B3 C3 D3 by an
enlargement centre (0,0) and enlargent factor k = -2
k = O A3 / OA = 4.4 / 2.2 = 2 because the object and
image are on diffent
sides, k is negative
y
x
12
10
8
6
4
2
-2
-4
-6
-6
-4
-2
0
2
4
6
8
10
A
B
C
D
A1
B1
C1
D1
A2
B2
C2
D2
A3
B3
C3
D3

5. **(a) (i)** To find the centre of enlargement, join any corresponding two points of the object and the image, the intersection point is the centre of enlargement.

∴ the centre of enlargement is **(1, 2) Ans**

(ii) Scale factor = $\frac{3.2}{1.6} = \mathbf{2}$ **Ans**

(b) Let the matrix be $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$, then

$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 1 & 3 \\ 4 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 3 \\ 1 & 5 \end{pmatrix}$

$a + 4b = 1$ ....(i) $c + 4d = 1$ ....(iii)

$3a + 4b = 3$ ....(ii) $3c + 4d = 5$......................(iv)

Solving the equations (i) and (ii) simultaneously yields $a = 1 \ and \ b = 0$.

Similarly solving equations (iii) and (iv) simultaneously yields $c = 2$ and $d = -\frac{1}{4}$

∴ the required matrix is $\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{2} & -\frac{\mathbf{1}}{\mathbf{4}} \end{pmatrix}$ **Ans**

(c) Triangle **ABC** is mapped onto triangle $\mathbf{A_3B_3C_3}$ by an anticlockwise rotation of 90° about (0, 0) or by a clockwise rotation of 270° about (0, 0).

(d) (i) Comparing the matrices $\begin{pmatrix} k & 0 \\ 0 & 1 \end{pmatrix}$ and $\begin{pmatrix} -3 & 0 \\ 0 & 1 \end{pmatrix}$, we have $k = -3$

∴ the scale factor of this transformation is $\boldsymbol{k} = \mathbf{-3}$ **Ans**

(ii) To find the coordinates of $\mathrm{A_4, B_4}$ and $\mathrm{C_4}$ (image) we need to multiply the given matrix with the coordinates of triangle ABC (object)

$\begin{pmatrix} -3 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 3 & 1 \\ 4 & 4 & 5 \end{pmatrix}$

$\begin{pmatrix} -3+0 & -9+0 & -3+0 \\ 0+4 & 0+4 & 0+5 \end{pmatrix}$

$\begin{pmatrix} -3 & -9 & -3 \\ 4 & 4 & 5 \end{pmatrix}$

∴ the coordinates of $\mathbf{A_4, B_4}$ and $\mathbf{C_4}$ are $(\mathbf{-3, 4})$, $(\mathbf{-9, 4})$ and $(\mathbf{-3, 5})$ respectively

THE END OF SOLUTIONS TO ALL QUESTIONS: ENJOYOUR REVISION & GOD BLESS YOU!!!!!!