GRADE TEN(10) MATHEMATICS MODULE II

UNIT ONE

APPROXIMATIONS

Introduction

Welcome to unit one of this module. In this unit you shall look at interesting ways of approximating measures. It is not easy to have an exact measure or an accurate measure. This unit has 4 topics:

- Rounding off to
    a) nearest unit
    b) nearest fraction of a unit
    c) specific number of decimal places
- Standard form
- Significant figures
- Estimations

It is estimated that you will need eight to nine hours to do all the activities in this unit. However, if you spend more or less time on studying is okay because what is important is to understand the topic.

Objectives

By the end of this unit you should be able to:

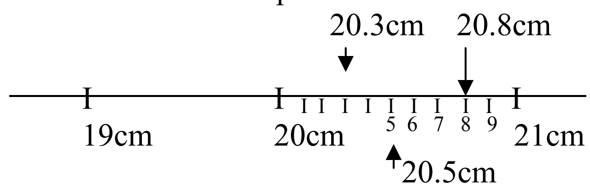
a) round off to the nearest unit and fraction of a unit;
b) round off to specific number of significant figures;
c) express numbers in correct standard form; and
d) estimate numbers or measurements correctly.

Topic 1 Rounding off

(a) Nearest Unit

Rounding off to the nearest unit simply means rounding off to the either the nearest centimetres (cm), kilometre (Km) or Kilogramme.

Let us look at an example of 20.3cm to the nearest centimetre.



You notice that 20.3cm is nearer to 20cm than to 21cm on the ruler so we can conclude that 20.3cm to the nearest cm will be 20cm. However 20.8cm is nearer to 21cm so to the nearest centimetre it will be 21cm.

Now let us look at 20.5 cm which is half way between 20cm and 21cm. for such a situation we shall use a standard rule to guide us in rounding off such figures. This rule is that if you have a number 5 or more after the rounding off digit, then increase the rounding off digit by 1.

For example 20.5 cm
5 or more, add one to 0 to get 21cm.
20.7 becomes 21cm

However, if a number after the rounding off digit is less than 5 like 20.4cm or 20.3, you just ignore it and 20.4cm to the nearest centimetre will be 20cm.

18.1 Kg to the nearest kilogram will be 18kg.

Example

Round off 27.857Kg to the nearest Kg.

Answer: 27.857Kg to the nearest kilogramme is 28kg.

(b) Nearest fraction of a Unit.

You can also round off measurements to the nearest fraction of a unit. If you measure the length of a book and you get 30.37cm you can round off to the nearest tenth of a centimetre which means 1 decimal place.

30.37cm becomes 30.4cm
2.07Kg becomes 2.1Kg

Example

Round off 29.395m to the (≈ means approximately)

(a) nearest tenth of a metre Ans. 29.395m ≈ 29.4m
(b) nearest hundredth of a metre Ans. 29.395m ≈ 29.40m

(c) Number of decimal places

You can also express numbers to a specific number of decimal places.

Examples

Round off 3.03864 to

(a) 3 decimal places Ans. is 3.039
(b) 2 decimal places Ans. is 3.04
(c) 1 decimal place Ans. is 3.0

Notice that when rounding off you simply look at the immediate number after number of decimal places you want. For example, in 20.0346

(a) to 3 decimal places is 20.035
(b) to 2 decimal places is 20.03

if b we round off the original question 20.0346 where the next number after two decimal places is less than 5 so we ignore.

Activity 1

(a) Round off 17.86cm to the nearest

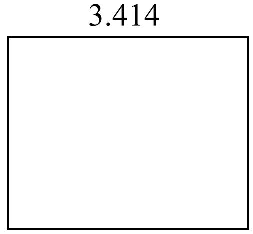
(i) centimetre (ii) tenth of a centimetre

(b) Round off 2997.5 to the nearest

(i) 100 (ii) 1000 (iii) tenth

(c) Find the perimeter of a square to the nearest hundredth of a kilometre

3.414



Activity 2

(a) Write 0.34562 to (i) 4 decimal places (ii) 2 decimal places

(b) Round off to 1 decimal place
(i) 0.807 (ii) 0.1245 (iii) 28.047

(c) Evaluate and give the answer to 3 decimal places

(i) $\frac{1}{2} \times \frac{1}{9}$ (ii) 0.0075 x 75 (iii) $\frac{3}{200} \times \frac{1}{2}$ (iv) 14.37 x 0.0035

Topic 2 Standard form

This is also known as scientific notation. It is simply a method of expressing a number in the form $a \times 10^n$ where $1 \leq a < 10$ and n is an integer. I hope you remember the integers from the earlier module before this module.

Numbers greater than 1

Examples

(1) Express $20700m^2$ in standard form

Ans. $207000 = 207000 \times 100000$
$= 2.07000 \times 10^5$
2.070000×10^5 is correct standard form or scientific form.

In 207000, the decimal point is at the end of the last number, so you move it to the left until it becomes less than 10 i.e. 2.07000. The decimal point moves 5 times so in $a \times 10^n$, n = 5 and a = 2.07000
So we have 2.07000×10^5
2.07×10^5

(2) Express 740 in standard form
$740 = 7.40 \times 10^2$
$= 7.4 \times 10^2$

(3) 730.35 in standard form

$730.35 = 7.3035 \times 10^2$

(4) 8 in standard form 8×10^0 because the decimal point has not moved.

(5) Evaluate $7.5 \div 0.00005$ and express the answer in standard form.

Answer

$$7.5 \div 0.00005 = \frac{7.5}{0.00005} = \frac{750000}{5} = 250000$$

$$250000 = 2.5 \times 10^5$$

Activity 3

(1) Express the following in standard form
(a) 100
(b) 3070
(c) 42070
(d) 333
(e) 1

(2) Calculate and give answers in standard form
(i) 20×320
(ii) $96 \div 0.012$
(iii) $900 \div 0.0018$

(3) The equator is of length 6371Km, write this length in standard form

Numbers less than 1

We can also express numbers less than one in standard form

Let us take 0.01

$$0.01 = \frac{1}{100} = \frac{1}{10 \times 10} = \frac{1}{10^2} = 1 \times 10^{-2}$$

This is so because from our earlier ones less on indices $\frac{1}{10^2} = 10^{-2}$

So 1×10^{-2} is correct standard form.

Examples

(1) Express 0.342 in standard form

$$0.342 = \frac{3.42}{10} = \frac{3.42}{10}$$
$$= 3.42 \times \frac{1}{10^1}$$
$$= 3.42 \times 10^{-1}$$

or you can just count how many times the decimal point moves to make the number lie between 1 and 10. so 0.342 it has moved once to the right so the power will be negative i.e. 10^{-1}

So we have 3.42×10^{-1}

Express

(2) n0.00034 in standard form 0.00034
$= 3.4 \times 10^{-4}$

Activity 4

(1) Write the following in standard form

(a) 0.0022
(b) 0.010003
(c) 0.409
(d) 0.000001

(2) Evaluate giving your answer in standard form

(a) 0.34×0.3
(b) 0.013^2
(c) $\dfrac{8.4 \times 10^{-3} \times 0.00064}{0.00096}$

(3) A square has sides 0.009m. Find its area and give your answer in standard form.

Topic 3 Significant figures

In any numbers all digits are significant apart from zero where significance depends on its position. Let us look at the following examples

(a) 23000 has 2 significant figures, because zeros at the end of a whole number are not significant.
(b) 200300 has 4 significant figures the zeros in between non zero digits are significant
(c) 0.00030 has 2 significant figures. The first 4 zeros are not significant but the zero at the end is significant because zeros at the end of a decimal number are significant. Zeros in front of a decimal number are not significant. The zero at the end is significant because it shows that the number is to specific decimal places and in this case 5 decimal places.
(d) 0.0207 has 3 significant figures, the zeros in front are not significant but they show the position of the decimal point.

Rounding off to a number of significant figures.

Examples

(1) Express 4445 to 1, 2 and 3 significant figures.

4445 = 4000 to 1 significant figure
4445 = 4400 to 2 significant figure
4445 = 4450 to 3 significant figure

(2) State the number of significant figures in (a) 279000 (b) 0.0004203
(c) 25.0304 (d) 19000000

(a) Ans. 3 sig. fig. (b) Ans. 4 sig. figures (c) Ans. 6 sig. fig.
(d) Ans. 2 sig. figures

Activity 5

(1) State the number of significant figures

(a) 334
(b) 0.0043
(c) 0.0501
(d) 0.0000400

(2) Round off 1378040 to

(a) 1 sig. fig.
(b) 2 sig. figs.

(c) 3 sig. figs
(d) 4 sig. figs
(e) 5 sig. figs

(3) Express the following in standard form correct to the number of significant figures indicated:

(a) 61283 (3)
(b) 0.00101 (1)
(c) 12.003 (3)

Topic 4 Estimations

We have already learnt that a measurement can never be exact though there are true measurements. The difference between the true measurement and the one obtained is what we call the error. Using the error we shall determine the upper and lower limits of a measure.

Let us look at an example of 20cm. If we draw a line 20cm long, the length of the line after drawing will lie between 19.5 and 20.5cm. 20.5 cm is called the upper limit and 19.5cm is the lower limit.
The difference between lower limit and the upper limit is called the least unit of measurement. Therefore the least unit of measurement for 20cm is 1cm since 20.5cm – 19.5cm = 1cm. There is also the term called the absolute error. This is half of the least unit of measurement i.e. ½ x 1cm = 0.5cm. So the absolute error = $\frac{\text{least unit of measure}}{2}$

So 20cm has 0.5 cm as the absolute error. When you have the absolute error you can get the upper limit and lower limit of the measurement.

Examples

Find the (i) absolute error
(ii) least and upper limits of 39m

39m has least unit of measurement 1m (all whole numbers has least unit of 1)

(i) Absolute error = ½ = 0.5m
(ii) Upper limit (UL) = (39 + 0.5)m = 39.5m
(iii) Lower limit (LL) = (39 – 0.5)m = 38.5m

For decimal numbers the least unit of measurement (LUM) depends on the number of places. For example 20.3 has least unit of measurement 0.1.

20.33 has LUM of 0.01
1.339 has LUM of 0.001
40.4567 has LUM of 0.0001

Example

Find the absolute error upper and lower limit in 13.09cm

Ans. LUM = 0.01

Absolute error (AE) = $\frac{0.01}{2}$ = 0.005

Upper limit = 13.09 + 0.005 = 13.095cm
Lower limit = 13.09 – 0.005 = 13.085cm

Activity 6

Find the (i) absolute error
(ii) upper and lower limits in
(a) 25cm (b) 4.07Km (c) 0.0003m

Relative Error and percentage Error

The relative error is the ratio of the absolute error to the true measurement.

$$\text{Relative error} = \frac{\text{Absolute error}}{\text{True measurement}}$$

$$\text{Percentage error} = \frac{\text{Absolute error}}{\text{True measurement}} \times 100\%$$

Example

Find the relative error and percentage error of 7.7.

First find absolute error.

Ans. 7.7 has 0.1 as LUM

Then absolute error = $\frac{0.1}{2}$

= 0.05

Relative error = $\frac{0.05}{7.7}$

$$\text{Percentage error} = \frac{0.05}{7.7} \times 100$$

Activity 7

Find the relative error and percentage error correct to 2 significant figures

(a) 3cm (b) 602kg (c) 28.06Km (d) 0.002m

Tolerance

What is tolerance. This is simply the difference between the greatest and least acceptable measurements.

Examples

(1) Find the tolerance in each of the following:

(a) UL = 7Km LL = 4m

Ans. Tolerance = (7 – 4)m
= 3m

(b) (8.5 ± 1.2)

Tolerance = (8.5 + 1.2) – (8.5 – 1.2)
= 9.7 – 7.3
= 2.4

(2) Find the least and greatest possible measurements

(a) (7 ± 1)g (b) (0.7 ± 0.2)s

(a)Ans. Greatest 7 + 1 = 8g
Least 7 – 1 = 6g

(b)Ans. Greatest 0.7 + 0.2 = 0.9s
least 0.7 – 0.2 = 0.5s

(3) Find the absolute error of the difference between 7.2 and 3.1

Answer.

Get absolute errors of individual numbers, then add them.

For 7.2 LUM = 0.1
$$AE = \frac{0.1}{2} = 0.05$$

For 3.1 LUM = 0.1
$$AE = \frac{0.1}{2} = 0.05$$

Add the two 0.05 + 0.05 = 0.10

So the absolute error of the difference between 7.2 and 3.1 is 0.10

In general when adding or subtracting measurement the absolute is the sum of errors in the original measurements.

You also need to know how to get the maximum and minimum difference.

7.2	and	3.1
UL_1 7.25		UL_2 3.15
LL_1 7.15		UL_2 3.05

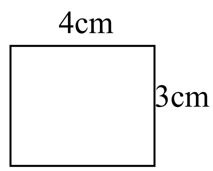
Maximum difference = UL – LL
= 7.25 – 3.05
= 4.20

Minimum difference = LL_1 - UL_2
= 7.15 – 3.15
= 4.00

The product and quotient of measurements

Examples

(1) Find the limits between which the areas of the rectangle with sides 4cm by 3cm should lie.



First get upper and lower limits of individual measurements

For 4cm
LUM = 1cm
AE = ½ = 0.5cm
UL = 4.5cm
LL = 3.5cm

For 3cm
LUM = 1cm
AE = ½ = 0.5cm
UL = 3.5cm
LL = 2.5cm

Maximum area

A = l x b
You get upper limits
A = 4.5cm x 3.5cm
A = 15.75cm^2

Minimum area

A = l x b
You get lower limits
A = 3.5cm x 2.5cm
A = 8.75cm^2

So the limits are 8.75cm^2 and 15.75cm^2

(2) Find the maximum and minimum mean for the following measurements 3L 4L 7L
Answer
Least U.M. = 1 Absolute error for each is 0.5
Upper limits 3.5L, 4.5L, 7.5L
Lower limits 2.5L, 3.5L, 6.5L

Maximum mean = $\frac{\text{sum of upper limits}}{3}$

$$= \frac{3.5 + 4.5 + 7.5}{3} = \frac{15.5}{3}$$

$$= 5.1667L$$

Minimum mean = sum of lower limits

$$= \frac{2.5 + 3.5 + 6.5}{3} = \frac{12.5}{3}$$

$$= 4.1667$$

Activity 8

(a) Find the maximum and minimum sum of (i) 5m and 12m (ii) 18.36l and 21.7l

(b) Find the maximum and minimum difference of (i) 40g and 60g (ii) 7.025cm and 0.025cm.

Activity 9

(1) Find the limits for the areas of the following:
 (a) square of sides 16cm
 (b) a right angled triangle of base 3cm and height 5cm

(2) Find the maximum and minimum average for the following
 20.1L, 137L and 15.9L

Summary

In this unit we looked at 4 different Topics which are rounding off standard form, significant figures and estimations. You should appreciate the fact that approximation should not be very difficult from an original measurements. You should also pay attention to the rules that govern the rounding of numbers.

UNIT TWO

MAPPING & FUNCTIONS

Introduction

Welcome to unit two of this module. In this unit you shall look at mappings and then you will move on to functions which are the most important part of this unit. This unit has two topics.

- Mappings
- Functions

If you do all activities in this unit, we estimate you will need four to five hours. However, if you take more, there is no need to worry because what is important is to understand the topic so well.

Objectives

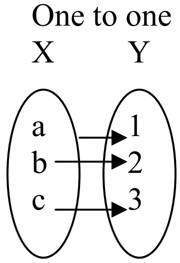
By the end of this unit, you should be able to:

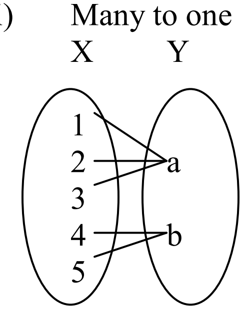
a) differentiate between domain and range;
b) solve questions involving functions; and
c) find the inverse functions.

Topic 1: Mappings

A mapping is a relation which has the following properties

(a)(i) One to one or (ii) Many to one

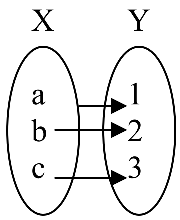




(b) Every element of domain has an image

Domain and Range

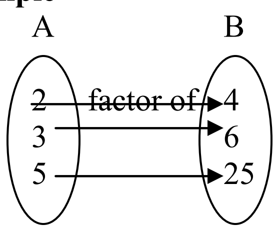
For the diagram a(i)



elements in X are mapped onto elements in Y. The set X is referred to as Domain and the set Y is Image. The elements of domain are called <u>objects</u> while elements of Y are called images.

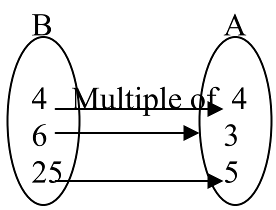
Example

(c)



In (a) A is the Domain and B is the range.

(b)



In (b) B is the domain and A is the range.

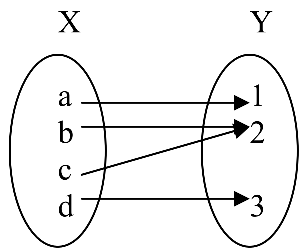
Topic 2 Functions

A function is a mapping with only one image for each and every member of the domain.

We usually use the letter 'f' to denote a function for example $f{:}x \rightarrow y$ is read as "f maps x onto y" or $f(x) = y$ which is read as "f of x is equal to y"

Example 1

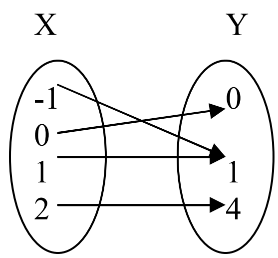
In the diagram below



f:x → y or f(x) = y

This is an example of a function

Example 2



- This is a function with formular $y = x^2$
- In the notation $f{:}x \rightarrow y$ it can be written as $f{:}x \rightarrow x^2$
- In the notation $f(x) = y$ it can be written as $f(x) = x^2$
- In set builder notation, $f = \{(x, y)\ y = x^2, x \in X, y \in Y\}$

We have four different ways of expressing a function.

(1) Arrow diagram
(2) Function notation $f{:}x \rightarrow y$
(3) Formular $f(x) = y$
(4) Set builder notation $f\{(x, y){:}y=fx)\}$

Refer to example 2 for clarity as all these have been used.

Example 3

If $f(x) = 3x + 1$, find the range for domain {1, 2, 3, 4}

The range will be

For $x = 1$ $\quad f(x) = 3x + 1$

$F(1) = 3(1) + 1$

$= 4$

$x = 2$ $\quad f(2) = 3(2) + 1$

$= 6 + 1$

$= 7$

$x = 3$ $\quad f(3) = 3(3) + 1$

$$= 9 + 1$$
$$= 10$$

$x = 4$ $\qquad f(4) = 3(4) + 1$
$$= 12 + 1$$
$$= 13$$

So the range is (4, 7, 10, 13)

Example 4

If f(x) = 3x + 1

Find the domain if the range is {4, 7, 10, 13}

$F(x) = 4$	what is x		
$3x + 1 = 4$	$f(x) = 7$	$f(x) = 10$	$f(x) = 13$
$3x = 4 - 1$	$3x + 1 = 7$	$3x + 1 = 10$	$3x + 1 = 13$
$\frac{3x}{3} = \frac{3}{3}$	$3x = 7 - 1$	$3x = 10 - 1$	$3x = 13 - 1$
$x = 1$	$\frac{3x}{3} = \frac{6}{3}$	$\frac{3x}{3} = \frac{9}{3}$	$\frac{3x}{3} = \frac{12}{3}$
	$x = 2$	$x = 3$	$x = 4$

Therefore the domain is {1, 2, 3, 4}.

From example 3 and 4, we see that we can get the range given the domain or we can get the domain given the range.

Activity 1

(1) For $f(x) = 4x - 3$, find the value of (a) f(1) (b) f(0)
(2) For $f(x) = 4x - 3$, find the x for $f(x) = 5$
(3) If $f(x) = bx + 2$ find b given that $f(3) = -4$
(4) If $x \rightarrow 3x - 1$, had domain {-1, 0, 1, 2, 3} find the range
(5) If $x \rightarrow 4x + 2$, has range {10, 14, 18, 22} find the domain.

Inverse functions

This is denoted by $f^{-1}(x)$ for a function f(x), $f^{-1}(x)$ is the inverse.

Examples

(1) Find the inverse function for $f(x) = 2x + 4$

Solution
First you let $\qquad f(x) = y$
Then you have $\qquad y = 2x + 4$, then make x the subject of the formula
$$y = 2x + 4$$

$$\frac{y-4}{2} = \frac{2x}{2}$$

$$x = \frac{y-4}{2}$$

Then $$f^{-1}(x) = \frac{x-4}{2}$$ This is the inverse function of $f(x) = 2x + 4$.

(2) For $f(x) = \frac{2x-2}{x}$ where $x \neq 0$ because if $x = 0$ the function will be undefined.

Find $f^{-1}(2)$

Solution

First you have to find $f^{-1}(x)$

So we let $f(x) = y$

$$\frac{y}{1} = \frac{2x-2}{x}$$

$yx = 2x - 2$

$yx - 2x = -2$

$$\frac{x(y-2)}{(y-2)} = \frac{-2}{y-2}$$

$$x = \frac{-2}{y-2}$$

So $f^{-1}(x) = \frac{-2}{x-2}$ where $x \neq 2$ because $2 - 2 = 0$ and if the denominator is zero the function is undefined.

Activity 2

(1) Find the inverse functions for the following:

- (a) $f(x) = 4x + 2$
- (b) $f(x) = 7x$
- (c) $f(x) = ½x + 1$
- (d) $f(x) = -\frac{3x}{2} - 2$

(2) For each question in 1 find (a) $f^{-1}(1)$ (b) $f^{-1}(0)$

(3) Find x if $f^{-1}(x) = 2$ for the function $f(x) = 4x + 2$

Summary

In this unit you looked at mappings and function. You identified the domain of a function and the range of the function. You also looked at the inverse function and how to get or derive the inverse function.

UNIT THREE

GRAPHS OF POLYNOMIALS

Introduction

Welcome to unit three of this module. In this unit you shall look at three types of graphs and the most interesting part is how to draw the graphs. The three types of graphs are all called polynomials. A polynomial is an algebraic expression in the form
$a_nx^n + a_{n-1} + x^{n-1} + \ldots. +a_2x^2 + ax_1\ ^{1+a}0x^0$ where an, an-1, a2 a are positive integers

This unit has 3 Topics

- Linear functions/equations
- Quadratic functions
- Cubic functions

To understand well all the concepts in this unit, you will need eleven to twelve hours. However, you may take more or less hours but what is important is to understand the topic so well.

Objectives

By the end of this unit, you should be able to:

a) draw a linear function graph;
b) calculate the gradient of a line function and identify the y intercept;
c) draw the quadratic function graph and calculate the gradient of the function of any given point; and
d) draw a cubic function.

Topic 1 Linear function

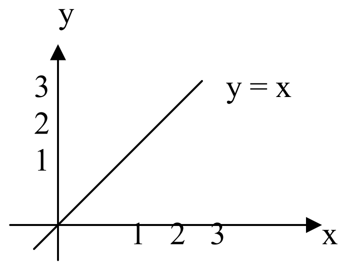
A linear function is of the form $f(x) = ax + b$ where a and b are constants. A linear function is a straight line.

Examples

(1) Draw the graph of
(a) $y = x$
Ans. First you have to make a table of values of x and the corresponding values of y.

x	0	1	2	3
y	0	1	2	3

$y = x$
when $x = 0$ $x = 1$ $x = 2$
$Y = 0$ $y = 1$ $y = 2$

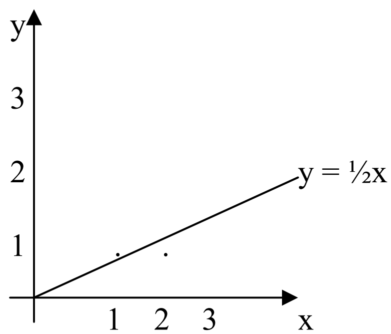


The x to y planes should be drawn to scale e.g. 1cm to 1 unit on each axis.

(b) $y = ½x$

x	0	1	2	3
y	1	½	1	$^3/_2$

$y = ½x$
When $x = 0$ $x = 1$ $x = 3$
$Y = 0$ $y = ½$ $y = {}^3/_2$



Activity 1

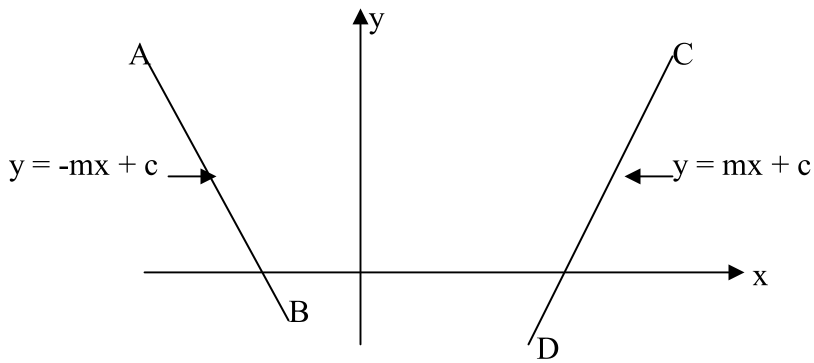
Draw the graphs of

(a) $y = 2x$ (b) $y = 3x + 1$ for values of x from -2 to 2

<u>Gradient or slope</u>

The gradient is the measure of the inclination of a line. It is also known as the slope and its relative to the positive x – axis.

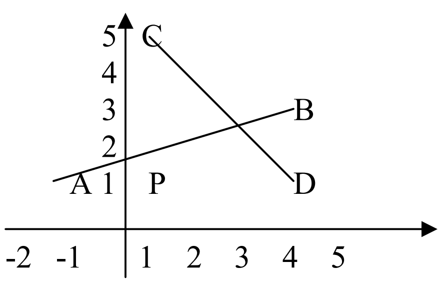
Let us look at the two lines below:



Line AB has negative gradient while line CD has positive gradient. So all lines in the slope of AB have negative gradient and all lines in the slope of CD have positive gradient. In the standard equation $y = mx + c$, m is the gradient.

Examples

Calculate the gradient of the following lines AB and CD



1cm to 1 unit on each axis

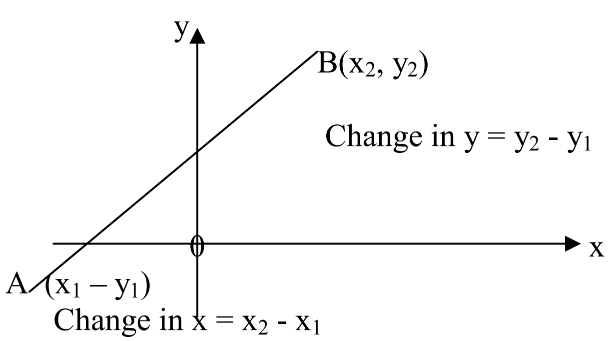
Gradient of AB $= \frac{BD}{AD} = \frac{2}{5}$

Positive gradient because its inclination is upwards to the right.

Gradient of CD $= \frac{CP}{PD} = \frac{4}{3} = \frac{-4}{3}$

But gradient of CD is negative because of Its inclination downward to the right.

If you are given two points $A(x_1\ y_1)$ and $B(x_2\ y_2)$ you can calculate the gradient of AB as the change in y with respect to change in x. Look at the graph below.



Therefore the gradient $m = \frac{y_2 - y_1}{x_2 - x_1}$

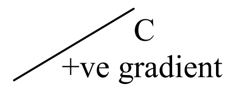
Example

Find the gradient of the line CD with points C(4, 2), D(1, 1).

Then the gradient $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 2}{1 - 4} = \frac{-1}{-3} = \frac{1}{3}$

$\ x_1 \ \ y_1 \ \ x_2 \ \ y_2$
C(4, 2) D(1, 1)

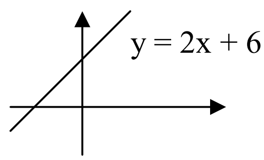
CD has the following slope



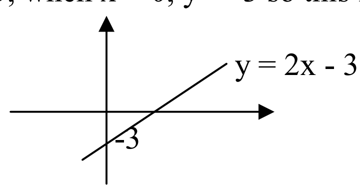
(3) The y intercept of the graph

The y intercept is the point where the graph cuts the y axis. In the standard equation of a straight line y = mx + c. C is the y-intercept.

For example in y = 2x + 6 when x = 0, y = 6 means the graph will cut the y – axis at (0, 6).



In y = 2x – 3, when x = 0, y = -3 so this line cuts the y-axis at (0, -3).



The graph will cut the y-axis when x = 0 always.

Example

Draw the graph of x + 2y -4 = 0

First write the equation to the standard form of y = mx + c.

So it will be $\frac{2y}{2} = \frac{-x}{2} + \frac{4}{2}$

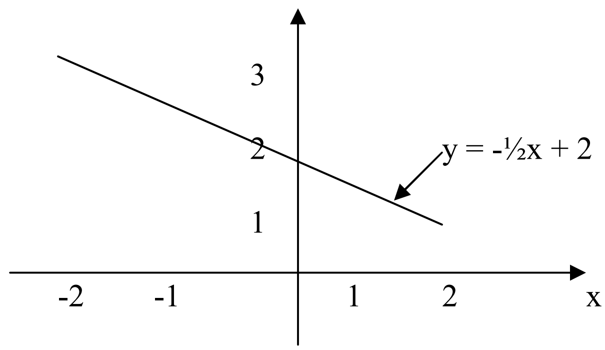
$y = -\frac{1}{2}x + 2$

In $y = -\frac{1}{2}x + 2$

$-\frac{1}{2}$ is the gradient and 2 is the y intercept.

Then draw a table of values

x	-2	-1	0	1	2
y	3	2½	2	1½	1



Activity 2

(1) State the gradient and the y intercept of (a) $y = 3x + 4$ (b) $3x - y = 4$

(2) Find the gradient of the lines that join the following points
(a) (3, 0) (4, 7) (b) (-3, -2) and (4, 2)

(3) Draw the graphs of the following:
(a) $4y = 2x - 4$ for values of x $\{x: -4 \leq x \leq 4\}$

(i) Write down the gradient and the y-intercept.

(b) $y = -3x + 3$ for the domain $\{-2 \leq x \leq 2\}$.

Topic 2 Quadratic function

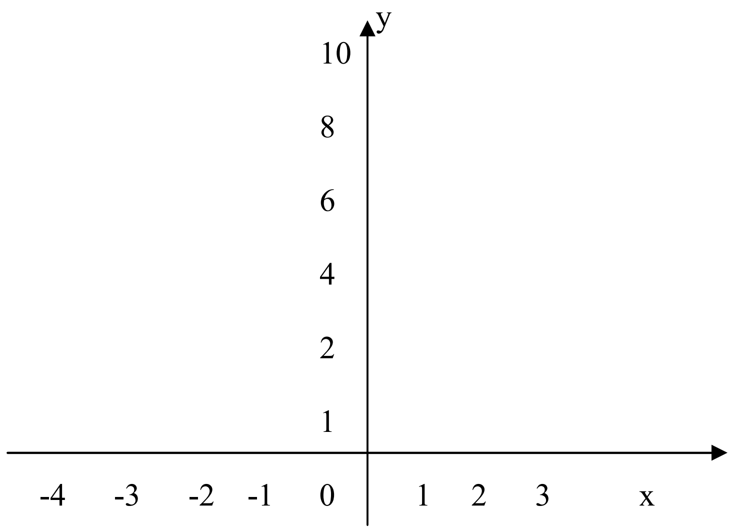
These are functions of the form $ax^2 + bx + c$ where a, b and c are constants.
Graphs of quadratic functions are curves and not straight lines.

Again we have to use a table of values to draw quadratic curves. Let us look at two examples.

Example 1

Draw the graph of $y = x^2$
First you draw a table of values for x from -3 to 3

x	-3	-2	-1	0	1	2	3
x = y	9	4	1	0	1	4	9



- in this graph the y intercept is 0 because the graph cuts the y-axis at (0, 0) so the y-intercept is 0.
- It also has a turning point at (0, 0) called the minimum point. The values $-3 \leq x \leq 3$ are called the domain and $0 \leq y \leq 9$ as the range.

Example 2

Draw the graph of $y = x + 3x - 2$ for $-5 \leq x \leq 4$
Use scale 1cm to represent 1 unit on the x-axis and 1 cm to represent 4 units on the y – axis.

From your graph establish

(i) The coordinates of the turning point
(ii) The minimum value
(iii) The equator of axis of symmetry of the graph
(iv) The range of the function y
(v) The value of x when $x^2 + 3x - 2 = 0$
(vi) The value of x when $x^2 + 3x - 5 = 0$

Solution

First we make a table of values $-5 \leq x \leq 4$

x	-5	-4	-3	-2	-1	0	1	2	3	4
x^2	25	16	9	4	1	0	1	4	9	16
3x	-15	-12	-9	-6	-3	0	3	6	9	12
-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2
y	8	2	-2	-4	-4	-2	2	8	16	26

(vii) $x^2 + 3x – 2 = y$
$x^2 + 3x – 5 + 3 = 0 + 3$
$x^2 + 3x – 2 = 3$
$x^2 + 3x – 2 = y$
$y = 3$

So we draw a new graph y = 3 and where it meets the curve, we read the x coordinates to give us the solutions x = 1.2 and x = -4.1.

(Insert graph)

(ii) The minimum value
This is the y coordinate of the turning point which is -4½

(iii) The equation of the axis of symmetry is the x – coordinate of the turning point
$X = -\frac{3}{2}$ is the equation of the line of symmetry.

(iv) The range of the function y. This is from the least value of y on the graph to the highest value of y on the graph. The least value is -4½ and the highest is 26.

So the range $-4\frac{1}{2} \leq y \leq 26$.

(v) The value of x when $x^2 + 3x - 2 = 0$. In this question you compare with the equation of the graph $y = x^2 + 3x - 2$
you notice that $0 = x^2 + 3x - 2$, the right hand side is equal to the left hand side and so y = 0.
So we draw a new graph y = 0 on the same graph. Where they meet, you read the x – coordinates that will be the solution to $x^2 + 3x - 2 = 0$. From the graph x = 0.6 and x = -3.45

(vi) The value of x when $x^2 + 3x - 5 = 0$, you add something to both side so that it comes to $x^2 + 3x - 2$ so it will be $x^2 + 3x - 5 + 3 = 0 + 3$

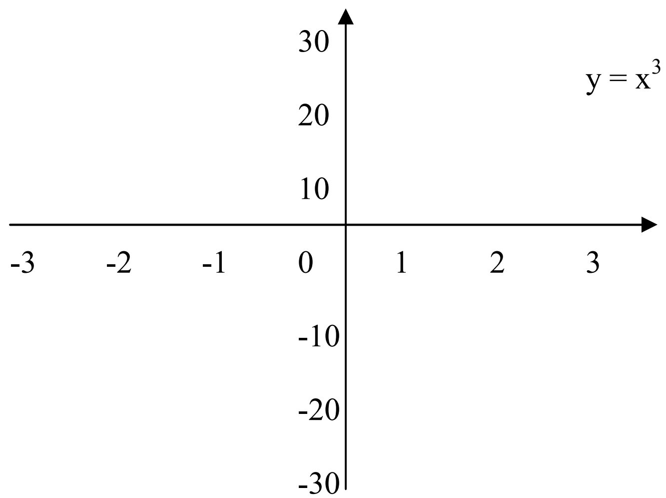
Topic 3 Cubic function

These are functions of the form $ax^3 + bx^2 + cx + d$
The highest power is 3.
Let us look at two examples.

Example 1

Draw the graph of y = x for y < x ≤ 3. First you draw a table of values.

X	-3	-2	-1	0	1	2	3
Y	-27	-3	-1	0	1	8	27

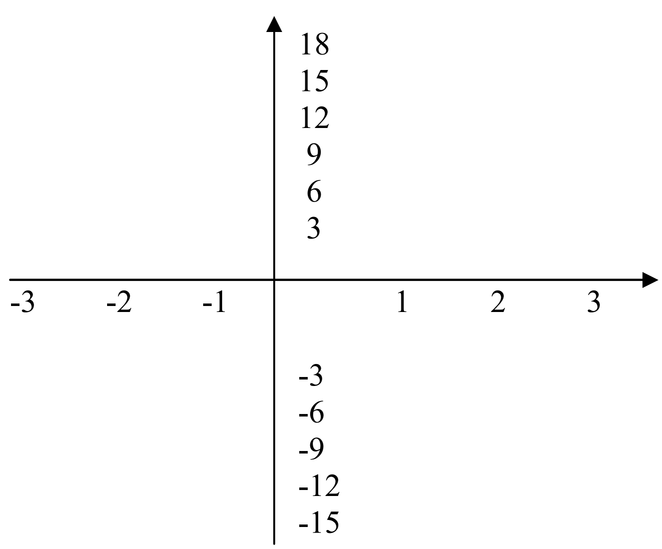


Example 2

Draw the graph of $y = x(x + 2)(x - 2)$ for values of x from -3 to +3. From the graph Deduce

(a) The coordinates of the maximum and minimum points.
(b) The values of x for which $x(x + 2)(x - 2) - 0$

X	-3	-2	-1	0	1	2	3
Y	-15	0	3	0	-3	0	15



$y = x(x + 2)\,(x - 2)$

It has two turning points, one is a maximum and one is a minimum.

Activity 3

(1) Draw the graphs of the following

(a) $y = x^2 + 2$ for $-2 \leq x \leq 2$ take scale 2cm to 1 unit (cm x-axis and 1cm to 1 unit on y – axis)

(b) $y = -2x^2 - 4x + 1$ for $-4 \leq x \leq 3$
Use scale 2cm to 1 unit on x-axis and 1cm to 1 unit of y-axis.

(2) From your graph find the values of the following where possible

(a) Coordinates of the turning point
(b) Values of x for $-2x^2 - 4x + 1 = 0$
(c) Values of x for $-2x^2 - 4x - 4 = 0$

(3) Draw the graph of $x^2 - 6x^2 + 9x$

For values of x from -2 to 6.
Use a scale of 2cm to 1 unit on the x-axis and 2cm to 10 units on the y-axis and find the following:

(a) The values of x $x^2 - 6x^2 + 9x = 0$
(b) The coordinates of the maximum and minimum points
(c) The range of the function of y.

Summary

This has been an interesting unit as it involves manipulation of our motor skills. You looked at linear functions and their graphs as straight lines. You also looked at quadratic functions and their graphs which are curves. Then finally you looked at the cubic functions and there graphs being curves.

UNIT FOUR

QUADRATIC EQUATIONS

Introduction

Welcome to unit four of this module. In this unit you shall discuss quadratic equation. You will look at solving quadratic equations algebraically (factorization, completing the square, and the formula methods). Then finally at solving quadratic equations graphically.

In this unit, there are 4 Topics:

- Solving quadratic equations using factorization method.
- Solving quadratic equations using completing the square method.
- Solving quadratic equations using the formula method.
- Solving quadratic equations using the graphical method.

If you do all the activities in this unit, we estimate you will need to study between 4 to 5 hours.

But do not worry if it takes you more or less time than this – we do not all work at the same pace. It is important that you follow all the activities and fully understand the topic.

Objectives

By the end of this unit you should be able to:

a) solve quadratic equations algebraically i.e. factorizing, completing the square and the formular; and
b) solve quadratic equations graphically.

Topic 1 Solving quadratic equations by factorization

If you remember in the previous module, we looked at factorizing quadratic expressions. This time we shall go a step further and solve quadratic equations.

How does a quadratic expression look like? This is in the form $ax^2 + bx + c$ where a, b and c are constants.

A quadratic equation in one variable is in the form $ax^2 + bx + c = 0$, where a, b and c are constants and $a \neq 0$

Some of the examples of quadratic equations are:

$x^2 + 3x + 2 = 0$
$-4x^2 + 4x - 1 = 0$
$2x^2 = -3x + 7$
$\frac{1}{2} x^2 - 8x + \frac{1}{2} = 0$
$x^2 - 9 = 0$
$2x^2 - x = 0$

In order to solve the quadratic equation $x^2 + 3x + 2 = 0$,we must first factorise the expression that is, $x^2 + 3x + 2 = 0$ will become
$x^2 + 2x + x + 2 = 0$
$x (x + 2) + (x + 2) = 0$
$(x + 2) (x + 1) = 0$
So $x^2 + 3x + 2 = 0 \Rightarrow (x + 2)(x + 1) = 0$

NOTE: In general if a and b are two real numbers such that ab = 0, then either a = 0 or b = 0 so, in the above example $(x +2)(x +1) = 0$ then either $x+ 2 = 0$ or $x + 1 = 0$
$x = -2$ $x = -1$

Therefore the roots are $x = -2$ or $x = -1$. The roots can then be checked in the equation as shown below
When $x = -2$
$x^2 +3x + 2 = 0$
Gives $(-2)^2 +3 (-2) + 2 = 0$
$-2 + 2 = 0$
$0 = 0$
When $x = -1$
Gives $(-1)^2 + 3(-1) + 2 = 0$
$-2 + 2 = 0$
$0 = 0$

Example 1
Solve the equation $x^2 = 7x- 12$.

Solution

$x^2 = 7x - 12$
We have to collect all the terms on to one side of the equation,
$x^2 + 7x + 12 = 0$
Now we can factorise the expression on the left hand side and solve as follows :

$x^2 + 7x + 12 = 0$
$x^2 + 4x + 3x + 12 = 0$
$x(x + 4) + 3(x + 4) = 0$
$(x + 4)(x + 3) = 0$

$x + 4 = 0$ or $x + 3 = 0$
$x = -4$ $x = -3$

Example 2
Solve for x in the equation $-2x^2 + 7x - 3 = 0$

Solution
$-2x^2 + 7x - 3 = 0$
$-2x^2 + 6x + x - 3 = 0$
$-2x(x - 3) + 1(x - 3) = 0$
$(-2x + 1)(x - 3) = 0$

$-2x + 1 = 0$ or $x - 3 = 0$
$-2x = -1$ $x = 3$
$x = ½$

Example 3

Solve $4/5\ x^2 + 1/5 - 1 = 0$

Solution

$4/5\ x^2 + 1/5\ x - 1 = 0$
First we have to remove the fractions by multiplying both sides of the equation by 5

$5(4/5)\ x^2 + 5(4/5)\ x + 5(1) = 0$

$4x^2 + x - 5 = 0$

Now we factorise and solve as follows

$4x^2 + 5x - 4x - 5 = 0$
$x(4x + 5) - 1(4x + 5) = 0$

$(4x + 5) (x – 1) = 0$

$4x + 5 = 0$ or $x – 1 = 0$
$4x = 5$ $x = 1$
$x = - 5/4$

Activity 1

Solve each of the following quadratic equations using the factorization method.

1. $x^2 + 2x – 3 = 0$
2. $x^2 + 3x + 2 = 0$
3. $2x^2 + x – 3 = 0$
4. $-3x^2 – 8 = 10x$
5. $½ x^2 = 5/6 x – 1/3$
6. $3/4x^2 – x + ¼ = 0$
7. $x^2 – 4$
8. $x^2 – 81$

Topic 2: Solving quadratic equations using the completing the square method .

If $x^2 = 4$, then $x = +$ or $- \sqrt{4}$
$x = + \; – 2$
That is $x = 2$ or $x = -2$
If $(x – 3)^2 = 4$, then either
$x – 3 = 2$
$x = 2 + 3$
$x = 5$
or
$x – 3 = -2$
$x = -2 + 3$
$x = 1$

Example 4

Solve $(x + 3)^2 = 7$, giving the answer correct to two decimal places.

Solution

$(x+3)^2 = 7$
$(x + 3) = \sqrt{7}$
$x + 3 = \underline{+} \sqrt{7}$ use a calculator to find $\sqrt{7}$
$x + 3 = \underline{+} 2.65$
Therefore, either
$x + 3 = 2.65$
$x = 2.65 - 3$
$x = - 0.35$
or $x + 3 = -2.65$
$x = -2.65 - 3$
$x = - 5.65$

Activity 2

Solve each of the following equations. Where necessary give the answers correct to 2 decimal places.

1. $(x + 2)^2 = 4$
2. $(x + 5)^2 = 9$
3. $(a - 5)^2 = 6$
4. $(y + ¼)^2 = 3$
5. $(x - 0.34)^2 = 7$
6. $(b - 8)^2 = 2 ¼$

Completing the square

The following expressions are examples of perfect squares

a. $x^2 + 2x + 1 = (x + 1)^2$
b. $x^2 + 6x + 9 = (x + 3)^2$
c. $x^2 - 12x + 36 = (x - 6)^2$
d. $x^2 - x + ¼ = (x - ½)^2$

Consider the expression $x^2 + 2x$. What should be added to this expression to make it a perfect square?
(Hint : Look expression (a) above)
If we add the square of ½ of the coefficient of x, that is we add $(½ x 2)^2$, we will get the following equation
$X^2 + 2x + 1^2 = x^2 + 2x + 1$
$= (x + 2)^2$
Similarly , to make $x^2 - 6x$ a perfect square add $(½x - 6) = (-3) = 9$. That is
$x^2 - 6x + (-3)^2 = x^2 - 6x + 9$
$= (x - 3)^2$

Example 5

Complete the square and factorise $x^2 - {}^5/_4X$

Solution

$x - {}^5/_4\, x$

The coefficient of x is $-{}^5/_4$

$\tfrac{1}{2}$ of $-{}^5/_4 = \tfrac{1}{2} \times -{}^5/_4$

$= -{}^5/_8$

So

$x^2 - {}^5/_4\, x + (-{}^5/_8)^2 = x - {}^5/_4\, x + {}^{25}/_{64}$

$= (x - {}^5/_8)^2$

The above method can be used to solve quadratic equations which cannot be factorised easily.

Example 6

Solve the equation $x^2 - 2x - 2 = 0$, giving the answer correct to two decimal places.

Solution

$x^2 - 2x - 2 = 0$

The expression can not be factorised easily . Therefore we must try to complete the square. First we take the constant term to the right hand side of the equation.

$x^2 - 2x = 2$

Now we add the square of ½ the coefficient of x on the left hand side and, to balance the equation add the same to the right hand side,

$x^2 - 2x + (-1)^2 = 2 + (-1)^2$

$x^2 - 2x + 1 = 2 + 1$

$x^2 - 2x + 1 = 3$

$(x - 1)^2 = 3$

$x - 1 = \underline{+}\sqrt{3}$ using the calculator $\sqrt{3} = 1.73$

$x - 1 = \underline{+}\ 1.73$

$x - 1 = 1.73 + 1$ or $x - 1 = -1.73 + 1$

$x = 2.73$ $x = 0.73$

Example 7

Solve the equation $2x^2 - 4x + 1 = 0$, giving your answer correct to two decimal places.

Solution

$2x^2-4x + 1=0$

First divide each term of the equation by 2 in order to make the coefficient of x^2 be 1. Thus

$x^2-2x+1/2=0$
Now take the constant term to the right side of the equation

$x^2-2x=-1/2$

Then add $(1/2x – 2)^2 = (-1)^2$ to both sides of the equation.

$x^2-2x +1=-1/2 +1$
$(x-1)^2=1/2$
$\sqrt{(x-1)^2} =1/2$
$x-1=\sqrt{1/2}$
$x-1=-\sqrt{0.5}$ use calculator
$x-1=-0.71$
$x= 1+0.71$
$x= 1.71$
or $x= 1-0.71$
$x = 0.29$

Example 8

Solve $½x^2 -5x = 3$

Solution

$½x - 5x = 3$

First we remove the fraction by multiplying through out by 2. So we have
$x-10x = 6$

Then we can now complete the square and solve the equation
$x-10x + (-5)^2 =6+(-5)^2$
$(x-5) = 6+25$
$(x-5) =31$
$x-5 =-\sqrt{31}$
$x – 5 = -5.57$
$x = 5 + 5.57$
$x = 5 +5.57$ Or $x = 5-5.57$
$x =10.57$ $x = 0.57$

Activity 3

Solve each of the following equation by using the method of completing the square. Give answers correct to two decimal places where necessary

1. $x - 4x - 5 = 0$
2. $x- 2x + 3 = 0$
3. $5x + 12x = -3$
4. $9x = -12y + 6$
5. $1/3x - x + 4 = 0$
6. $1/4x = -5x + 2$
7. $2x = 1/x = 5 -1/x$
8. $6/x = \frac{5x - 1}{3}$

Topic 3 Solving Quadratic Equation using the Formula Method

Let us solve the equation $ax + bx + c = 0$ using the completing the square method. Since we see that the coefficient of x is a, then we have to divide throughout by a, we have
$x + b/a\ x = -c/a$

Add to both sides of the equation with

$$(\tfrac{1}{2} \text{ of } {}^{b}/_{a})^2 \Rightarrow ({}^{b}/_{2a})^2 = \frac{b^2}{4a^2}$$

$$x^2 + \frac{bx}{a} + \frac{b}{4a^2} = \frac{-c}{a} + \frac{b^2}{4a^2}$$

Factorise the left hand side, you have

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a}$$

$$\left(\sqrt{x} + \frac{b}{2a}\right)^2 = \pm \sqrt{\frac{b^2}{4a^2} - \frac{c}{a}}$$

$$X = \frac{-b}{2a} \pm \sqrt{\frac{b^2}{4a^2} - \frac{c}{a}}$$

$$X = \frac{-b}{2a} \pm \sqrt{\frac{b - 4ac}{4a^2}}$$

$$X = \frac{-b \pm \sqrt{b - 4ac}}{2a}$$

Therefore, $x = \frac{-b + \sqrt{b - 4ac}}{2a}$

$x = -{}^{b}/_{2a} + \frac{\sqrt{b - 4ac}}{2a}$

Therefore $x = \frac{-b + \sqrt{b - 4ac}}{2a}$

The above formula can then be used to find the roots of the quadratic equations
$ax + bx + c = 0$ where
a is the coefficient of x
b is the coefficient of x
c is the constant

Example 9

Solve the equation $x + 5x + 3 = 0$

Solution

$x + 5x + 3 = 0$

The equation is in the form $ax + bx + c = 0$
Where a = 1, b = 5 and c = 3 and therefore its roots can be found using the formula

$$x = \frac{-b + \sqrt{b - 4ac}}{2a}$$
$$= \frac{-5 + \sqrt{5^2 - 4 \times 1 \times 3}}{2 \times 1}$$
$$= \frac{-5 + \sqrt{25 - 12}}{2}$$
$$= \frac{-5 + \sqrt{13}}{2}$$
$$= \frac{-5 + 3.606}{2} \text{ (correct to 3 decimal places)}$$

$x = \frac{-5 + 3.606}{2}$ or $x = \frac{-5 - 3.606}{2}$

$= \frac{-1.394}{2}$ $= \frac{-8.606}{2}$

Therefore $x = -0.697$ or $x = -4.303$

Example 10

Solve the equation $-2x^2 + 8x - 4 = 0$

Solution

$-2x^2 + 8x - 4 = 0$. We see that a = -2, b = 8 and c = -4

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-8 + \sqrt{8^2 - 4x - 2x - 4}}{2 x -2}$$

$$= \frac{-8 + \sqrt{64-32}}{-4}$$

$$= \frac{-8 + \sqrt{32}}{-4}$$

$$= \frac{-8 + 5.657}{-4} \quad \text{or } x = \frac{-8 - 5.657}{-4}$$

$$x = \frac{-2.343}{-4} \quad \text{or } x = \frac{-13.657}{-4}$$

$x = + 0.586$ $\quad$ $x = +3.414$

Correct to 3 d. p.

Example 11

Solve the equation $\frac{1}{3x^2} - 2x + 2 = 0$ giving the answer to two decimal places

Solution

$$\frac{1x^2}{3} - 2x + 2 = 0$$

Multiply on both sides of the equation by 2 so as to remove the fraction. So,

$$2 \text{ x } \frac{1}{2}x^2 - 2 \text{ x } 2x + 2 \text{ x } 2 = 0$$

$x^2 - 4x + 4 = 0$, Now expressing the equation in standard form,

$x^2 + bx + c = 0$

then $x^2 - 4x + 4 = 0$ has $a = 1$, $b = -4$ and $c = 4$

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-4) + \sqrt{(-4)^2 - 4(1)(4)}}{2 \text{ x } 1}$$

$$= \frac{4 + \sqrt{16-16}}{2}$$

$$= \frac{4 + \sqrt{0}}{2}$$

$$= \frac{4}{2}$$

$x = 2$

Activity 4

Solve each of the following quadratic equations by using the quadratic formula, where necessary give answers correct to two decimal places.

1. $x^2 + 3x + 2 = 0$
2. $6x^2 - x - 2 = 0$
3. $x^2 - 8x + 12 = 0$
4. $-2x^2 + 3x - 1 = 0$
5. $-5x^2 + 4x + 2 = 0$
6. $3x^2 - 4x - 2 = 0$
7. $3x^2 = 13x - 4$
8. $4x^2 = 7x - 3$
9. $\frac{1}{3}x^2 - 2x + 2 = 0$
10. $\frac{5}{2}x^2 = x + \frac{1}{2}$
11. $\frac{6}{x} = \frac{5x - 1}{3}$
12. $(3x + 2)(4x - 3) = 10x(x + 1)$

Equation with no real roots

The equation $x^2 = -4$ has no real roots. That is there is no real number x which gives -4 when it is squared.

When $x = -2$ $x^2 = (-2)^2 = 4$

Similarly, the equation $x^2 + 3x + 4 = 0$ has no real roots.

$x^2 + 3x + 4 = 0$

$x^2 + 3x = -4$

Now we can complete the square and solve the equation

$x^2 + 3x + (^3/_2)^2 = -4 + (^3/_2)^2$

$(x + ^3/_2)^2 = -4 + \frac{9}{4}$

$(x + \frac{3}{2})^2 = \frac{-7}{4}$

$x + \frac{3}{2} = \sqrt{\frac{-7}{4}}$

$x + \frac{3}{2} = \frac{\sqrt{-7}}{2}$

But the $\sqrt{-7}$ is not possible and therefore the equation has no real roots.

Activity 5

Solve the following equations using any method of your choice. Where necessary give your answer correct to two decimal places. Where there are no roots indicate this fact.

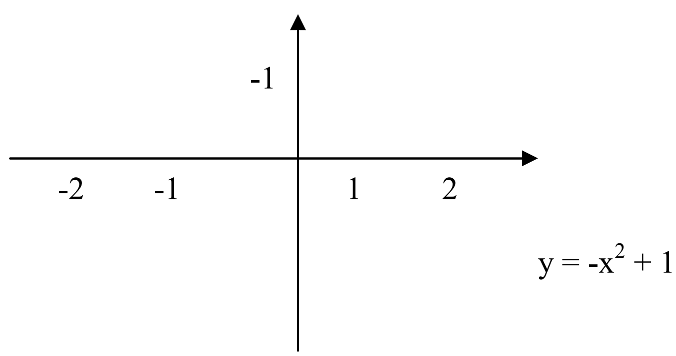
1. $x^2 - 2x + 1 = 0$
2. $x^2 - 2x + 4 = 0$
3. $2x^2 + 15x + 7 = 0$
4. $x^2 = 6(x + 3)$

5. $8 + \sqrt{3}x = 3x^2$

Topic 4: Solving quadratic equations using the graphical method.

This was partly covered in the previous unit when you were asked to draw graphs of quadratic functions (which are in the form $f(x) = ax^2 + bx + c$)
For example if $y = x^2 + 1$ you can be able to find $-x^2 + 1 = 0$ from the graph
Firstly you have to make a table, probably ranging from $-2 \le x \le 2$, $x \in R$ and then sketch the graph.

x	-2	-1	0	1	2
y	-3	0	1	0	-3



So if we are to find $-x^2 + 1 = 0$, this means that we have to read on the x-axis, those points which the x- axis intercept with the graph $y = -x^2 + 1$, i.e. the roots are -1 and 1

Activity 6

Solve the following quadratic equations using the graphical method.

1. $x^2 - 2x = 0$
2. $x^2 = -2x + 3$
3. $2x^2 + x - 3 = 0$

Summary

In this unit you have looked at four methods of solving quadratic equations, and these are
1. factorization

2. completing the square
3. formula and
4. graphical methods

- Remember that the coefficient of x^2 should always be one before completing the square
- When completing the square, always take the constant on the other side of the equation.
- Always add the square of ½ of the x coefficient to both sides of the equation then factorise.
- If the expression has a fraction in it, make sure you multiply throughout the expression by it reciprocal so as to get rid of the fraction.

Hope you enjoyed the unit and managed to get the activities correct, if not you can still go through the unit until you understand it fully.

UNIT FIVE

ATIO, PROPORTION AND RATE

Introduction

Welcome to unit 5 of this module. In this unit you shall discuss ratio, proportion and rate. This also include common measures of rate and environmental problems in relation to ratio, proportion and rate.

This unit has three Topics:

- Ratio direct and inverse proportion.
- Common measures of rate and

If you do all the activities in this unit, we estimate you will need to study between two to three hours.
Do not worry if it takes you more or less time than this – we do not all work at the same pace. It is important that you follow all the activities and fully understand the topic.

Objectives

By the end of this unit you should be able to;

a) express ratios in their simplest form;
b) solve questions involving direct and inverse proportions;
c) convert common measures of rate to the stated units; and
d) solve questions involving environmental problems.

Topic 1: Direct and Inverse proportions

Ratio

If Mary has 4 sweets and Jane has 6 sweets, who has more sweets than the other? Definitely Jane has more sweets.
These can also be compared in terms of their ratios ie. Mary : Jane and is read as "Mary to Jane". M: J can also be written in a fraction form $\frac{M}{J}$

We can safely say that, a ratio is a comparison between two or more like quantities. From the above example, the ratio between Mary's sweets to Jane's sweets is

4: 6 or $\frac{4}{6} = \frac{2}{3}$
2: 3

Example 1

The areas of two similar triangles are in the ratio 3 : 4 . If the area of the smaller triangle is $15cm^2$, What is the area of the of the larger triangle .
Solution
3: 4 and 15: a where a is the area of the larger triangle.
3: 4 = 15: a

$\frac{3}{4} = \frac{15}{a}$

$\frac{3a}{3} = \frac{4 \times 5}{3}$

a = 4 x 5
a = 20

Therefore the area the larger triangle is $20cm^2$

Example 2

The number of shoes sold by two agents in a month were in the ratio 8: 3 . At the end of the month the company gave out K638,000 bonus to be divided between the two agents on the basis of their sales. How much did each agent get as bonus?

Solution

The amount of bonus is K 638,000
Total ratio is 8 + 3 = 11, that is one will get $\frac{8}{11}$ of the bonus and the other $\frac{3}{11}$ of the bonus. Therefore, one agent gets
$^8/_{11}$ x K638,000 = 8 x 58,000 = K464,000 and other agent gets
$^3/_{11}$ x K638,000 = 3 x 58,000 = K174,000

Proportion

A proportion is a statement of the equality of ratios between two pairs of quantities.
For example, the statement $^a/_b = {}^x/_z$ or az = bx

Example 3

If $^3/_{10} = {}^{12}/_x$, find x

Solution

$^3/_{10} = {}^{12}/_x$ we cross multiply, so we have

$3x = 120$
$x = \frac{120}{3}$
$x = 40$

Example 4

The electrical resistance of a piece of wire is proportional to its length. Given that the resistance of a length of wire 3m long is 2.25 ohms. What is the length of the wire whose resistance is 5.5 ohms?

Solution

Let l be the length of the wire whose resistance is 5.5 ohms
$\frac{5.5}{L} = \frac{2.25}{3}$
$2.25l = 5.5 \times 3$
$L = \frac{5.5 \times 3}{2.25}$
$L = 7.33$m. (Correct to two decimal places)

Direct Proportion

Two quantities are in direct proportion with each other, if both increases or decreases in the same ratio.
For example, 1 apple costs K1200, then 4 apples will cost K4800, and will cost K7,200. As the number of apples increases so does the cost.

Example 5

A man uses 10 Litres of fuel to cover a distance of 15km. How much fuel would be needed to cover a distance of 24km?

Solution

$^{10}/_{15} = {}^{x}/_{24}$
$15x = 10 \times 24$
$x = \frac{10 \times 24}{15}$
$x = 16$ Litres Therefore the man needs 16 Litres of fuel to cover a distance of 24km.

Inverse Proportions

Two quantities are in inverse proportion with each other when one quantity increases and the other decreases in the same ratio.

Example 6

It takes 4 days for a group of 8 men to dig a trench. How long would it take a group of 16 men dig the same trench?

Solution

4 days --- 8 men
x days --- 16 men

We invert one of the ratios since it is an inverse proportion
(not $^{8}/_{16}$)

$^{4}/_{x} = {}^{16}/_{8}$
$^{16x}/_{16} = \frac{4 \times 8}{16}$
x = 2 days

Therefore it would take 2 days for a group of 16 men dig the same trench.

Activity 1

1. Find in simplest form the ratio of
 (a) 3cm to 3km
 (b) 50mins to 1hr 30mins

2. A plan has a scale of 1: 20,000. Find the actual measurement on the ground in meters and square meters for
 (a) Length of 15cm and 7.2cm on the map.
 (b) Area of a field whose measurements are 4cm and 6cm the map.

3. A piece of material 45cm long can be out into 9 pieces. How long will each piece of material be if 15 pieces were cut?

4. There is enough food in a camp to last 12 men for 10 days. How long would the food last, if there were 15 men?

Topic 2: Common Measure of Rate

When a car covers a distance of x km in 1 hr then its speed is expressed as x km/h. The km per hour is known as the rate of measure are m/s, (metres per second), m/s^2 (metres per second squared, as in acceleration.

We shall concentrate on how we can convert one rate of measure from one form to another.

Example

Convert the following in m/s (meters per second)

(a) 10m/min
(b) 360km/h

Solution

(a) 10m/min is the same as 10m/1 min, since 10 is the already in meters, then we have to convert 1 min to seconds.
10m /1 x 60 seconds =10 / 60 =0.167m/s

(b) 360 km/h = 360km/ 1 hour = (360 x 1000) m / (1 x 60 x 60) seconds.
= 10 x 1000 / 60 x 60
= 100m/s

Activity 2

1. Convert the following to m/s.
   (a) 20m/min
   (b) 35km/h
   (c) 30m/h
   (d) 60km/min

2. Convert the following to km/h.
   (a) 540km/h
   (b) 72km/h
   (c) 700m/min
   (d) 360km/min

Summary

In this unit you discussed proportion which included direct and inverse proportion.
It was also noted that ratio has no units hence it is expressed as a:b or a/b.
Hope you enjoyed studying this unit and managed to answer all the questions in the activities. You may move to the next unit.

UNIT SIX

VARIATION

Introduction

In the last unit we looked at ratio and proportion. We hope you enjoyed your study. We shall now look at variation. This will include direct, inverse, joint and partial variations.

In this unit, there are two Topics:

- Direct and inverse variation , and their graphs.
- Joint and partial variation

If you do all the activities in this unit, we estimate you will need to study between two to three hours. Do not worry if it takes you more or less time than this – we do not all work at the same pace. It is important that you follow all the activities and fully understand the topic.

Objectives

By the end of this unit you should be able to;

- Solve questions on direct and inverse variations.
- Draw graphs of direct and inverse variations
- Solve questions on joint variation and partial variation

Topic 1 Direct and Inverse Variation

1. Direct Variation

Consider the table below

A	4	6	8	10	12	x
B	2	3	4	5	6	10

Table1

If we take the ratio A:B or $\frac{A}{B}$ we shall get $\frac{4}{2} = 2$, $\frac{6}{3} = 2$, $\frac{12}{6} = 2$, and therefore $\frac{x}{10} = 2$

Hence x = 20.

Since the ratios in each case are the same, we can deduce that A is proportional to B. This means that there is a definite relationship between A and B and the relationship is, $\frac{A}{B} = 2$, and therefore A = 2B

This is expressed as $\frac{A}{B} = k$ or $A = kB$

From the table above, we can also notice that as A increases, B also increases. This kind of relationship is called direct variation and is expressed as A α B, where the symbol "(α)" is read as 'varies directly as' or 'is directly proportion to.' This A α B implies that $\frac{A}{B} = k$. The constant k is called the constant of variation. Notice that since A α B, then B B α A and B = kA.

Example 1

If x α y and x = 16 when y = 20, find

(a) an equation connecting x and y
(b) the value of y when x = 20

Solution

(a) x α y implies $\frac{x}{y} = k$

therefore, $x = ky$

$16 = k \times 20$

$k = {}^{16}/_{20} = {}^{4}/_{5}$

(b) when x = 20

$20 = ky$

$20 = {}^{4}/_{5}y$

$\frac{100}{4} = \frac{4y}{4}$

Therefore, y = 25

Example 2

If y α 2x + 4 and y = 6 when x = 3, show that 5y – 12 = 6x.

Solution

y α 2x + 4

$y = k(2x + 4)$

$6 = k(2 \times 3 + 4)$

$6 = k(6 + 4)$

$\frac{6}{10} = \frac{k(10)}{10}$

$k = {}^{3}/_{5}$ Therefore, $y = {}^{3}/_{5}(2x + 4)$

Since $y = {}^{3}/_{5}(2x + 4)$, we can cross multiply, we have

$5y = 3(2x + 4)$

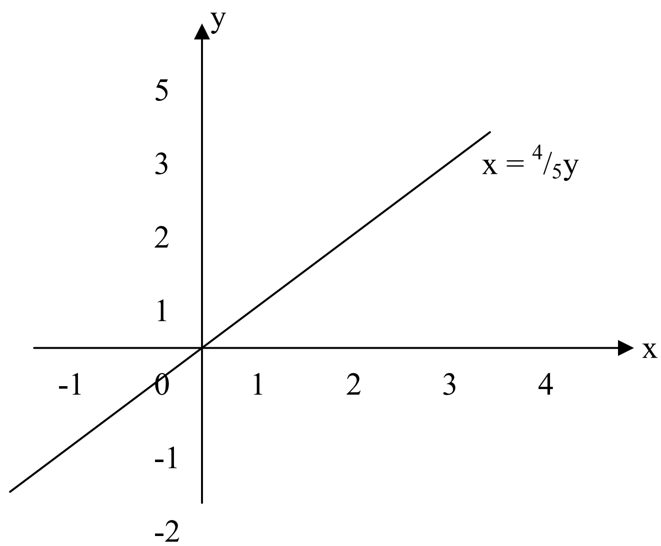
$5y = 6x + 12$
$6x = 5y – 12$ hence shown
$5y – 12 = 6x$

If we are to draw these graphs on the Cartesian plane, these graphs would be straight lines, as shown below. These equations are linear. For example, the graph in example 1 $x = {}^{4}/_{5}\,y$. In order to draw this graph, we have to make a table.

x	- 1	0	1	2	4
y	$-{}^{5}/_{4}$	0	${}^{5}/_{4}$	${}^{5}/_{2}$	5

That is, replace x with intergers from -1 to 4 to get the value of y.

Then you have to plot on an XOY plane.



1.2 <u>Inverse Variation</u>

Let us consider the table below.

X	1	2	3	4	5	6	10
Y	5	2.5	1.67	1.25	1	0.83	0.5

What is happening to the values of x and y?

The values of x are increasing while the values of y are decreasing.

From the above, we can say that if two quantities x and y vary in such a way that when one increases and the other decreases, or vice versa, then the two are said to be showing <u>inverse variation</u>.

This is written as x α $^1/_y$ read as "x varies inversely as y, and written as x = $^k/_y$ where k is a constant.

Example 3

Given that y varies inversely as X^2, and y = 48 when x = 3, calculate y when x = 4.

Solution:

$y \propto \frac{1}{x^2}$

$y = \frac{K}{x^2}$

$48 = \frac{k}{3^2}$

$48 = \frac{k}{9}$

$k = 432$

Therefore, $y = \frac{432}{X^2}$

$y = \frac{432}{4^2}$

$y = \frac{432}{16}$

$y = 27$

Example 4

If Y varies inversely as x, find for the table below

(a) The equation connecting x and y
(b) The value of a and b

X	0.6	0.9	b
Y	30	a	9

Solution

Let $y = \frac{k}{x}$

When x = 0.6 and y = 30, then

$30 = \frac{k}{0.6}$

$k = 30 \times 0.6$

$k = 18$

(a) $y = \frac{18}{x}$ is the equation connecting x and y.

(b) $y = \frac{18}{X}$ when a = y, then $a = \frac{18}{0.9} \Rightarrow a = \frac{180}{9} = 20$

And $y = \frac{18}{x}$, then from the table x = b when

$y = 9$

$9 = \frac{18}{b}$

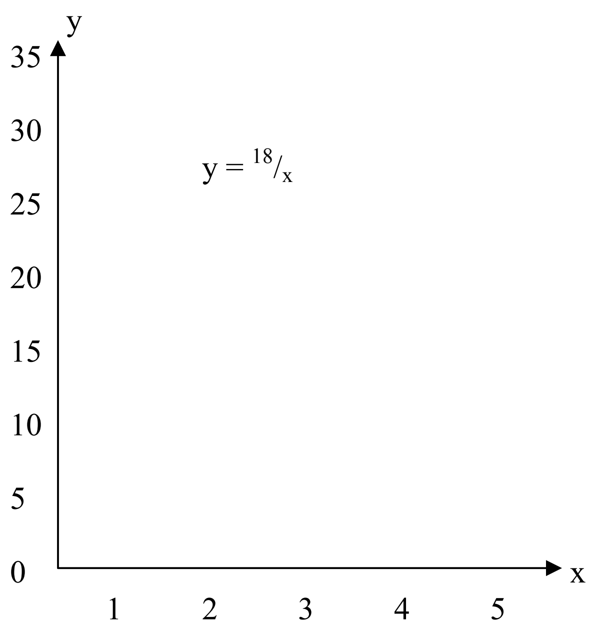
$b = \frac{18}{9}$

$b = 2$

From example 4, we can draw the graph for $y = \frac{18}{x}$

Using the same table

X	0.6	0.9	2	3	4
Y	30	20	9	6	4.5



We notice that when x increases, y decreases. This is an example of a graph of inverse variation.

Activity 1

In this activity, the questions are on either direct or inverse variation.

1. y varies directly as x and y = 1 when x = 6. Find the value of y when x = 4.
2. Given that y varies as x^2, and y = 36 when x = 4, find:

(a) y when $x = 6$
(b) x when $y = \frac{1}{4}$

3. Y varies as the square root of x, and $y = 5$ when $x = 10$. Find:
   (a) y when $x = 40$
   (b) x when $y = 15$

4. Given that y varies inversely as x, and $y = 8$ when $x = 3$, find:
   (a) y when $x = 84$
   (b) x when $y = 9$

5. Given that y varies inversely as x^2, and $y = 48$ when $x = 3$, calculate y when $x = 4$
6. Y is inversely proportional to $\sqrt{x}$. If $y = 5$ when $x = 16$, find:
   (a) y when $x = 100$
   (b) x when $y = 60$

7. When a railway train rounds a curve of Rm, the outer rail has to be raised above the level of the inner rail by a height hcm h varies inversely as R, and when $h = 12.8$, $R = 500$. Find the formula relating h and R and calculate the height for a radius 800m.

8. The air resistance, R newtons to the motion of a train varies as the square root of its speed, Vm/s. If the resistance is 3,600 newtons when the speed is 15m/s, calculate R when $V = 60$.

Topic 2

1. JOINT VARIATION

A variation in which the variable depends on two or more other variables is called Joint variation.

The statement "y varies jointly as x and Z" is expressed as $y \propto x\,Z$ or $y = kxz$.

Example

x varies directly as y and inversely as the square root of z. If $x = 16$ when $y = 3$ and $z = 36$, the value of x when $y = 12$ and $z = 4$.

Solution

$x \;\alpha\; \frac{y}{\sqrt{z}}$ $\qquad$ $x = \frac{32y}{\sqrt{z}}$

$x = \frac{ky}{\sqrt{z}}$ $\qquad$ Thus $x = \frac{32 \times 12}{\sqrt{4}}$

$16 = \frac{k \times 3}{\sqrt{36}}$ $\qquad$ $x = \frac{32 \times 12}{2}$

$16 = \frac{k \times 3}{6}$

$k = 32$

Activity 2

1. x varies directly as y and inversely as the square of z. If $x = 2^2/_3$ when y = 6 and z = 3, find the value of x when y = 12 and x = 4.

2. If x varies directly as y and z and x = 12 when y = 8 and z = 9, find the value of

   (a) x when y = 6 and z = 20
   (b) z when x = 20 and y = 15

3. If y varies directly as x and inversely as z and y = 25 when x = 4 and z = 6, find the value of (a) y when x = 3 and z = 8, (b) z, when y = 15 and x = 2.

4. If y varies directly as x^2 and the square root of z and y = 8 when x = 4 and z = 9, find the value of y, when x = 3 and z = 36.

5. The pressure P on a disc immersed in a liquid varies as the depth, d and as the square of the radius, r, of the disc. The pressure is 5000 Pa (Newton's per square metre) when the depth is 4.5m and the radius is 4m.

(a) Find a formula for P.
(b) Calculate (i) P when the depth is 3m and the radius is 2.8m
(ii) d when pressure is 2500 Pa and radius is 3m.

2. PARTIAL VARIATION

A variation in which one variable depends partly on a constant and partly varies on any other variable is called a partial variation.

Total cost of food at Mayela School is given as C = a + KN. This implies that the cost of food was partly constant and partly varies i.e.
C $\longrightarrow$ total cost of food

N $\longrightarrow$ number of people
a and k $\longrightarrow$ constants.

Example

C is partly constant and varies as N. C = 45 when N = 10 and C = 87 when N = 24.

(a) Find the formula connecting C and N
(b) Find C when N = 18

Solution

(a) $C \quad a + N$
$C = a + kN$
$45 = a + k \times 10 \Rightarrow 45 = a + 10k$ ….(i)
$87 = a + k \times 24 \Rightarrow 87 = a + 24k$ ….(ii)

Solving equations (i) and (ii) simultaneously we have

$\frac{-42}{-14} = \frac{-14k}{-14}$ Therefore, k = 3

Replacing k with 3 in any of the two equations

$45 = a + kN$
$45 = a + 3(10)$
$45 = a + 30$
$A = 15$

The formula connecting C and N is $C = 15 + 3N$

(b) $C = 15 + 3N$ when $N = 18$, we get
$C = 15 + 3(18)$
$= 15 + 54$
$C = 69$

Activity 3

1. x is partly constant and partly varies with y when y = 3, x = 1 and when y = 4, x = 14
   (a) find the relationship between x and y
   (b) Find x when y = 10

2. D is partly constant and partly varies with V, when v = 40, D = 150 and when D = 192, V = 54.
   (a) Find the formula connecting D and V.
   (b) Find D when v = 73.

3. The cost of a table is the variation of two parts. One is proportional to the area, and the other square of the length. If the cost of 2m by 3m is k50 and the cost of 1.5m by 4m is K64. Find the cost of a table 2.5m square.

Summary

In this unit you looked at direct and inverse variation, where you noticed that direct variation involves two variables where when one variable increases, the other also increases. This was not the case with inverse variation. In inverse variation, one variable increases while the other variable decreases or vice-versa.

You also went on to discuss the other variations and these are joint variation and partial variation. Joint variation involved three variables of which one variable depends on the other two variables either directly or inversely. As for partial variation, there is usually two constants and one of the constants is either directly or inversely varied.

I hope you enjoyed studying this unit and you were able to do all the activities given to you.

UNIT SEVEN

DISTANCE – TIME GRAPHS AND SPEED-TIME GRAPHS

Introduction

In the last unit you discussed the different types of variations. These included the direct, inverse, joint and partial variations. We hope you enjoyed and you are looking forward to learn more in this unit.

In this unit, there are two topics:

- Distance-Time graphs
- Speed-Time graphs

If you do all the activities in this unit, we estimate you will need four to six hours. Do not worry if it takes you more or less time than this. Remember we can not all work at the same pace. It is important you follow all the activities and fully understand the topic.

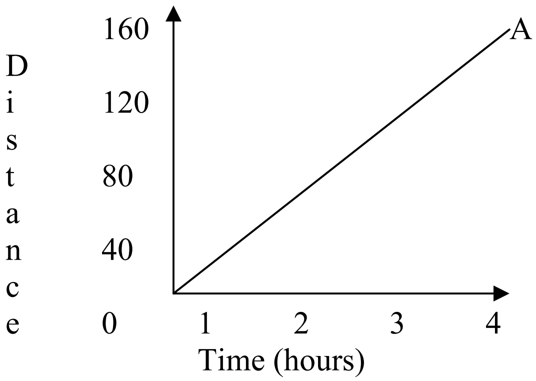
Objectives

By the end of this unit you should be able to:-

a) draw and interpret distance-time and speed-time graphs;
b) calculate the distance covered from a speed-time graph (area under a curve); and
c) Determine acceleration and retardation from gradient of speed time graph.

Topic 1 Distance Time Graph

If you remember simple problems on time, distance and speed were covered in Grade 9. For example,



(fig. 1)

This shows a car traveling at 40km/h. This means that the car will cover a distance of 120km in 3 hours and 160km in 4 hours. How far would it travel in 6 hours.

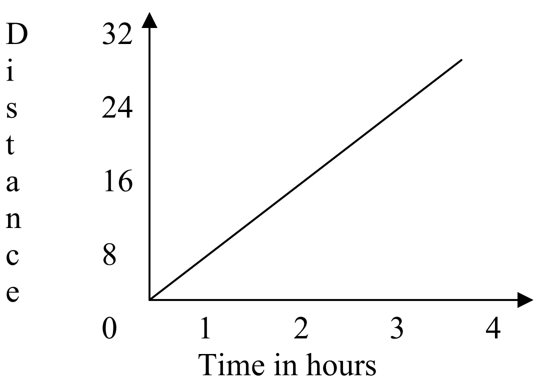
It will travel 240 Km. we see that the gradient in the figure above represents the velocity.

Example 1.

Chileshe walks at a speed of 8km/h. Draw a distance time graph to show this journey using a scale of 1cm to represent 8km on the vertical axis, and 1 cm to represent 1 hour on the horizontal axis.

Solution

Distance	8	16	24	32
Time	1	2	3	4



Example 2

Malaika was sent to the market to buy some vegetables, 12 Km away. She left home at 12.00 hours and arrived at the market at 13.30 hours. She left the market at 14.30 hours and arrived at home at 16.15 hours.

(a) Draw a distance time graph for Malaika's journey
(b) Using your graph find:-
  (i) How long she was in the market
  (ii) How far she was from home at 15.00 hours
  (iii) How far she was from the market at 12.45 hours.
(c) Find her average speed for the whole journey

Solution

(a) On a graph paper

(b)
(i) She as 1 hour in the market
(ii) She was about 8.6 km away from home at 15.00 hours (see graph)
(iii) She was about (12 – 5.8) 6.2Km away from the market at 13.45 hours (see graph)

(c) Average Speed = $\dfrac{\text{Total distance}}{\text{Total Time}}$

$$= \frac{24\text{Km}}{1\tfrac{1}{2} + 1\tfrac{3}{4}} = \frac{24}{{}^{13}/_{4}} = \frac{24 \times 4}{13} = 7.38\text{Km/h}$$

Example 2

Space for a Graph

Example 3

Space for a Graph

Example 3

A boy sets off on a bicycle at 07.00 hours from town P and to town Q (which is 100km away) at an average speed of 20Km/h. After cycling for 2 hours he stops for an hour for a meal and then goes on as before. At 10.00 hours his father sets off from town A to twon B in his car traveling at 50Km/h. Draw a distance – time graph and find when each reaches twon B. When and where does the father overtake the boy?

Solution

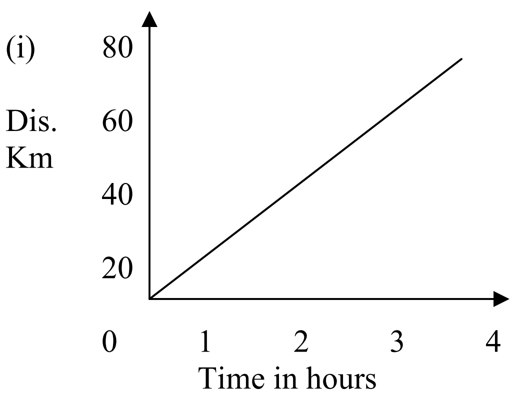
See graph (Example 3) for the distance time graph.

- The boy reaches town B at 13.00 hours.
- The father reaches town B at 12.00 hours.
- The father overtakes the boy at 11.24 hours.

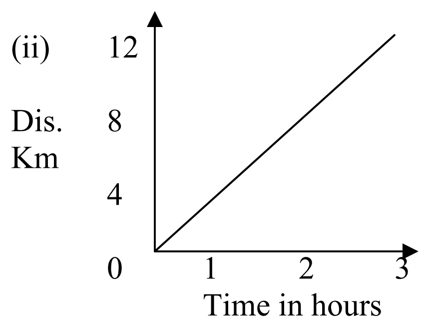
Activity 1

1. For each of the following journeys, find
   (a) the total distance traveled
   (b) the time taken
   (c) the speed

(i)



(ii)



2. A train leaves town P for town Q (200km away) at 08.00 hours and moves at a speed of 100 km/h. After two hours it stops for another 2 hours then sets off back at the same speed as before.

   (a) Draw the distance time graph for the train.
   (b) How did the journey take?
   (c) What was the average speed from P to Q?
   (d) At what time did the train leave for town B.
   (e) What was the average speed for the whole journey.

3. A man starts off from twon A at 09.00 hours to walk to twon B at 6 Km/h. After an hour, he meets the bus which runs from B to town A. The bus waits for 15 minutes at

town A and then returns to town B. The bus travels at 24 Km/h. Find where and when the man is overtaken by the returning bus. (Using the graph).

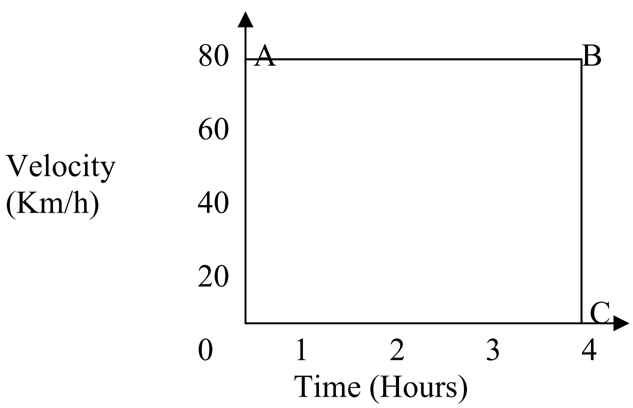
Topic 2 Speed – Time Graphs/Velocity-Time

What is velocity?

Velocity is a measure of the rate of change of distance with respect to time in a specific direction.

Speed, on the other hand, measures the rate of change of distance with respect to time with no regard to direction.

Usually on the velocity time graph, the velocity is represented on the vertical axis and time on the horizontal axis. Let us consider a car traveling at a constant speed of 80km/h. The graph is a straight line parallel to the x-axis as shown in the figure below.



Suppose the car traveled for 4 hours and we want to find the total distance it has travelled. We would use the formula D = ST, where S is the speed, t is the time and D is the Distance.

Total distance traveled = 60Km/h x 5 hours
= 300 Km

But this is actually the area of rectangle OABC
Area OABC = OA x OC = 60 x 5 = 300

In general, the total distance travelled by an object is equal to the area under the velocity-time graph.

Acceleration

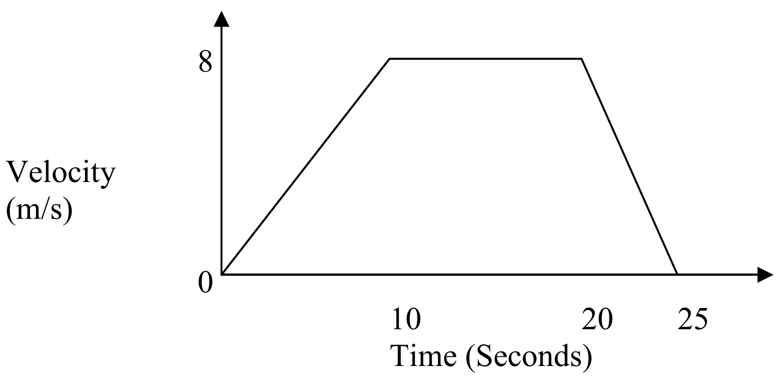
If a moving object increases its speed, we refer to this increase as the acceleration of the object.

Acceleration is the rate of change of velocity with respect to time.

Acceleration is usually measured in m/s^2 or cm/s^2 (metres per second or centimetres per second).

Example 1

The figure below shows the velocity time graph of an object.



The object under consideration in the graph above increases its speed from 0 to 8 m/s for the first 8 seconds. It then travels at a constant speed of 8m/s for a further 10 seconds before coming to rest in another 5 seconds. Find:

(a) the acceleration for the first 10 seconds
(b) the total distance travelled
(c) the acceleration for the last 5 seconds of its motion

Solution

(a) The acceleration for the first 10 seconds is given by the gradient of the line OA. Therefore, acceleration = $^8/_{10} = {}^4/_5$ m/s^2 or 0.8 m/s^2

(b) The total distance traveled = area under the speed time graph for 25 seconds.

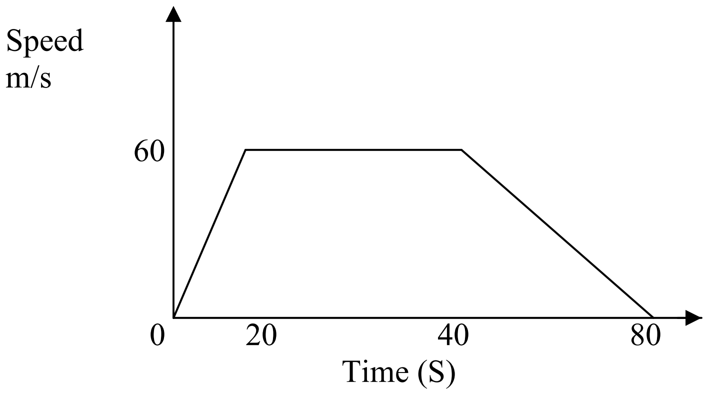
Total distance travelled = Area of trapezium OABC
$= ½ (25 + 10)8$
$= 35 \times 8$
$= 280m$

(c) The acceleration for the last 5 seconds of this motion is given by the gradient of BC. Therefore, Acceleration = $-^8/_5 = -1^3/_5 m/s^2$ or $-1.6m/s^2$

The negative sign indicates that there is a decrease in velocity. This acceleration is sometimes called retardation. Therefore, retardation = $1.6 m/s^2$.

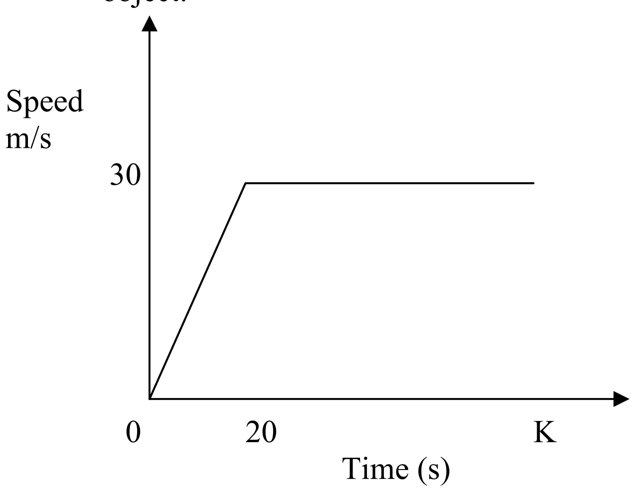
Activity 2

1. The diagram below is the speed time graph of a journey.



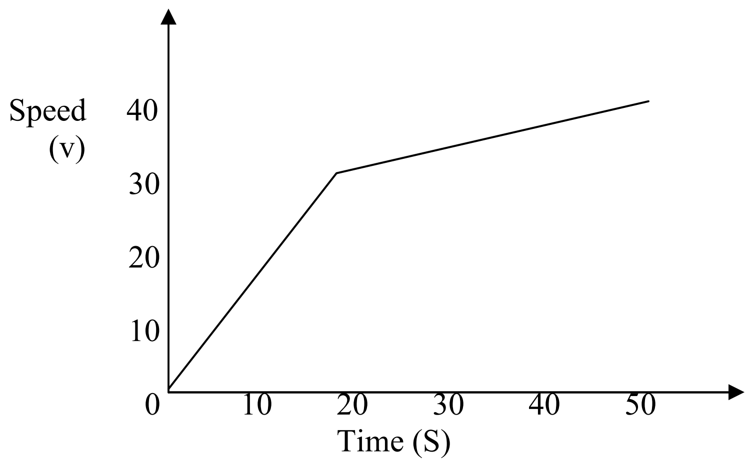
Calculate (a) the distance covered in the first 20 seconds
(b) the average speed for the whole journey
(c) the speed at 50 seconds

2. The diagram is the speed time graph for the first k seconds of the motion of an object.



(a) Find the acceleration when t = 10.
(b) Find the distance traveled in the first 20 seconds
(c) The distance traveled in the first k seconds is 720m. Find the value of k.

3.



The diagram is the speed time graph of part of a motor cycle journey.

(i) The motorcycle accelerates uniformly from rest until it reaches a speed of 30m/s after 20 seconds. Calculate the acceleration of the motor cycle.

(ii) The motorcycle continues to accelerate, but at a slower rate, for a further 30 seconds, until it reaches a speed of 40m/s. Calculate the distance it travels during the period $t = 20$ to $t = 50$.

(iii) When $t = 50$, the motor cycle begins to slow down with a uniform retardation of 3½ m/s^2. Calculate how long it will take for the motor cycle to come to rest.

Summary

In this unit you looked at the Distance time graph and Speed time graph. The two are different in that in the speed time graph, distance is calculated by getting the area under the graph, while in the distance time graph we read from the graph or we use the formular speed is equal to distance divided by the time $S = {}^{D}/_{T}$, if you have been given speed and time you just change the subject to $D = ST$ and solve.

UNIT EIGHT

SYMMETRY

Introduction

May I welcome you to unit 8 of this module. In this unit you shall look at imaginary lines that separate objects into two identical shapes about a point.

This unit has 2 topics:

- Line symmetry
- Rotation symmetry

If you do all activities in this topic, it is estimated that you will need four to five hours. Sometimes you may take more than the stated period but that should not worry you, what is important is to understand the topic very well.

Objectives

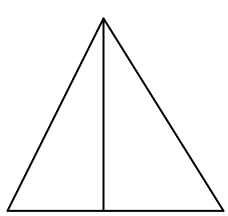
By the time you reach the end of this unit, you should be able to:

identify shapes with lines of symmetry;
draw lines of symmetry; and
find the order of rotational symmetry of an object or shape.

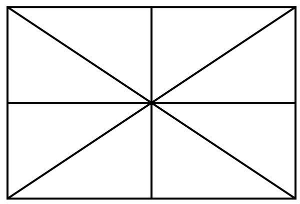
Topic 1 Line of Symmetry

The line of symmetry is an imaginary line that divides the shape into two identical shapes; so that one half fits on the other half.

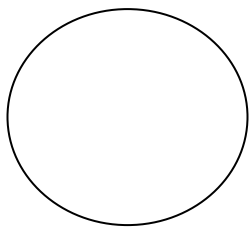
Example



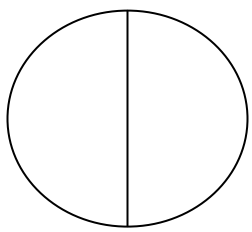
This is a line of symmetry



A square has four lines of symmetry



A circle has uncountable lines of symmetry



A circle with the shown figure has one line of symmetry through the same figure.

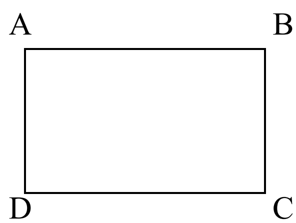
Activity 1

(1) How many lines of symmetry has the following shapes
(a) rectangle (b) equilateral triangle (c) regular hexagon

(2) How many lines of symmetry has the following letters
(i) H (ii) E (iii) N (iv) W (v) V (vi) X

Topic 2 Rotational Symmetry

Rotational symmetry is all about rotating a figure about a fixed point and fitting on its outline. For example if we take a square of vertices ABCD

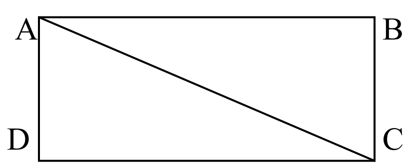


It can be rotated about a fixed point O 4 times fitting its outline through an angle of 90^0.

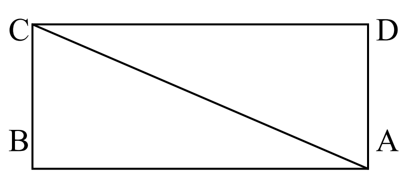
We can say that a shape has rotational symmetry with respect to a particular point if it fits outline when rotated about that point through a given angle. The fixed point is called the centre of rotational symmetry.

Example

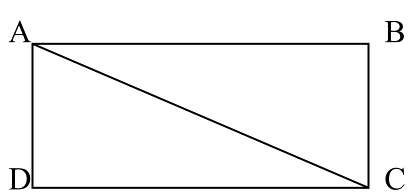
Let us look at a rectangle, if rotated about O through 180^0, A will move to C



At this point it has fit its outline once.



If rotated again through 180^0, A will move to its original position making a second fitting.



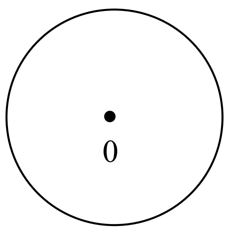
This means a rectangle fits its outline twice when rotated through an angle of 180^0 through the fixed point O.

The number of times an object or shape fits its out line when rotated is called the **order of rotational symmetry**. A rectangle has order of rotational symmetry of 2. A square has 4 because it can fit its outline 4 times when rotated at 90^0 through a fixed point.

A regular polygon with n sides has rotational symmetry about its centre through an angle of $\frac{360^0}{n}$ when n is the order of rotational symmetry.

For example, in a rectangle, the order of rotational symmetry is 2 so the angle through which it is rotated is $\frac{360^0}{n} = \frac{360^0}{2} = 180^0$

What about in a circle?

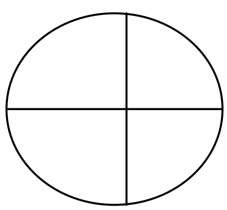


A circle has rotational symmetry of an infinite order.

However, a circle with certain features may have specific order of rotational symmetry.

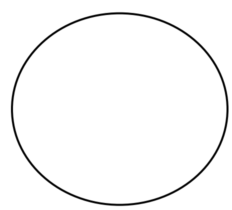
For example,

(a)



This has order of 4

(b)

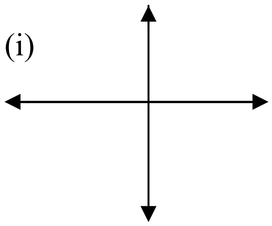


This has order of 1

Activity 2

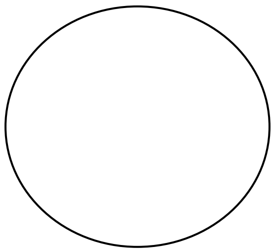
(1) (a) State the order of rotational symmetry
(b) Calculate the angle of rotational symmetry

(i)

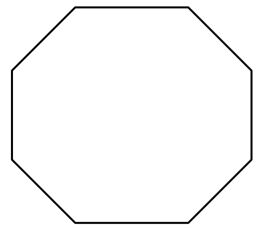


(ii)

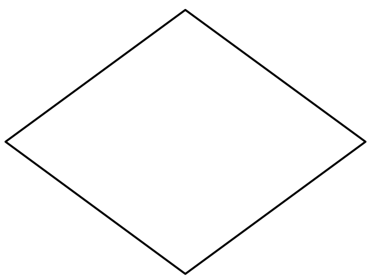
(iii)



(iv)



(v)



Summary

In this unit we have looked at line symmetry and rotational symmetry. We appreciated that a line symmetry is an imaginary line that divides an object into two identical shapes. Under rotational symmetry it involves a centre of rotation which is a fixed point and also the order of rotational symmetry which the number of times an object fits its outline when rotated about a fixed point.

UNIT NINE

SIMILARITY AND CONGRUENCY

Introduction

In the last unit we discussed symmetry. In this unit you shall look at another topic, that is, similarity and congruency.

In this unit, there are two topics:

- Topic 1: Similarity
  - 1.1 Similar figures
  - 1.2 Similar triangles
  - 1.3 Areas of similar figures
  - 1.4 Volume of similar figures.

- Topic 2: Congruent Figures

If you do all the activities in this unit, we estimate you will need to study between eight to nine hours.

Objectives

By the end of this unit you should be able to:

a) identify ratio of sides of similar figures;
b) calculate ratio of sides of similar figures;
c) calculate unknown sides or angles in similar figures;
d) calculate areas and volumes of similar figures; and
e) relate similarity and congruency in the natural environment.

Topic 1 Similarity

1.1 Similar figures

What are similar figures?

These are figures which have the same shape but do not necessarily have the same size.

For example:

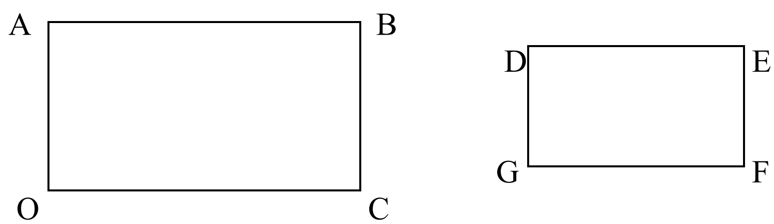


Figure 1

In figure 1, square OABC and square DEFG have same shapes. One figure is an enlargement of the other figure.

Two plane figures are similar if:

(a) corresponding angles are equal or
(b) corresponding sides are in the same ratio.

1.2 Similar triangles

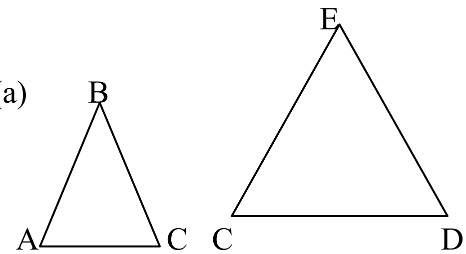
Two given triangles are similar if the corresponding angles are equal or if the corresponding sides are proportional.

From the above statement, we can also say that if the corresponding angles are equal then the corresponding sides will be proportional. If the corresponding sides are proportional then the corresponding angles will be equal.

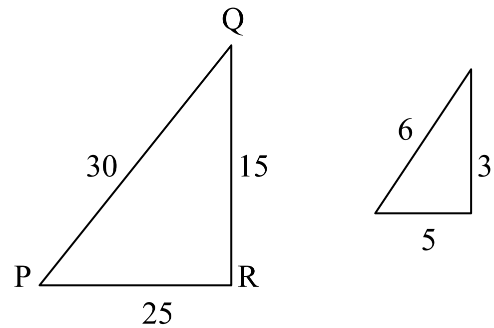
Example 1

Show whether or not the following pairs of triangles are similar.

(a)



(b)



Solution

(a) $<A = <C$, $<B = <D$, $<C = <E$. Since the corresponding angles are equal, ΔABC is similar to ΔCDE.

(b) Since $\frac{PQ}{ST} = \frac{30}{6} = \frac{5}{1}$ $\frac{QR}{TU} = \frac{15}{3} = \frac{5}{1}$ and $\frac{PR}{SU} = \frac{25}{5} = \frac{5}{1}$

The corresponding sides are proportional and the sides are proportional.

Therefore, ΔPQR is similar to ΔSTU.

In short, triangles which have two sides in the same ratio and have the included angle equal are also similar.

For example,

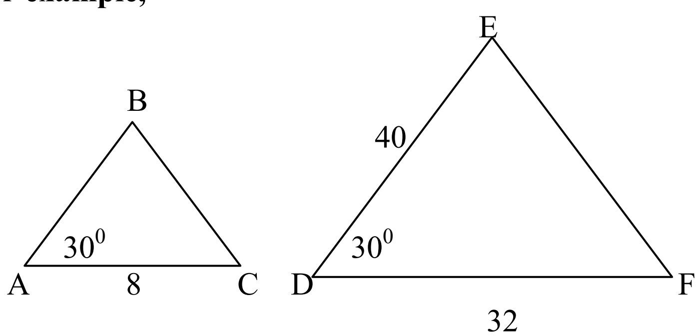
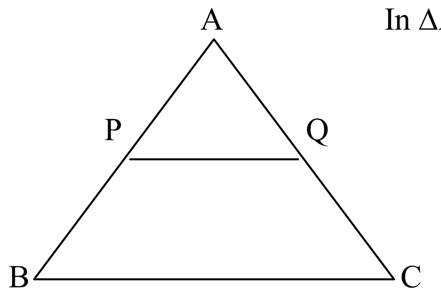


Figure 2

In figure 2 $\frac{AB}{DE} = \frac{10}{40} = \frac{1}{4} = \frac{AC}{DF}$ and $<A = <D$

Therefore, ΔABC is similar to ΔDEF.

Example 2

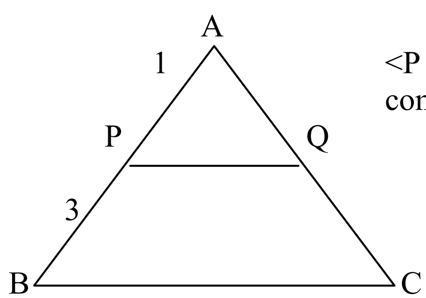


In ΔABC, PQ is parallel to BC. AB = 4AP and BC = 20cm

(a) Show that ΔABC is similar to ΔAPQ
(b) Find the length of PQ

Solution

(a) From the given in formation in the question we have



<P = <B, <Q = <C, are corresponding angles, <A is common. Therefore, ΔABC is similar to ΔAPQ.

AB = 4AP

$\frac{AB}{AP} = \frac{4}{1}$

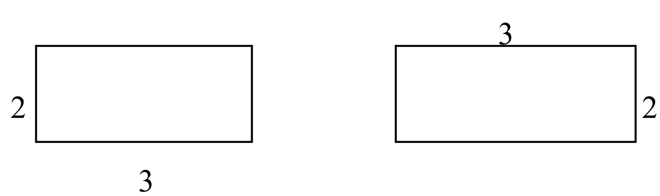
$\frac{PQ}{BC} = \frac{1}{4}$ BC

$PQ = \frac{1}{4} \times 20 = 5cm$

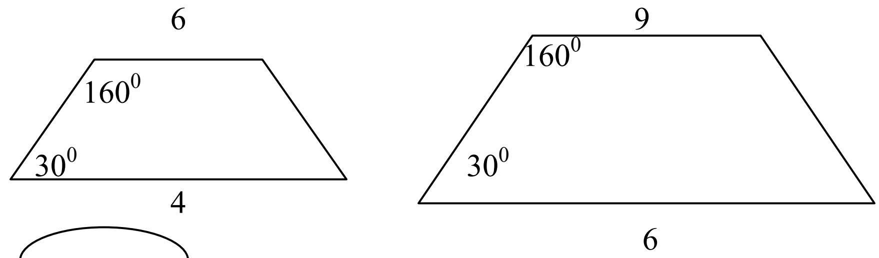
Activity 1

1. State whether or not the following pairs of figures are similar:

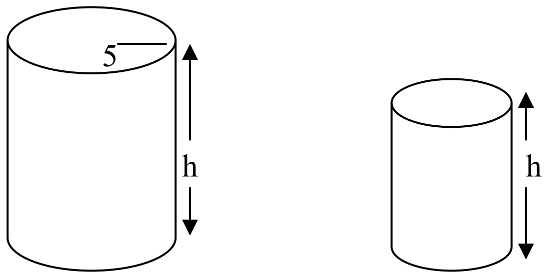
(a)



(b)

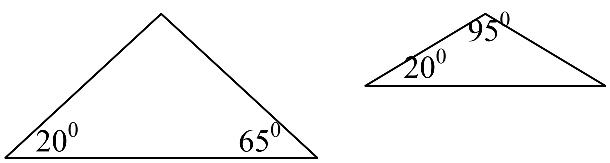


(c)

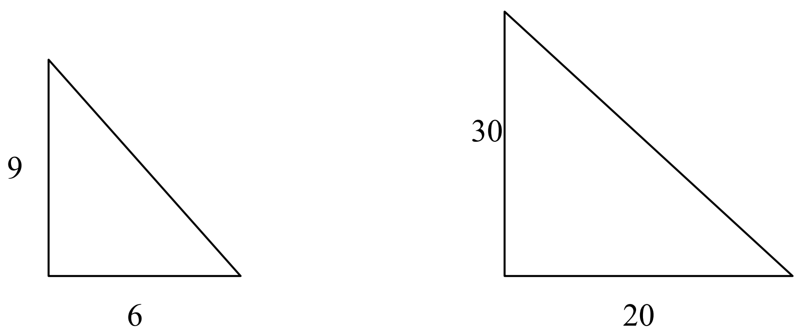


2. Show whether or not the following pairs of triangles are similar

(a)

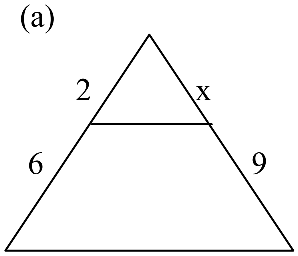


(b)

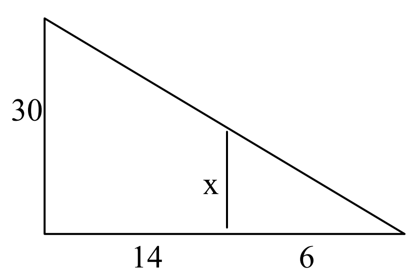


3. Calculate the value of x in each of the following triangles.

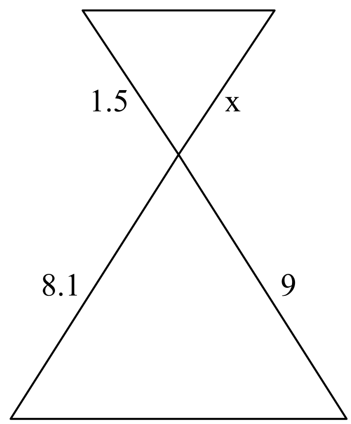
(a)



(b)

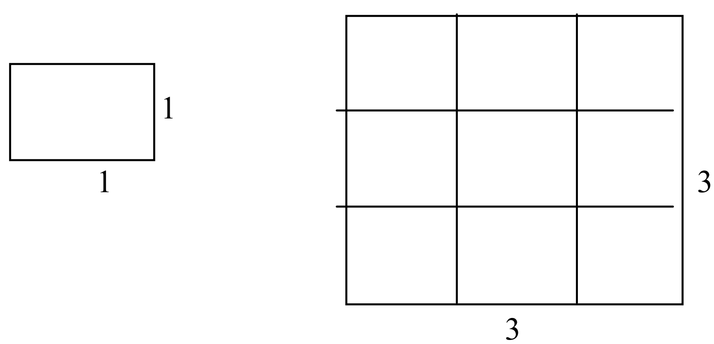


(c)



1.3 Areas of Similar figures

Let us consider the following squares

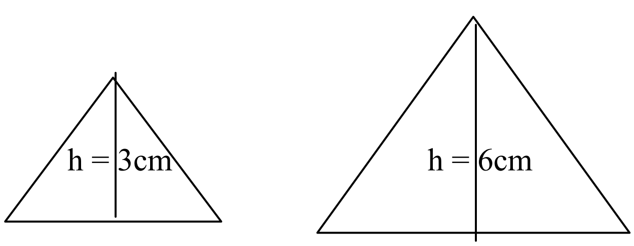


The lengths of the sides are in the ratio 1:3. The areas are 1 square unit and 9 square units respectively. Therefore, the ratio of their areas are in the ratio 1:9. What have you noticed about the ratio of the areas in relation to the ratio of the sides?

We are simply squaring the ratio of the ratio of the sides ie. $1^2:3^2 = 1:9$

Example 3

The given triangles are similar. Find the area of the smaller triangle, if the area of the larger triangle is $64cm^2$.



Solution

Ratio of sides = 3:6 In this case the sides are actually the heights.
= 1:2

Therefore, ratio of areas = $1^2 : 2^2$
= 1 : 4

Now, let us let x be the area of the smaller triangle.

1 : 4 = x : 64

$\frac{1}{4} = \frac{x}{64}$ By cross multiplying,

4x = 1 x 64

$x = \frac{1 \times 64}{4}$

x = 16

Therefore, area of the smaller triangle is $16cm^2$

Example 4

A rectangle of area 16cm has a length of 8cm. A similar rectangle has an area of 36cm. Calculate its length.

Solution

Ratio of areas = 16 : 36
= 4 : 9

Therefore, Ratio of sides = $\sqrt{4} : \sqrt{9}$
= 2 : 3

Let x be the length of the similar rectangle. Then,

2 : 3 = 8 : x

$\frac{2}{3} = \frac{8}{x}$

$\frac{2x}{2} = \frac{3 \times 8}{2}$

$x = \frac{24}{2}$

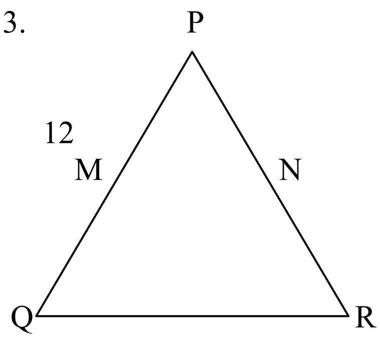
x = 12cm.

Activity 2

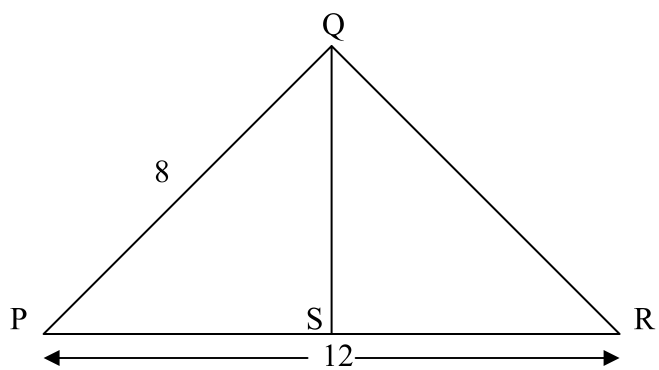
1. Two similar trapeziums have corresponding heights 4cm and 6cm, find the ratio of their areas.

2. Two similar triangles have corresponding sides in the ratio 4 : 7. Find the ratio of their areas:

3. In the diagram, PQ = 12cm and the area of PMN Is $^9/_{16}$ of the area of ΔPQR. Calculate the length of PM.



4.

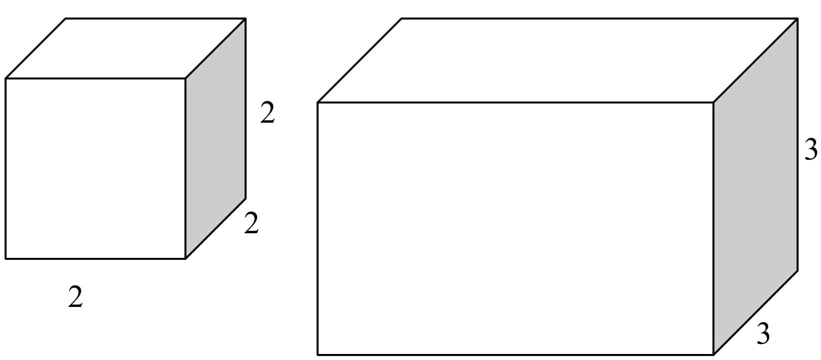


In the triangle PQR, PQ = 8cm, PR = 12cm and S is a point on the side PR such that <PQS = <PRQ.

(i) Write down the other pair of equal angles.
(ii) Use similar triangles to calculate the length of PS.
(iii) Given that the area of ΔPQS is $24cm^2$. Calculate the area of ΔPQR.

1.4 Volume of similar objects

Let us consider the volumes of two cubes of lengths 2cm and 3cm respectively.



3

Ratio of there sides = 2 : 3
Ratio of there sides = 4 : 9
Volume of the smaller cube = 2 x 2 x 2 = 8cm^3
Volume of the larger cube = 3 x 3 x 3 = 27cm^3
Ratio of the volumes = 8 : 27
What have you noticed about ratio of the volumes in relation to the ratio of the sides?
The ratio of the volume is found by cubing the ratio of the sides.

In general, if the ratio of the sides of two similar solids is a : b, the ratio of volumes is $a^3 : b^3$.

Example 5

A cylinder of radius 4cm has a volume of 256cm^3. Find the volume of a similar cylinder of radius 3cm^3.

Solution

Ratio of radii = 4 : 3
Therefore, ratio of volume = 64 : 27

Let V be the volume of cylinder of radius 3cm. Then
64 : 27 = 256 : V

$$\frac{64}{27} = \frac{256}{v}$$

$$64v = 27 \times 256$$

$$v = \frac{27 \times 256}{64}$$

$$v = 108.$$

Therefore, the volume of the cylinder of radius 3cm is 108cm^3.

Example 6

A solid has height of 5cm and a volume of 120cm^3. A similar solid has 3240cm^3. Find its height.

Solution

Ratio of volumes = 120 : 3240
= 1 : 27

Therefore, ratio of sides = $\sqrt[3]{1} : \sqrt[3]{27}$
= 1 : 3

Let h be the height of solid with volume $3240cm^3$.

1 : 3 = 15 : h

$$\frac{1}{3} = \frac{15}{h}$$

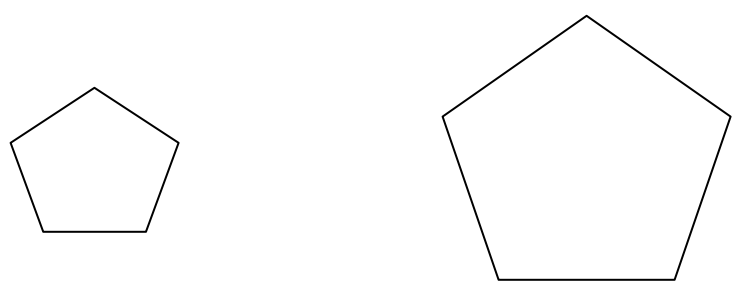
h = 45cm

Activity 3

1. Two similar cones have radii in the ratio 2 : 5. Find the ratios of there:
   (a) heights
   (b) surface areas
   (c) volumes.

2. Two similar blocks have corresponding edges of length 13cm and 39 cm. Find the ratio of their masses.

3. Two plant pots are geometrically similar. The height of the smaller pot is 5cm. The height of the larger pot is 15cm.



   (a) The distance of the base of the larger pot is 7cm. Find the distance of the base of the smaller pot.
   (b) Find the ratio of the volume of the smaller pot to that of the larger one. Give your answer in the form 1 : a (Dec. 1997 P1, Question 5)

Topic 2 Congruent Figures

Two plane figures are congruent if they have exactly the same shape and the same size. Congruent shapes can be superimposed exactly on to one another.

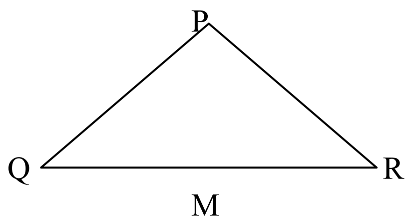


Figure 3

In fig. 3, PM is a line of symmetry. ΔPQM is the image of the triangle PRM. The two triangles have exactly the same shape and size.

We say that ΔPQM is congruent to ΔPRM or ΔPQM ≡ ΔPRM, where the symbol ≡ means "is congruent to".

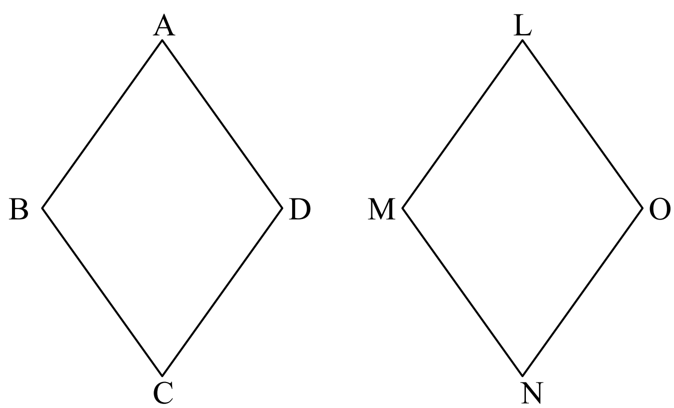


Figure 4

In figure 4, <A = <L, <B = <M, <C = <N, <D = <O.
AB = LM, BC = MN, AD = LO.
Therefore, Kite ABCD ≡ Kite LMNO

Congruent figures have both corresponding lengths and corresponding angles equal.

2.1 Conditions for congruence of triangles

Condition 1

Two triangles are congruent if : 3 sides of one triangle are equal to the corresponding 3 sides of the other triangle. This condition is abbreviated as SSS (Side, Side, Side).

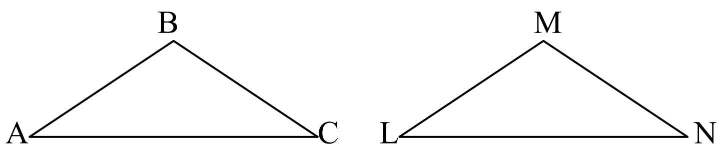


Figure 5

In fig 5, AB = LM, BC = MN, AC = LN and so <ABC ≡ <LMN.

Condition 2

Two triangles are congruent if:

Any two sides of the first triangle are equal to the corresponding two sides of the second triangle.
The included angle of one must be equal to the included angle of the other. This condition is abbreviated as S.A.S. (Side, Angle, Side).

Figure 6

In fig. 6, <B = <E, BC = EG and BD = EF.
Therefore, <BCD ≡ <EFG. (SAS).

Condition 3

Two triangles are congruent if

- Any two angles of the first triangle are equal to any two angles of the second and
- One side is equal to the corresponding side of the other. This condition is abbreviated as AAS (Angle, Angle, Side).

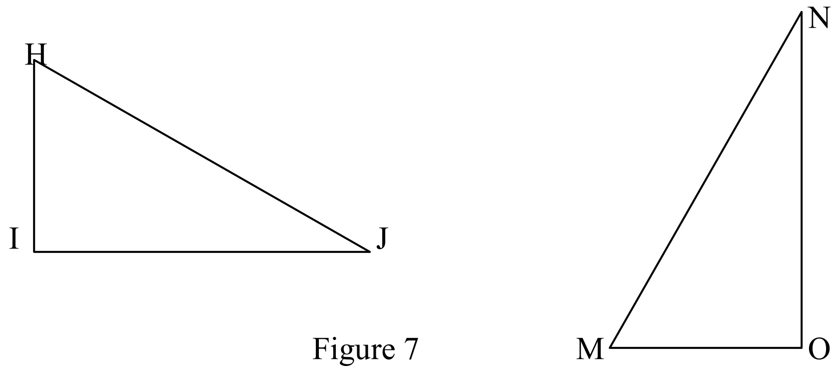


Figure 7

In figure 7, <H = <M, <J = <N and HI = MO.

Therefore, <HIJ ≡ <MNO (AAS).

Condition 4

Two right-angled triangles are congruent if

- The Hypotenuse and one side of the first triangle are equal to the hypotenuse and one side of the other triangle. This condition is abbreviated as RHS (Right angle, Hypotenuse, Side).

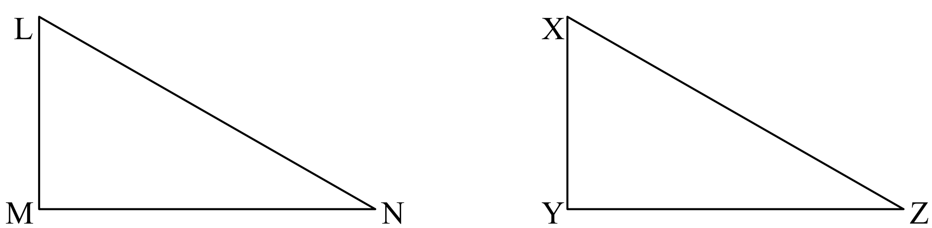
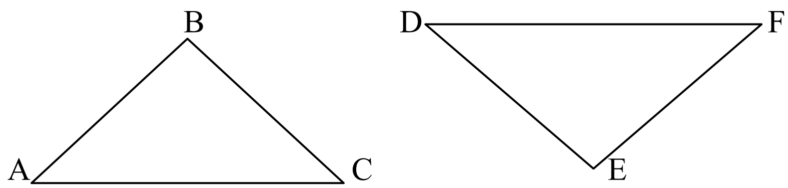


Figure 8, LN = XZ, MN = XY, <M = <Y = 90^0
Therefore, <LMN ≡ <XYZ

Example

Show that the two triangles below are congruent.



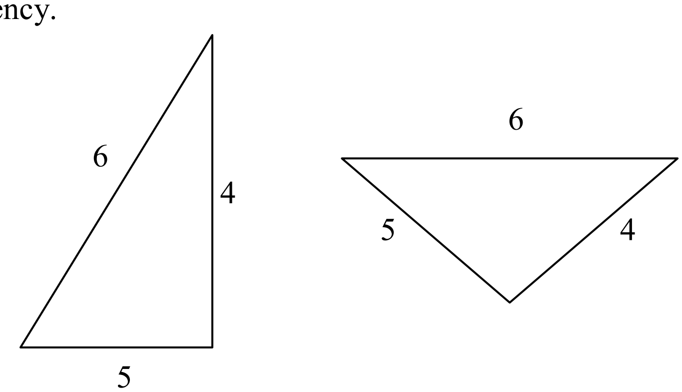
Solution

AB = EF, BC = DE and <B = <E
Therefore, <ABC ≡ <DEF (SSA)
Since <ABC ≡ <DEF, it follows that corresponding sides and angles are equal. So <B = EF, BC = DE, <B = <E, <A = <F and <C = <D.

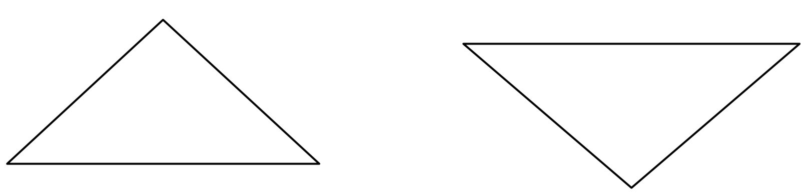
Activity 4

State whether or not the pairs of triangles are congruent, if they are state the condition for congruency.

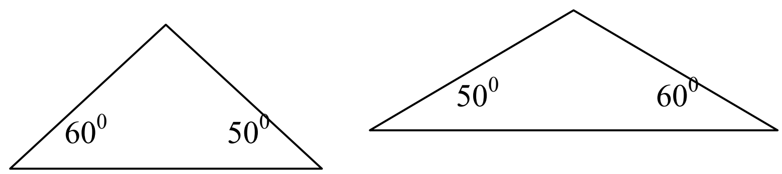
1.



2.



3.



Summary

In this unit, you have looked at so many definitions and conditions of congruency. We hope you also managed to solve the activities given to you. You may now move on to the next topic which is Circle Theorem.

UNIT TEN

CIRCLE THEOREM

Introduction

Welcome to unit ten of this module. In this unit you shall look at the angle properties of a circle and polygons. It is also an interesting topic because it involves various circle properties related to angles.

This unit has 5 topics:

- Definitions of parts of a circle
- Properties of the chord
- Properties of the angles of a circle
- Cyclic quadraleterals
- Properties of the tangent to a circle.

If you do all the activities in this unit, it is estimated that you will need seven to eight hours. However, you may take less or more hours but this should not worry you. What is important is to understand the topic very well.

Objectives

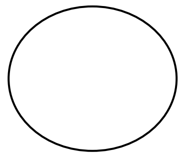
By the end of this unit you should be able to:

a) apply the properties of the chord in solving questions;
b) apply the properties of the angles of a circle to solve questions;
c) solve questions involving cyclic quadrilateral; and
d) apply the properties of a tanget when solving questions.

Topic 1 Definitions of parts of a circle

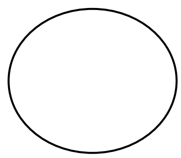
A circle has various parts and we can define them individually as follows:

Circumference – This is the distance around the circle.



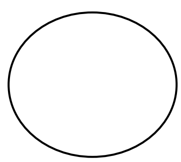
Arc – An arc is a fractional part of the circumference.

AB is an arc and its known as the Minor arc.



This is called the Major arc ABC

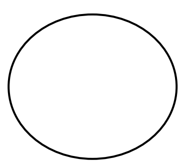
Diameter – This is the line that joins to points on the circumference and passing through the centre of the circle.



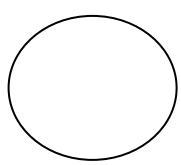
AB = diameter

Radius – This is half way the diameter or the point on the circumference joining the centre of the circle.

CA is a radius plural for radius is Radii.

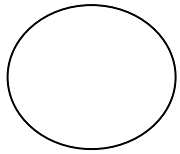


Chord – This is a line which joins any two parts on the circumference.



AB = Chord
The largest or longest chord is the diameter because it passes through the centre

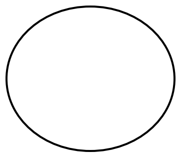
Segment – This is the area bounded by the chord and an arc.



Major segment

Minor segment

Sector – This is the region bounded by 2 radii and an arc.



Sector

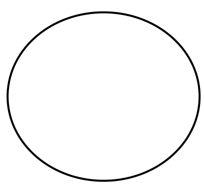
The two radii are OA and OB and the arc make the sector AOB

The above definitions are very important as we shall be using them in the next Topics.

Topic 2: Properties of the chord

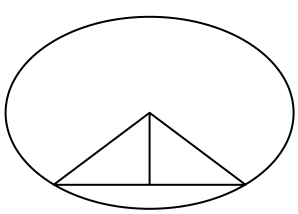
(a) The Perpendicular to a chord.

The perpendicular line is a line at 90^0 with a line.



ON is the line perpendicular to the chord AB. This line ON bisects the chord AB into AN and NB and AN = NB.

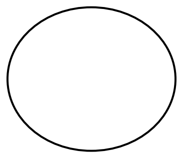
Look at the circle below and observe the triangle formed.



OA and OB are radii and are equal so they form an isoceles triangle OAB

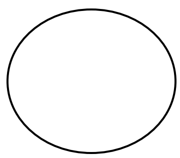
In general line drawn from the centre of the circle perpendicular to a chord bisects the chord.

(b) Chords equidistant from the centre of the circle



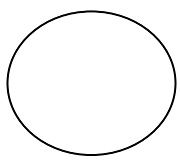
Two chords which are equidistant or have the same distance from the circle are equal.
Since OP = OQ then AB = CD.

You also have two equal chords subtending equal angles at the centre of the circle.



Since AB = CD the <AOB = <COD

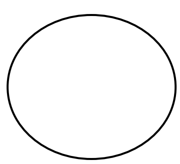
The equal chords also make a subtends equal arcs.



Arc APB = Arc CQD since AB = CD from our earlier explanations.

Examples

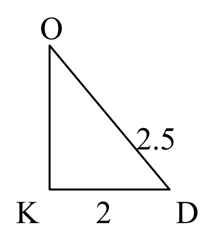
(1) In the circle below we have two chords



AB = CD which are 4cm and radius is 2.5cm. Find OK.

Solution

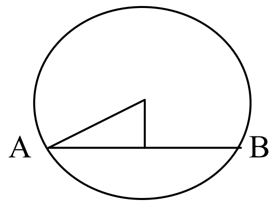
Use triangle OKD



Use Pythagoras to find OK

$OK^2 + 2^2 = 2.5^2$
$OK + 4 = 6.25$
$OK = 6.25 - 4$
$OK = 2.25$

(2) Find the length of the chord which is 10cm from the centre of a circle with radius 16cm.



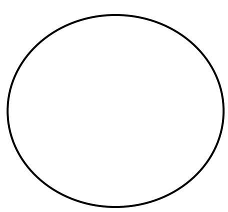
Using Pythagoras we find AC and multiply by 2

$16^2 = AC^2 + CO^2$
$16^2 = AC^2 + 10^2$
$16^2 - 100 = AC^2$
$AC^2 = 256 - 100$
$AC^2 = 156$
$AC = \sqrt{156}$
$AC = 12.49$cm

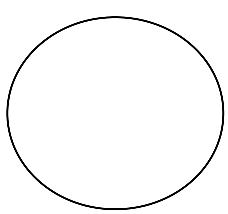
Activity 1

(1) Find how far the chord is from the centre of the circle whose radius is 26cm and the chord is 20cm.

(2) Find AB



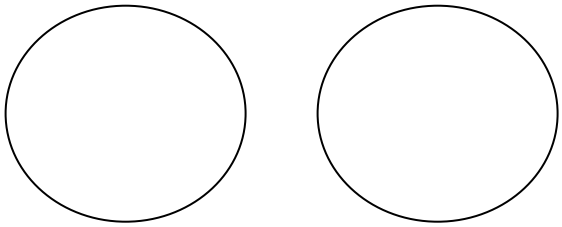
(3) In this diagram ON = OM = 4m. PN = 7m. Find RS



(4) A circle centre O has a chord 8cm long. It is 1cm from the centre what is the radius of the circle.

Topic 3 The properties of the angles of a circle

(a) Angle at the centre is twice the angle at the circumference. Let us look at the circle below.



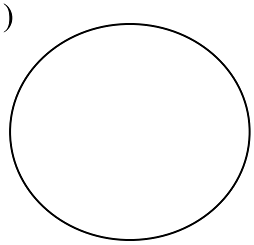
Obtuse angle POX = 2x
= 2(100)
PRX = ½ x 200
= 100^0

The two angles should touch the ends of the same chord.

Example

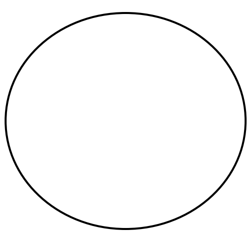
Find y

(1)



y = 2 x 20
y = 40^0

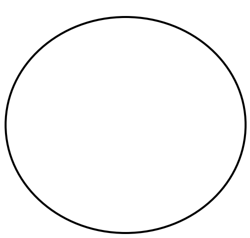
(2)



First find the angle at the centre by considering ΔOBC which is an isosceles triangle.
180 – (35 + 35)
180 – 70 = 110

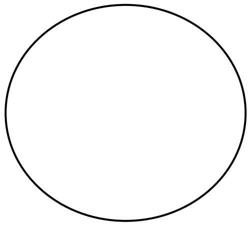
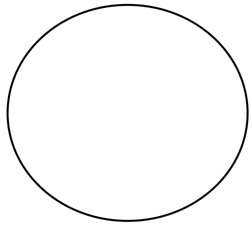
So the angle at the centre is 110^0. Therefore, on the circumference $\frac{110}{2} = 55^0$

(b) The angle in the semi circle is 90



If XZ is a diameter then <XYZ = 90^0

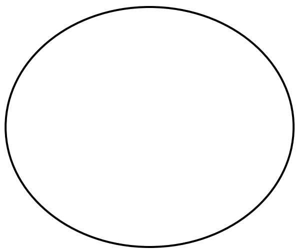
(c) Angles in the same segment are equal.

Angle AOE = $2y^0$

Example

If the diagram QR is a diameter, O is the centre. Find a and b.



Solution

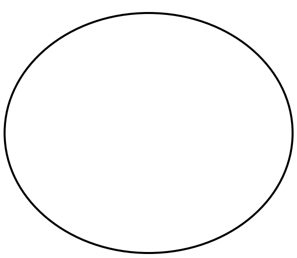
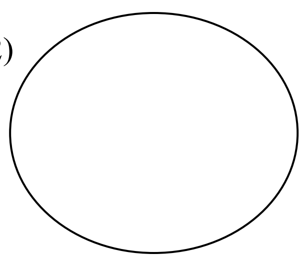
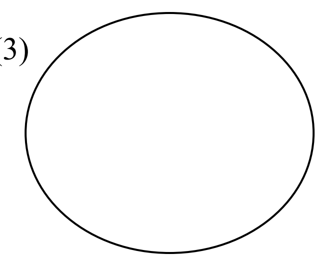
Using the property of angle at the centre is twice the angle on the circumference. We have 60^0 at the centre and so $a^0 = \frac{60^0}{2}$ $a = 30^0$.

For b, we use angles in the same segment <PRQ is in the same segment with <PSQ from ΔROP (Isosceles) <PRO = 60^0

Therefore, <PSQ = b = 60^0.

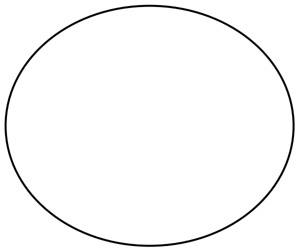
Activity 2

Find the angle marked Z.

(1)  (2)  (3) 

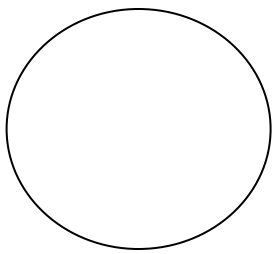
Topic 4 Cyclic Quadrilateral

A cyclic quadrilateral is one which has all its vertices on the circumference of a circle.



ABCD is an example of a cyclic quadrilateral.

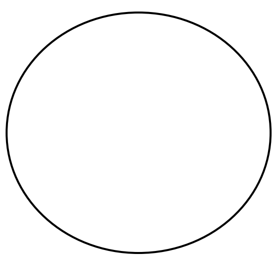
(a) Opposite angles of a cyclic quadrilateral are supplementary.
Supplementary means two angles adding up to 180^0



$x^0 + y^0 = 180^0$
$a^0 + b^0 = 180^0$

Examples

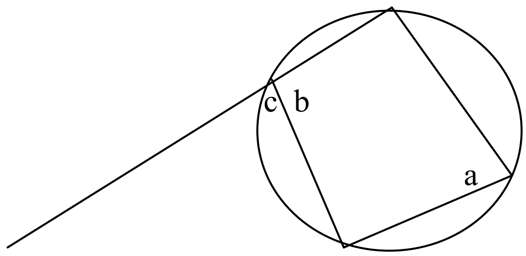
(a) Find angles marked p, 2p and K.



$2p + 70^0 = 180^0$ (Opposite angle in a cyclic quadrilateral add up to 180^0)
$2p = 180^0 - 70^0$
$\frac{2p}{2} = \frac{110^0}{2}$
$P = 55^0$

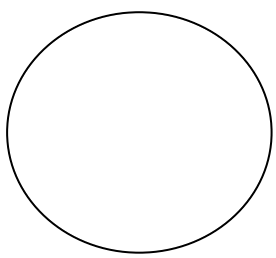
The $P + K = 180^0$
$B5 + k = 180^0$
$k = 180^0 - 55^0$
$K = 125^0$

(b) Exterior angle of a cyclic quadrilateral is equal to the opposite interior angle.



(opposite angles of cyclic quadrilateral)
a + b = 180 and b + c = 180 (straight line angle)
b = b
Therefore, a = c

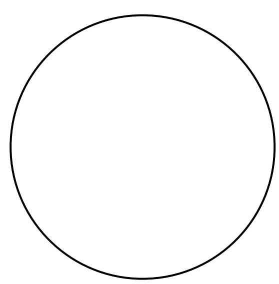
Example



Find P.
P = <CDX
CDX = $180^0 - 80^0$
CDX = 100^0
Therefore, P = 100^0

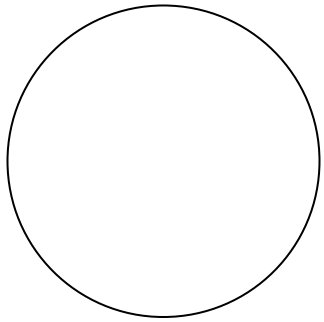
Activity 3

(1) Find the angle marked Z.

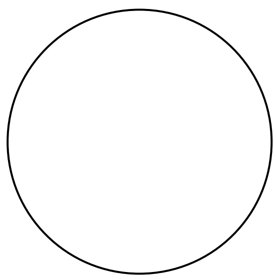


In the figure above ABCD is a cyclic quadrilateral. O is the centre, DC produced meets X. If <XCB = 66^0 and <CBO = 58^0

(2) If <SRT = 15^0 and <PQR = 40^0, Find Z

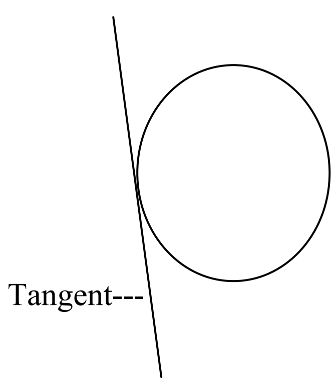


(3) The figure below has ABCD on the circumference. AD = DC and $<DOC = 25^0$. find z and y

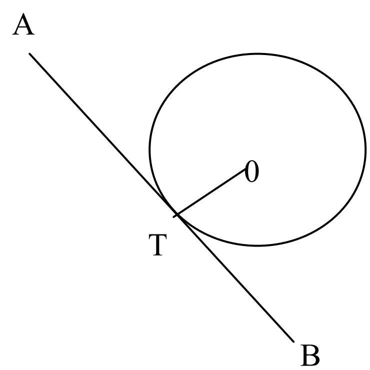


Topic 5 Properties of the tangent to a circle.

A tangent is a line that touches a circle at one point.

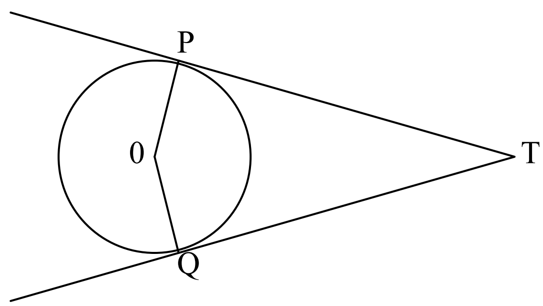


(a) A tangent to a circle is perpendicular to the radius, when its drawn at its point of contact.

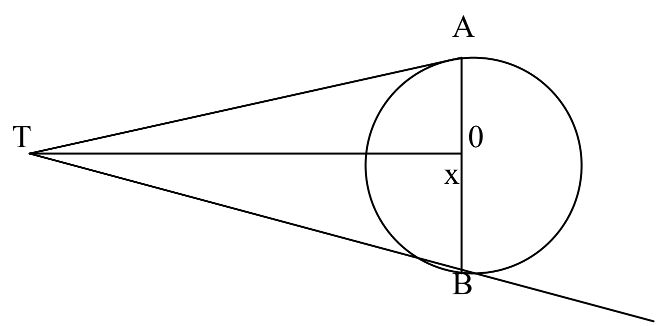


(b) Tangents from an external point are equal upto the point of contact with the circle.

PT = QT



Example

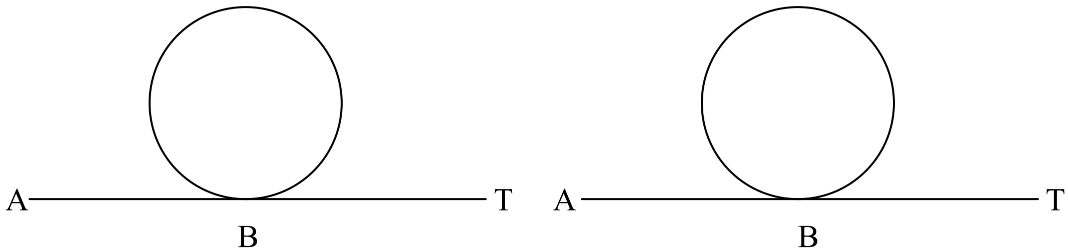


Find <TAX if <ATO = 35 and TA, TB are tangents where O is the centre of the circle.

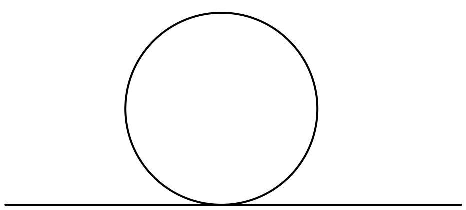
Solution.
A x T = 90
Therefore, TAX = 180 – (90 + 35)
= 180 – 125
TAX = 55

(c) Alternate segments

Let us have a look at the circles below

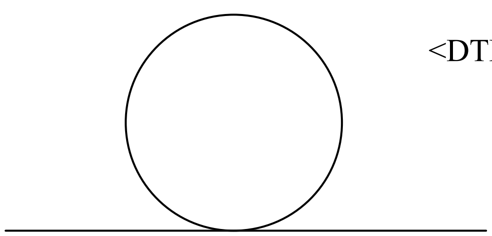


These circles have tangents at B. CB is a chord when it divides the circle into two segments BCX and BCY. Segment BCX is alternate to angle TBC. This means that all angles formed in segment BCX are equal to angle BTC.



Example

In the diagram TE is a tangent.



<DTE = 80 and BD = BT. Calculate <BAT

Using alternate segments <BDT = 80 and so $BTD = \frac{180 - 80}{2}$

$$= \frac{100}{2}$$

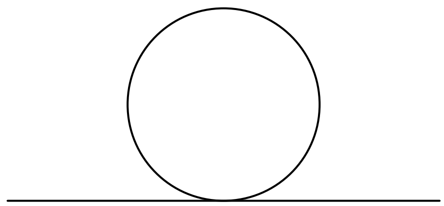
$$= 50$$

Again using alternate segment TAB and angle BTE.

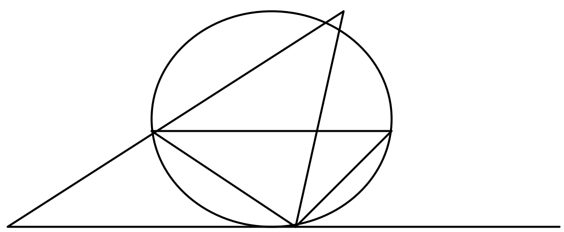
We find <BTE = 80 + 50 = 130. So BAT = 130

Activity 4

(1) In the diagram find QRT, RQP = 21



(2) In the diagram, find <PQT and <PUQ where PRQ = 30 and <PTQ = 42



Summary

This has been an interesting unit where you have learnt various properties of circle in relation to angles you looked at chord properties, angle properties, tangent properties and also the cyclic quadrilateral. Remember to understand all these properties so that you can answer any question on circle theorem.

UNIT ELEVEN

MENSURATION

Introduction

Welcome to the next unit of this module. In the previous unit you discussed the angle properties of a circle and polygons. In this unit you shall look at mensuration.

In this unit we shall look at 3 topics:

- Perimeter and areas of plane figures.
- Area and circumference of a circle.
- Surface area and volumes of three dimensional figures.

You would be expected to study a minimum of eight to nine hours after having carried out all the activities in this unit.

Objectives

By the end of this unit you should be able to:

a) find perimeter and area of plane figures;
b) calculate surface areas and volumes of three dimensional figures; and
c) calculate circumference and area of circles.

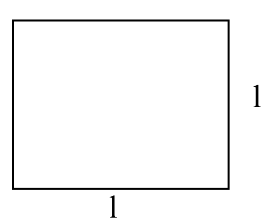
Topic 1 Perimeter and Area of Plane figures.

You looked at some perimeters and areas of plane figures in Grade 8 and 9. In this unit you shall revise these formulae and then extend our knowledge to other more complex, geometrical figures.

Note that in all our calculations in this unit we should take π as 3.142

1.1. **Rectangles and Squares**

(a)



(b)

In figure 1, we see that (a) is a rectangle in which the length is l and breadth is b and (b) is a square of side l.

Let us use P to represent Perimeter and A to represent Area, then

Perimeter of rectangle	$P = 2(l + b)$
Perimeter of square	$P = 4l$
Area of rectangle	$A = lb$
Area of square	$A = l^2$

1.2 Triangles

Triangles can be in any of the following forms.

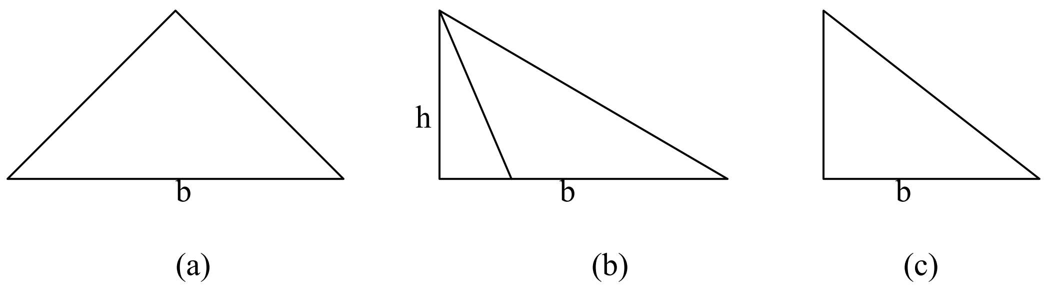


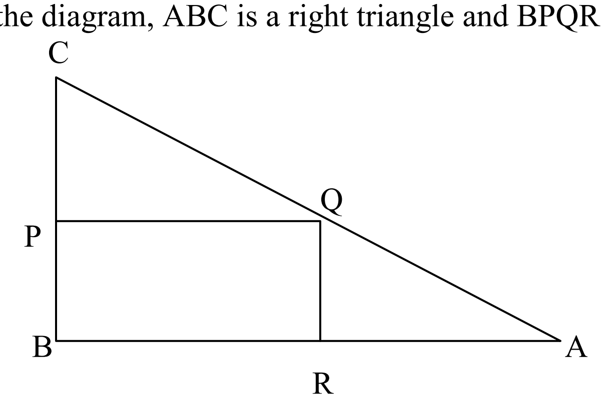
Figure 2

All the triangles in figure 2, have h as the height which is perpendicular to the base b. the perimeter of a triangle is the sum of all the three sides. To find the area of a triangle, the height and the base must be either given or calculatable. The base is the side from which the perpendicular height is measured.

Area of triangle = ½bh.

Example 1

In the diagram, ABC is a right triangle and BPQR is a rectangle.



AR = BR = 5cm and BP = PC = 3.5cm. Calculate

(a) The perimeter and area of BPQR
(b) The perimeter and area of triangle ABC.

Solution

(a) Perimeter of BPQR $= 2(l + b)$
$= 2(5 + 3.5)$
$= 2(8.5)$
$= 17cm$

Area of BPQR $= lb$
$= 5 \times 3.5$
$= 17.5cm^2$

(b) Perimeter of $\Delta ABC = AB + BC + CA$
i.e. $AB = 2BR$
$= 2(5)$
$= 10cm$

$BC = 2BP$
$= 2(3.5)$
$= 7cm$

$CA = \sqrt{AB^2 + BC^2}$ Since ΔABC is right angled. Hence we use the Pythagoras Theorem.

$= \sqrt{10^2 + 7^2}$
$= \sqrt{100 + 49}$

$= \sqrt{149}$
$= 12.21cm$

Perimeter of ΔABC $= 10 + 7 + 12.21$
$= 29.21cm$

Area of $\Delta ABC = \frac{1}{2} bh$
$= \frac{1}{2} \times 10 \times 7$ ($b = AB$, $h = BC$)
$= 35cm^2$

1.3 Parallelogram

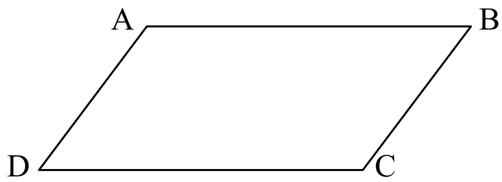


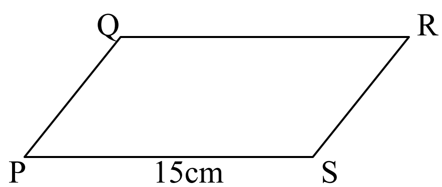
Figure 3

Figure 3, shows a parallelogram. OABC of height h and base b.
Area of a parallelogram is given by the formula

Area = base x height
= bh

Example 2

The diagram shows a parallelogram PQRS with sides of 15cm and height of 6cm. Find its area.



Solution

We have been given that b = 15cm and h = 6cm. Therefore, A = 15 x 6 = 90cm^2.

1.4 Trapezium

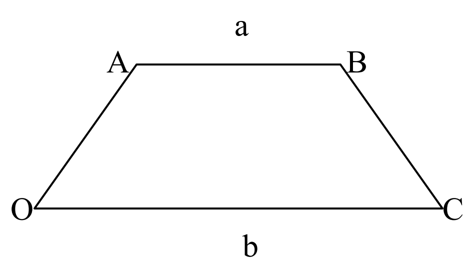


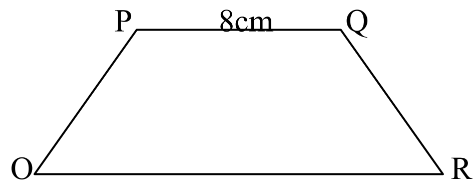
Figure 4

Figure 4 shows a trapezium OABC in which h is the height and b as the base.

The area of a trapezium is given by the formula
Area = ½ x sum of the parallel sides of height.
$A = \frac{1}{2}(a + b)h$

Example 3

The diagram shows the trapezium OPQR in which the parallel sides are 8cm and 12cm long and height is 5cm.



Solution

$A = ½ (a + b)h$
$= ½ (8 + 12)5$
$= ½ (20)5$
$A = 50cm^2$.

Activity 1

1. A rectangle has length 12cm and breadth 8cm. Find (a) its perimeter (b) its area.
2. The length of a square is 4cm. Calculate (a) the perimeter (b) the area.
3. The area of a square is $256cm^2$. Find the length of its diagonal correct to 1 decimal place.
4. An isosceles triangle has length of 4cm long and the third side 10cm. Calculate (a) its perimeter (b) its area.
5. The sides of a parallelogram are 15cm and 12cm. If the height is 10cm long, find the distance between the sides which are 12 cm long.
6. Calculate the area of a trapezium whose parallel sides are 12 cm and 16 cm long. The height between the parallel sides is 10cm.

Topic 2 Area and Circumference of a Circle

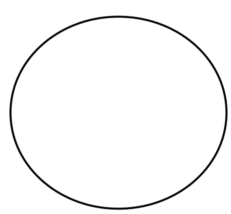


figure 5

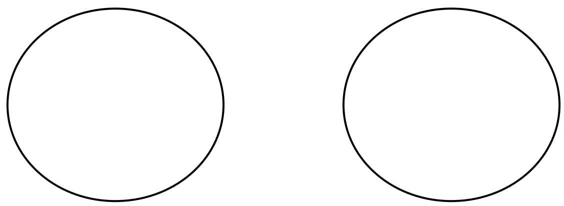
Figure 5 shows a circle with radius r and centre O. The distance around a circle is called the circumference and it is denoted by C.

Therefore, the circumference of a circle is calculated using the formula

$C = 2\pi r$ or $C = \pi d$, where d is the diameter and $\pi = 3.142$ or $\frac{22}{7}$

Therefore, the area of a circle is calculated using the formula $A = \pi r^2$

1.5 Sectors and Segments



(a) (b)

Figure 6

In figure 6 (a) OAXB is the sector of the circle with radius r and centre O. The angle of the sector is marked x.

1.5.1 Area of Sector

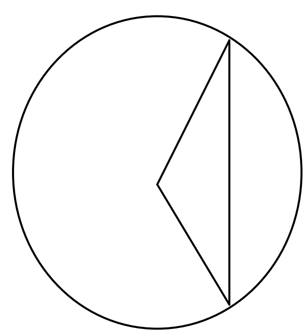
The shade area of a sector is a portion of the area of a circle. We also know that a complete circle is 360^0 (a revolution). The fact that the sector is x then we are saying $\frac{x}{360^0}$ is the portion of a revolution. Therefore the area of Sector = $\frac{x}{360^0} \times \pi r^2$ (Area of circle).

Figure 6(b) shows a segment A x B of the circle with radius r and centre O. Area of segment = Area of sector OA x B – Area of ΔOAB.

Example 4

The diagram shows a circle with radius 7cm and the sector OXYZ which subtends an angle of 150^0 at the centre O. The length of the chord XZ is 10cm. Calculate

(a) The circumference of the circle
(b) The area of the circle
(c) The Area of the sector OXYZ
(d) The area of the segment XYZ



Solution

(a) Circumference = $2\pi r$
$= 2 \times 3.142 \times 7$
$= 14 \times 3.142$
$= 43.988$cm.

(b) Area $= \pi r^2$
$= 3.142 \times 7 \times 7$
$= 3.142 \times 49$
$= 153.958$cm.

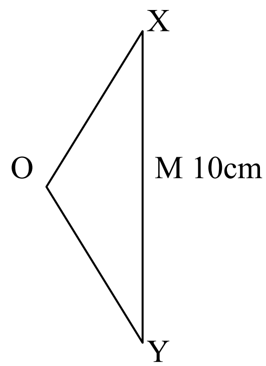
(c) Area of sector OXYZ = $\frac{X^0}{360^0}$ x πr^2

$= \frac{150}{360}$ x 153.958

$= 64.149cm^2$

(d) Area segment XYZ = Area of circle – Area of triangle OXY

Area of ΔOXY



We see that Area of <XYZ = 2 x area of <OXM

Area of <OXM = ½ $\left(\frac{XY}{2}\right)$ x h

XY = 10cm and XM = 5cm

Area of sector OXYZ = $64.149cm^2$ and we can calculate the value of h as follows:

$h = \sqrt{7^2 - 5^2}$

$\sqrt{49 - 25}$

$\sqrt{24}$

$4.899cm^2$.

Now that we have all the information required to calculate the area of segment XYZ. Thus,

Area of ΔOXM = ½ x 5 x 4.899

$= 12.248cm^2$.

Area of ΔOXZ = 2 x $12.248cm^2$

$= 24.496cm^2$.

Therefore, area of segment XYZ = $(64.149 – 24.496)cm^2$

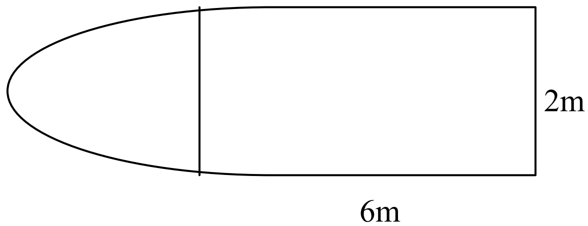
$= 39.653cm^2$.

Example 5.

The diagram shows a vegetable bed consisting of a rectangle and a semi circle. The length of a rectangle is 6cm and the breadth is 2m. Calculate

(a) the perimeter of the vegetable bed

(b) the area of the vegetable bed.



Solution

(a) The perimeter is the sum of three sides of the rectangle and half the circumference.

Perimeter $= 6 + 2 + 6 + (\frac{1}{2} \times 2\pi r)$
$= 6 + 2 + 6 + (\frac{1}{2} \times 2 \times 3.142 \times 1)$
$= 14 + 3.142$
$= 17.142$cm

(b) Area of the vegetable bed = Area of rectangle + Area of semi - circle

Area of rectangle $= 6 \times 2$
$= 12m^2$.

Area of semi circle $= \frac{1}{2}\pi r^2$
$= \frac{1}{2} \times 3.142 \times 1 \times 1$
$= 1.571$m

Therefore, area of vegetable bed $= 12 + 1.571$
$= 13.571m^2$

Activity 2

(in this exercise take $\pi = 3.142$)

1. The circumference of a circle is 94cm. Find the radius correct to 2 decimal places.
2. Find the area of a circle with radius 6cm.
3. The area of a circle is $29.4cm^2$. Find its radius.
4. A fruit garden is in the form a circle of radius 6m. It is surrounded by a circular path 2m wide. Sketch the fruit garden and its path and then calculate
   (a) The area of the fruit garden
   (b) The area of the path.
5. An arc of a circle of radius 7cm subtends an angle of 108^0 at the centre of a circle.
   (a) Calculate the area of the sector bounded by the arc and two radii, giving your answer correct to 2 decimal places.
   (b) If the chord joining the two end points of the arc is 8cm long, find the area of the segment bounded by the arc and the chord. Give your answers correct to 3 decimal places.

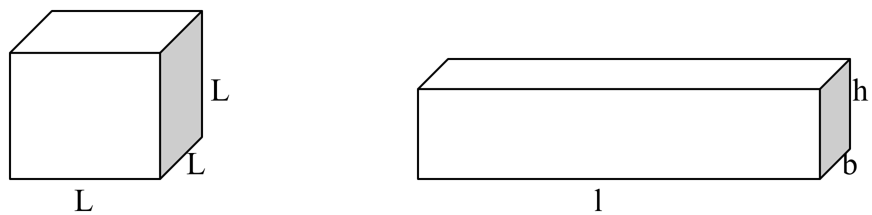
Topic 3. Surface Area and Volumes of Solids

Any geometric figure with three dimensions is called a solid. Some examples of solids we dealt with them in Grade 8 and 9. These include cubes, cuboids, pyramids and spheres. In our study we shall look at solids such as prisms and cylinders.

The surface area of any solid refers to the outer area of that particular solid.

<u>Prisms</u>

What is a prism? This is a solid which has two end faces that are congruent and parallel and the other faces are parallelograms. Examples of prisms are cubes, cuboids, triangular prisms and hexagonal prisms.



(a) Figure 7 (b)

Figure 7 (a) shows a cube with faces of side l. A cube has 6 equal faces and the area of each face = l^2. Therefore, total surface area of cube = 6 x l^2 = $6l^2$.
The volume of a cube is $V = l^3$.

Figure 7(b) shows a cuboid of length l, breadth b and height h. A cuboid has three pairs of congruent faces. The area of these three pairs of faces are as follows:

Area of 1st pair = 2hb
Area of 2nd pair = 2bl
Area of 3rd pair = 2hl

Therefore, total surface area = 2hb + 2bl + 2hl
= 2(hb + bl + hl)

The volume of a cuboid is V = lbh.

Example 6

The sides of a cube is 4cm. Find
(a) Its total surface area
(b) Its volume

Solution

(a) Total surface area = $6l^2$.
$= 6 \times 4 \times 4$
$= 96cm^2$.

(b) Volume = l^3
$= 4 \times 4 \times 4$
$= 64cm^3$.

Example 7

A box has length 25cm, breadth 9cm and height 8cm. Find:

(a) Its total surface area
(b) Its volume

Solution

(a) Total surface area = $2(lb + bh + lh)$
$= 2(25 \times 9 + 9 \times 8 + 25 \times 8)$
$= 2(225 + 72 + 200)$
$= 2(497)$
$= 994cm^2$.

(b) Volume $= lbh$
$= 25 \times 9 \times 8$
$= 1800cm^3$.

Triangular Prism

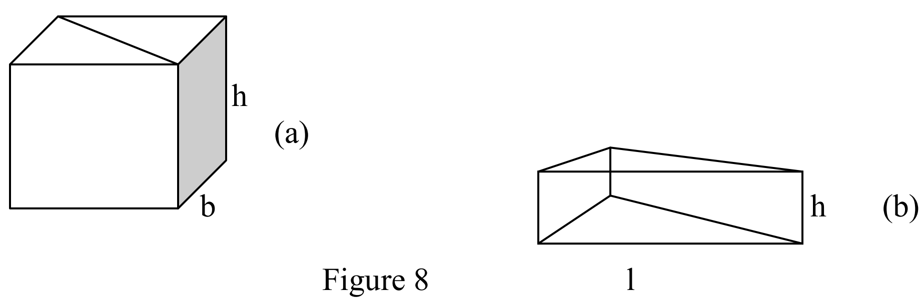


Figure 8

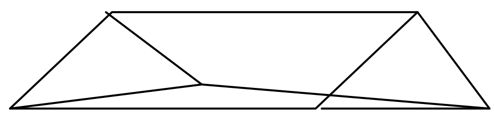
Figure 8 shows a cuboid which has been cut across one of the diagonal planes to give a triangular prism. A triangular prism has two congruent triangular faces and three rectangular faces. The total surface area of a triangular prism is the sum of the areas of the triangles and the three rectangles.

The volume of the triangular prism in figures (b) is half the volume of the cuboid in figure (a).
i.e. $V = \frac{1}{2}lbh$
$= \text{Area of triangular base x height}$

In general, the area of a triangular prism is given by the formula

$V = \text{Area of base x height}$

h **Figure 9** In this case, the base is a triangle

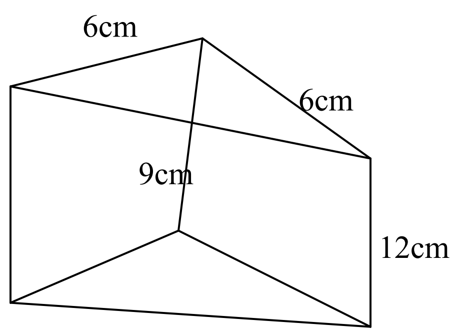


Example 8

A prism has a base in the form of an isosceles triangle with two equal sides of 6cm each and the third side 9cm long, the height of the prism is 12cm. Calculate

(a) the volume
(b) the total surface area of the prism.

Solution



(a) Volume = Area of base x height of prism
Let the height of triangle be x. Then

$X = \sqrt{6^2 - 4.5^2}$
$= \sqrt{36 - 20.25}$
$= \sqrt{15.75}$
$X = 3.97\text{cm}$

Area of base $= \frac{1}{2} \times 9 \times 3.97$
$= 17.865\text{cm}^2$

Therefore, volume = Area of base x h
= 17.865 x 12
= 214.38cm^3.

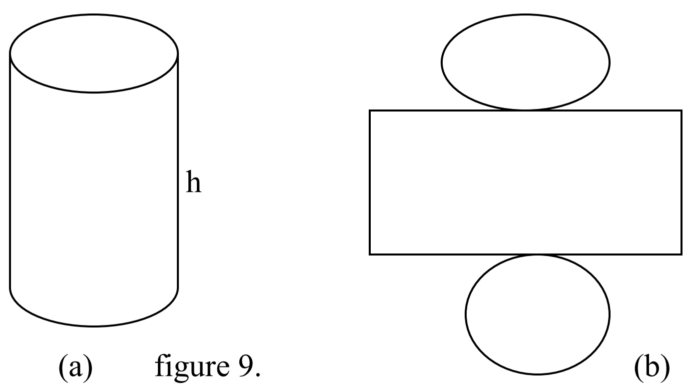
(b) Total surface area = Area of two Triangles + Area of three rectangles.
Area of two Triangles = 2 (½ x 9 x 3.97)
= 35.73cm^2.

The total area of the two equal rectangles are
Area = 2(12 x 6)
= 2 x 32
= 64cm^2.

The area of the third rectangle
Area = 9 x 12
= 108cm^2.

Therefore, total surface area = 35.73 + 64 + 108
= 207.73cm^2.

2.2 Cylinders



(a) figure 9. (b)

Figure 9 (a) shows a closed cylinder and figure 9 (b) shows the net of the cylinder.

The curved surface area of a cylinder = $2\pi rh$
The total surface area of a closed cylinder = $2\pi rh + 2\pi r^2$
The volume of a cylinder = $\pi r^2 h$.

Example 9

A closed cylinder has height 6cm and the diameter of the base is 12cm. Calculate

(a) its curved surface area
(b) its total surface area
(c) its volume

Solution

(a) Curved surface area $= 2\pi rh$
$= 2 \times 3.142 \times 6 \times 6$
$= 226.224cm^2$.

(b) Total surface area $= 2\pi r^2 + 2\pi rh$
$= 2\pi r\,(r + h)$
$= 2 \times 3.142 \times 6\,(6 + 6)$
$= 452.448cm^2$.

(c) Volume $= \pi r^2 h$
$= 3.142 \times 6 \times 6 \times 6$
$= 678.672cm^3$.

Activity 3

1. A cuboid is 5cm long, 3cm broad and 4cm height. Calculate
   (a) The total length of its edges.
   (b) Its total surface area
   (c) Its volume

2. The faces of a cube are of side 6cm long. Calculate
   (a) The total length of the edges of the cube
   (b) Its total surface area
   (c) Its volume

3. A triangular prism has a base in the form of right-angled triangle, with sides 8cm, 10cm and 12cm. If the height of the prism is 6.5cm sketch the prism and calculate

   (a) its total surface area
   (b) its volume

4. A cylinder has radius x and height z. Write down the formula for each of the following:

   (a) the area of one end
   (b) the total outside area if the cylinder is open at one end
   (c) the total surface area if both ends are closed

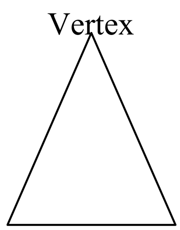
5. A cylinder has base radius 6cm and volume $296cm^3$. Find its height.

6. A closed cylindrical tank can hold 10,000 litres of water when quarter full. If the radius of the tank is 120cm, calculate its surface area in cm^3.

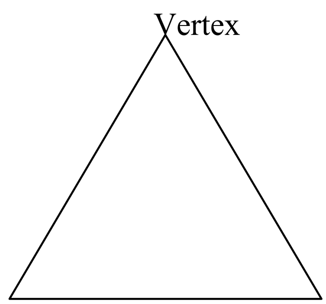
3.0 Pyramids and Cones

3.1 Pyramid

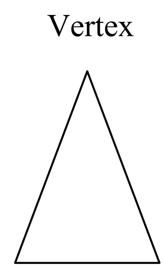
This is a solid with a base in the form of a polygon and faces in triangular form and they all meet at one point. The points where the faces meet is called the vertex. Let us look at examples of pyramids.



This is a square base pyramid



This is a rectangular base Pyramid



This is a triangular base pyramid and it has a special name called tetrahedrom

Volume of a pyramid

Volume is got by getting $^1/_3$ of the area of the base of a pyramid and multiplying by the height.

V = $^1/_3$ x area of base x height

3.2 <u>Cones</u>

A cone is a special kind of a pyramid with a circular base.

Volume = $^1/_3$ x area of base

We can also get the curved surface area of a cone using the formular.

Curved surface area of cone = πrS. S is the slant height.

Examples

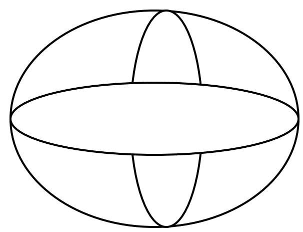
(1) Calculate the volume of a square base pyramid with sides 4cm and height 6cm.
V = base area x height
= 4 x 4 x 6
= 16 x 6
= 96cm^3.

(2) A given cone has height 16cm and radius of base 12cm. Calculate, its slant height, volume and the curved surface area.

$S^2 = 16^2 + 12^2$	Volume	Curved Surface area
$S^2 = 256 + 144$	V = $^1/_3$ base area + height	= πrS
$S^2 = \underline{400}$	V = $^1/_3\,\pi r^2h$	= 3.142 x 12 x 20
$S = \sqrt{400}$	= $^1/_3$ x 3.142 x 12^2 x 16	= 754.29cm^2.
S = 20cm	V = 2413.7cm^2.	

4.0 The Sphere

A sphere is a solid which has all the points on its surface equidistant from the centre. As in the figure below.



Since the methods of deriving the formula for surface area and volume is beyond the scope of the module. We shall accept these formulae as given and simply apply them in our examples.

The surface area of a sphere is given by the formula $A = 4\pi r^2$
The volume of a sphere is given by the formula $V = {}^4/_3\, \pi r^3$

Example

A sphere has radius 6cm. Calculate its (a) surface area and (b) volume.

Solution

(a) Surface area $= 4\pi r^2$
$= 4 \times 3.142 \times 6 \times 6$
$= 452.448cm^2$.

(b) Volume $= {}^4/_3\, \pi r^3$
$= {}^4/_3 \times 3.142 \times 6 \times 6 \times 6$
$= 904.896cm^3$.

Example

Find the radius of a sphere of volume $40cm^3$.

Solution

$V = {}^4/_3\, \pi r^3$
$40 = {}^4/_3 \times 3.142 \times r^3$
$r^3 = \dfrac{40 \times 3}{4 \times 3.142}$

$r^3 = 9.548$
$r = \sqrt[3]{9.548}$
$r = 3.183$cm.

Activity 4

1. A pyramid has a right-angled triangular base and a volume of $125cm^3$. If the shorter sides of the base are 4cm and 8cm long, find the height of the pyramid correct to 2 decimal places.

2. A cone has a base of radius 6cm and slant height of 24cm. Calculate
   (a) its height
   (b) its total surface area including the base
   (c) its volume

3. Calculate the radius of a sphere of surface area $651.3cm^3$ correct to 3 significant figures.

Summary

We have just finished discussing area and volume of solids.

The following are the formulae we have discussed:

Shape/Solid	**Perimeter/Area**	**Volume**
Rectangle	$P = 2(l + b)$ $A = lb$	$V = lbh$ (cuboid)
Square	$P = 4l$ $A = l^2$	$V = l^3$ (cube)
Triangle	P = sum of three sides $= x + y + z$ $A = \frac{1}{2} bh$	V = Area x height (Prism)
Parallelogram	$A = bh$	-
Trapezium	$A = \frac{1}{2}(a + b)h$	-
Circle	$P = C = 2\pi r$ $A = \pi r^2$	-
Cylinder	T.S.A. $= 2\pi r(h + r)$ (closed) $= 2\pi rh + \pi r^2$ (open)	
Pyramid		$V = \frac{1}{3}$ x Area of base height
Cone	$A = \pi rS + \pi r^2$	$V = \frac{1}{3}$ x Area of base
Sphere	$A = 4\pi r^2$	$V = \frac{4}{3}\pi r^3$

UNIT TWELVE

LOCUS

Introduction

Locus is a topic which involves a lot of constructions. One needs to recall the simple constructions learnt in grade 10. We shall look at locus in two dimension and also three dimension. The plural for locus is loci.

This unit has two topics:

- Locus in two dimension
- Locus in three dimension

If you do all activities in this unit, you will need seven to eight hours. It is also possible to do all activities in less or more hours. This should not worry you as what is important is to understand the concepts of property.

Objectives

By the end of this unit, you should be able to:

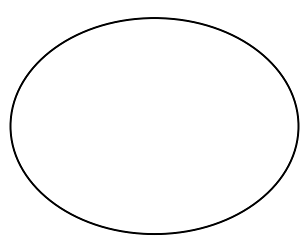
a) construct locus of points in two dimension; and
b) identify and describe the locus of points in three dimension.

Topic 1 Locus in two Dimension

(a) Locus of points equidistant (same distance) from a point. This is a circle.

Example

Draw the locus of points (P) 3cm from a point A. This is a circle with radius 3cm and centre A.



The distance of P_1, P_2, P_3 from centre A is 3cm. In set builder notation is written as {P:AP = 3cm} where A is a fixed point and P is the moving point.

(b) Locus of points equidistant from two points

The locus of points equidistance from two points say A and B is simply a perpendicular bisector of the line joining A and B.

Using construction rules, bisect the line.

Locus of points equidistant from two points A and B.

If you pick any point on the locus, the distance from that point to A will be equal to the distance from B. For example, for point Q, AQ = BQ

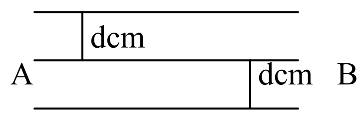
(c) Locus of points equidistant from two intersecting lines

Example

The locus of points equidistant from two intersecting lines is simple, the bisector of the angles formed by the two lines. If the two lines are RO and OP bisect angle O or <ROP. If the two lines are RO and OQ bisect angle O or <ROQ and this line will be perpendicular to OK.

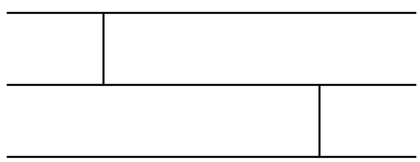
(d) Locus of points equidistant from a straight line.

This is a pair of two parallel lines to the given straight line



Example

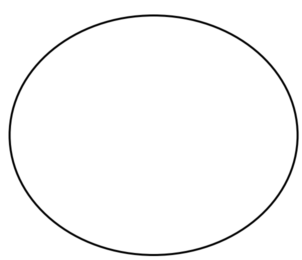
Draw the locus of points 4cm from a line AB



You construct two parallel lines to the line AB with compass radius 4cm.

(e) Constant angle locus

These are angles in the same segment. If you recall under circle properties we learnt of this property say that angles in the same segment are equal.



Example

Construct a triangle ABC in which AB = 10cm, BC = 6cm and angle ABC = 70^0. Then construct a circumcircle of the triangle ABC. A circumcircle is a circle that pass through all the vertices of a triangle.

Take ADC to be the major arc and ABC = 70^0. All angles in this segment will be 70^0 e.g. ADC.

Activity 1

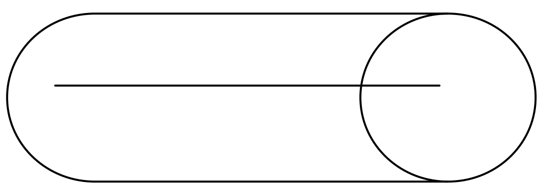
(1) Draw the locus of points which a 5cm from a fixed point A.
(2) Draw or construct the locus of 3cm equidistant from and straight line AB
(3) Construct the locus of point equidistant from two points X and Y.
(4) On the line AB which 5cm long construct the locus of points which subtend an angle of 30^0 on AB.
(5) Draw the triangle XYZ with XY = 10cm and <ZXY = 35 and <ZYX = 45. Draw the locus of points with 6cm from X. Lable the points of interTopic between the two loci as R and S.

Topic 2 Locus in three dimension

It is not easy to draw the locus in three dimensions. Some loci in this dimension form the Surface of shapes which we can recognize. In two dimension the locus from a fixed point is a circle. In three dimension the locus of points equidistant from a fixed point is a sphere.

This is the locus of points rcm from a fixed pint in three dimension. It is a sphere with radius rcm.

The locus of point equidistant from a straight line in three dimension is a cylindrical surface while in two dimension it’s a pair of parallel lines.



This shape is the locus of points equidistant from a given straight line in three dimension. It is a cylindrical shape.

Summary

In this unit you have looked at locus in two dimension and locus in three dimension. Remember from a fixed point in two dimension it is a circle while in three dimension it is a sphere. Also from a fixed line in two dimension it is a pair of parallel lines to a fixed line while in three dimension it is a cylindrical surface. From two intersecting lines, you bisect the angles, the two lines are formed.

Hope you enjoyed the unit. After you have mastered all the topics in grade 10 and 11 modules then you are ready to proceed to grade 12 modules. Keep it up. You have come all this way.