PASS MATHEMATICS WITH A DISTINCTION

MATHEMATICS SYLLABUS D (4024/2) EXAMINATION QUESTIONS FROM (2014-2017) WITH ANSWERS:

Mathematical Formulae

$$y = ax^n \; then \; \frac{dy}{dx} = nax^{n-1}$$

$$\int ax^n dx = \frac{ax^{n+1}}{n+1} + c$$

$$\Delta \textbf{ABC}: \frac{a}{sinA} = \frac{b}{sinB} = \frac{c}{sinC}$$

$$\Delta\textbf{ABC}: a^2 = b^2 + c^2 - 2bcCosA$$

$$\textbf{Mean} = \bar{x} = \frac{\sum fx}{\sum f}, \quad SD = \sqrt{\left\{\frac{\sum f(x-\bar{x})^2}{\sum f}\right\}} = \sqrt{\left\{\frac{\sum fx^2}{\sum f} - (\bar{x})^2\right\}}$$

$$\textbf{AP}: \quad T_n = a + (n-1)d, \qquad \textbf{GP}: T_n = ar^{n-1}$$

$$\textbf{AP}: \boldsymbol{S_n} = \frac{n}{2}[2a + (n-1)d], \quad \textbf{GP}: \boldsymbol{S_n} = \frac{(1-r^n)}{1-r}, \; r < 1$$

Contents: Sets, Probability, Sequences and Series, Pseudo code & flow charts, Vectors, Trigonometry, Mensuration, Earth Geometry and Statistics.

Compiled & Solved by Kachama Dickson. C

Contacts: 0966295655/0955295655

Email: kachamadickson@gmail.com

Kachama Dickson C is a final Year student at the Copperbelt University (CBU) who is currently pursuing a bachelors of Science in Mathematics education (BSc.MA.Ed).To show his eligibility and competence in mathematics, he has produced many pamphlets for mathematics with this being one of his best pamphlets.

This pamphlet consists of final Examination questions from 2014-2017 for both School Certificate and General Certificate ordinary levels. Answers with all necessary working methods are shown at the end of questions.

"I believe, this pamphlet will be of great help to you even as you prepare for your final Examinations:"

CONTENTS

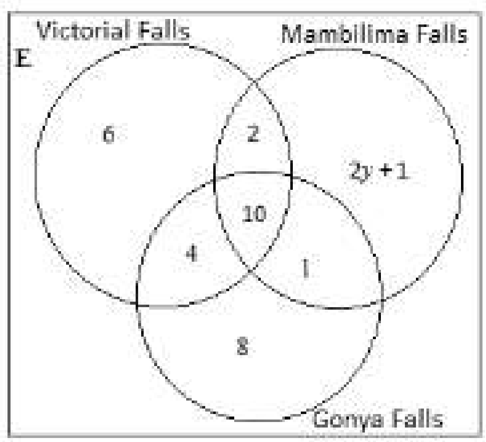
TOPIC 1: SETS

[2017 P2 NOVE, Q1 (b)]

1. A survey carried out at Kamulima Farming Block showed that 44 farmers planted maize, 32 panted sweet potatoes, 37 planted cassava, 14 planted both maize and sweet potatoes, 24 planted both sweet potatoes and cassava, 20 planted both maize and cassava, 9 planted all the three crops and 6 did not plant any of these crops.
   (i) Illustrate this information on a Venn diagram.
   (ii) How many farmers
      (a) Where at this farming block,
      (b) Planted maize only
      (c) Planted two different crops

[2017 P2 JULY, Q3 (b)]

2. The Venn diagram below shows tourist attractions visited by certain students in a certain week.



(i) Find the value of y if 7 students visited Mambilima Falls only.
(ii) How many students visited
   (a) Victoria falls but not Gonya Falls,
   (b) Two tourist attractions only,
   (c) One tourist attraction only?

[2016 P2 NOV, Q2 (b)]

3. Of the 50 villagers who can tune in to Kambani Radio Station,29 listen to news,25 listen to sports, 22 listen to music, 11 listen to both news and sports,9 listen to both sports and music,12 listen to both news and music,4 listen to all the three programmes and 2 do not listen to any programme.

(i) Draw a Venn diagram to illustrate this information.

(ii) How many villages

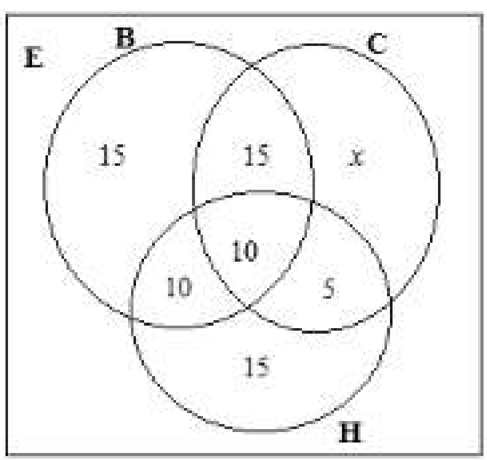
(a) Listen to music only,

(b) Listen to one type of programme only

(c) Listen to two types of programs only

[2015 P2 NOV, Q4]

4. The Venn diagram below shows the number of students who took Business (B), Human Resources (H), and community development(C) at Mafundisho College. 100 students took all these three courses.



(a) Find

(i) the value of x

(ii) the number of students who took Human Resources,

(iii) $n(B \cap C) \cap H'$

(iv) $n(B \cup C) \cap H'$

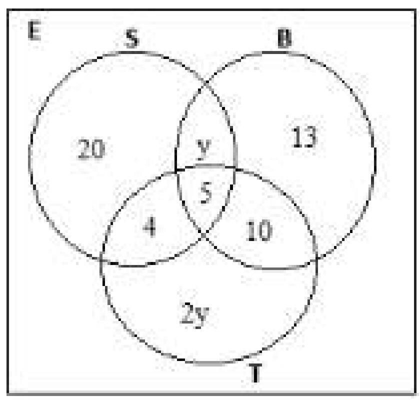
(b) If a student is chosen at random, what is the probability that he took

(i) one course,

(ii) at least two course?

[2014 P2 NOV, Q5 (a)]

5. Tokhozani sports club offers Squash(S), Badminton (B) and Tennis (T).The Venn diagram shows choices of the 73 members of the club.



(i) Calculate the value of y

(ii) Find the number of member s who played Squash or tennis but not Badminton.

(iii) How many members played two different sports only?

(iv) Find the number of members who played one sport only.

TOPIC 2: PROBABILITY

[2017 P2 JULY, Q3 (a)]

1. In a box of 10 bulbs, 3 are faulty. If two bulbs are drawn at random one after the other, find the probability that
   (i) Both are good.
   (ii) One is faulty and the other one is good.

[2017 P2 NOV, Q2 (a)]

2. A box of chalk contains 5 white, 4 blue and 3 yellow pieces of chalk. A piece of chalk is selected at random from the box and not replaced. A second piece of chalk is then selected.
   (a) Draw a tree diagram to show all the possible outcomes.
   (b) Find the probability of selecting pieces of chalk of the same colour.

[2016 P2 NOV, Q2 (b)]

3. A survey was carried out at certain hospital indicated that the probability that patient tested positive for malaria is 0.6. What is the probability that two patients selected at random
   (i) one tested negative while the other positive,
   (ii) both patients tested negative.

[2014 P2 NOV, Q11 (b)]

4. Two pupils are to present a school at a Human Rights Conference. If the two are chosen at random from a group of 8 girls and 6 boys, calculate the probability that the two pupils picked
   (i) are both girls,
   (ii) at least one is a boy.

TOPIC 3: SEQUENCES AND SERIES

[GENERAL QUESTIONS 1-2]

1. For the sequence 25, 22, 19, 16, …,. Find
   (a) Formula for the n^{th} term
   (b) Sum of the first 20 terms

2. The 6th and the 13th terms of an arithmetic progression are 17 and 38 respectively. Find
   (a) the first term "a" and the common difference "d" and hence list the first 5 terms of this sequence,
   (b) the general formula for the n^{th} term and the 19th term,
   (c) the sum of the first 40 terms of this sequence.

3. (a) for the sequence 11, 13, 15, 17, . . . , find the 13th term
   (b) If the arithmetic mean of 5 and c is 11, what is the value of c?
   (c) If 16, x, 6, y form an arithmetic progression, find the value of x and y

[2017 P2 JULY, Q2 (b)]

4. The first three terms of a geometric progression are $6 + n, 10 + n$ and $15 + n$. Find
   (i) the value of n,
   (ii) the common ratio,
   (iii) the sum of the first 6 terms of this sequence.

[2017 P2 NOV, Q5 (b)]

5. For the geometric progression 20, 5, $1\frac{1}{4}$, . . . , find
   (i) the common ratio,
   (ii) the n^{th} term,
   (iii) the sum of the first 8 terms.

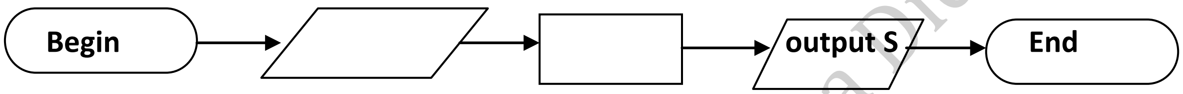
[2016 P2 NOV, Q5 (a)]

6. The first three terms of a geometric progression are $x + 1, x - 3$ and $x - 1$.
   (i) the value of x,
   (ii) the first term,
   (iii) the sum to infinity.

TOPIC 4: PSEUDO CODE AND FLOW CHARTS

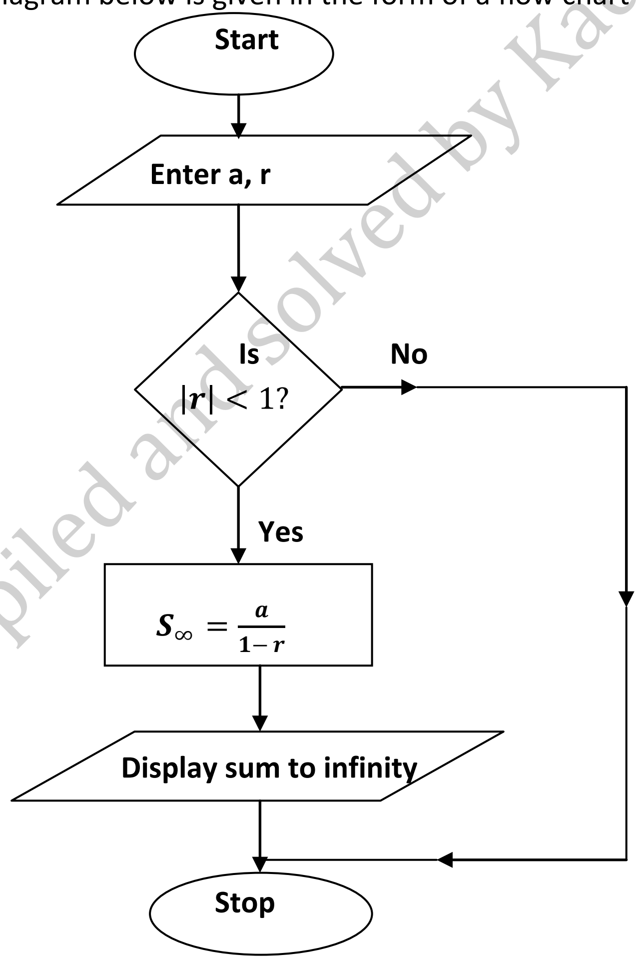
[2017 P1 NOV, Q20 (b)]

1. The diagram below is an incomplete flow chart to calculate the curved surface area, S of a cone with base radius r and slant height l. complete the flow chart below by writing appropriate statements in the blank symbols



[2017 P2 JULY, Q6 (b)]

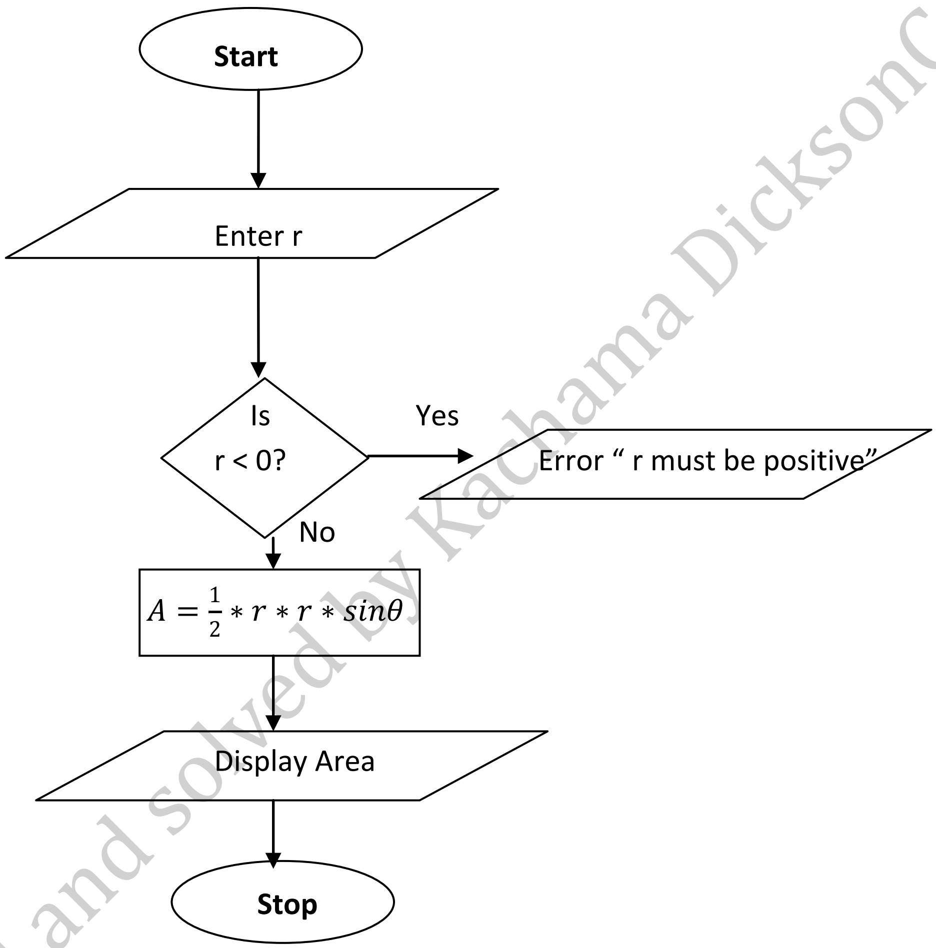
2. The diagram below is given in the form of a flow chart



Write a pseudo code corresponding to the flow chart program above

[2017 P2 NOV, Q6]

3. Study the flow chart below



Write a pseudo code corresponding to the flow chart program above

[2016 P2 NOV, Q3 (a)]

4. The program below is given in the form of a pseudo code.

```
Start
        Enter radius
           If radius < 0
           Then display "error message" and re-enter positive radius
        Else enter height
           If height < 0
           Then display "error message" and re-enter positive height
        Else Volume = 1/3 * π * square radius * height
            End if
        Display volume
Stop
```

Draw the corresponding flowchart for the information given above.

TOPIC 5: CALCULUS

1. (a) Find the derivative of $y = 2x^3 - 2x^2 - 3x + 1$ with respect to x

   (b) Evaluate $\int_2^5 (3x^2 + 2)dx$

   (c) Evaluate $\int_{-1}^3 (3x^2 - 2x)dx$

2. Find the equation of the tangent to the curve $y = x^2 - 3x - 4$ at a point where $x = 2$

3. Find the coordinates of the points on the curve $y = 2x^3 - 3x^2 - 36x - 3$ where the gradient is zero.

4. The equation of the curve is$y = x^3 - \frac{3}{2}x^2$. Find

   (a) equation of the normal where $x = 2$

   (b) the coordinates of the stationary points

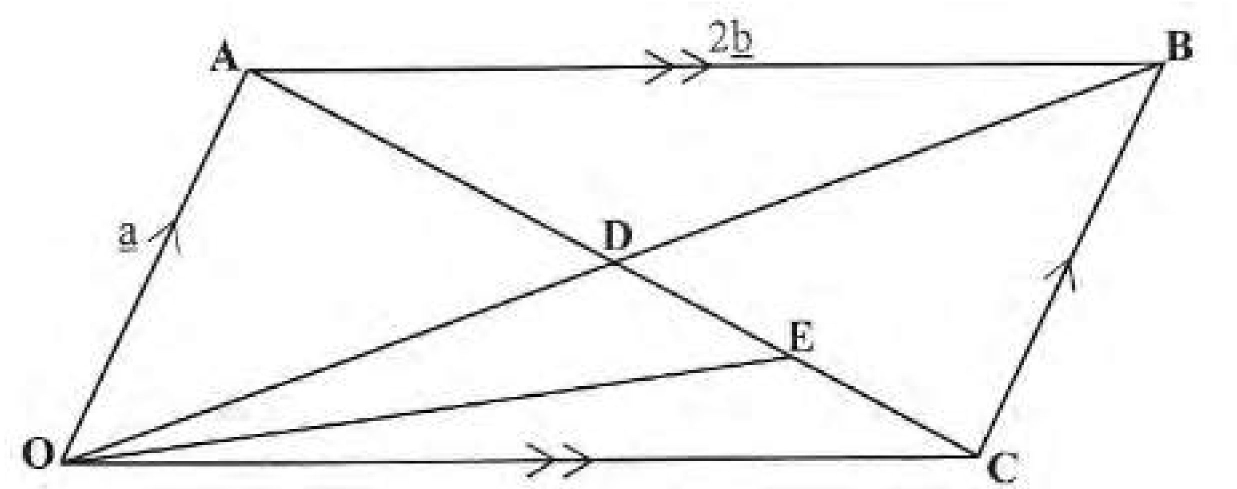
5. Find the equation of the tangent to the curve $y = x^2 - 2x + 3$ at the point (3, 0)

6. A curve is such that $\frac{dy}{dx} = 15x^2 - 12x$. Given that it passes through (1, 3), find its equation.

TOPIC 6 : VECTORS

[2017 P2 JULY, Q6 (a)]

1. I n the diagram below, OABC is a parallelogram in which $\overrightarrow{OA} = \underline{a}$ and $\overrightarrow{OB} = 2\underline{b}$ OB and AC intersect at D. E is the midpoint of CD. E is the mid - point of CD.

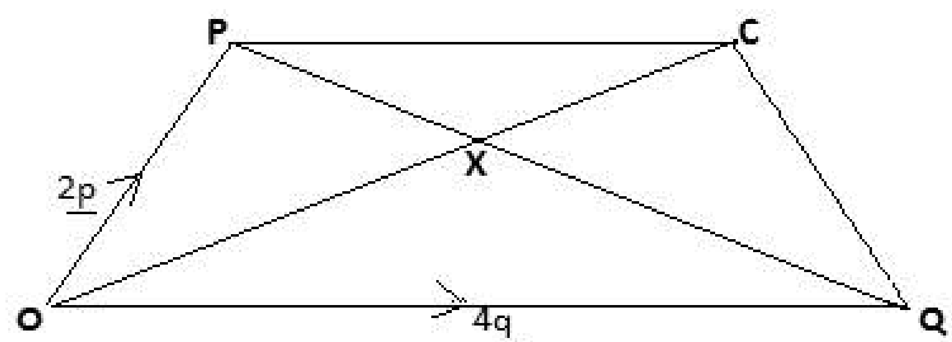


Express in terms of $\underline{a}$ and / or $\underline{b}$.

(i) $\overrightarrow{OB}$, (ii) $\overrightarrow{OE}$ (iii) $\overrightarrow{CD}$

[2017 P2 NOV, Q2 (a)]

2. In the diagram below, $\overrightarrow{OP} = 2\underline{p}$, $\overrightarrow{OQ} = 4\underline{q}$ and $PX : XQ = 1:2$



(i) Express in terms of p and / or q.

(a) $\overrightarrow{PQ}$

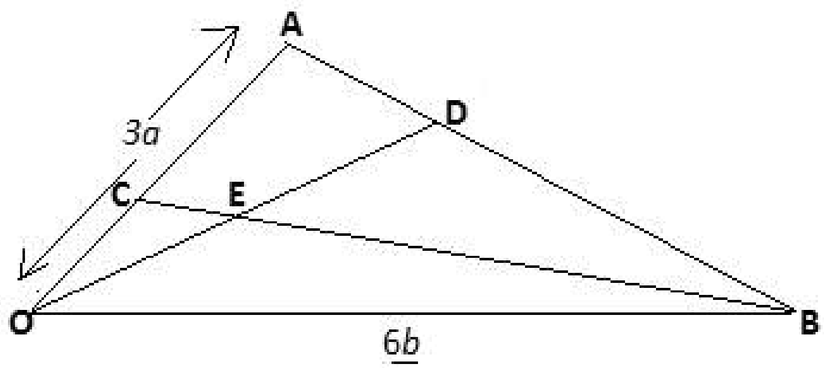
(b) $\overrightarrow{PX}$

(c) $\overrightarrow{OX}$

(ii) Given that $\overrightarrow{CO} = h\overrightarrow{OX}$, show that $\overrightarrow{CQ} = 4\left(1 - \frac{h}{3}\right)\underline{q} - \frac{4h}{3}\underline{p}$.

[2016 P2, NOV, Q3 (b)]

3. In the diagram below, OAB is a triangle in which $\overrightarrow{OA} = 3\underline{a}$ and $\overrightarrow{OB} = 6\underline{b}$.
OC : CA = 2 : 3 and AD : DB = 1 : 2. OD meets CB at E.



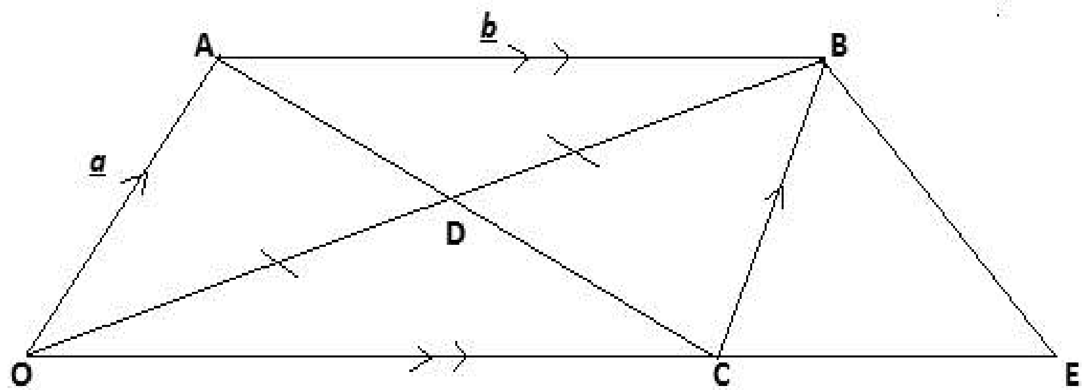
(i) Express each of the following in terms of $\underline{a}$ and / or $\underline{b}$

(a) $\overrightarrow{AB}$ (b) $\overrightarrow{OD}$ (c) $\overrightarrow{BC}$

(ii) Given that $\overrightarrow{BE} = h\overrightarrow{BC}$, express $\overrightarrow{BE}$ in terms of h, $\underline{a}$ and $\underline{b}$

[2015 P2 NOV, Q6]

4. In the diagram below, **OABC** is a parallelogram in which $\overrightarrow{\boldsymbol{OA}} = \underline{a}$ and $\overrightarrow{\boldsymbol{OB}} = \underline{b}$. **CA** and **OB** meet at **D** such that **OD** = **DB**. **OC** is produced to **E**, such that **CE** = $\frac{1}{2}$**OC.**

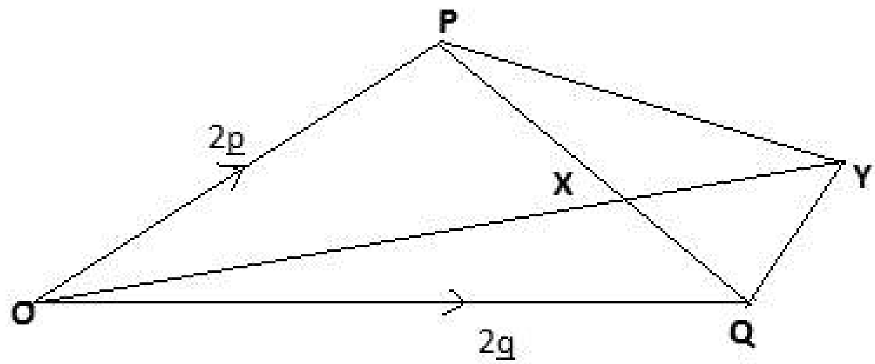


Express each of the following in terms of $\underline{a}$ and/ or $\underline{b}$

(i) $\overrightarrow{OB}$ (ii) $\overrightarrow{AD}$ (iii) $\overrightarrow{BE}$

[2014 P2 NOV, Q6]

5. In the diagram, $\overrightarrow{OP}$= 2p and $\overrightarrow{OQ}$ = 2q. X is a point on PQ such that $\frac{PX}{PQ} = \frac{2}{3}$



(i) Express in terms of p and/ or q

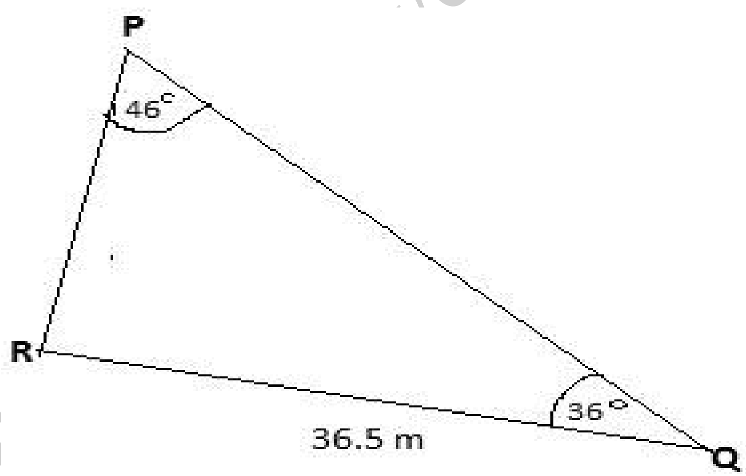
(a) $\overrightarrow{PQ}$ (b) $\overrightarrow{QX}$

(ii) Given also that $\overrightarrow{OY} = h\overrightarrow{OX}$, express $\overrightarrow{OY}$ in terms of p, q and h

TOPIC 7: TRIGONOMETRY

[2017 P2 JULY, Q 10]

1. (a) In Triangle PQR below, QR = 36.5, angle PQR = 36° and angle QPR = 46°.



Calculate

(i) PQ

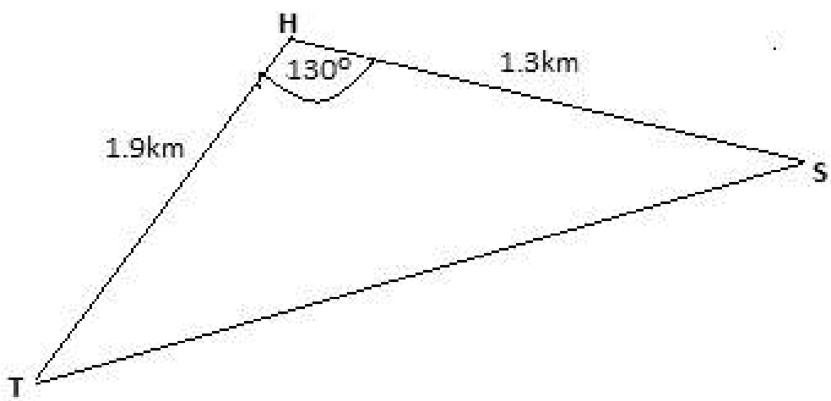
(ii) the area of triangle PQR

(iii) the shortest distance from R to PQ

(b) Solve the equation $\sin\theta = 0.6792$ for $0° \leq \theta \leq 360°$.

[2015 P2 NOV, Q7]

2. **(a)** The diagram below shows the Location of houses for a village Headman (H), his Secretary (S) and a Trustee (T). H is 1.3 km from S, T is 1.9 km from H and angle THS = 130°



Calculate

(i) the area of triangle THS

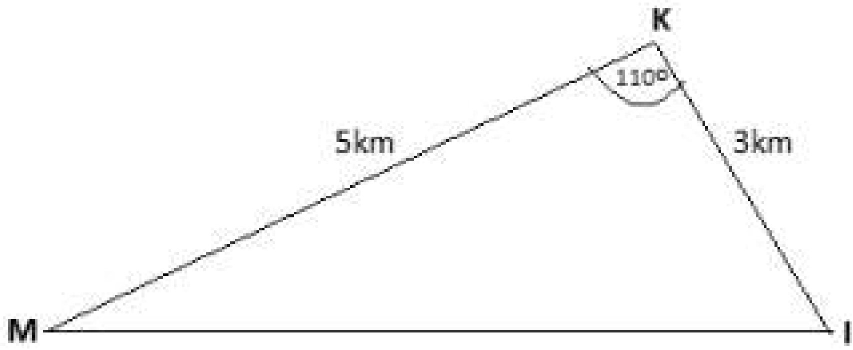
(ii) the distance TS

(iii) the shortest distance from H to TS

(b) Find the angle between 0° and 90° which satisfies the equation $\cos\theta = \frac{2}{3}$.

[2016 P2 NOV, Q 10]

3. **(a)** The diagram below shows the location of three secondary schools, namely Mufulira (M), Kantanshi (K) and Ipusukilo (I) in Mufulira district. M is 5km from K, I is 3km from K and $M\hat{K}I$ is 110°



Calculate

(i) MI

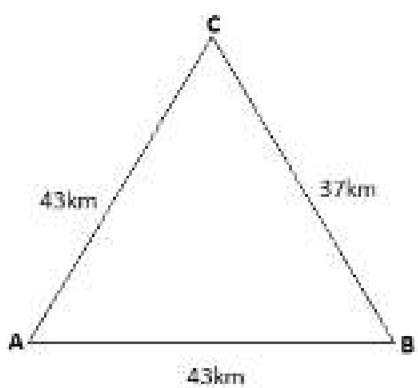
(ii) the area of triangle MKI

(iii) the shortest distance from K to MI

(b) Solve the equation $\tan\theta = 0.7$ for $0° \leq \theta \leq 180°$.

[2015 P2 NOV, Q10 (a)]

4. (a) In Votani Constituency, **A**, **B** and **C** are polling stations as shown on the diagram



Calculate

(i) angle **BAC**

(ii) angle **ACB**,

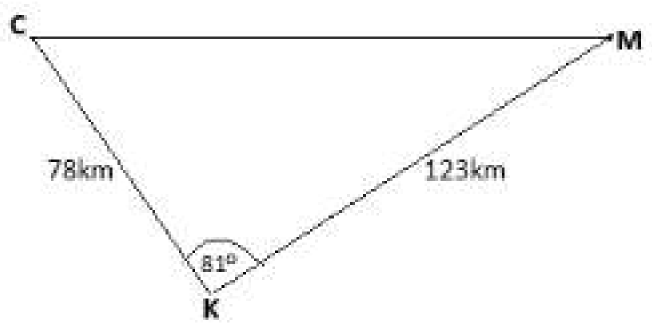
(iii) the area of triangle **ABC** correct to 1 decimal place,

(iv) the shortest distance from **C** to **AB**

(b) Solve the equation $\cos\theta = 0.5$ for $0° \leq \theta \leq 360°$

[2014 P2 NOV, Q7 (a)]

5. Positions of Kabwela (K), Chopa (c) and Muzi (M) are as shown in this diagram below, Chopa is 78km from Kabwela and Muzi is 123km from Kabwela.



(i) Given that $C\hat{K}M = 81°$, calculate the area of triangle CMK.

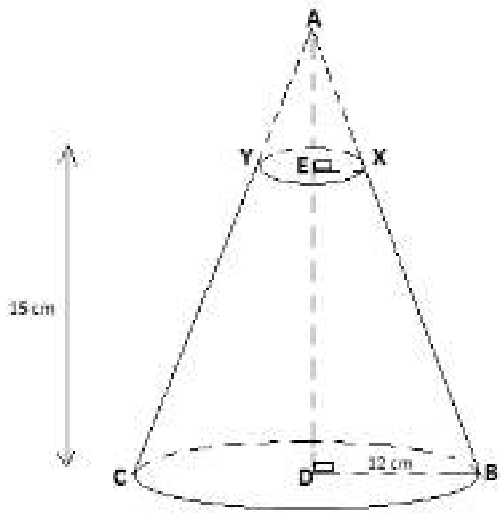
(ii) Calculate the distance CM

(iii) A company has been contracted to construct a road from Muzi (M) to Chopa (C). Find the total cost of constructing this road, if the company charges K21500.00 per kilometer.

TOPIC 8: MENSURATION

[2017 P2 JULY, Q 12 (a)]

1. The figure below is a cone ABC form which BCXY remained after the small cone AXY was cut of [Take $\pi = 3.142$]



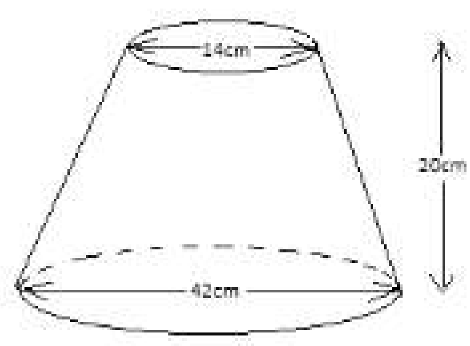
Given that EX = 4cm, DB = 12cm and DE = 15cm, calculate

(i) the height AE, of the smaller con AXY.

(ii) the volume of XBCY, the shape that remained.

[2017 P2 NOV, Q4 (b)]

2. The figure below is a frustum of a cone. The base diameter and top diameter are 42cm and 14 cm respectively, while the height is 20cm. (**Take** $\boldsymbol{\pi = 3.142}$)



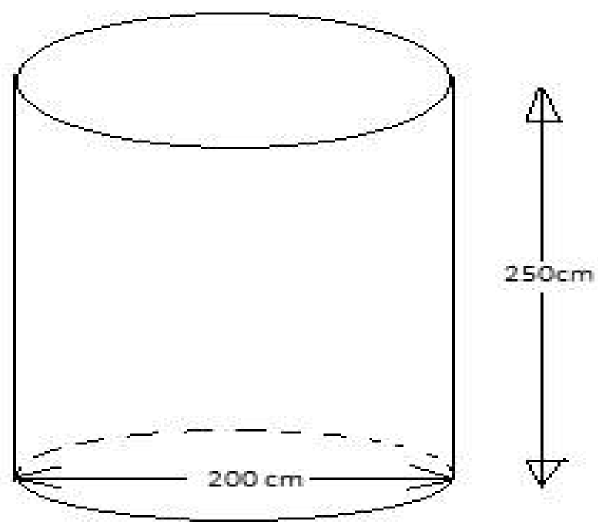
Calculate its volume.

[2016 P2 NOV, Q9 (b/c)]

3. **(a)** The cross section of a rectangular tank measures 1.2m by 0.9. if it contains fuel to a depth of 10m, find the number of litres of fuel in the tank. ($1\text{m}^3 = 1000\text{litres}$)
(b) A cone has a perpendicular height of 12 cm and slant height of 13cm, calculate its total surface area. (Take$\pi = 3.142$)

[2015 P2 NOV, Q11 (b)]

4. A cylindrical water tank at Mwaiseni Lodge has diameter of 200cm and height of 250cm as shown below



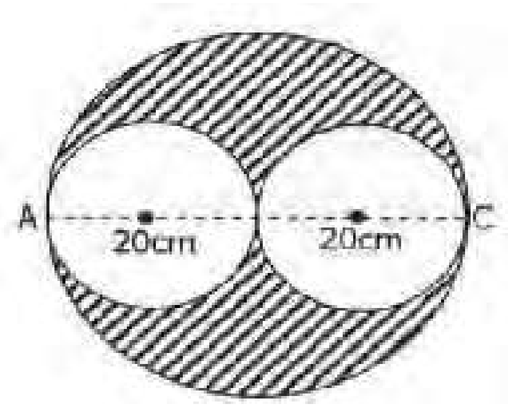
Taking π to be 3.142, find

(i) The total surface area of the tank if it is closed,

(ii) The number of litres of water the tank can hold.

[2014 P2 NOV, Q9 (a)]

5. A pattern on a chitenge material consists of three circles as shown in the diagram below. AC is the diameter of the bigger circle.



Given that the diameter of each of the small circles is 20cm and taking $\pi = 3.142$, calculate the

(i) total perimeter of the two small circles

(ii) area of the shaded part

TOPIC 9: EARTH GEOMETRY

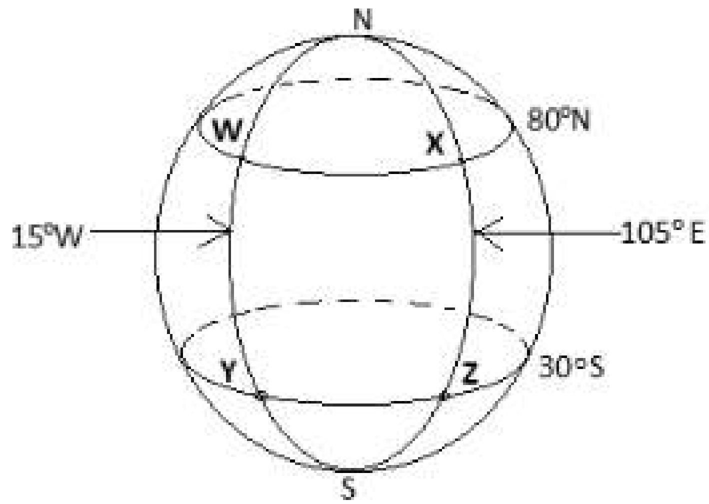
[2017 P2 JULY, Q12 (b)]

1. **P** $(80°N, 10°E)$, $\mathbf{Q}(80°N, 70°E)$, $\mathbf{R}(85°S, 70°E)$ and $\mathbf{S}(85°S, 10°E)$ are the points on the surface of the earth.
   (i) Show the points on a clearly labeled sketch of the surface of the earth
   (ii) Find in nautical miles
   (a) The distance QR along the longitude,
   (b) The circumference of latitude 85°S.

   [Take $\pi = 3.142$ and $R = 3437nm$]

[2017 P2 NOV, Q9 (a)]

2. W, X, Y and Z are four points on the surface of the earth as shown in the diagram below. **(Take $\pi = 3.142$ and $R = 3437$)**



(i) Calculate the difference in latitudes between W and Y.

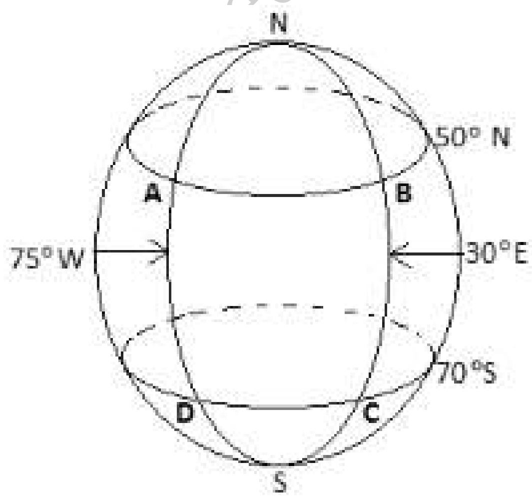
(ii) Calculate the distance in nautical miles between

(a) X and Z along the longitudes 105°E

(b) Y and Z along the circle of latitude 30°S

[2015 P2 NOV, Q 9(a)]

3. The points A, B, C and D are on the surface of the earth.
(Take $\pi = 3.142$ and $R = 3437\text{nm}$)



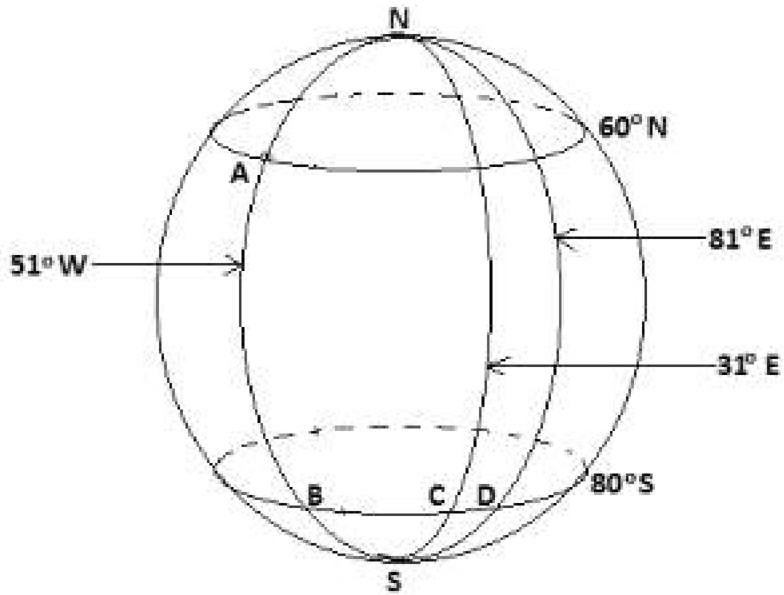
(i) Find the difference in latitude between points C and B

(ii) Calculate the length of the circle of latitude 50°N in nautical miles

(iii) Find the distance AD in nautical miles

[2015 P2 NOV, Q11 (a)]

4. **A**, **B**, **C** and **D** are points on the surface of the earth as shown in the diagram below.



(i) Using latitudes and longitudes, write the positions of points **A** and **B.**

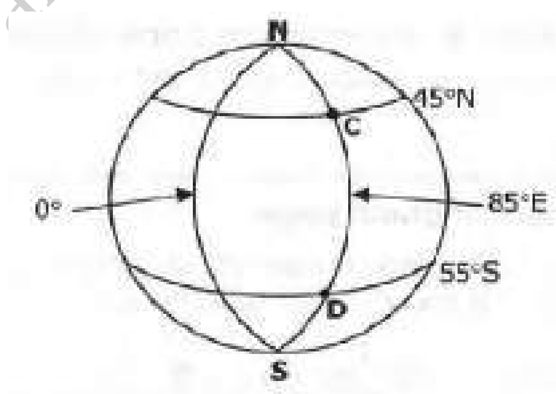
(ii) Find the difference in longitudes between points **C** and **D**

(iii) Calculate the distance **CD** in nautical miles $[\pi = 3.142 \text{ and } R = 3437\text{nm}]$.

(iv) Given that the local time at **D** is 13 05 hours, find the time at **C**

[2014 P2 NOV, Q9 (b)]

5. The diagram below shows a model of the earth. The points C and D are on the same longitude. The latitudes of C and D are $45°N \text{ and } 55°S$ respectively.
(Take$\pi = 3.142$ and $R = 3437$nm)



(i) Write the position of the point C

(ii) Calculate the difference in latitude between C and D.

(iii) Find the distance CD in nautical miles.

(iv) Calculate the circumference of the latitude $45°N$ in nautical miles.

TOPIC 10: STATISTICS (PART A)

[2017 P2 JULY, Q8]

1. The frequency table below shows the number of copies of newspapers allocated to 48 newspaper vendors.

Number of copies	$25 < x \leq 30$	$30 < x \leq 35$	$35 < x \leq 40$	$40 < x \leq 45$	$45 < x \leq 50$	$50 < x \leq 55$	$55 < x \leq 60$
Number of vendors	5	4	7	11	12	8	1

Calculate the standard deviation.

[2017 P2 NOV, Q8]

2. The table below shows the amount of money spent by 100 learners at school on a particular day.

Amount in Kwacha	$0 < x \leq 5$	$5 < x \leq 10$	$0 < x \leq 15$	$15 < x \leq 20$	$20 < x \leq 25$	$25 < x \leq 30$
Frequency	13	27	35	16	7	2

Calculate the standard deviation.

[2016 P2 NOV, Q7]

3. The ages of people living at Pamodzi Village are recorded in the frequency table below.

Ages	$0 < x \leq 10$	$10 < x \leq 10$	$20 < x \leq 30$	$30 < x \leq 40$	$40 < x \leq 50$	$50 < x \leq 60$
Number of people	7	22	28	23	15	5

Calculate the standard deviation

[2015 P2 NOV, Q9]

4. On a particular day, a tuck shop owner recorded the expenditure of 350 boys and the results were as shown in the table below.

Amount (K)	$10 < x \leq 20$	$20 < x \leq 30$	$30 < x \leq 40$	$40 < x \leq 50$	$50 < x \leq 60$	$60 < x \leq 70$	$70 < x \leq 80$	$80 < x \leq 90$	$90 < x \leq 100$
No. of Boys	20	50	55	70	60	45	35	10	5

Calculate the mean amount of money spent

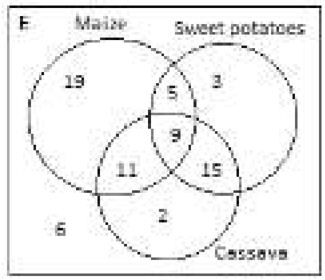
END OF QUESTIONS

ANSWERS TO THE QUESTIONS

TOPIC 1 SOLUTION: SETS

1.

(i) Hint: To illustrate the information on the Venn diagram, start by filling the intersection of all the three sets (9) and followed by the intersection of two sets.



(ii) (a) Total number of farmers = 19 + 5 +3 + 9 +11 + 15 + 2 + 6 = 70 farmers

(b) Maize only = 19 farmers

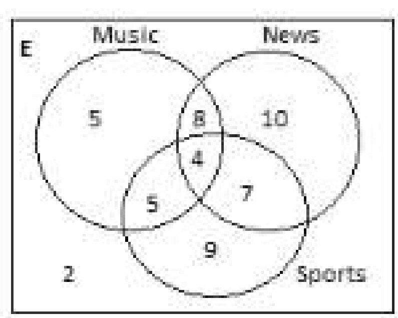
(c) Two different crops = 11 +5 + 15 = 31.

2. (i) $2y + 1 = 7 \rightarrow \quad 2y = 7 - 1 \rightarrow 2y = 6 \rightarrow y = 3$

(ii (a) 6 + 2 = 8 (b) 4 +1 +2 = 7 (b) 6 + 2(3) + 1 + 8 = 21

3. (i) Total = 50

Hint: start with the intersection of all the three sets, followed by the intersection of any of the two sets



(ii) **(a)** Music only = **5** **(b)** One type of programme only = 5 + 9 +10 = **24**

(c) Two types of programmes only = 8 + 7 + 5 = 20

4. **(a)** **(i)** 15 + 15 + 10 + 10 + x + 15 + 5 = 100

70 + x =100

$\boldsymbol{x}$ **= 30**

(ii) Human Resource = 10 + 10 + 15 + 5 = 40

(iii) n(B ∩ C) n H'

(B ∩C) = {15, 10}

H' = {15, 15, 30}

(iv) n(B ∪C) n H'

(B U C) = {15, 30, 10, 5}

H' = {15, 15, 30}

∴ n (B ∩ C) n H' = 15   ∴ n (BUC) n H' = 15 + 30 = 45.

(b) (i) P(one course) = $\frac{Totat\ number\ \ of\ those\ who\ took\ one\ course}{Total\ number\ \ of\ all\ students}$

$= \frac{15+15+30}{100} = \frac{60}{100} = \mathbf{\frac{3}{3}}$

(ii) P(at least two courses) = $\frac{10+10+5+15}{100} = \frac{40}{100} = \mathbf{\frac{2}{5}}$

5. **(i)** y + 2y + 4 +20 +5 +10 + 13 = 73

3y + 52 = 73

3y = 21

$\boldsymbol{y}$ **= 7**

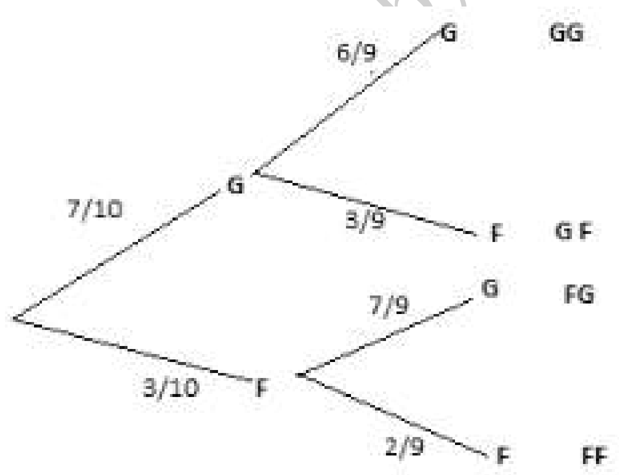
(ii) played S or T but not B = 20 + 4 + 2(7) = **38**

(iii) Two different sports = 4 + 10 + 7 = 2**1**

(iv) played one sport only = 14 + 13 + 20 = **47**

TOPIC 2 SOLUTIONS: PROBABILITY

1. Faulty = 3 and good = 10 – 3 = 7
   Use a tree diagram



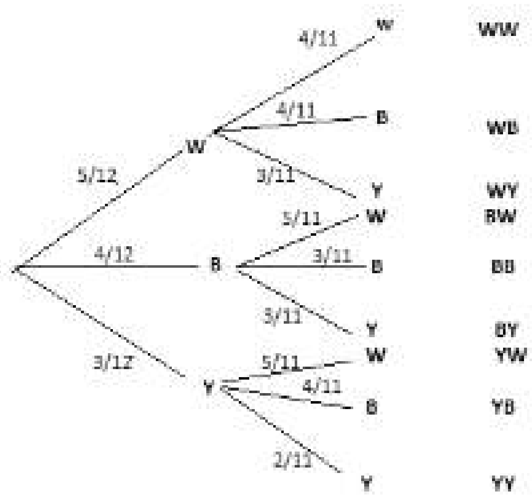
(i) P (Both good) = P(G, G) = $\left(\frac{7}{10} \times \frac{6}{9}\right) = \frac{56}{90} = \frac{7}{15}$ Ans

(ii) P (one is faulty and the one is good) = P(G, F) + P(F,G)

$= \left(\frac{7}{10} \times \frac{3}{9}\right) + \left(\frac{3}{10} \times \frac{7}{9}\right)$

$= \frac{21}{90} + \frac{21}{90} = \frac{42}{90} = \mathbf{\frac{7}{15}}$

2. (a) Total 5 + 4 + 3 =12



(b) P(Same colour) = P(WW) + P(BB) + (YY)

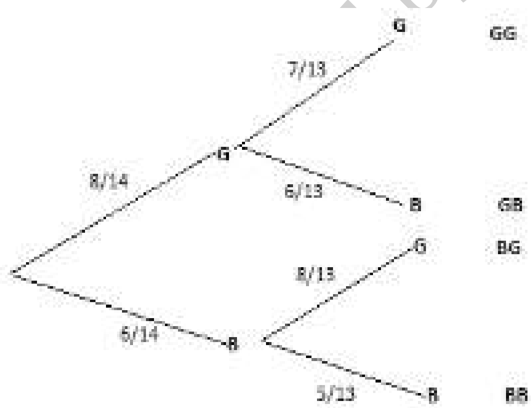
$$= \left(\frac{5}{12}\times\frac{4}{11}\right)+\left(\frac{4}{12}\times\frac{3}{11}\right)+\left(\frac{3}{12}\times\frac{2}{11}\right)=\frac{20}{132}+\frac{12}{132}+\frac{6}{132}=\frac{38}{132}=\mathbf{\frac{19}{69}}$$

3. **(i)** P(+) = 0.6 and P(-) = 1- 0.6 = 0.4

P (one (-) and the other (+)) = $(0.6\times 0.4)+(0.4\times 0.6)=0.24+0.24=\mathbf{0.48}$

(ii) P (Both negative) = $(0.4\times 0.4)=0.16\ \ or\,\frac{16}{100}=\mathbf{\frac{4}{25}}$

4. Total = 8 + 6 = 14: Hint, use a tree diagram for easy calculation of probabilities.



(i) P (Both girls) = $\left(\frac{8}{14}\times\frac{7}{13}\right)=\frac{56}{182}=\mathbf{\frac{56}{182}}$

(ii) P (at least one is a boy) = P(BB) + P(BG) + P(GB)

$$=\left(\frac{6}{14}\times\frac{5}{13}\right)+\left(\frac{6}{14}\times\frac{8}{13}\right)+\left(\frac{8}{14}\times\frac{6}{13}\right)$$

$$=\frac{30}{182}+\frac{48}{182}+\frac{48}{182}$$

$$=\frac{126}{182}=\mathbf{\frac{9}{13}}$$

TOPIC 3 SOLUTIONS: SEQUENCES AND SERIES

1. **(a)** a = 25, d = $T_2 - T_1$ = 22 – 25 = -3

$\mathbf{T_n}$ = **a + (−1) d**

$\mathbf{T_n}$ = 25 + (n – 1)−3

T_n = 25 – $3n$ + 3

∴ $\boldsymbol{T_n}$ **= 28 −3n**

(b) $\mathbf{S_n = \frac{n}{2}[2a + (n-1)d]}$

$S_{20} = \frac{20}{2}[2(25) + (20-1) - 3]$

S_{20} = 10 [50 + (−19) −3]

S_{20} = 10 (50− 57)

∴ $\mathbf{S_{20} = -70}$

2. **(a)** $\mathbf{T_n}$ **= a + (n − 1)d**

$\mathbf{T_6}$ **= a + (6−1)** d

17 = a + 5d

a = 17 −5d ……………. Equation (1)

$\mathbf{T_{13}}$ **= a + (13 -1) d**

38 = a + 12d ---------------Equation (2)

Substitute a by 17 – 5d in (2) yields

17 – 5d + 12d = 38

$7d$ = 21

$\boldsymbol{d}$ **= 3** and a = 17 – 5(3) = 2

∴ the required AP the first five terms are; **2, 5, 8, 11, 14, . . .**

(b) T_n =a + (n-1) d

= 2 + (n – 1)3

= 2 + 3n −3

∴ $\mathbf{T_n}$ **= 3n −1**

The 19th term = T_{19} = 3n -1

T_{19} = 3(19) -1

T_{19} = 57 - 1

∴ $\boldsymbol{T_{19}}$ **= 56**

(c) $S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{40} = \frac{40}{2}[2(2) + (40-1)3]$

S_{40} = 20 [4 + (39)3]

S_{40} = 20 (121)

∴ $\boldsymbol{S_{40} = 2420}$

3. (a) a = 11, d = 13 – 11 = 2

$T_n = a + (n-1)d$

$T_{13} = 11 + (13-1)2$

= 11 + 24

∴ $\boldsymbol{T_{13} = 35}$

(b) b = $\frac{a+c}{2}$

11 = $\frac{5+c}{2}$

5 + c = 22

c = 22 – 5

$\boldsymbol{c = 17}$

(c) 16, x, 6, y

x is the arithmetic mean of 16 and 6

∴ $x = \frac{16+6}{2} = \frac{22}{2} = 11$

The first three terms are 16, 11,6. Thus d = -5

∴ $y = 6 + (-5) = 1$

4. (i) to find n, we use the common ratio formula

That is r= $\frac{T_2}{T_1} = \frac{T_3}{T_2} = \cdots \frac{T_n}{T_{n-1}}$

$\frac{10+n}{6+n} = \frac{15+n}{10+n}$

(10+n) (10+n) = (6+n) (15+n)

100 + 20n + n^2 = 90 + 21n + n^2

100−90 = 21n – 20n

10 = n

∴ $\boldsymbol{n}$ **=10**

The GP is: 16, 20, 25 . . .

(ii) **r** = $\frac{20}{16} = \frac{5}{4}$ $or\ \mathbf{1.25}$

(iii) $S_n = \frac{a(r^n-1)}{r-1}$ for r > 1

$S_6 = \frac{16((1.25)^6-1)}{1.25-1}$

$S_6 = \frac{16(3.814697266-1)}{0.25}$

$S_6 = \frac{16(2.814697266)}{0.25}$

$S_6 = \frac{45.03515625}{0.25}$

S_n = 180.140625

∴ $S_6 \approx \mathbf{180}$

5. (i) r = $\frac{5}{20} = \frac{1}{4} = 0.25$

(ii) $T_n = ar^{n-1}$

$T_n = 20\left(\frac{1}{4}\right)^{n-1}$

$T_n = 20\frac{1^{n-1}}{4^{n-1}}$

$\therefore \boldsymbol{T_n = \frac{20}{4^{n-1}}}$

(iii) $S_n = \frac{a(1-r^n)}{1-r}$ for r < 1

$S_8 = \frac{20(1-(0.25)^8)}{1-0.25}$

$S_8 = \frac{20\ (1-0.00001558906)}{0.75}$

$S_8 = \frac{20(0.9999847412)}{0.75}$

$\boldsymbol{S_8}$ **= 26.66625977** $\approx \mathbf{27.7}$

6. (i) We know that r= $\frac{T_2}{T_1} = \frac{T_3}{T_2} = \cdots \frac{T_n}{T_{n-1}}$

$\frac{x-3}{x+1} = \frac{x-1}{x-3}$

$(x-3)\,(x-3) = (x-1)\,(x+1)$

$x^2 - 6x + 9 = x^2 - 1$

$-6x = -1 - 9$

$-6x = -10$

$x = \frac{10}{6} = \mathbf{\frac{5}{3}}$

Hence the GP is; $\frac{5}{3} + 1, \frac{5}{3} - 3, \frac{5}{3} - 1 \ldots \ldots$

$\frac{8}{3}, \frac{-4}{3}, \frac{2}{3}, \ldots$

(ii) the first term "a" = $\mathbf{\frac{8}{3}}$

(iii) $S_\infty = \frac{a}{1-r}$

$S_\infty = \frac{8/3}{1-(-\frac{1}{2})}$

$S_\infty = \frac{8/3}{\frac{3}{2}}$

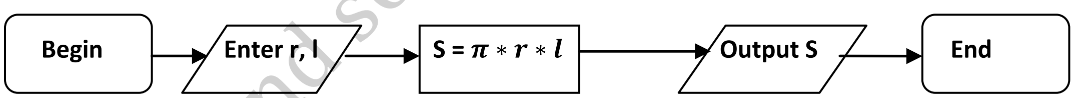
$S_\infty = \frac{8}{3} \div \frac{3}{2}$

$S_\infty = \frac{8}{3} \times \frac{2}{3}$

$\therefore \boldsymbol{S_\infty = \frac{16}{9}}$

TOPIC 4 SOLUTION: PSEUDO CODE & FLOW CHARTS

1.



2. Start
   Enter a and r
   If r < 1
   Then display "error message "and re-enter positive r
   Else sum to infinity = $\frac{a}{1-r}$
   End if
   Display sum to infinity
   Stop

3. Start

Enter radius

If radius < 0

Then display "error message "and re-enter positive radius

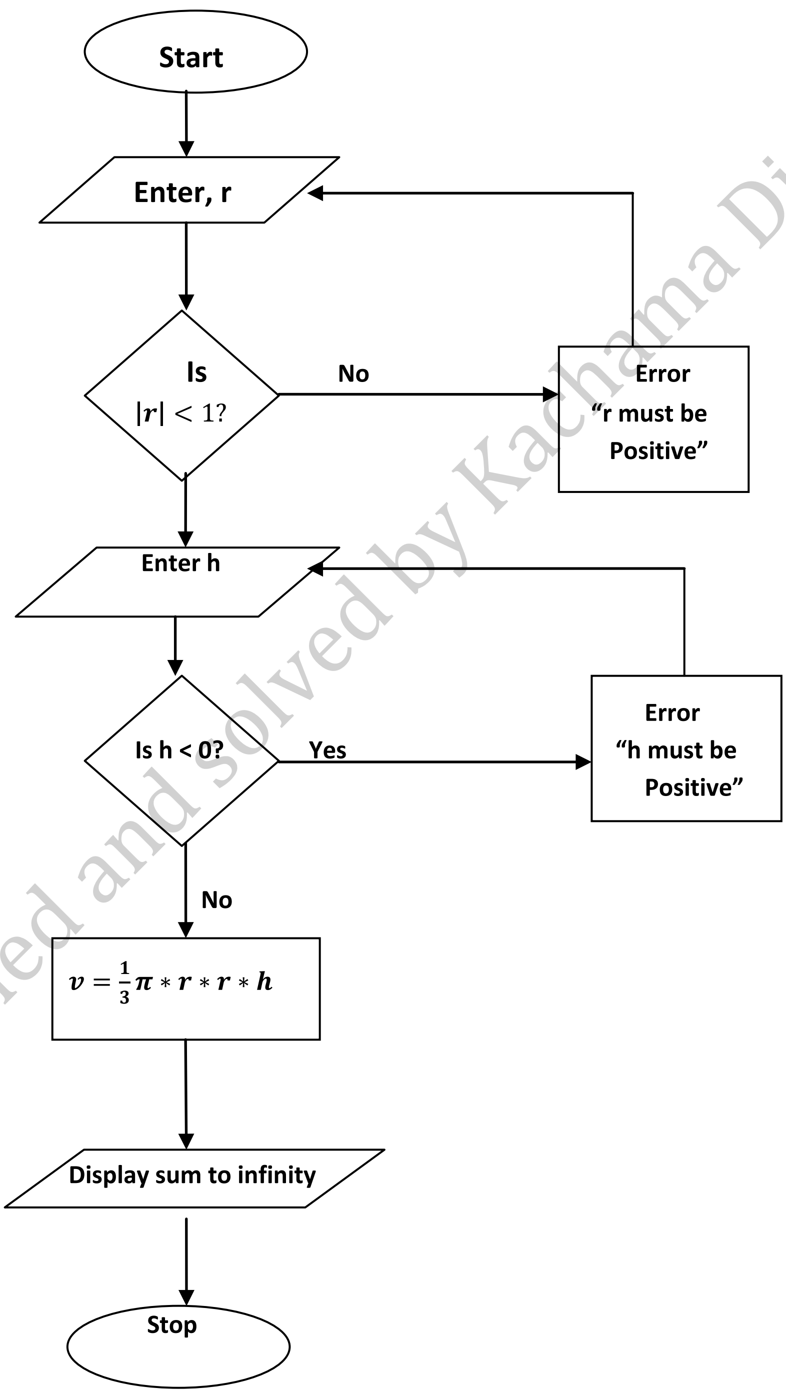
Else Area $= \frac{1}{2} * r * r * sin\theta$

End if

Display Area

Stop

4.



TOPIC 5 SOLUTIONS: CALCULUS

1. (a) $\frac{dy}{dx} = (3)2x^{3-1} - (2)2x^{2-1} - (1)3x^{1-1}$

$\frac{dy}{dx} = 6x^2 - 4x - 3$

(b) $\int_2^3 (3x^2 + 2)\, dx$

$\left[\frac{3x^{3+1}}{2+1} + \frac{2x^{0+1}}{0+1}\right]_2^3$

= $\left[\frac{3x^3}{3} + \frac{2x^2}{1}\right]_2^3$

= $[x^3 - 2x]_2^3$

= ($(3)^3 + 2(3)$) – ($2^2 + 2(2)$

= (26 + 6) – (8+4

= 21

(c) $\int_{-1}^3 (3x^2 - 2x)dx$

= $\left[\frac{3x^3}{3} - \frac{2x^2}{2}\right]_{-1}^3$

= $[x^3 - x^2]_{-1}^3$

= $[(3)^3 - (3)^2] - [(-1)^3 - (-1)^2]$

= (29−9) – (-1-1)

= 18 – (−2)

= 18+2

= 20

2. $m = \frac{dy}{dx} = 2x - 3$ at $x = 2$

m = 2(2) −3 = 1

$y - y_1 = m(x - x_1)$ passes through (2, −6)

$y - (-6) = 1(x - 2)$

$y + 6 = x - 2$

$\boldsymbol{y = x - 8}$

3. $y = 2x^3 - 3x^2 - 36x - 3$

$\frac{dy}{dx} = 6x^2 - 6x - 36$

0= $6x^2 - 6x - 36$ dividing through by 6

$x^2 - x - 6 = 0$

$(x + 2)(x - 3) = 0$

$x = -2$ *or* $x = 3$

when $x = -2$

$y = 2(-2)^3 - 3(-2)^2 - 36(-2) - 3$

$y = -16 - 12 + 72 - 3$

y = 41

when $x = 3$

$y = 2(3)^3 - 3(3)^2 - 36(3) - 3 = -84$

∴ the coordinates on curve are $(-\mathbf{2}, \mathbf{41})$ and $(\mathbf{3}, -\mathbf{84})$

4. $y = x^3 - \frac{3}{2}x^2$

$\frac{dy}{dx} = 3x^2 - 3x = 3(2)^2 - 3(2) = 6$

∴ $m_1 = 6$ which is the gradient of the curve

we know that tangent is perpendicular to the curve,

$m_1 m_2 = -1$, at $x = 2, y = 2$

$m_2 = -\frac{1}{6}$ Which is the gradient of the normal?

$y - y_1 = m_2(x - x_1)$ (2,2)

$y - 2 = -\frac{1}{6}(x - 2)$

$y = -\frac{1}{6}x + \frac{1}{3} + 2$

$\boldsymbol{y = -\frac{1}{6}x + \frac{7}{3}}$

at the stationary points, $\frac{dy}{dx} = 0$

$3x^2 - 3x = 0$

$3x(x - 1) = 0$

$x = 0$ *or* $x = 1$

for $x = 0$.

$y = (0)^3 - \frac{3}{2}(0)^2 = 0$

for $x = 1$

$y = (1)^3 - \frac{3}{2}(1)^2 = -\frac{1}{2}$

therefore the stationary points are;

(0, 0) and (1, $-\frac{1}{2}$)

5. $m = \frac{dy}{dx} = 2x - 2$ $at\ x = 3, m = 4$

$y - y_1 = m(x - x_1)$

$y - 0 = 4(x - 3)$

$\boldsymbol{y = 4x - 12}$

6. $\frac{dy}{dx} = 15x^2 - 12x$ at $x = 1, \frac{dy}{dx} = m = 3$

$y - y_1 = m(x - x_1)$

$y - 3 = 3(x - 1)$

$y = 3x - 3 + 3$

$\boldsymbol{y = 3x}$

TOPIC 6 SOLUTIONS: VECTORS

1. (i) $\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB}$

$\boldsymbol{\overrightarrow{OB} = \underline{a} + \underline{b}}$

(ii) To find $\overrightarrow{OE}$, first find $\overrightarrow{CA}$ and $\overrightarrow{CD}$

$\overrightarrow{CA} = \overrightarrow{CO} + \overrightarrow{OA}$

$\overrightarrow{CA} = -2\underline{b} + a$

$\overrightarrow{CD} = \frac{1}{2}\overrightarrow{CA}$

$\overrightarrow{CD} = \frac{1}{2}(-2\underline{b} + \underline{a})$

$\overrightarrow{CD} = -\underline{b} + \frac{1}{2}\underline{a}$

$\therefore \overrightarrow{OE} = \overrightarrow{OC} + \overrightarrow{CE}$

$\overrightarrow{OE} = \overrightarrow{OC} + \frac{1}{2}\overrightarrow{CD}$

$\overrightarrow{OE} = 2b + \frac{1}{2}(-b + \frac{1}{2}\underline{a})$

$\overrightarrow{OE} = 2b - \frac{1}{2}b + \frac{1}{4}\underline{a}$

$\overrightarrow{OE} = \frac{4b-b}{2} + \frac{1}{4}\underline{a}$

$\overrightarrow{OE} = \frac{3}{2}b + \frac{1}{4}\underline{a}$

$\boldsymbol{\overrightarrow{OE} = \frac{1}{4}(6\underline{b} + \underline{a})}$

(iii) $\overrightarrow{CD} = \frac{1}{2}\overrightarrow{CA}$

$\overrightarrow{CD} = \frac{1}{2}(-2\underline{b} + \underline{a})$

$\overrightarrow{CD} = -b + \frac{1}{2}a$

$\boldsymbol{\overrightarrow{CD} = \frac{1}{2}\underline{a} - \underline{b}}$

2. (i) (a) $\overrightarrow{PQ} = \overrightarrow{PO} + \overrightarrow{OQ}$

$\overrightarrow{PQ} = -2\underline{p} + 4\underline{q}$

$\overrightarrow{PQ} = 4\underline{q} - \underline{p}$

$\therefore \overrightarrow{PQ} = 2(2\underline{q} - \underline{p})$

(b) $\overrightarrow{PX} = \frac{1}{3}\overrightarrow{PQ}$

$\boldsymbol{\overrightarrow{PX} = \frac{1}{3}(4q - p)}$

(c) $\overrightarrow{OX} = \overrightarrow{OP} + \overrightarrow{PX}$

$\overrightarrow{OX} = 2\underline{p} + \frac{4}{3}q - \frac{2}{3}\underline{p}$

$\overrightarrow{OX} = 2p - \frac{2}{3}p - \frac{4}{3}\underline{q}$

$\overrightarrow{OX} = \frac{6p-2p}{3} - \frac{4q}{3}$

$\overrightarrow{OX} = \frac{4}{3}\underline{p} - \frac{4}{3}\underline{q}$

$\overrightarrow{OC} = h\overrightarrow{OX}$

$\overrightarrow{OC} = h(\frac{4p}{3} + \frac{4q}{3})$

$\overrightarrow{OC} = \frac{4h}{3}\underline{p} + \frac{4h}{3}\underline{q}$

$\therefore \overrightarrow{CQ} = \overrightarrow{CO} + \overrightarrow{OQ}$

$\overrightarrow{CQ} = -(\frac{4h}{3}\underline{P} + \frac{4h}{3}\underline{q}) + 4\underline{q}$

$\overrightarrow{CQ} = 4\underline{q} - \frac{4h}{3}\underline{q} - \frac{4h}{3}p$

$\boldsymbol{\overrightarrow{CQ} = 4(1 - \frac{h}{3})\underline{q} - \frac{4h}{3}\underline{p}}$

$\therefore \overrightarrow{\mathbf{OX}} = \frac{4}{3}(\underline{p} - \underline{q})$

3. **(i)** (a) $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$

$\overrightarrow{AB} = -3\underline{a} + 6\underline{b}$

$\overrightarrow{AB} = 6\underline{b} - 3\underline{a}$

$\therefore \overrightarrow{\mathbf{AB}} = 3(2\underline{b} - \underline{a})$

(b) $\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{AD}$

$\overrightarrow{OD} = \overrightarrow{OA} + \frac{1}{3}\overrightarrow{AB}$

$\overrightarrow{OD} = 3a + \frac{1}{3}(6\underline{b} - 3\underline{a})$

$\overrightarrow{OD} = 3\underline{a} + 3\underline{b} - \underline{a}$

$\overrightarrow{\mathbf{OD}} = 2\underline{a} + 2\underline{b} = 2(\underline{a} + \underline{b})$

(c) $\overrightarrow{BC} = \overrightarrow{BO} + \overrightarrow{OC}$

$\overrightarrow{BC} = -\overrightarrow{OB} + \frac{2}{5}\overrightarrow{OA}$

$\overrightarrow{BC} = -6\underline{b} + \frac{2}{5}(3\underline{a})$

$\overrightarrow{BC} = -6b + \frac{6a}{5}$

$\overrightarrow{\mathbf{BC}} = \frac{6}{5}\underline{a} + 6\underline{b}$

(ii) $\overrightarrow{BE} = h\overrightarrow{BC}$

$\overrightarrow{BE} = h(\frac{6}{5}\underline{a} - 6\underline{b})$

$\overrightarrow{\mathbf{BE}} = 6h(\frac{1}{5}\underline{a} - \underline{b})$

4. (i) $\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB}$

$\overrightarrow{OB} = \underline{a} + \underline{b}$

(ii) $\overrightarrow{AD} = \overrightarrow{AO} + \overrightarrow{OD}$

$\overrightarrow{AD} = -\overrightarrow{OA} + \frac{1}{2}\overrightarrow{OB}$

$\overrightarrow{AD} = -a + \frac{1}{2}\underline{a} + \frac{1}{2}\underline{b}$

$\overrightarrow{AD} = \frac{-2a+a}{2} + \frac{b}{2}$

$\overrightarrow{AD} = -\frac{1}{2}\underline{a} + \frac{1}{2}\underline{b}$

$\overrightarrow{AD} = -\frac{1}{2}(\underline{a} - \underline{b})$

(iii) $\overrightarrow{BE} = \overrightarrow{BC} + \overrightarrow{CE}$

$\overrightarrow{BE} = \overrightarrow{BC} + \frac{1}{2}\overrightarrow{OC}$

$\overrightarrow{BE} = -a + \frac{1}{2}b$

$\overrightarrow{BE} = \frac{1}{2}\underline{b} - \underline{a}$

5. (i) (a) $\overrightarrow{PQ} = \overrightarrow{PO} + \overrightarrow{OQ}$

$\overrightarrow{PQ} = -2\underline{p} + 2\underline{q}$

$\overrightarrow{PQ} = 2\underline{q} - 2\underline{p}$

(b) $\overrightarrow{QX} = \frac{2}{5}\overrightarrow{PQ}$

$\overrightarrow{\mathbf{QX}} = \frac{2}{5}(2\underline{q} - 2\underline{p})$

(ii) $\overrightarrow{OY} = h\overrightarrow{OX}$

$\overrightarrow{OX} = \overrightarrow{OQ} + \overrightarrow{QX}$

$\overrightarrow{OX} = 2q + \frac{4q}{5} - \frac{4p}{5}$

$\overrightarrow{OX} = \frac{10q+4q}{5} - \frac{4p}{5}$

$\overrightarrow{OX} = \frac{14}{5}\underline{q} - \frac{4}{5}\underline{p}$

$\overrightarrow{\mathbf{OX}} = \frac{1}{5}(14\underline{q} - 4\underline{p})$

$\therefore \overrightarrow{OY} = h(\frac{14}{5}q - \frac{4}{5}p)$

$\overrightarrow{OY} = \frac{14h}{5}\underline{q} - \frac{4h}{5}\underline{p}$

$\overrightarrow{\mathbf{OY}} = \frac{1}{5}(14h\underline{q} - 4h\underline{p})$

TOPIC 7 SOLUTIONS: TROGONOMTRY

1. (a) (i) to find PQ = r, first find angle R

$\hat{R} = 180° - (46° + 36°)$

$= 98°$

$\therefore \frac{\sin R}{r} = \frac{\sin P}{p}$

$\frac{\sin 98°}{r} = \frac{\sin 46°}{36.5}$

$r\sin 46° = 36.5 \sin 98°$

$r = \frac{36.5 \sin 98°}{\sin 46°}$

$r = 50.24716343$

$\therefore \mathbf{PQ \approx 50.2km}$

(ii) $A = \frac{1}{2} \times p \times r \times \sin Q$

$= \frac{1}{2} \times 36.5 \times 50.2 \times \sin 36°$

$= 916.15 \sin 36°$

$\therefore A = 538.4994589 \approx \mathbf{538.5\ km^2}$

(ii) Shortest distance $= \frac{2A}{b} = \frac{2 \times 538..2}{50.2} \approx \mathbf{10.7\ km}$

(b) $\sin\theta = 0.6792$

$\theta = \sin^{-1}(0.6792)$ → press shift / 2nd f and then sin

$\theta = 42.8°$

In the 1st quadrant θ = 42.8°

In the 2nd quadrant θ = 180 − 42.8 = 137.2°

$\therefore\ \boldsymbol{\theta = 42.8° \text{ or } \theta = 137.2°}$

2. **(a) (i)** $A = \frac{1}{2} ts \sin H$

$A = \frac{1}{2} \times 1.3 \times 1.9\ sin130°$

$A = 1.235 \sin 130°$

$A = 0.946$

$\mathbf{A \approx 0.95\ km^2}$

(ii) $h^2 = t^2 + s^2 - 2tscosH$

$h^2 = (1.3)^2 + (1.9)^2 - 2(1.3)(1.9)\cos 130°$

$h^2 = 5.3 - (-3.175370792)$

$h^2 = 5.3 + 3.17537$

$h^2 = 8.475370792$

$h^2 = \sqrt{8.475370792}$

$h = 2.911249009$

$\therefore\ \mathbf{TS \approx 2.9km}$

(iii) Shortest distance $= \frac{2A}{b}$

$= \frac{2 \times 0.95}{2.9}$

$= 0.65517$

$\approx \mathbf{0.66km}$

(b) $\cos\theta = \frac{2}{3}$

$\theta = \cos^{-1}\left(\frac{2}{3}\right)$

$\theta = 48.1896851$

$\therefore\ \theta \approx 48.2°$

3. (i) $k^2 = i^2 + m^2 - 2im\cos K$

$k^2 = 5^2 + 3^2 - 2(5)(3)\cos 110°$

$k^2 = 34 - (-10.2606043)$

$k^2 = 44.26060643$

$k = \sqrt{44.26060643}$

$k = 6.656864368$

$\therefore\ \mathbf{MI \approx 6.65km}$

(ii) $A = \frac{1}{2} \times i \times m\ sinK$

$A = \frac{1}{2} \times 5 \times 3\ sin110°$

$A = 7.5 \sin 110°$

$A = 7.047694656$

$\mathbf{A \approx 7.05km^2}$

(iii) shortest. Distance $= \frac{2A}{b} = \frac{2 \times 7.05}{6.65}$

$\approx \mathbf{2.12km}$

(b) $\tan\theta = 0.7$

$\theta = \tan^{-1}(0.7)$

$\theta = 34.9920202$

$\mathbf{\theta \approx 35°}$

4. (a) (i) $\text{Cos } A = \frac{b^2 + c^2 - a^2}{2bc}$

$\text{Cos } A = \frac{43^2 + 43^2 - 37^2}{2(43)(43)}$

$\text{Cos } A = \frac{2329}{3698}$

$A = \cos^{-1}\left(\frac{2329}{3698}\right)$

$A = 50.96463954$

$\therefore\ \mathbf{B\hat{A}C \approx 51°}$

(ii) $A\hat{C}B = (180° - 51) \div 2$

$A\hat{C}B = 129° \div 2$

$\therefore\ \mathbf{A\hat{C}B = 64.5°}$

(iii) $A = \frac{1}{2} absinC$

$A = \frac{1}{2} \times 37 \times 43 \times sin64.5$

$A = 795.5 \sin 64.5°$

$\mathbf{A = 718.0km^2}$

(iv) S. distance $= \frac{2A}{b}$

$= \mathbf{\frac{2 \times 718.0}{43}}$

$= 33.39565552$

$\approx \mathbf{33\ km}$

(b) $\cos\theta = 0.5$

$\theta = \cos^{-1} 0.5$

$\theta = 60°$

In the 1st quadrant $\theta = 60°$

in the 4th quad $\theta = 360 - 60°$

$\boldsymbol{\theta = 60° \text{ or } \theta = 300°}$

5. **(i)** $A = \frac{1}{2} mcsinK$

$A = \frac{1}{2} \times 78 \times 123 \times sin81°$

$A = 4797 \sin 81°$

$A = 4737.94097$

$\mathbf{A \cong 4700km^2}$

(ii) $k^2 = c^2 + m^2 - 2cmcosK$

$k^2 = (123)^2 + (78)^2 - 2(123)(78)\ \text{Cos}81°$

$k^2 = 21213 - (3001.664515)$

$k^2 = 18211.33548$

$k = \sqrt{18211.33548}$

$k = 134.9493812$

$\therefore\ \mathbf{CM \approx 135km}$

$(\boldsymbol{iii})$ K 215 000 × 135 = $\boldsymbol{K29025000}$

was the total cost for constructing road CM

TOPIC 8 SOLUTIONS: MENSURATION

1. (i) Let height EA $= x$, then

$\frac{x}{4} = \frac{15+x}{12}$

$12x = 4(15 + x)$

$12x = 60 + 4x$

$12x - 4x = 60$

$8x = 60$

$x = 7.5$

$\therefore$ **AE = 7.5** And AD = 7.5 +15 = **22.5 cm**

(ii) Volume that remained is the that of the frustum

$V = \frac{1}{3}\pi R^2 H - \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi(R^2 H - r^2 h)$

$V = \frac{1}{3} \times 3.142(12^2 \times 22.5 - 4^2 \times 7.5)$

$V = \frac{3.142}{3}(3240 - 120)$

$V = \frac{3.142}{3}(3120)$

V = 3267.68cm^3

2. First find the height of the small cone that was cutoff.

$\frac{h}{7} = \frac{20+h}{21}$

$21h = 7(20+h)$

$21h = 140 + 7h$

$21h - 7h = 140$

$14h = 140$

$h = 10$cm and H = 10 + 20 = 30cm

$\therefore V = \frac{1}{3}\pi R^2 H - \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi(R^2 H - r^2 h)$

$V = \frac{3.142}{3}(21^2 \times 30 - 7^2 \times 10)$

$V = \frac{3.142}{3}(13230 - 490)$

$V = \frac{3.142}{3}(12740)$

V = 13343.02667

$\mathbf{V \cong 13343 cm^2}$

3. (a) V = lbh

V = 1.2 × 0.9 × 10

V = 10.8cm^3

1cm^3 → 1000l

10.8cm^3 → x

$x = 10.8 \times 1000l$

$x = 10800l$

$\therefore \mathbf{V = 10800l}$

(b) first find the radius of a cone

$r^2 = 13^2 - 12^2$

$r^2 = 25$

$r = \sqrt{25}$

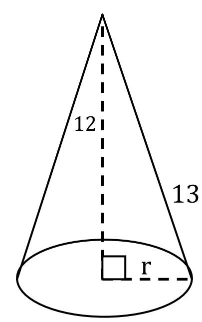
r = 5cm

$\therefore T.S.A = \pi r^2 + \pi r l$

T.S.A = $\pi r(r + l)$

T.S.A = 3.142 × 5(5 + 13)

T. S. A = 282.78cm^2



4. **(i)** T. S. A = $2\pi r^2 + 2\pi r h$

T. S. A = $2\pi r(r + h)$

T.S.A = (2× 3.142 × 10)(10 × 250)

T. S. A = 62.84(260)

T. S. A = 16338.4cm^2

(ii) $V = \pi r^2 h$

V = 3.142 × 10^2 × 250

V = 78550 cm^3

1 000 000cm^3 → 1000l

78550cm^3 = x

$x = \frac{78550 \times 1000}{1\,000\,000}$

x = 78.55l

$\therefore \mathbf{V = 78.55l}$

5. (i) Perimeter of the two small circles

P = $2\pi r + 2\pi r$

P = $4\pi r$

P = 4× 3.142 × 10

P = 125.68 cm

(ii) Area of the shaded part

A = area of large circle – area of two small circles

A = $\pi R^2 - 2\pi r^2$

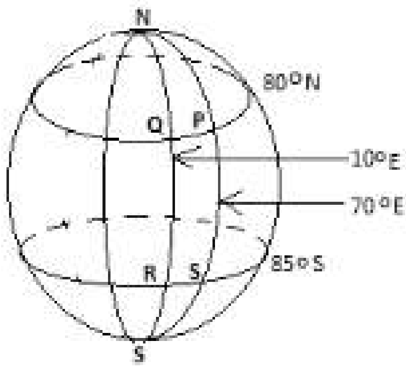
A = 3.142× 20^2 − 2(3.142 × 10^2)

A = 1256.8 – 628.4

A = 628.4 cm^2

TOPIC 9 SOLUTIONS: EARRTH GEOMETRY

1. (i)



(ii)(a) Distance QR $= \frac{\theta}{360°} \times 2\pi R \quad \theta = 80° + 85° = 165°$

QR $= \frac{165°}{360°} \times 2 \times 3.142 \times 3437$

QR $= 9899.132833$

QR $\approx$ 9900*nm*

(b) $C = 2\pi R cos\theta$

$C = 21600 \text{ Cos } \theta$

$C = 21600 \times cos\, 85°$

$C = 1882.564043$

C $\approx$ 1900*nm*

2. **(i)** Difference in latitudes

$\boldsymbol{\theta} = 80° + 30° = \mathbf{110°}$

(ii) (a) XZ $= \frac{\theta}{360} \times 2\pi R$

XZ $= \frac{110°}{360°} \times 2 \times 3.142 \times 3437$

XZ $= 6599.421889$

XZ $\approx$ 6600*nm*

(b) YZ $= \frac{\propto}{360°} \times 2\pi R cos\theta$ where $\propto = 15° + 105 = 120°$

YZ $= \frac{120°}{360°} \times 2 \times 3.142 \times 3437 \times cos 30°$

YZ $= 6234.836734$

YZ $\approx$ 6200nm

3. **(i)** Difference in latitudes

$\theta = 50° + 70° = \mathbf{120°}$

(ii) $C = 2\pi R cos\theta$

$C = 21600 \cos 50°$

C=13884.21237nm

C $\approx$ 19000*nm*

(ii) AD $= \frac{\theta}{360°} \times 2\pi R$

AD $= \frac{120°}{360°} \times 2 \times 3.142 \times 3437$

AD $= 7199.369333$

AD $\approx$ 7200*nm*

4. **(i)** A (60°N, 61°W)

B (80°S, 61°W)

(ii) Difference in longitudes

$\propto = 81° - 31° = 50°$

(iii) CD $= \frac{\propto}{360°} \times 2\pi R cos\theta$

CD $= \frac{50°}{360°} \times 2 \times 3.142 \times 3437 \times cos 80°$

CD $= 520.8989021$nm

CD $\approx$ 521*nm*

(iv) local time at C

$1h = 15°$

$x = 50°$

$x = \frac{50°}{15°}$

$x = 3\frac{5}{15}$

$\frac{5}{15} \times 60 = 20$minutes

3:20hrs

13 : 05

$-$ 3 : 20

09 55 hrs

5. (i) C (45°N, 85°E)

(ii) Difference in latitudes

$\theta = 45° + 55° = \mathbf{100°}$

(iii) $CD = \frac{\theta}{360°} \times 2\pi R$

$CD = \frac{100°}{360°} \times 2 \times 3.142 \times 3437$

$CD = 6999.47444$

$\mathbf{CD \approx 6000\ nm}$

(iv) $C = 2\pi R cos\theta$

$C = 21600 \cos\theta$

$C = 15273.50647$

C = 15000nm

TOPIC 10 SOLUTIONS: STATISTICS

1.

Class marks	Midpoints (x)	Frequency (f)	fx	x^2	fx^2
$25 < x \le 30$	27.5	5	137.5	756.25	3781.25
$30 < x \le 35$	32.5	4	130	1056.25	4225
$35 < x \le 40$	37.5	7	262.5	1406.25	9843.75
$40 < x \le 45$	42.5	11	467.5	1806.25	19868.75
$45 < x \le 50$	47.5	12	570	2256.25	27075
$55 < x \le 55$	52.5	8	420	2256.25	22050
$55 < x \le 60$	57.5	1	57.5	3306.25	3306.25
Totals		$\sum f = 48$	$\sum fx = 2045$		$\sum fx^2 = 90150$

$\bar{x} = \frac{\sum fx}{\sum f} = \frac{2045}{48} = \mathbf{42.6}$

$SD = \sqrt{\left\{\frac{\sum fx^2}{\sum f} - (\bar{x})^2\right\}}$

$SD = \sqrt{\frac{90150}{48} - (42.6)^2}$

$SD = \sqrt{1878.125 - 1814.76}$

$SD = \sqrt{63.365}$

SD = 7.96 ≈ 8

2.

Class marks	x	f	fx	$(x - \bar{x})^2$	$f(x - \bar{x})^2$
$0 < x \le 5$	2.5	13	32.5	12.25	159.25
$5 < x \le 10$	7.5	27	205.5	17.2225	465.0
$10 < x \le 15$	12.5	35	437.5	0.7225	25.288
$15 < x \le 20$	17.5	16	280	34.2225	547.58
$20 < x \le 25$	22.5	7	157.5	117.7225	824.058
$25 < x \le 30$	27.5	2	55	251.2225	502.445
		$\sum f = 100$	$\sum fx = 1168$		$\sum f(x - \bar{x})^2 = 2523.621$

$$\bar{x} = \frac{\sum fx}{\sum f} = \frac{1165}{100} = \mathbf{11.65}$$

$$\text{SD} = \sqrt{\frac{\sum f(x-\bar{x})^2}{\sum f}} = \sqrt{\frac{2523.621}{100}} = \sqrt{25.23621} = 5.023565467$$

$\text{SD} \approx \mathbf{5}$

3.

Class marks	x	f	fx	x^2	fx^2
$0 < x \leq 10$	5	7	35	25	175
$10 < x \leq 20$	15	22	330	225	4950
$20 < x \leq 30$	25	28	700	625	17500
$30 < x \leq 40$	35	23	805	1225	28175
$40 < x \leq 50$	45	15	675	2025	30375
$50 < x \leq 60$	55	5	275	3025	15125
		$\sum f = \mathbf{100}$	$\sum fx = \mathbf{2820}$		$\sum f x^2 = \mathbf{96300}$

$\text{Mean} = \bar{x} = \frac{\sum fx}{\sum f} = \frac{2820}{100} = 28.2$

$$\therefore SD = \sqrt{\frac{\sum f x^2}{\sum f} - (\bar{x})^2}$$

$\text{SD} = \sqrt{\frac{96300}{100} - (28.2)^2}$

$\text{SD} = \sqrt{963 - 795.24}$

$\text{SD} = \sqrt{167.76}$

$\text{SD} = 12.95221989$

$\approx \mathbf{13}$

4. $\text{Mean} = \bar{x} = \frac{\sum fx}{\sum x}$

$$\sum fx = (15 \times 20) + (25 \times 50) + (35 \times 55) + (45 \times 70) + (55 \times 60) + (65 \times 40) + (75 \times 35) + (10 \times 85) + (95 \times 5)$$

$\sum fx = 16475$

$\sum f = 350$

$\therefore \bar{x} = \frac{16475}{350}$

$\bar{x} = 47.0742857$

$\approx \mathbf{47.1}$

WISH YOU ALL THE BEST DURING YOUR REVISIONS