TOPIC 1: BASIC ARITHMETIC

NUMBERS AND ORDER OF OPERATIONS

Order of operation refers to an established order in which operations must be done. Anything in brackets has to be done first, then multiplication and division and then addition and subtraction. An aid to remember this is BODMAS. Brackets of division, multiplication, addition and subtraction [BODMAS].

PAST EXAMINATION EXAMPLES

2000 P1

1. Evaluate **6 + 4 ÷ 2**

SOLUTION

$6 + 4 \div 2$

$= 6 + (4 \div 2)$

$= 6 + 2$

$= 8$

2002 P1

2. Evaluate **5.5 + 5 × 0.4**

SOLUTION

$5.5 + 5 \times 0.4$

$= 5.5 + (5 \times 0.4)$

$= 5.5 + 2.0$

$= 7.5$

2012 P1

3. Evaluate **7 + 21 ÷ 3 × 7 - 7**

SOLUTION

$7 + 21 \div 3 \times 7 - 7$

$= 7 + (21 \div 3) \times 7 - 7$

$= 7 + 7 \times 7 - 7$

$= 7 + 49 - 7$

$= 49$

PAST EXAMINATION QUESTIONS

2015 G.C.E P1

1. Evaluate **1 + 7 × 3**

2014 G.C.E P1

2. Evaluate **18 – 3 × 2 + 4**

2014 P1

3. Evaluate **14 + 3(7 – 2) – 2 × 5**

2013 G.C.E P1

4. Evaluate **9 + 5 × 2**

2004 P1

5. Evaluate **24.9 - 7.5 ÷ 0.3**

2006 P1

6. Evaluate **– 5 × 7 –24 ÷ 3**

2001 P1

7. Evaluate **7 – 2 + 3 × 4**

8. Find the values of

(a) **16 – 2 × 2**

(b) **24 ÷ 6 + 2 × 9**

9. Find the values of

(a) **0.2 × 0.45**

(b) **8.4 ÷ 0.02**

TOPIC 2: PERCENTAGES, DECIMALS AND FRACTIONS

A) CONVERTING A PERCENTAGE TO A FRACTION

When converting the percentage to a fraction, divide by 100 and remove the %.

EXAMPLE 2.1

1. Express **5%** as a fraction in its lowest term.

SOLUTION

5%

$= \frac{5}{100}$

$= \frac{1}{20}$

2. Express **35%** as a fraction in its lowest term.

SOLUTION

35%

$= \frac{35}{100}$

$= \frac{7}{20}$

3. Express **60%** as a fraction in its lowest term.

SOLUTION

60%

$= \frac{60}{100}$

$= \frac{3}{5}$

B) CONVERTING A FRACTION TO A PERCENTAGE

When converting a fraction to a percentage multiply by 100%.

EXAMPLE 2.2

1. Express $\mathbf{\frac{3}{4}}$ as a percentage.

SOLUTION

$\frac{3}{4}$

$= \frac{3}{4} \times 100\%$

$= 75\%$

2. Express $\mathbf{\frac{2}{5}}$ as a percentage.

SOLUTION

$\frac{2}{5}$

$= \frac{2}{5} \times 100\%$

$= 40\%$

3. Express $\mathbf{\frac{1}{5}}$ as a percentage.

SOLUTION

$\frac{1}{5}$

$= \frac{1}{5} \times 100\%$

$= 20\%$

TOPIC 2: PERCENTAGES, DECIMALS AND FRACTIONS

C) CONVERTING A DECIMAL TO A PERCENTAGE

When converting a decimal to a percentage, shift 2 decimal places to the right by multiplying by 100.

EXAMPLE 2.3

1. Express **0.25** as a percentage.

SOLUTION

0.25

= 0.25×100

= 25%

2. Express **0.0125** as a percentage.

SOLUTION

0.0125

= 0.0125×100

= 1.25%

3. Express **0.03925** as a percentage.

SOLUTION

0.03925

= 0.03925×100

= 3.925%

D) CONVERTING A PERCENTAGE TO A DECIMAL

When converting a percentage to a decimal, shift 2 decimal places to the left by dividing by 100.

EXAMPLE 2.4

1. Express $\mathbf{78}\%$ as a decimal.

SOLUTION

78%

$= \frac{78}{100}$

= 0.78

2. Express $\mathbf{12}\%$ as a decimal.

SOLUTION

12%

$= \frac{12}{100}$

= 0.12

3. Express $\mathbf{6.5}\%$ as a decimal.

SOLUTION

6.5%

$= \frac{6.5}{100}$

= 0.065

D) CONVERTING A FRACTION TO A DECIMAL

EXAMPLE 2.5

1. Express $\mathbf{\frac{2}{5}}$ as a decimal.

SOLUTION

$\frac{2^{0}}{5}$ = 0.4

2. Express $\mathbf{\frac{3}{4}}$ as a decimal.

SOLUTION

$\frac{3^{020}}{4}$ = 0.75

3. Express $\mathbf{\frac{13}{20}}$ as a decimal.

SOLUTION

$\frac{13^{0100}}{20}$ = 0.65

TOPIC 2: PERCENTAGES, DECIMALS AND FRACTIONS

PAST EXAMINATION QUESTIONS

2015 P1

1. Express **15%** as a fraction in its lowest term.

2006 P1

2. Express **28%** as a fraction in its lowest term.

2013 P2

3. Express **0.041** as a percentage.

2001 P1

4. Express $\frac{27}{40}$ as a percentage.

2002 P1

5. Express $\frac{9}{25}$ as a percentage.

2010 P1

6. Express $\frac{21}{40}$ as a percentage.

2004 P1

7. Express $\frac{3}{25}$ as a decimal fraction.

2007 P1

8. Express $\frac{3}{8}$ as a decimal fraction.

2015 P1

9. Express $\frac{14}{125}$ as a decimal fraction.

2010 P1

10. Express **0.52** as a fraction in its lowest term.

2002 P2

11. Express **0.05** as a fraction in its lowest term.

2012 P2

12. Express $3\frac{2}{5}\%$ as a decimal

2002 P1

13. Subtract **0.5625** from **0.6875**

14. Express **0.03** as a fraction in its lowest term.

TOPIC 3: OPERATIONS OF FRACTIONS

ADDING AND SUBTRACTING OF FRACTIONS

EXAMPLE 3.1

1. Calculate $\frac{1}{3} + \frac{2}{5} - \frac{1}{4}$

SOLUTION

$\frac{1}{3} + \frac{2}{5} - \frac{1}{4}$

$= \frac{20+24-15}{60}$

$= \frac{29}{60}$

2. Calculate $1\frac{1}{5} - 2\frac{3}{4} + 3\frac{1}{2}$

SOLUTION

$1\frac{1}{5} - 2\frac{3}{4} - 3\frac{1}{2}$

$= \frac{6}{5} - \frac{11}{4} + \frac{7}{2}$

$= \frac{24-55+70}{20}$

$= \frac{39}{20}$

$= 1\frac{19}{20}$

MULTIPLYING AND DIVIDING OF FRACTIONS

When multiplying fractions, multiply the numerator and denominator together.
When dividing fractions, multiply the 1st fraction by the reciprocal of the 2nd fraction.

EXAMPLE 3.2

1. Calculate $\frac{3}{4} \times \frac{1}{2} \times \frac{5}{6}$

SOLUTION

$\frac{3}{4} \times \frac{1}{2} \times \frac{5}{6}$

$= \frac{15}{48}$

$= \frac{5}{16}$

2. Calculate $\frac{3}{5} \div \frac{2}{3}$

SOLUTION

$= \frac{3}{5} \div \frac{2}{3}$

$= \frac{3}{5} \times \frac{3}{2}$

$= \frac{9}{10}$

COMBINED OPERATIONS OF FRACTIONS

When calculating fractions where there are different signs we apply BODMAS and follow the normal rules.

Note: when there are mixed fractions we first change mixed fractions to improper ones.

TOPIC 3: OPERATIONS OF FRACTIONS

COMBINED OPERATIONS OF FRACTIONS

EXAMPLES

2015 GCE MATHS P2

1. Calculate $2\frac{1}{4} + 4\frac{1}{2} \div \frac{2}{3}$

SOLUTION

$2\frac{1}{4} + 4\frac{1}{2} \div \frac{2}{3}$

$= \frac{9}{4} + \frac{9}{2} \div \frac{2}{9}$

$= \frac{9}{4} + (\frac{9}{2} \times \frac{9}{2})$

$= \frac{9}{4} + \frac{81}{4}$

$= \frac{90}{4}$

2013 MATHS P2

2. Calculate $2\frac{5}{8} - 2\frac{1}{6} \div 1\frac{1}{12}$

SOLUTION

$2\frac{5}{8} - 2\frac{1}{6} \div 1\frac{1}{12}$

$= \frac{21}{8} - \frac{13}{6} \div \frac{13}{12}$

$= \frac{21}{8} - (\frac{13}{6} \times \frac{12}{13})$

$= \frac{21}{8} - \frac{2}{1}$

$= \frac{21-16}{8} \quad = \frac{5}{8}$

2004 MATHS P1

3. Calculate $\frac{7}{8} - \frac{7}{11}$

SOLUTION

$\frac{7}{8} - \frac{7}{11}$

$= \frac{77-56}{88}$

$= \frac{21}{88}$

PAST EXAMINATION QUESTIONS

2001 MATHS P1

1. Evaluate **7-(-4.5)**

2006 MATHS P1

2. Evaluate $\mathbf{1.5 + 3\frac{1}{4}}$

2007 MATHS P1

3. Evaluate $\frac{6}{11} + \frac{4}{7}$

2015 GCE MATHS P1

4. Evaluate $\frac{2}{3} - \frac{3}{4}$

2012 MATHS P1

5. Evaluate $2\frac{1}{5} - \frac{3}{4}$

2014 GCE MATHS P2

6. Evaluate $2\frac{1}{4} - 1\frac{5}{6} + 4$

2006 MATHS P1

7. Divide $7\frac{1}{2}$ By $1\frac{1}{2}$

2012 MATHS P1

8. Evaluate $\frac{2}{5} \div \frac{7}{12}$

2011 MATHS P2

9. Evaluate $3\frac{1}{4} \div 5\frac{1}{5}$

2007 MATHS P1

10. Simplify $3\frac{1}{3} + \frac{4}{5} \div \frac{8}{15}$

2012 MATHS P2

11. Evaluate $2\frac{1}{3} - 2\frac{1}{4} \div 1\frac{1}{2}$

2009 MATHS P2

12. Evaluate $3 + \frac{1}{2} \div \frac{1}{4}$

TOPIC 4: BASIC ALGEBRA

ALGEBRAIC EXPRESSION

An algebraic expression is one that contains a combination of numbers and letters.e.g. 2a,3b+5d,6x-5.

SIMPLIFICATION OF ALGEBRAIC EXPRESSION

Simplification is the combination of algebraic terms. This is achieved by adding or subtraction of like terms. Examples of like terms are 2x,6x and 10x.Unlike terms can't be simplified.

EXAMPLE 4.1

1. Simplify as far as possible

$2k - 6 + 3k + 4$ [2]

Solution

$2k - 6 + 3k + 4$

$2k + 3k - 6 + 4$

$5k - 2$

2. Simplify as far as possible

$3[2c - 5] - 5[4c + 3]$

Solution

$3[2c - 5] - 5[4c + 3]$

$6c - 15 - 20c - 15$

$6c - 20c - 15 - 15$

$6c - 30$

3. Simplify as far as possible

$2e + 3f - 2[e + f]$

Solution

$2e + 3f - 2[e + f]$

$2e + 3f - 2e - 2f$

$2e - 2e + 3f - 2f$

f

PAST EXAMINATION QUESTIONS

2016 SPECIMEN P1

1. Simplify

$5(2a-3b) - (6a - 2)$ [2]

2014 P2

2. Simplify

$5(2y - 3) - 2(5 - 2y)$ [2]

2014 P1

3. Simplify

$7a + 2b - 4(a - b)$ [2]

2012 P1

4. Simplify

$3x - 7[2x - 5]$ [2]

2011 P2

5. Simplify

$3x - 4y - 2[x - 4y] - 2y$ [2]

1987 MATHS P2

6. Simplify as far as possible

$3[2y - 5] - 2[7 - 2y]$ [2]

TOPIC 4: BASIC ALGEBRA

SUBSTITUTION

Substitution is the replacement of the letters in an expression with specific values.

EXAMPLE 4.1

2001 MATHS P1

1. Given That
a=3 b=-4 c=2
Evaluate:

$$\frac{\mathbf{4a-(b+c)}}{\mathbf{c}}$$

SOLUTION

$$\frac{4a-(b+c)}{c}$$

$$= \frac{4(3)-(-4+2)}{2}$$

$= 7$

2006 MATHS P1

2. Given That
a=3 b=-5 c=4
Evaluate:

$\mathbf{a - 2b - c^2}$

SOLUTION

$a - 2b - c^2$

$= (3) - 2(-5) - (4)^2$

$= 3 + 10 - 16$

$= -3$

2004 MATHS P2

3. Given That
a=3 b=2 c=4
Evaluate:

$$\frac{\mathbf{a}}{\mathbf{b}} + \frac{\mathbf{b}}{\mathbf{c}}$$

SOLUTION

$$\frac{a}{b} + \frac{b}{c}$$

$$= \frac{3}{2} + \frac{2}{4}$$

$= 2$

PAST EXAMINATION QUESTIONS

2005 MATHS P2

1. Given That
$\mathbf{a} = \frac{\mathbf{3a+b}}{\mathbf{b}}$
Find the Value of a When
b = 2

2007 MATHS P2

2. Given That
x = 8 y = -3
Evaluate:

$\mathbf{x - y^2}$

2009 MATHS P2

3. Given that
p = 7 q = -3
Evaluate:

$\mathbf{p^2 - q^3}$

2015 GCE MATHS P2

4. Given That
a = -1 b = 3 c = -9
Evaluate:

$\mathbf{a^2 - bc}$

2004 MATHS P1

5. Given That
$\mathbf{A = \frac{x}{2} + y - 1}$

Find the Value of x When

A = 11 and Y = 5

2001 MATHS P1

6. Given That
a – b = 10 and
$a^2 - b^2 = 64$,

Find the value of **a + b**

TOPIC 4: BASIC ALGEBRA

FACTORISATION

To factorize is to single out that which is common.

EXAMPLE 4.3.1

1. Factorise Completely

$4x^2 - 12x$

Solution

$4x^2 - 12x$

$= 4x [x – 3]$

2. Factorise Completely

3ax + 6ab + 4x + 8b

Solution

3ax + 6ab + 4x + 8b

= 3a[x+2b] +4[x+2b]

= [x+2b] [3a+4]

3. Factorise Completely

3ac – 6ad – 2bc + 4bd

Solution

3ac – 6ad – 2bc + 4bd

= 3a[c-2d] -2b[c-2bd]

= [c-2d] [3a-2bd]

PAST EXAMINATION QUESTIONS

2015 GCE P2

1. Factorise Completely

$2x^2 – 3x + 2cx – 3c$ [2]

2014 P2

2. Factorise Completely

2xy + 8x – 3y – 12 [2]

2013 P1

3. Factorise Completely

3ap – 8bq – 4aq + 6bq [2]

2012 P2

4. Factorise Completely

2xy + x – 10y – 5 [2]

2007 P1

5. Factorise Completely

2ax + 6bx – 4ay – 12by [2]

6. Factorise Completely

3an – 6ac – 2bd + 4cd [2]

7. Factorise Completely

3x – 6y + dx -2dy [2]

8. Factorise Completely

ac -3c +2ab -6b [2]

9. Factorise Completely

cx -2cy -3dx +6dy [2]

ii) FACTORISATION USING DIFFERENCE OF TWO SQUARES

Difference of two squares is an expression of the form $a^2 - b^2$ Which can be factorized to $(a + b)(a - b)$. HENCE $a^2 - b^2 = (a + b)(a - b)$

EXAMPLE 4.3.2

1. Factorise Completely

$x^2 - 4$

Solution

$x^2 - 4$

$= [x + 2][x - 2]$

2011 P2

2. Factorise Completely

a) $3x^2 - 3$ [2]

Solution

$3x^2 - 3$

$= 3[x^2 - 1]$

$= 3[x + 1][x - 1]$

2007 MATHS P2

3. Factorise Completely

$3x - 12x^3$

Solution

$3x - 12x^3$

$= 3x[1 - 4x^2]$

$= 3x[1 + 2x][1 - 2x]$

PAST EXAMINATION QUESTIONS

2004 MATHS P2

1. Factorise Completely

$4 - 16x^2$

2006 MATHS P1

2. Factorise Completely

$at^2 - 4a + 2t^2 - 8$

2010 P2

3. Factorise Completely

$5 - 20x^2$ [2]

2013 P2

4. Factorise Completely

$3y^2 - 12$ [2]

2014 GCE P2

5. Factorise Completely

$125a^2 - 5$ [2]

2015 GCE P1

6. Simplify

$9x^3 - 81x$ [2]

2016 SPE P1

7. Factorise Completely

$3x^2 - 27$ [2]

2009 P2

8. Simplify

$\frac{a-2}{a^2-4}$ [3]

2015 GCE P2

9. Simplify

$\frac{h^2-k^2}{h+k}$ [3]

2016 P2

10. Simplify

$\frac{x-1}{x^2-1}$ [3]

2017 GCE P2

11. Simplify

$\frac{\mathbf{m^2-1}}{\mathbf{m-m}}$ [2]

12. Simplify

$\frac{\mathbf{y^2-25}}{\mathbf{y+5}}$ [2]

iii) FACTORISATION OF QUADRATIC EXPRESSIONS

A quadratic expression $(ax^2 + bx + c)$ can be factorized into 2 pairs of brackets by using the method below.

Step 1: find the product of a and c.
Step 2: find the factors of a and c whose sum is b
step 3: express the term bx the sum of terms using the factors found in step 2.

EXAMPLES ON FACTORISATION

1. Factorise Completely

$x^2 + 5x + 6$

Solution

$x^2 + 5x + 6$

P=6 s=5 f=2,3

$= x^2 + 3x + 2x + 6$

$= x[x+2] + 3[x+2]$

$= [x+2]\,[x+3]$

2. Factorise Completely

$3x^2 - x - 10$

Solution

$3x^2 - x - 10$

P=-30 s=-1 f= -6,5

$= 3x^2 - 6x + 5x - 10$

$= 3x[x-2] + 5[x-2]$

$= [x-2]\,[3x+5]$

2005 MATHS P2

3. Simplify

$$\frac{1-x^2}{5-3x-2x^2}$$

Solution

$$\frac{1-x^2}{5-3x-2x^2}$$

$$= \frac{[1+x][1-x]}{[2x+5][1-x]}$$

$$= \frac{[1+x]}{[2x+5]}$$

EXERCISE ON FACTORISATION

1. Factorise Completely

$x^2 - 7x + 12$ [2]

2. Factorise Completely

$a^2 + 5a - 6$ [2]

2007 P1

3. Factorise Completely

$6x^2 - x - 2$ [2]

2006 P2

4. Simplify

$$\frac{2x^2-2}{2x^2-x-3}$$ [3]

2010 P2

5. Simplify

$$\frac{3y^2 - 5y-12}{y^2-9}$$ [3]

2014 P1

3. Factorise Completely

$5x^2 - x - 4$ [2]

TOPIC 4: BASIC ALGEBRA

ALGEBRAIC FRACTIONS

SUBTRACTION AND ADDITION OF ALGEBRAIC FRACTIONS

A fraction which contains at least one letter is called an algebraic fraction.
Algebraic fractions can be added or subtracted exactly the same way as arithmetical fractions by expressing them as fractions with the lowest lcm.

EXAMPLE 4.3.1

1. Express as a single fraction and simplify where possible.

$$\frac{x+1}{3} + \frac{x+3}{2}$$

Solution

$$\frac{x+1}{3} + \frac{x+3}{2}$$

$$= \frac{2(x+1)+3(x+3)}{6}$$

$$= \frac{5x+11}{6}$$

2. Express as a single fraction and simplify where possible.

$$\frac{3}{x-2} - \frac{2}{x-3}$$

Solution

$$\frac{3}{x-2} - \frac{2}{x-3}$$

$$= \frac{3(x-3)-2(x-2)}{(x-2)(x-3)}$$

$$= \frac{x-5}{(x-2)(x-3)}$$

2009 P2

3. Express as a single fraction and simplify where possible.

Solution

$$\frac{5}{2x-1} - \frac{7}{3x-2}$$

$$= \frac{5(3x-2)-7(2x-1)}{(2x-1)(3x-2)}$$

$$= \frac{x-3}{(2x-1)(3x-2)}$$

D) MULTIPLICATION AND DIVISION OF ALGEBRAIC FRACTIONS

Algebraic fractions can be multiplied or divided in exactly the same way as arithmetical fractions.

1. Express as a single fraction and simplify where possible.

$$\frac{2z}{xy} \times \frac{y}{4z}$$

Solution

$$\frac{2z}{xy} \times \frac{y}{4z}$$

$$= \frac{1}{2x}$$

2. Express as a single fraction and simplify where possible.

$$\frac{4}{a+b} \times \frac{a^2 - b^2}{12}$$

Solution

$$\frac{4}{a+b} \times \frac{a^2 - b^2}{12}$$

$$= \frac{4}{a+b} \times \frac{(a+b)(a-b)}{12}$$

$$= \frac{(a-b)}{3}$$

3. Express as a single fraction and simplify where possible.

$$\frac{xy}{6} \div \frac{y}{3x}$$

Solution

$$\frac{xy}{6} \div \frac{y}{3x}$$

$$= \frac{xy}{6} \times \frac{3x}{y}$$

$$= \frac{x^2}{2}$$

TOPIC 4: BASIC ALGEBRA

ALGEBRAIC FRACTIONS

EXERCISE

2004 P2

1. Express as a single fraction in its simplest form.

$\frac{3}{2} - \frac{1-2x}{4x}$ [3]

2005 P2

2. Express as a single fraction in its simplest form.

$\frac{3}{p-1} - \frac{2}{1-p}$ [3]

2007 P1

3. Express as a single fraction in its simplest form.

$\frac{3}{x} + \frac{2}{x-1}$ [2]

2007 P2

4. Express as a single fraction.

$\frac{x+5}{3} - \frac{x+2}{4}$ [3]

2010 P2

5. Express as a single fraction in its simplest form.

$\frac{2}{1-3x} + \frac{4}{5+x}$ [3]

2011 P2

6. Express as a single fraction in its simplest form.

$\frac{x+2}{3} - \frac{2x-3}{4}$ [3]

2012 P2

8. Express as a single fraction in its simplest form.

$\frac{4}{2x-1} - \frac{3}{x-1}$ [3]

2013 P2

9. Express as a single fraction in its simplest form.

$\frac{2a}{x-1} - \frac{a}{x-2}$ [3]

2014 P2

10. Express as a single fraction in its simplest form.

$\frac{5}{2y-1} - \frac{6}{3y-1}$ [3]

2014 GCE P2

11. Express as a single fraction in its simplest form.

$\frac{5}{1-2b} - \frac{7}{2-3b}$ [3]

2015 GCE P2

12. Express as a single fraction in its simplest form.

$\frac{2}{b-2} - \frac{3}{1-2b}$ [3]

2016 P2

13. Express as a single fraction in its simplest form.

$\frac{2}{2x-1} - \frac{1}{3x+1}$ [3]

TOPIC 4: BASIC ALGEBRA

ALGEBRAIC FRACTIONS

EXERCISE

1. Simplify

$\frac{3a}{5c^2} \times \frac{10c^3}{a^2}$ [3]

2. Simplify

$\frac{7y}{25a^2} \div \frac{21y^4}{125a^6}$ [3]

3. Simplify

$\frac{2a}{6y^3} \times \frac{36y^7}{a^5}$ [3]

2016 SPECIMEN P2

4. Simplify

$\frac{13k}{20a^2} \times \frac{5}{39k^2}$ [3]

2016 P2

5. Simplify

$\frac{17k^2}{20a^2} \div \frac{51k^2}{5a}$ [3]

2017 GCE P2

6. Simplify

$\frac{p^2q^3}{4} \times \frac{8}{pq} \div 2p^2q$ [3]

TOPIC 5: EQUATIONS

1) CHANGING THE SUBJECT OF AN EQUATION

An equation is a mathematical statement that 2 algebraic expressions are equal.

Changing the subject of an equation is rearranging an equation.

EXAMPLE 5.1

1987 P2

1. Given That
3w – g = hw + 5,

Express **w** in terms of g and h.

Solution

3w – g = hw + 5,

3w - hw = 5 + g

W (3-h) = 5 + g

w = $\frac{g+5}{3-h}$

2. Given That

f = $\frac{u+v}{uv}$

Express **u** in terms of v and f.

Solution

f = $\frac{u+v}{uv}$

fuv= u + v

fuv – u = v

u (fv - 1) = v

u = $\frac{v}{fv-1}$

2015 GCE P1

3. Given That

$\mathbf{k^2}$ **- bc = b + h.**

Make **b** the subject of the formula.

Solution

k^2 - bc = b + h

bc + b = k^2 - h

b(c + 1)= k^2 - h

b = $\frac{k^2-h}{c+1}$

PAST EXAMINATION QUESTIONS

2005 MATHS P2

1. Given that
a = $\frac{3a+b}{b}$

Express b in terms of a.

2010 MATHS P2

2. Given that
d = $\frac{p+y}{5y}$

Express y in terms of d and p.

2013 MATHS P1

3. Given that
m = $\frac{2my-5x}{2}$
Express x in terms of m and y.

2014 MATHS P1

4. Given that
a = $\frac{d-b}{cd-1}$
Make d the subject of the formula.

5. Given that
c = $\frac{2d+1}{3d-1}$

Express d in terms of c.

6. Given that

2 + gw = h - 3kw,

Express w in terms of h, a and k.

TOPIC 5: EQUATIONS

2) LINEAR EQUATIONS

A linear equation is an equation involving an expression of the first degree (highest power of the variable is 1).

2.1) LINEAR EQUATIONS IN ONE VARIABLE

A linear equation is two variables can be written in the form ax + b= 0 where a ≠ 0. To solve a linear equation, rearrange and it for that variable

EXAMPLE 5.2.1

2004 P2

1. Solve the equation

3[m – 5] = 7 – 2[m – 3]

Solution

3[m – 5] = 7 – 2[m – 3]

3m –15 = 7 – 2m + 6

3m + 2m = 7 + 15 + 6

5m = 28

m = 5.6

2006 P2

2. Solve the equation

2x – 6 = 4 – 3[x – 5]

Solution

2x – 6 = 4 – 3[x – 5]

2x – 6 = 4 – 3x + 15

2x + 3x = 4 + 6 + 15

5x = 25

x = 5

2010 P2

3. Solve the equation

2[2x – 5] + 2= x + 7

Solution

2[2x – 5] + 2= x + 7

4x – 10 + 2= x + 7

4x – x = 8 + 7

3x = 15

x = 5

4. Solve the equation

$$\frac{\mathbf{x+1}}{\mathbf{2}} = \frac{\mathbf{3x-6}}{\mathbf{4}} \qquad \textbf{[3]}$$

Solution

$$\frac{x+2}{2} = \frac{-3x-6}{5}$$

4 (x + 2) = 2 (-3x -6)

4x + 8 = -6x -12

4x + 6x = - 8 -12

10x = -20

x = -2

5. Solve the equation

$$\frac{\mathbf{2x+2}}{\mathbf{2}} = \frac{\mathbf{x+5}}{\mathbf{3}} \qquad \textbf{[3]}$$

Solution

$$\frac{2x+2}{2} = \frac{x+5}{3}$$

3 (2x + 2) = 2 (x + 5)

6x + 6= 2x + 10

6x - 2x = 10 - 6

4x = 4

x = 1

TOPIC 5: EQUATIONS

LINEAR EQUATIONS IN ONE VARIABLE

Exercise

2001 P1

1. Solve the equation

$$\frac{x+6}{3} = \frac{3x-6}{5} \qquad [3]$$

2004 P1

2. Solve the equation

$$\frac{1}{3}(3k+4) = \frac{3}{2}(2k-5) \qquad [2]$$

2006 P1

3. Solve the equation

$6x - 7 = 2x + 5$

2011 P2

4. Solve the equation

$$\frac{12}{X+2} = \frac{3}{5}$$

2012 P2

5. Solve the equation

$$\frac{6}{X+2} = \frac{2}{3}$$

2013 P2

6. Solve the equation

$$3\left(\frac{x}{5} - 4\right) = 6 - 3x \qquad [3]$$

2013 P1

7. Solve the equations

$(x - 1)(x + 1) = 8$

2014 P2

8. Solve the equations

$5x - 8 - 3(x + 1) = -7$

2015 GCE P2

9. Solve the equation

$$\frac{x+2}{2} = \frac{2x-1}{3}$$

2015 GCE P1

10. Solve the equations

$5x + 19 = 23 - 7x$

11. Solve the equations

$2(x - 3) - (4 - x) = 5$

12. Solve the equation

$$\frac{x-1}{3} + \frac{x+5}{2} = 8 \qquad [3]$$

13. Solve the equation

$3(2x - 7) = 6 - 4(2 - x)$

14. Solve the equations

$7y - 4(y + 5) = 1$

TOPIC 5: EQUATIONS

2.2) LINEAR EQUATIONS IN TWO VARIABLE

A linear equation in two variables can be written in the form $ax + by + c = 0$ where a and b are ≠ 0.

2.2.1) SIMULTANEOUS EQUATIONS OF LINEAR EQUATIONS IN TWO VARIABLES

There are mainly two methods to solve simultaneous equations.

1. Substitution method

2. Elimination method

1. SUBSTITUTION METHOD

One variable is expressed in terms of the other in either of the given equations and this expression is substituted into the other equation**.**

1. Solve the simultaneous equations by substitution

$\mathbf{2x - y = 5}$

$\mathbf{X - 2y = 4}$

Solution

$2x - y = 5$-----------i)

$X - 2y = 4$----------ii)

From equation ii) $x = 2y + 4$

Substitute equ i into equ ii)

$2(2y + 4) - y = 5$

$4y + 8 - y = 5$

$3y = -3$

$Y = -1$

$x = 2[-1] + 4$

$x = 2$

2001 P1

2. Solve the simultaneous equations by substitution

$\mathbf{2x - y = 3}$

$\mathbf{3x - 2y = 4}$

Solution

$2x - y = 3$-----------i)

$3x - 2y = 4$---------ii)

From equation i) $y = 2x + 3$

Substitute equ i into equ ii)

$3x - 2(2x + 3) = 4$

$3x - 4x - 6 = 4$

$x = -10$

$Y = 2x + 3$

$Y = 2(-10) + 3$

$Y = -17$

TOPIC 5: EQUATIONS

2. ELIMINATION METHOD

One variable is eliminated in order to remain with another variable.

1. Solve the simultaneous equations by elimination.

$2x - 3y = 4$

$x + 3y = 11$

Solution

$$\begin{array}{rl} & [2x - 3y = 4] \text{ ----------i)} \\ + & [x + 3y = 11] \text{ --------ii)} \\ \hline & 3x = 15 \\ & x = 5 \end{array}$$

Substitute x = 5 into equ ii)

$3y = 11 - 5$

$y = 2$

2. Solve the simultaneous equations by elim

$2x - y = 5$

$x - 4y = 6$

Solution

$$\begin{array}{l} 1\,[2x - y = 5] \text{ ----------i)} \\ 2\,[x - 4y = 6] \text{ --------ii)} \\ \hline [2x - y = 5] \\ -[2x - 8y = 12] \\ \hline 7y = -7 \\ y = -1 \end{array}$$

Substitute y = -1 into equ ii)

$x = 4[-1] + 6 \rightarrow x = 2$ and $y = -1$

PAST EXAMINATION QUESTIONS

2007 P1

1. Solve the simultaneous equations.

$y = 2x$

$x - y = 10$

2010 P1

2. Solve the simultaneous equations.

$2x + y = 4$

$x - 2y = 2$

2012 P1

3. Solve the simultaneous equations.

$2x + 5y = 16$

$3x - 2y = 5$

2013 P1

4. Solve the simultaneous equations.

$b = 6 - a$

$2a + 3b = 13$

2014 P1

5. Solve the simultaneous equations.

$3x = 4y + 1$

$7x + y = 23$

TOPIC 5: QUADRATIC EQUATIONS

A quadratic equation is an equation which has the form $ax^2 + bx + c = 0$ where

a is the coefficient of x^2

b is the coefficient of x

c is the coefficient of c

Quadratic equations can be solved using the quadratic equation which states:

$$X = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

EXAMPLE 5.1

2004 MATHS P2

1. Solve the equation $\mathbf{3x^2}$ **- x -1 = 0**

Giving your answer correct to 2 decimal places.
[5]

Solution

$3x^2$ -x -1=0 [where a=3 b=-1 c=-1]

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(3)(-1)}}{2(3)}$$

$$x = \frac{1 \pm \sqrt{1+12}}{6}$$

$$x = \frac{1 \pm \sqrt{13}}{6}$$

$$x = \frac{1 \pm 3.60555}{6}$$

$x = \frac{1 + 3.60555}{6}$ **or** $x = \frac{1 - 3.60555}{6}$

$x = 0.77$ **or** $x = -0.43$

2005 MATHS P2

2. Solve the equation $-\mathbf{2p^2}$**- 5p +1 = 0**

Giving your answer correct to 3 significant figures.
[5]

Solution

$-2p^2$-5P +1 =0 [a=-2 b=-5 c=1]

$$P = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$P = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(-2)(1)}}{2(3)}$$

$$P = \frac{5 \pm \sqrt{25+8}}{6}$$

$$P = \frac{5 \pm \sqrt{33}}{6}$$

$$P = \frac{5 \pm 5.74456}{6}$$

$P = \frac{5 + 5.74456}{6}$ **or** $x = \frac{5 - 5.74456}{6}$

$P = 2.69$ **or** $x = -0.186$

TOPIC 5: QUADRATIC EQUATIONS

PAST EXAMINATION QUESTIONS

2007 MATHS P2

1. Solve the equation $2x^2 - 3x - 1 = 0$

Giving your answer correct to 2 decimal places. [5]

2009 MATHS P2

2. Solve the equation $x^2 + x - 1 = 0$

Giving your answer correct to 2 decimal places. [5]

2010 MATHS P2

3. Solve the equation $-x^2 - 5x + 7 = 0$

Giving your answer correct to 2 decimal places. [5]

2011 MATHS P2

4. Solve the equation

$(2x - 1)(3x - 2) = 0$

giving your answer correct to 2 decimal places. [5]

2013 MATHS P2

5. Solve the equation $x^2 + 6x = -2$

Giving your answer correct to 2 decimal places. [5]

2014 MATHS P2

6. Solve the equation $1 - 2m - 5m^2 = 0$

Giving your answer correct to 2 decimal places. [5]

2015 GCE MATHS P2

7. Solve the equation $m^2 - m - 5 = 0$

Giving your answer correct to 2 decimal places. [5]

2015 GCE MATHS P1

8. Solve the equation $2x^2 - 3x - 9 = 0$

[2]

2016 MATHS P2

9. Solve the equation $x^2 + 2x = 7$

Giving your answer correct to 2 decimal places. [5]

2017 GCE MATHS P2

10. Solve the equation $3z^2 = 7z - 1$

Giving your answer correct to 2 decimal places. [5]

TOPIC 6: INDICES, STANDARD FORM AND APPROXIMATION

A) INDICES

Indices indicate how many times a number is multiplied by itself.e.g $2 \times 2 \times 2 = 2^3$

Laws of indices: for any real number a

A) $a^1 = a$ Any number to the power 1 is itself. E.g. $5^1 = 5$
B) $a^0 = 1$ Any number to the power 0 is 1. E.g. $6^0 = 1$
C) $a^{-n} = \frac{1}{a^n}$ A neg power indicates the reciprocal of the number with a positive power.Eg $7^{-2} = \frac{1}{7^2}$
D) $a^m \times a^n = a^{m+n}$ To multiply powers of the same base add the powers. E.g. $3^2 \times 3^4 = 3^{2+4} == 3^6$
E) $a^m \div a^n = a^{m-n}$ To divide powers of the same base subtract the powers. E.g. $3^5 \div 3^2 = 3^{5-2} == 3^3$
F) $(a^m)^n = a^{mn}$ To raise a power to a powers multiply the powers. E.g. $(5^2)^3 = 5^6$
G) $a^{\frac{m}{n}} = (\sqrt[n]{a})^m$ A fractional power indicates a root. E.g.$8^{\frac{1}{3}} = \sqrt[3]{8}$

H) $(\frac{a}{b})^{-n} = (\frac{b}{a})^n$

EXAMPLE 6.1

1. Find the value of

4^{-2}

Solution

4^{-2}

$= \frac{1}{4^2}$

$= \frac{1}{16}$

2. Find the value of

$(\frac{2}{3})^{-2}$

Solution

$(\frac{2}{3})^{-2}$

$= (\frac{3}{2})^2$

$= \frac{3^2}{2^2}$

$= \frac{9}{4}$

3. Find the value of

$2^3 \times 2^5 \div 2^6$

Solution

$2^3 \times 2^5 \div 2^6$

$= 2^{3+5-6}$

$= 2^2$

$= 4$

4. Find the value of

$27^{\frac{1}{3}}$

Solution

$27^{\frac{1}{3}}$

$= (\sqrt[3]{27})^1$

$= 3$

PAST EXAMINATION QUESTIONS

2001 P1

1. Evaluate

a) $5^2 + 5^1$

b) $(\frac{2}{3})^2 \times (\frac{2}{3})^1 \times (\frac{2}{3})^0$

2004 & 2016 GCE P1

2. Evaluate

a) $16^{\frac{-3}{4}}$

b) $2^3 + 2^0$

2007 P1

3. Evaluate

a) $5^3 \times 5^1 \times 8^0$

b) $8^{\frac{2}{3}}$

2016 SPE & 2012 P1

4. Evaluate

a) $(\sqrt[3]{64})^2$

b) $216^{\frac{1}{3}}$

TOPIC 6: INDICES, STANDARD FORM AND APPROXIMATION

B) STANDARD FORM AND APPROXIMATION

Standard form is a method of expressing numbers in the form $a \times 10^n$, where $1 \le a < 10$ and n is an integer.

EXAMPLE 6.2

1. Express in standard form

452

Solution

452

= 4.52×100

= 4.52×10^2

2. Express in standard form

0.000268

Solution

0.000268

= 2.68×0.0001

= 2.68×10^{-4}

2001 P1

3. Express in standard form

0.0000056

Solution

0.0000056

= 5.6×0.000001

= 5.6×10^{-5}

C) APPROXIMATION

An approximation is a stated value of a number which is close to but not equal to the true value of that number. Rounding off is the process of approximating a number.

To round off a number, find the place in the number where the rounding off must be done and look at the digit to the right.
if this is 5 or greater add 1 to rounded off digit.
if this is 4 or less, leave the rounded digit the same.

To a number of decimal places (dp): indicates that rounding off has been to leave only the number of digits required after the decimal point.

To a number of significant figures (sf): indicates that rounding off has been to leave only the number of significant figures required.

2007 P1

1. The number of orphans and vulnerable children in one province in Zambia is **599 900**
a) write this number in standard form. [1]
b) Express this number correct to one sig fig [1]

2015 GCE P1

2. Express in standard form

$(\mathbf{5} \times \mathbf{10^7}) \times (\mathbf{4} \times \mathbf{10^{11}})$ [2]

2010 P1

3. Express **4 995 257** in scientific notation correct to 3 significant figures. [2]

2012 P1

4. The population of African country in 2010 was **13 046 508**.Express this population in standard form, correct to 3 significant figures.

TOPIC 7: EQUATIONS INVOLVING INDICES

There are two types of equations involving indices.

i) EQUATIONS OF THE $x^a = b$

This is an equation where x is a variable and a and b are constants which can be solved by raising both sides of the equation to the reciprocal of the power of a.

EXAMPLE 7.1

1. Solve

$\mathbf{x^3 = 8}$

Solution

$x^3 = 8$

$(x^3)^{\frac{1}{3}} = (8)^{\frac{1}{3}}$

$x = (8)^{\frac{1}{3}}$

$x = 2$

2. Solve

$\mathbf{x^{\frac{1}{2}} = 3}$

Solution

$x^{\frac{1}{2}} = 3$

$(x^{\frac{1}{2}})^2 = 3^2$

$x = 9$

3. Solve

$\mathbf{x^{\frac{-2}{3}} = 16}$

Solution

$x^{\frac{-2}{3}} = 16$

$(x^{\frac{-2}{3}})^{\frac{-3}{2}} = (16)^{\frac{-3}{2}}$

$x = 16^{\frac{-3}{2}}$

$x = \frac{1}{64}$

ii) EQUATIONS OF THE $a^x = b$

This is an equation where x is a variable and a and b are constants and can be solved by raising both sides of the equation in index form with the same base and equating the powers.

EXAMPLE 7.2

1. Solve

$\mathbf{2^x = 8}$

Solution

$2^x = 8$

$2^x = 2^3$

$x = 3$

2. Solve

$\mathbf{25^x = 5}$

Solution

$25^x = 5$

$5^{2x} = 5^1$

$x = \frac{1}{2}$

3. Solve

$\mathbf{27^x = \frac{1}{9}}$

Solution

$27^x = \frac{1}{9}$

$3^{3x} = 3^{-2}$

$x = \frac{-2}{3}$

TOPIC 7: EQUATIONS INVOLVING INDICES

PAST EXAMINATION QUESTIONS

1. Solve

$x^{\frac{2}{3}} = 4$

2. Solve

$\mathbf{x^4 = \frac{1}{16}}$

3. Solve

$\mathbf{x^{-3} = 8}$

4. Solve

$\mathbf{2^x = 16}$

5. Solve

$\mathbf{10^x = 1}$

6. Solve

$\mathbf{25^x = \frac{1}{5}}$

7. Solve

$\mathbf{(\frac{1}{16})^x = 8}$

8. Solve the equation

$\mathbf{(x+1)^2 = 81.}$ **[2]**

9. Solve the equation

$\mathbf{(x-2)^2 = 9.}$ **[2]**

10. Solve the equation

$\mathbf{(x-5)^2 = 100.}$ **[2]**

11. Solve the equation

$\mathbf{\sqrt{2y-1} = 5}$

12. Solve the equation

$\mathbf{\sqrt{y-1} = 6}$

1987 MATHS P2

13. Simplify as far as possible.

$2p^2q\,[3q - p^3]$ [2]

2005 P2

14. Solve the equation

$\mathbf{(x+3)^2 = 64.}$ **[2]**

2014 P1

15. Solve the equation

$\mathbf{2(x-3)^2 = 18.}$ **[2]**

2006 P1

16. Solve the equation

a) $\mathbf{\sqrt{3y-2} = 1}$

2016 SPE P1

17. Solve the equation

$\mathbf{2^{2m-3} = 8^m}$

2016 SPE P1

18. Solve the equation

$\mathbf{(2x-1)^2 = 9.}$ **[2]**

TOPIC 8: SET NOTATION AND PRESENTATION

A set is denoted by capital letters. The objects of a set are enclosed in curly brackets.

Elements/members: are objects which belong to a set. The symbol ε means "is a member of. "

Universal set: is the set which contains all elements under discussion.
The universal set is denoted by e.

Empty set: is the set which has no elements. The empty set is denoted by { } or ф .

Number of elements: the number of elements in the set a is denoted by n(a).

Subset: is a set which belongs to another set. The symbol c means is "a subset of ".

OPERATIONS OF SETS

Intersection of sets: is the set common to all original sets. The intersection set is denoted by ∩.

e.g. if a= {1,2,3,4,5} and b= {2,4,6} then a ∩ b = {2,4}

Union of sets: is the set of all elements in of the original sets. The union set is denoted by ∪.

e.g. if a= {1,2,3,4,5} and b= {2,4,6} then a ∪ b = {1,2,3,4,5,6}

Complement of a set: is the set of all elements which are not in that set but are in the universal set. The complement of a set is denoted by '.
e.g. if e= {natural numbers less than 10} and a= {prime numbers} then a '= {1,4,6,8,9}

Venn diagram: are used to show the relationships between sets.in a Venn diagram the universal set is represented by a rectangle and sets by circles.

Examples

1. Given that Set A has 5 elements and B has 128 subsets.
   (a) Find the number of subsets of A
   (b) Find the number of elements of Set B.

Solution

(a) Number of subsets $= 2^n$
$= 2^5$
$= 32$, A has 32 subsets

b) No. of subset $= 2^n$
$128 = 2^n$
$2^7 = 2^n$
$n = 7$, B has 7 elements

TOPIC 8: SET NOTATION AND PRESENTATION

Examples

1. If E = { Natural numbers less than 13}
   P = {x: x is a prime number}
   O = {x: x is an old number}
   S ={x: x is a square number}

List Sets E, P, O and S and hence find the following:

(a) P'

(b) $(P \cap O)'$

(c) $(P \cup S \cup O)'$

Solutions

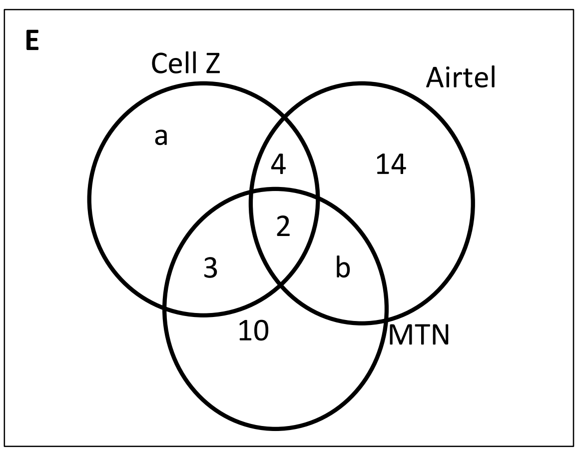
E = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}
P = {2, 3, 5, 7, 11}
O = {1, 3, 5, 7, 9, 11}
S = {1, 4, 9}

a) P' elements in the universal set that are not in P
   $\therefore P' = \{1, 4, 6, 8, 9, 10, 12\}$

b) $(P \cap O)'$ elements in the universal set that are not in $P \cap O = \{3, 5, 7, 11\}$
   $\therefore (P \cap O)' = \{1, 2, 4, 6, 8, 9, 10, 12\}$

c) First list $P \cup S \cup O = \{1, 2, 3, 4, 5, 7, 9, 11\}$ then list elements of universal that are not $P \cup S \cup O$
   $\therefore (P \cup S \cup O)' = \{6, 8, 10, 12\}$

2. A survey was conducted on 60 women connecting the types of Sim cards used in their cell phones for the past 2 years. Their responses are given in the diagram below.



(a) Given that 23 women have used Cell Z Sim cards, find the values of a and b
(b) How many women have used only two different Sim cards?
(c) If a woman is selected at random from the group, what is the probability that
   i. She has no cell phone
   ii. She used only type of a Sim card
(d) How many women did not use MTN and Cell Z Sim Cards?
(e) How many women used either Airtel or MTN Sim Cards but not Cell Z?

TOPIC 9: FUNCTIONS

A function is an operation that is applied to a given set of values (domain) to give another set of values (range). A function is represented by the letter f.

2001 P1

1. for the function

g (x) = $\mathbf{\frac{1}{2x-5}}$

Find **g (2).**

Solution

g (2) = $\frac{1}{2(2)-5}$

g (2) = $\frac{1}{4-5}$

g (2) = - 1

2004 P1

2. for the function

f(x) = $\mathbf{\frac{3-5x}{2x}}$

Find **f (3).**

Solution

f (3) = $\frac{3-5(3)}{2(3)}$

f (3) = $\frac{3-15}{6}$

f (3) = - 2

2006 P1

3. for the function

h(x) = $\mathbf{\frac{3}{2x+1}}$

Find the value of

a) x for which h is not a function

b) h (-5)

Solution

a) 2x+1 = 0… $x = \frac{-1}{2}$

b) h (-5) = $\frac{3}{2(-5)+1}$

h (-5) = $\frac{-1}{3}$

INVERSE FUNCTION

An inverse function is an operation that reverses a function.it is written as f^{-1}(x).
It can be found by making x the subject of the function.

2001 P1

1. for the function

g(x) = $\mathbf{\frac{1}{2x-5}}$

Find $\mathbf{g^{-1}}$**(x)**

Solution

Let y = g(x)

y = $\frac{1}{2x-5}$

2xy – 5y = 1

x = $\frac{1+5y}{2y}$

g^{-1}(x) = $\frac{1+5x}{2x}$

2004 P1

2. for the function

f(x) = $\mathbf{\frac{3-5x}{2x}}$

Find $\mathbf{g^{-1}}$**(x)**

Solution

Let y = f(x)

y = $\frac{3-5x}{2x}$

2xy = 3 – 5x

x = $\frac{3}{2y+5}$

$f^{-1}(x)$ = $\frac{3}{2x+5}$

2007 P1

3. for the function

f(x) = $\mathbf{\frac{2x-3}{x}}$

Find $\mathbf{f^{-1}}$**(x)**

Solution

Let y = f(x)

y = $\frac{2x-3}{x}$

xy = 2x - 3

x = $\frac{-3}{y-2}$

$f^{-1}(x)$ = $\frac{-3}{x-2}$

TOPIC 9: FUNCTIONS

COMPOSITE FUNCTIONS

PAST EXAMINATION QUESTIONS

2010 P1

1. for the function

g(x) = $\frac{4x-3}{2x-5}$

Find

a) g (-1) **[1]**

b) g^{-1}**(x)** **[1]**

b) g^{-1}**(0)** **[1]**

2012 P1

2. Given that

f (x) = $\frac{3x-5}{2}$ **and g(x) =** $\frac{x-4}{6}$

Find

a) f (-9) **[1]**

b) f^{-1}**(x)** **[1]**

c) The value of x for which

f (x)= 3 g (x) **[2]**

2015 P1

3. Given that

f(x) = $\frac{5x}{x-1}$

Find

a) f (6) **[1]**

b) f^{-1}**(x)** **[1]**

c) The value of x for which

f(x)= 0 **[2]**

2016 SPE P1

4. Given that

f(x) = 2x-3 and g(x) = $\frac{3x+1}{x+2}$

Find

a) f^{-1}**(x)** **[1]**

b) f^{-1}**(11)** **[1]**

c) gf (x) **[2]**

TOPIC 10: CORDINATE SYSTEM AND GRAPHS

CARTESIAN CORDINATES

This gives the position of a point in a plane (two dimensions) by reference to two coordinate axes (the x-axis and y-axis) at right angles.

GENERAL GRAPH TERMS

1. Gradient: this is the rate of change of y with respect to x. Gradient is usually written as m

Gradient (m) = $\frac{\text{change in y}}{\text{change in x}} = \frac{y_2 - y_1}{x_2 - x_1}$

A positive gradient slopes upward to the right.
A negative gradient slopes downward to the right.

The gradients of parallel lines are equal. ($m_1 = m_2$)

The product of two perpendicular line is -1. ($m_1 \times m_2 = -1$)

2.X-intercept: this is the point where the line or curve cuts across the x-axis.
at the x-intercept y=0

3.Y-intercept: this is the point where the line or curve cuts across the y-axis.
at the y-intercept x=0

LINEAR GRAPHS

linear graphs: the equation of a linear graph (straight line) is given by:
y= mx + c [where m is the gradient and c is the y-intercept] or $y - y_1 = m(x - x_1)$

EXAMPLES

1. State the gradient and y-intercept of the line **x+ 2y = 2**

SOLUTION

Firstly, we make y the subject of the formula

$x + 2y = 2$

$2y = -x + 2$

$y = -\frac{1}{2}x + 1$ compared to Y=mX+C

Gradient = m = $-\frac{1}{2}$

Y-Intercept = C = 1

2004 P2

2. A straight line is given by the equation **2x + 3y = 3.** Write down its gradient.

SOLUTION

$2x + 3y = 3$

$3y = -2x + 3$

$y = -\frac{2}{3}x + 1$ compared to Y=mX+C

Gradient = m = $-\frac{2}{3}$

TOPIC 10: CORDINATE SYSTEM

LINEAR GRAPHS

EXAMPLES

2009 P2

3. If B is a point **(6, 3)** and **C** is **(-2, 1),**

Find

a) The gradient of line BC

SOLUTION

B (6,3) C (-2,1)

m = $\frac{y_2 - y_1}{x_2 - x_1}$

m = $\frac{3-1}{6-(-2)}$

m = $\frac{2}{8} = \frac{1}{4}$

2009 P2

b) The equation of line BC

SOLUTION

B (6,3) C (-2,1

m = $\frac{1}{4}$

y = $\frac{1}{4}$x + c

1 = $\frac{1}{4}$(-2) + c

C = $\frac{3}{2}$

y = $\frac{1}{4}$x + $\frac{3}{2}$

PAST EXAMINATION QUESTIONS

2009 P2

1. Find the equation of line l passing through a Point (0,5) whose gradient is 3.
[2]

2010 P2

2. Given find that **2x – 7y – 1 = 0**, find the gradient of the equation.
[1]

2010 P1

3. If A is a point (4,0) and B is (0,8),

Find the equation of AB. [2]

2012 P1

4. Find the gradient of a straight line whose equation **4x + 2y = 9**.
[1]

2013 P1

5. Find the equation of the line joining the point A(0,7) to the point B(7,0)
[1]

2016 SPE P1

6. Find the gradient of a straight line passing through (-2, -6) and (8,5).
[2]

CURVE GRAPHS

QUADRATIC GRAPHS: ALL QUADRATIC GRAPHS CAN BE WRITTEN IN THE FORM $y = ax^2 + bx + c$.
ITS SHAPE IS THAT OF A PARABOLA

FOR A QUADRATIC GRAPH ($y = ax^2 + bx + c$)

i) THE Y-INTERCEPT= (0, C)
ii) THE X-INTERCEPT= (C, 0) (IF IT EXISTS AND THE EXPRESSION CAN BE FACTORISED).
iii) THE TURNING POINTS (BOTTOM OR TOP OF PARABOLA).

EXAMPLES

1. The table below shows some of the values of x and the corresponding values of y for equation $\mathbf{y = -2x^2 - x + 8}$.

x	-3	-2	-1	-0.5	1	2	3
y	-7	2	7	8	5	-2	-13

a) draw the graph

b) By drawing a tangent find a gradient at the point (1,5). (a tangent is a line which touches the curve at the point)

c) Estimate the area bounded by the curve the x-axis, x = - 1 and x = 1

SOLUTION

a) plot these points and draw a smooth curve around them.

TOPIC 11: INEQUATIONS AND LINEAR PROGRAMMING

An inequation or inequality is a mathematical statement.

BASIC INEQUATION NOTATION

The sign $<$ means "less than."
The sign $>$ means "greater than."
The sign $\leq$ means "less than or equal to."
The sign $\geq$ "greater than or equal to."
e.g. $x > y$ means that x is greater than y.

$a \leq b$ means that a is less than or equal to b.

RULES FOR INEQUATIONS

If an inequation is multiplied or divided by a negative number, then the inequation sign must be reversed.

1. Solve the inequation

$3x – 4 > 5$

Solution

$3x – 4 > 5$

$3x > 5 + 4$

$3x > 9$

$x > 3$

2. Solve the inequation

$5b – 4 < 6$

Solution

$5b – 4 < 6$

$5b < 6 + 4$

$5b < 10$

$b < 2$

3. Solve the inequation

$2(b - 1) \leq 6$

Solution

$2(b - 1) \leq 6$

$2b – 2 \leq 6$

$2b \leq 6 + 2$

$b \leq 4$

4. Solve the inequation

$2 – 5x > 12$

Solution

$2 – 5x > 12$

$-5x > 12 - 2$

$-5x > 10$

$x < -2$ (the sign changes)

5. Solve the inequation

$– 2x + 3 > 3x + 2$

Solution

$– 2x + 7 > 3x + 2$

$– 2x – 3x > 2 - 7$

$-5x > -5$

$x < 1$

2011 Paper 2

6. Solve the inequation

$7 – 2t < 9$

Solution

$7 – 2t < 9$

$-2t < 9-7$

$-2t < 2$

$t > -1$

TOPIC 11: INEQUATIONS

Examples

7. Solve the inequation

$3b - 2 > 5b + 14$

Solution

$3b - 2 > 5b + 14$

$3b - 5b > 14 + 2$

$-2b > 16$

$b < -8$

8. Solve the inequation

$4(b - 2) > 5b + 1$

Solution

$4(b - 2) > 5b + 1$

$4b - 8 > 5b + 1$

$4b - 5b > 8 + 1$

$-b > 9$

$B < -9$

2009 Paper 2

9. Solve the inequation

$1 - \frac{3}{5}x < 4$

Solution

$1 - \frac{3}{5}x < 4 \rightarrow -\frac{3}{5}x < 4 - 1$

$-\frac{3}{5}x < 3 \rightarrow -3x < 15$

$x > -5$ (the sign changes)

PAST EXAMINATION QUESTIONS

2012 Paper 2

1. Solve the inequation

$4b - 3 < 6b + 4$. [2]

2012 Paper 1

2. Solve the inequation

$2y - 1 < 5$. [2]

2013 Paper 1

3. Solve the inequation

$3x > 4 - 2x$. [2]

2013 Paper 2

4. Solve the inequation

$-5x - 9 < 16$. [2]

2014 GCE Paper 2

5. Solve the inequation

$1 - 2x \leq 4 \; \frac{x}{2} - 9$. [2]

2014 Paper 2

6. Solve the inequation

$9t - 4 < 12t - 10$ [2]

7. Solve the inequality.

$4 - x < 6$ [2]

8. Solve the inequality.

$a - 2 > 4(5 - a)$ [2]

9. Solve the inequality.

$17 - 2p \leq 10$ [2]

TOPIC 12: PROPORTION AND VARIATION

PROPORTION

A proportion is a relationship of the equality of ratios between two pairs of quantities. The symbol means "is proportion to" or "varies as"

DIRECT PROPORTION: this is the relationship between quantities such that both quantities increase or decrease in the same ratio.

EXAMPLE ON DIRECT PROPORTION

INVERSE PROPORTION: this is the relationship between quantities such that when one quantity increases the other quantity decreases in the same ratio.

EXAMPLE ON INVERSE PROPORTION

1. It takes 8 hours for 5 people to paint a room. How long would take 4 people.

SOLUTION

8 Hours ------------- 5 People

? Hours--------------- 4 People

$= \frac{8 \times 5}{4}$

= 10 Hours

2013 CB MOCK P1

2. 3 pupils can sweep a certain classroom in 20 minutes. How many pupils can sweep the same classroom in 5 minutes?

SOLUTION

3 Pupils ------------- 20 Minutes

? Pupils--------------- 5 Minutes

$= \frac{3 \times 20}{5}$

= 12 Pupils

VARIATION

Variation requires you to find a constant k (called the constant of proportionality) in a formula that relates two or more variables in a ratio.

DIRECT VARIATION: If y is directly proportion to x then it increases or decreases in the same ratio. Then this relationship can be written as y=kx where k is a called a constant of proportionality.

1. It is given that y varies directly as the square of x, when y=8, x=2.find

a) The constant of variation K
b) The value of y when x=5
c) The value of x when y=32

Solution

a) $Y=kx^2$ $8=2^2k$ $k=\frac{8}{4}$ k=2

b) $y=kx^2$ $y=2x^2$ $y=2(5)^2$ y=50

c) $y=2x^2$ $32=2x^2$ $x^2=16$ x=4 or x=-4

2006 P1

2. Two variables x and y have corresponding values as shown, given that y varies directly as x. Find

X	6	9	12
Y	34		68

a) The constant of variation K
b) The value of y when x=9

Solution

a) Y=kx   34=6k $k=\frac{34}{6}=\frac{17}{3}$

b) Y=kx $y=\frac{17}{3}x$ $y=\frac{17}{3}(9)$ y = 51

INVERSE VARIATION: If y is inversely proportion to x, then this relationship can be written as $Y=\frac{K}{X}$ where k is a called a constant of proportionality.

1. It is given that y varies inversely as x and y=4 when x=6.find

a) The constant of variation K
b) The value of y when x=3
c) The value of x when y=12

Solution

a) $y=\frac{k}{x}$ $4=\frac{k}{6}$ k= 24

b) $y=\frac{k}{x}$ $y=\frac{24}{x}$ $y=\frac{24}{3}$ y=8

c) $y=\frac{24}{x}$ $12=\frac{24}{x}$ 12x=24   x=2

2. It is given that y varies inversely as (x-2) and y=2 when x=5.find

a) The constant of variation K
b) The value of y when x=8
c) The value of x when y=6

Solution

a) $y=\frac{k}{x-2}$ $2=\frac{k}{5-2}$ k= 6

b) $y=\frac{k}{x-2}$ $y=\frac{6}{x-2}$ $y=\frac{6}{8-2}$ y=1

c) $y=\frac{6}{x-2}$ $6=\frac{6}{x-2}$ 6x-12=6   x=3

TOPIC 12: PROPORTION AND VARIATION

VARIATION

JOINT VARIATION: this is a variation in which one variable depends on two or more variables.it combines direct and inverse variation. Then this relationship can be written as $Y = \frac{KZ}{X}$ where k is a called a constant of proportionality.

2004 P1

1. It is given that Y varies directly as z and inversely as the square of w, when y=6, z=2 and w=3 find

a) k

b) y when z=6 and w=9

c) w when y=3 and z=4

Solution

a) $y=\frac{kz}{w^2}$ $6=\frac{2k}{3^2}$ k=27

b) $y=\frac{27z}{w^2}$ $y=\frac{27(6)}{9^2}$ y=2

c) $y=\frac{27z}{w^2}$ $3=\frac{27(4)}{w^2}$ $w^2=36$ w=6 or w=-6

2010 P1

2. It is given that x varies directly as z and inversely as the square of y, when x=2, y=3 and z=4 find

a) k

b) y when x=3 and z=24

Solution

a) $x=\frac{kz}{y^2}$ $2=\frac{4k}{3^2}$ $k=\frac{9}{2}$

b) $x=\frac{kz}{y^2}$ $x=\frac{9z}{2y^2}$ $3=\frac{9(24)}{2y^2}$

$y^2=36$ y=6 or y=-6

PAST EXAMINATION QUESTIONS

1. 16 workers can build a wall in 25 days. How many workers are needed if the wall is to be built in 10 days ?

2. It takes 6 people 12 hours to paint a house. If the work has to be completed in 8 hours, how many people will be needed ?

2012 P1

3. Mr Hambwiimbwi planned to employ **20** men to build his house in **7** days. On the day work was to start, he decided to reduce the number of men so that work could now be completed in **28** days, working at the same rate.

a) How many men were needed for the work?

TOPIC 12: PROPORTION AND VARIATION

EXERCISE

2016 SPE P1

1. It is given that $y = \frac{kx}{z^2}$ and x=3 when y=6 and z=2, find the value of

a) The constant k

b) Y when x=4 and z=8

c) Z when x=9 and y=2

2015 GCE P1

2. Two variables p and q have corresponding values as shown in the table below.

p	3	5	7	
q	6/5		11/5	6

Given that q varies directly as p, find

a) The constant of variation, k.

b) The value of q when p=5

c) The value of p when q=6

2014 P1

3. It is given that $y = kx^2 - 1$ where k is a constant of variation and that y =17 when x = 3, find the

a) The value of the constant k

b) Value of Y when x = -5

c) Values of x when y = 7

2013 P1

4. Given that y varies directly as x and inversely as 2m -1. Given that **y = 5** when **x = 7** and **m = 4**, find

a) The constant of variation K

b) The value of y when x = 2 and m = 3

c) The value of m when y = 2 and x = 4

2012 P1

5. It is given y varies directly as x and z. Given that **y = 9** when **x = 6** and **z = 0.5**, find

a) The constant of variation K

b) The value of y when x = 4 and z = 3

c) The value of x when y = 4.5 and z = 5

6. It is given that $y = \frac{k}{\sqrt{x}}$ where k is a constant. Pairs of corresponding values are given in the table below

x	36	4	q
y	4	p	8

Find the value of p and q.

TOPIC 13: SEQUENCES

A sequence is a list of numbers which follow a mathematical rule. Each number in a sequence is called a term of the sequence.

ARITHMETIC PROGRESSION

An arithmetic progression is a sequence of numbers which increase or decrease by a fixed amount called common difference (d) and the starting point is called the first term (a).
The nth term of an arithmetic sequence is given by:

$a_n = a + (n-1)d$ where

a_n Is the position of the term.
a is the first term.
n is the number of terms.
d is the common difference given by (T_2- T_1).

2004 P1

1. Write the **nth** term of the sequence and hence find the 10th term.

8,12,16,20, …

Solution

a = 8 d = 12-8 = 16-12 = 20-16 = 4

$a_n = a + (n-1)d$
$a_n = 8 + (n-1)4$
$a_n = 8 + 4n - 4$
$a_n = 4n + 8 - 4$
$a_n = 4n + 4$

$a_{10} = 4(10) + 4$ = 40 + 4 = 44

2010 P1

1. For the sequence 7,9,11, 13……write down
a) the eleventh term.
b) The expression for the **nth** term.

Solution

a = 7 d = 9-7 = 11-9 = 13-11 = 2

$a_n = a + (n-1)d$
$a_n = 7 + (n-1)2$
$a_n = 7 + 2n - 2$
$a_n = 2n + 7 - 2$
$a_n = 2n + 5$

$a_{11} = 2(11) + 5$= 22 + 5 = 27

TOPIC 13: SEQUENCES

SUM OF AN ARITHMETIC PROGRESSION

Suppose you were asked to find the sum of the arithmetic progression below

8,12,16,20, it would be too slow (and difficult) to add them all up, hence we think of a quicker way.

For an AP with first term a, and n number of terms and last term l,

$$S_n = \frac{n}{2}(a + l)$$

However, we may not know the last term l.so we convert this formula into a more suitable one.L is the n^{th} term and so L = $a + (n-1)d$

Then, $S_n = \frac{n}{2}(a + l)$

$$S_n = \frac{n}{2}(a + a + (n-1)\ d)$$

$$S_n = \frac{n}{2}(2a + (n-1)\ d)$$

1. Find the **sum of the first 10** terms of the sequence and hence find the 10^{th} term.

8,12,16,20 …

Solution

a = 8 n = 10 d = 12 – 8 = 4

$S_n = \frac{n}{2}(2a + (n-1)\ d)$

$S_{10} = \frac{10}{2}(2(8) + (10-1)\ 4)$

$S_{10} = 5(16 + 36)$

$S_{10} = 260$

2. Find the **sum of the first 15** terms of the sequence and hence find the 10^{th} term.

11, 14,17,20,23 …

Solution

a = 11 n = 15 d = 14 – 11 = 3

$S_n = \frac{n}{2}(2a + (n-1)\ d)$

$S_{15} = \frac{15}{2}(2(11) + (15-1)\ 3)$

$S_{15} = 7.5(22 + 42)$

$S_{15} = 480$

GEOMETRIC PROGRESSION

A Geometric progression is a sequence of numbers where each term is multiplied by a fixed constant called common ratio (r) and the starting point is called the first term (a).
The nth term of a Geometric sequence is given by:

$a_n = ar^{n-1}$, Where

a_n Is the position of the term.
a is the first term.
n is the number of terms.
r is the common ratio given by ($T_2 \div T_1$).

1. State the common ratio of

32, 16, 8, 4, 2………

And hence find the n^{th} term.

Solution

a = 32 r = 32÷8 = 2

$a_n = ar^{n-1}$

$a_n = 32(2^{n-1})$

$a_n = 2^5\ (2^{n-1}) = 2^{5+n-1} = 2^{n+4}$

2. Find the 8^{th} term of the G.P

2, 6, 18, 54,………

Solution

a = 2 r = 6÷2 = 3

$a_n = ar^{n-1}$

$a_n = 2(3^{n-1})$

$a_8 = 2(3^{8-1})$

$= 2(3^7) = 4374$

3. If x + 1, x + 3 and x + 8 are the first three terms of a geometric progression, find,

(a) The value of x

(b) The common ratio

Solution

(a) $r = \frac{x+3}{x+1} = \frac{x+8}{x+3}$

Then (x + 3) (x + 3) = (x + 8) (x + 1)

Which gives $x^2 + 6x + 9 = x^2 + 9x + 8$

Which gives $x = \frac{1}{3}$

(b) $r = \frac{x+3}{x+1} = \frac{10}{3} \div \frac{4}{3} = \frac{5}{2}$

4. The 4^{th} and 8^{th} terms of a G.P are 3 and $\frac{1}{27}$ respectively. Find the value of the value of the first term and common ratio.

Solution

$T_n = ar^{n-1}$

$T_4 = ar^3 = 3$ and $T_8 = ar^7 = \frac{1}{27}$

Divide to eliminate a: $\frac{ar^7}{ar^3} = \frac{1}{27} \div 3$

$r^4 = \frac{1}{81}$ $\quad r = \sqrt[4]{\frac{1}{81}}$ $\quad r = \frac{1}{3}$

$a = \frac{3}{r^3} = \frac{3}{(\frac{1}{3})^3} = 81$

a = 81 and $r = \frac{1}{3}$

SUM OF A GEOMETRIC PROGRESSION

The sum of the first n terms of a geometric series is given by

$S_n = \frac{a(1-r^n)}{1-r}$ (if r < 1) or $S_n = \frac{a(r^n-1)}{r-1}$ (if r > 1)

1. Find the sum of the first 6 terms of the G.P

64, 16, 4,1 …

Solution

a = 64 r = 16÷64 = 0.25

$S_n = \frac{a(1-r^n)}{1-r}$ (Since r < 1)

$S_6 = \frac{64(1-0.25^6)}{1-0.25} = \frac{64(0.999)}{0.75} = \mathbf{85.3}$

2. Find the sum of the first 5 terms of the G.P

3, 12, 48,192 …

Solution

a = 3 r = 12÷3 = 4

$S_n = \frac{a(r^n-1)}{r-1}$ (Since r > 1)

$S_6 = \frac{3(4^5-1)}{4-1} = \frac{3(1023)}{3} = \mathbf{1023}$

SUM TO INFINITY

The sum to infinity is given by $S_\infty = \frac{a}{1-r}$ (if |r| < 1)

1. Find the sum to infinity of the geometric progression below

5, 2.5, 1.25, 0.625 …

Solution

a = 5 r = 2.5÷5 = 0.5

$S_\infty = \frac{a}{1-r}$

$S_\infty = \frac{5}{1-0.5}$

$S_\infty = \mathbf{10}$

2. Find the sum to infinity of the geometric progression below

$2, \frac{2}{3}, \frac{2}{9}, \frac{2}{27} \ldots$

Solution

a = 2 $r = \frac{2}{3} \div 2 = \frac{1}{3}$

$S_\infty = \frac{a}{1-r}$

$S_\infty = \frac{2}{1-\frac{1}{3}}$

$S_\infty = \mathbf{3}$

PAST EXAMINATION QUESTIONS

2013 P1

1 For the sequence 2, 5, 8, 11…find

(a) the 6^{th} term.

(b) An expression for the n^{th} term.

(c) The sum of the first 20 terms.

2015 GCE P1

2. (a) Find the next term in the sequence

1 , 2 , 4 , 7 , 11 , …

(b) For the sequence 1, 6, 11, 16…find

an expression for the n^{th} term.

2016 SPE P1

3. For the sequence 11 , 14 , 17 , 20 , … find the

(a) 15^{th} term,

(b) Sum of the first 20 terms.

2016 SPE P2

4. The 3^{rd} and 4^{th} term of a G.P are 4 and respectively. Find

(i) The first term and common ratio.

(ii) The sum of the first 10 terms

(iii) The sum to infinity of this geometric progression.

2017 GCE P2

5. The first three terms of a geometric progression are 6 + n, 10 + n and 15 + n.

Find,

(a) The value of n

(b) The common ratio

(c) The sum of the first 6 terms of this sequence.

6. The first three consecutive terms of an arithmetic progression are 5 – x , 3x + 1 and x + 9.

Find,

(a) The value of x

(b) The first term

(c) Common difference

(d) the sum of the first ten terms

TOPIC 13: MATRICES

A matrix is an array of numbers. The numbers of an array are called elements.

Order of matrices: this is given by the number of rows by the number of columns.

e.g. $\begin{pmatrix} 1 & 2 \end{pmatrix}$ is a 1×2 matrix $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$ is a 2×1 matrix

$\begin{pmatrix} 4 & 2 \\ 1 & 3 \end{pmatrix}$ is a 2×2 matrix $\begin{pmatrix} 1 & 2 \\ 3 & 7 \\ 8 & 9 \end{pmatrix}$ 3×2 matrix

Transpose of a matrix: This is the change of the order of a matrix. Rows become columns and columns becomes rows.e.g.

If Matrix A = $\begin{pmatrix} 1 & 2 \\ 3 & 7 \\ 8 & 9 \end{pmatrix}$ Then $A^T = \begin{pmatrix} 1 & 3 & 8 \\ 2 & 7 & 9 \end{pmatrix}$ and If Matrix B = $\begin{pmatrix} 4 & 2 \\ 1 & 3 \end{pmatrix}$ Then $B^T = \begin{pmatrix} 4 & 1 \\ 2 & 3 \end{pmatrix}$

Addition and subtraction of matrices

Matrices of the same order can be added or subtracted by adding or subtracting corresponding elements.

1. Simplify $\begin{pmatrix} \mathbf{1} & \mathbf{4} \\ \mathbf{2} & \mathbf{3} \end{pmatrix} + \begin{pmatrix} \mathbf{2} & \mathbf{3} \\ \mathbf{-3} & \mathbf{1} \end{pmatrix}$

Solution

$\begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix} + \begin{pmatrix} 2 & 3 \\ -3 & 1 \end{pmatrix}$

$= \begin{pmatrix} 1+2 & 4+3 \\ 2+(-3) & 3+1 \end{pmatrix}$

$= \begin{pmatrix} 3 & 7 \\ -1 & 4 \end{pmatrix}$

1. Simplify $\begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix} + \begin{pmatrix} 3 & -2 \\ 0 & 4 \\ -1 & 2 \end{pmatrix} - \begin{pmatrix} 6 & -4 \\ -3 & 2 \\ 2 & 0 \end{pmatrix}$

Solution

$\begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix} + \begin{pmatrix} 3 & -2 \\ 0 & 4 \\ -1 & 2 \end{pmatrix} - \begin{pmatrix} 6 & -4 \\ -3 & 2 \\ 2 & 0 \end{pmatrix}$

$= \begin{pmatrix} 1+3-6 & 4+(-2)-(-4) \\ 2+0-(-3) & 5+4-2 \\ 3+(-1)-2 & 6+2-0 \end{pmatrix}$

$= \begin{pmatrix} -2 & 6 \\ 5 & 7 \\ 0 & 8 \end{pmatrix}$

Scalar multiplication of a matrix

A scalar is a number written in front of a matrix. Each element of that matrix is multiplied by that number.e. g

$3\begin{pmatrix} 2 & 0 \\ 3 & -1 \end{pmatrix} = \begin{pmatrix} 3 \times 2 & 3 \times 0 \\ 3 \times 3 & 3 \times -1 \end{pmatrix} = \begin{pmatrix} 6 & 0 \\ 9 & -3 \end{pmatrix}$

TOPIC 13: MATRICES

Multiplication of matrices

Matrices can be multiplied if the number of columns in the first matrix is equal to the number of rows in the second matrix.it is done by multiplying each row of the first matrix against each column of the second matrix, and adding the results together to form a single matrix.

EXAMPLE

1. Simplify $\begin{pmatrix} \mathbf{1} & \mathbf{3} \\ \mathbf{-2} & \mathbf{2} \end{pmatrix} \times \begin{pmatrix} \mathbf{4} \\ \mathbf{-5} \end{pmatrix}$

Solution

$\begin{pmatrix} 1 & 3 \\ -2 & 2 \end{pmatrix} \times \begin{pmatrix} 4 \\ -5 \end{pmatrix}$

$= \begin{pmatrix} (1 \times 4) + (3 \times -5) \\ (-2 \times 4) + (2 \times -5) \end{pmatrix}$

$= \begin{pmatrix} -11 \\ -18 \end{pmatrix}$

2. Find $\mathbf{(1 \quad 2)} \times \begin{pmatrix} \mathbf{3} & \mathbf{0} \\ \mathbf{2} & \mathbf{4} \end{pmatrix}$

Solution

$(1 \quad 2) \times \begin{pmatrix} 3 & 0 \\ 2 & 4 \end{pmatrix}$

$= ((1 \times 4) + (3 \times -5) \quad (1 \times 4) + (3 \times -5)$

$= (7 \quad 8)$

3. find $\begin{pmatrix} \mathbf{1} & \mathbf{3} \\ \mathbf{2} & \mathbf{4} \end{pmatrix} \times \begin{pmatrix} \mathbf{0} & \mathbf{1} \\ \mathbf{1} & \mathbf{2} \end{pmatrix}$

solution

$\begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix} \times \begin{pmatrix} 0 & 1 \\ 1 & 2 \end{pmatrix}$

$= \begin{pmatrix} (\mathbf{1} \times \mathbf{0}) + (\mathbf{3} \times \mathbf{1}) & (\mathbf{1} \times \mathbf{1}) + (\mathbf{3} \times \mathbf{2}) \\ (\mathbf{2} \times \mathbf{0}) + (\mathbf{4} \times \mathbf{1}) & (\mathbf{2} \times \mathbf{1}) + (\mathbf{4} \times \mathbf{2}) \end{pmatrix}$

$= \begin{pmatrix} 3 & 7 \\ 4 & 10 \end{pmatrix}$

INVERSE MATRIX

Identity matrix: this is a square matrix (2×2) whose elements in the main diagonals are all “1” and the others are all “0” and is denoted by “I” e.g.

$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ is the 2×2 identity matrix

For any square matrix A, **AI = IA= A**

TOPIC 13: MATRICES

Determinant of a matrix

This is the product of elements in the main diagonal minus the elements in the minor diagonal. The determinant of matrix A is denoted "det A ".

In general if A = $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ det A = ad - bc

e.g. if A = $\begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$ det A = (2×4) - (3×1) = 8 – 3 = 5

A matrix whose determinant is zero is called a singular matrix.

Inverse of a matrix

This is another matrix such that when two matrices are multiplied together in any order, the result is the identity matrix.

In general if A = $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ then $A^{-1} = \frac{1}{\det A}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

A singular matrix has no inverse because the determinant is zero.

2006 P2

1. If M = $\begin{pmatrix} -1 & 3 \\ 1 & 2 \end{pmatrix}$, find

i) M^2 .

iii) The Determinate of M.

ii) M^{-1}.

solution

i) $M^2 = \begin{pmatrix} -1 & 3 \\ 1 & 2 \end{pmatrix} \times \begin{pmatrix} -1 & 3 \\ 1 & 2 \end{pmatrix}$

$M^2 = \begin{pmatrix} (-1\times -1)+(3\times 1) & (-1\times 3)+(3\times 2) \\ (1\times -1)+(2\times 1) & (1\times 3)+(2\times 2) \end{pmatrix}$

$M^2 = \begin{pmatrix} 4 & 3 \\ 1 & 7 \end{pmatrix}$

ii) $\text{Det M} = (-1\times 2) - (3\times 1) =$ - 5

iI) M = $\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} -1 & 3 \\ 1 & 2 \end{pmatrix}$

$M^{-1} = \frac{1}{\text{Det M}}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

$M^{-1} = \frac{1}{-5}\begin{pmatrix} 2 & -3 \\ -1 & -1 \end{pmatrix} = \begin{pmatrix} \frac{-2}{5} & \frac{3}{5} \\ \frac{1}{5} & \frac{1}{5} \end{pmatrix}$

2009 P2

2. Given that A = $\begin{pmatrix} 3 & 7 \\ 2 & 5 \end{pmatrix}$ find

i) The determinant of A.

ii) The inverse of A

iii) The value of $A^{-1}\begin{pmatrix} -2 \\ 1 \end{pmatrix}$

solution

i) A = $\begin{pmatrix} 3 & 7 \\ 2 & 5 \end{pmatrix}$

Det A = $(3\times 5) - (7\times 2) =$ 15 – 14 = 1

ii) $A^{-1} = \frac{1}{1}\begin{pmatrix} 5 & -7 \\ -2 & 3 \end{pmatrix}$

$= \begin{pmatrix} 5 & -7 \\ -2 & 3 \end{pmatrix}$

iii) $A^{-1}\begin{pmatrix} -2 \\ 1 \end{pmatrix}$

$= \begin{pmatrix} 5 & -7 \\ -2 & 3 \end{pmatrix} \times \begin{pmatrix} -2 \\ 1 \end{pmatrix}$

$= \begin{pmatrix} (5\times -2)+(-7\times 1) \\ (-2\times -2)+(3\times 1) \end{pmatrix}$

$= \begin{pmatrix} -17 \\ 7 \end{pmatrix}$

TOPIC 13: MATRICES

2007 P2

3. A = $\begin{pmatrix} 5 & -2 \\ 3 & x \end{pmatrix}$ **and B =** $\begin{pmatrix} 6 \\ 4 \end{pmatrix}$

i) Given that the determinant of A is 21.

find x.

ii) Hence find A^{-1}

iii) AB

solution

i) A = $\begin{pmatrix} 5 & -2 \\ 3 & x \end{pmatrix}$

Det A = $(5 \times x) - (3 \times -2) = 5x + 6$

Det A = $5x + 6 = 21 \quad x=3$

ii) $A^{-1} = \frac{1}{21}\begin{pmatrix} 3 & 2 \\ -3 & 5 \end{pmatrix}$

$= \begin{pmatrix} \frac{3}{7} & \frac{2}{21} \\ \frac{-3}{21} & \frac{5}{21} \end{pmatrix}$

iii) A = $\begin{pmatrix} 5 & -2 \\ 3 & 3 \end{pmatrix}$ B = $\begin{pmatrix} 6 \\ 4 \end{pmatrix}$

AB = $\begin{pmatrix} 5 & -2 \\ 3 & 3 \end{pmatrix} \times \begin{pmatrix} 6 \\ 4 \end{pmatrix}$

AB = $\begin{pmatrix} (5 \times 6) + (-2 \times 4) \\ (3 \times 6) + (3 \times 4) \end{pmatrix}$

AB = $\begin{pmatrix} 22 \\ 30 \end{pmatrix}$

2010 P2

4. If P = $\begin{pmatrix} 2 & 0 \\ 6 & 1 \end{pmatrix}$ and Q = $\begin{pmatrix} a & 0 \\ 1 & b \end{pmatrix}$**,** find

i) PQ. [2]

ii) The Values of a and b given that PQ = P – Q. [3]

iii) The determinate of P. [2]

solution

i) PQ = $\begin{pmatrix} 2 & 0 \\ 6 & 1 \end{pmatrix} \times \begin{pmatrix} a & 0 \\ 1 & b \end{pmatrix}$

PQ = $\begin{pmatrix} (2 \times a) + (0 \times 1) & (2 \times 0) + (0 \times b) \\ (6 \times a) + (1 \times 1) & (6 \times 0) + (1 \times b) \end{pmatrix}$

PQ = $\begin{pmatrix} 2a & 0 \\ 6a + 1 & b \end{pmatrix}$

ii) P - Q = $\begin{pmatrix} 2 & 0 \\ 6 & 1 \end{pmatrix} - \begin{pmatrix} a & 0 \\ 1 & b \end{pmatrix} = \begin{pmatrix} 2 - a & 0 \\ 5 & 1 - b \end{pmatrix}$

PQ = P - Q = $\begin{pmatrix} 2a & 0 \\ 6a + 1 & b \end{pmatrix} = \begin{pmatrix} 2 - a & 0 \\ 5 & 1 - b \end{pmatrix}$

Comparing corresponding elements on both matrices

6a+1=5 a = $\frac{2}{3}$

b = 1 – b b = $\frac{1}{2}$

iii) Det P = $(2 \times 1) - (6 \times 0)$

Det P = 2

EXERCISE

1987 P2

1. If M = $\begin{pmatrix} 1 & s \\ r & 2 \end{pmatrix}$ N = $\begin{pmatrix} 2 & -3 \\ 0 & 8 \end{pmatrix}$

i) Express **4M - 3N** in terms of r and s. [2]

ii) Find N^2

iii) Given that **NM = 8M,** Find the value of r and s.

2005 P2

2. If A = $\begin{pmatrix} 1 & x \\ -1 & 2 \end{pmatrix}$ **, find**

i) A^2

ii) Det A^2

iii) The values of x for which **Det** A^2**=9**

TOPIC 13: MATRICES

2011 P2

3. Given that matrix A = $\begin{pmatrix} \mathbf{1} & \mathbf{x} \\ \mathbf{-1} & \mathbf{2} \end{pmatrix}$

i) Write an expression in terms of for the determinant of A. [1]

ii) Find the value of x given that the determinant of A is 5. [2]

iii) write $\mathbf{A^{-1}}$. [1]

2012 P2

4. A = $\begin{pmatrix} \mathbf{2} & \mathbf{3} \\ \mathbf{-1} & \mathbf{5} \end{pmatrix}$ **and B =** $\begin{pmatrix} \mathbf{2} \\ \mathbf{3} \end{pmatrix}$, find the

i) Determinant of A. [1]

ii) The inverse of A. [1]

iii) Value of AB. [2]

Iv) The transpose of Matrix A. [1]

2013 P2

5. A = $\begin{pmatrix} 2 & 2 & -1 \end{pmatrix}$, P = $\begin{pmatrix} -1 & -1 \\ 4 & 2 \\ 2 & 0 \end{pmatrix}$ and

Q = $\begin{pmatrix} 2 & -1 \\ 4 & 1 \end{pmatrix}$, find

i) -2P. [1]

ii) The Determinate of Q. [1]

iii) AP. [2]

2014 GCE P2

6. Given that P = $\begin{pmatrix} \mathbf{-1} & \mathbf{2} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}$ and Q = $\begin{pmatrix} \mathbf{1} & \mathbf{2} \\ \mathbf{1} & \mathbf{-2} \end{pmatrix}$

Find the

i) The determinate of Q. [1]

ii) The matrix 2Q – 3P. [2]

iii) The matrix $\mathbf{P^2}$. [2]

iv) The transpose of P,P^T [2]

2015 GCE P2

7. Given that matrix B = $\begin{pmatrix} \mathbf{0} & \mathbf{-2} \\ \mathbf{2} & \mathbf{-8} \end{pmatrix}$, find

i) $\frac{-3}{2}$B, [1]

ii) The determinant of B. [1]

iii) B^2. [1]

iv) B^T [1]

2016 SPE P2

8. If matrix P = $\begin{pmatrix} \mathbf{4} & \mathbf{2} \\ \mathbf{0} & \mathbf{3} \end{pmatrix}$ and Q = $\begin{pmatrix} \mathbf{12} & \mathbf{4} \\ \mathbf{-9} & \mathbf{m} \end{pmatrix}$

Find the

i) Value of m for which the determinates of P and Q are equal. [2]

ii) The inverse of Q. [2]

iii) The matrix $\mathbf{P^2}$. [2]

TOPIC 13: MATRICES

Exercise

2001 P1

1. Given that A = $\begin{pmatrix} -5 & 4 \\ 3 & 2 \end{pmatrix}$ and B = $\begin{pmatrix} 6 & 4 \\ 3 & 2 \end{pmatrix}$

Find,

i) Matrix C such that C = 2A - B. [1]

ii) Det B. [1]

iii) A^T. [1]

2010 P1

2. Given that $\begin{pmatrix} 2 & x \\ -4 & 1 \end{pmatrix}\begin{pmatrix} 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 10 \\ -8 \end{pmatrix}$

Find the value of x. [2]

2012 P1

3. Given that A = $\begin{pmatrix} 1 & -3 \\ 4 & 5 \end{pmatrix}$ and B = $\begin{pmatrix} 9 & 3 \\ -12 & 6 \end{pmatrix}$

Find,

i) A - $\frac{1}{3}$B . [1]

ii) Matrix D such that D = 3A - 2B. [1]

2013 P1

4. Given that the matrix

$\begin{pmatrix} 3x & 2 \\ 1 & 4 \end{pmatrix}$ is singular. Find the value of x.

2015 GCE P1

5. Write the following as a single matrix

$$\begin{pmatrix} 1 & 2 & 5 \end{pmatrix}\begin{pmatrix} 4 & 4 \\ 0 & 1 \\ 3 & 2 \end{pmatrix}$$

2016 SPE P1

6. Find the transpose of the Matrix

A = $\begin{pmatrix} 1 & 3 & -1 \\ -2 & 0 & 4 \end{pmatrix}$.

2016 SPE P1

7. Given that M = $\begin{pmatrix} 4 & 1 & 3 \\ 1 & -5 & 4 \end{pmatrix}$ and N = $\begin{pmatrix} 3 & -2 \\ -1 & 4 \\ -2 & 0 \end{pmatrix}$.

Express MN as a single matrix.

TOPIC 14: PROBABILITY

This is the likelihood of an event to happen.

Probability is measured on a scale from 0 to 1.

A value of 0 means it is impossible whilst a value of 1 means it is certain.

The probability of an event is usually denoted by P(A).

Examples

1. A box contains 3 red balls, 2 blue balls and 1 white ball. If a ball is picked at random from the box. What is the probability that it is?

(a) Red

(b) White

(c) Black

(d) Not White

Solution

(a) Total number of balls is 3+2+1=6

P (Red) $= \frac{\text{Number of red balls}}{\text{Total number of balls}} = \frac{3}{6} = \frac{1}{2}$

(b) P (White) $= \frac{\text{Number of white balls}}{\text{Total number of balls}} = \frac{1}{6}$

(c) P (Black) $= \frac{\text{Number of black balls}}{\text{Total number of balls}} = \frac{0}{6} = 0$

(d) P (Not White) = 1 – P (White)

$= 1 - \frac{1}{6} = \frac{5}{6}$

Combined Events

These are events which involves one or two more events.

1. Mutually Exclusive Events: These are events which cannot occur at the same time.

In general, If A and B are mutually exclusive events

P (A or B) = P (A) + P (B).

1. A box contains 3 red balls, 2 blue balls and 1 white ball. If a ball is picked at random from the box. What is the probability that it is:

(a) Red or white

(b) Blue or white

(c) Blue or Red

Solution

(a) P (Red) $= \frac{3}{6}$ P (White) $= \frac{1}{6}$

P (Red or White) $= \frac{3}{6} + \frac{1}{6} = \frac{4}{6} = \frac{2}{3}$

(b) P (Blue) $= \frac{2}{6}$ P (White) $= \frac{1}{6}$

P (Red or White) $= \frac{2}{6} + \frac{1}{6} = \frac{3}{6} = \frac{1}{2}$

TOPIC 14: PROBABILITY

2. Independent Events: These are events which have no effect on each other.

In general, If A and B are independent events

P (A and B) = P (A) × P (B).

1. A box contains 2 red beads and 3 white beads. A bead is picked at random from the bag and replaced in the bag. Then a second bead is picked from the same bag. What is the probability that both beads were?

(a) Red

(b) White

Solution

(a) P (Red on 1st picking) = $\frac{2}{5}$

P (Red on 2nd picking) = $\frac{2}{5}$

P (Both Red) = $\frac{2}{5} \times \frac{2}{5} = \frac{4}{25}$

(b) P (white on 1st picking) = $\frac{3}{5}$

P (White on 2nd picking) = $\frac{3}{5}$

P (Both Red) = $\frac{3}{5} \times \frac{3}{5} = \frac{9}{25}$

3. Dependent Events: Two events are dependent if the first event affects the second event.

In general, If A and B are dependent events

P (A and B) = P (A) × P(B).

1. A box contains 2 red beads and 3 white beads. A bead is picked at random from the bag and **is not** replaced in the bag. Then a second bead is picked from the same bag. What is the probability that both beads:

(a) That both beads were **Red**.

(b) That both beads were **white**.

(c) The first was **Red** and second was **white**.

(d) The first was **White** and second was **Red**.

Solution

(a) P (Red on 1st picking) = $\frac{2}{5}$

P (Red on 2nd picking) = $\frac{1}{4}$

P (Both Red) = $\frac{2}{5} \times \frac{1}{4} = \frac{1}{10}$

(b) P (white on 1st picking) = $\frac{3}{5}$

P (White on 2nd picking) = $\frac{2}{4}$

P (Both Red) = $\frac{3}{5} \times \frac{2}{4} = \frac{3}{10}$

TOPIC 14: PROBABILITY

Exercise

2010 P1

1. A Grade One pupil has a certain number of Fanta, Coca-Cola and Sprite bottle tops in her bag. She takes one bottle top at random from the bag and the probability that it is a Fanta bottle top is 0.25 and the probability that it is a sprite bottle top is 0.4.

(a) Find the probability that it is

(i) a Coca-Cola bottle top. [1]

(ii) Not a Sprite bottle top. [1]

(b) Originally there were 16 sprite bottle tops in her bag. Find the total number of bottle tops that she had. [2]

2012 P1

2. A pack of eleven identical cards, are labelled 1 to 11. The cards are shuffled and placed upside down. If a card is picked at random from the pack, what is the probability that it is a

(a) Prime number.

(b) Even number

(c) Divisible by 3

(d) Divisible by 4

(e) Has a perfect square root.

2011 P2

3. A box contains 3 green apples and 5 red apples. An apple is picked from the box and not replaced then a second apple is picked. Expressing the answer as a fraction in its simplest form, Calculate,

(i) The probability that both apples picked are green. [2]

(ii) The probability that both apples are of different colours. [3]

2012 P2

4. A box has 7 identical sweets.3 of these are green and the rest are red. Kamwanga picks one sweet at random and eats it. After sometime he picks another one and eats it.

(i) Construct a tree diagram to illustrate the outcomes of the two sweets taken. [3]

(ii) The probability that the first sweet was red and the second was green. [2]

TOPIC 14: PROBABILITY

Exercise

2013 P2

5. Kapofu bought three oranges and two apples which she put in a bag. Later on she picked one fruit from the bag and ate it. After sometime she picked another fruit at random and ate it.

(i) Construct a tree diagram to illustrate the outcomes of the two sweets taken. [3]

(ii) The probability that the first sweet was red and the second was green. [2]

2014 GCE P2

6. A girl has 5 oranges and 2 lemons in her bag. She picks one fruit at random from the bag and eats it. After sometime, she picks another fruit at random and eats it as well.

Calculate the probability that

i) Both fruits picked were oranges, [1]

ii) The first fruit picked is a lemon, [2]

iii) Only the first fruit picked is a lemon. [2]

2015 GCE P2

7. A box has 14 identical balls, three of which are blue. Two balls are drawn at random from the box, one after the other without replacement.

Calculate the probability that

i) The two balls are blue, [2]

ii) At least one ball drawn is blue. [3]

2016 P2

8. A survey carried out at a certain hospital indicates that the probability that a patient tested positive for malaria is 0.6. What is the probability that two patients selected at random,

(i) One tested negative while the other positive, [3]

ii) Both patients tested negative. [2]

2016 SPE P2

9. There are 6 girls and 4 boys in a drama club. Two members are chosen at random to represent the club at a meeting. What is the probability,

i) Both members are boys, [2]

ii) One member is a girl? [3]

TOPIC 15: COMPUTERS

A computer is an Electronic device that can:

- Accept data, as input
- Process the data
- Store data and information
- Produce information, as output.

ALGORITHMS.

An algorithm is generally a set of logical steps that need to be followed in order to solve a problem.

METHODS OF IMPLEMENTING AN ALGORITHM.

There are two basic methods of implementing an algorithm and these are:

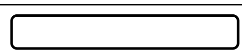
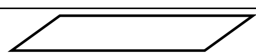
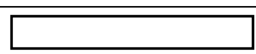
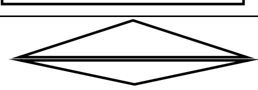
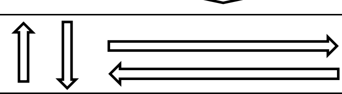
(i) FLOW CHARTS

(ii) PSEUDO CODE

FLOW CHARTS

A computer carries out all its tasks in a logical way.
A set of logical steps that need to be followed to solve a problem are also referred to as FLOW CHART.
How does one construct a flow chart?
To construct a flow chart one needs firstly to master the symbols and their meaning.

BASIC FLOWCHART SYMBOLS

SYMBOL	MEANING
	BEGIN/END OR START/ STOP
	INPUT/OUTPUT ORENTRY/DISPLAY
	PROCESS
	DECISION
	PROGRAM FLOW

BASIC FOUR OPERATORS

SYMBOL	MEANING
+	ADDITION
–	SUBTRACTION
*	MULTIPLICATION
/	DIVISION

Topic 15: Computers

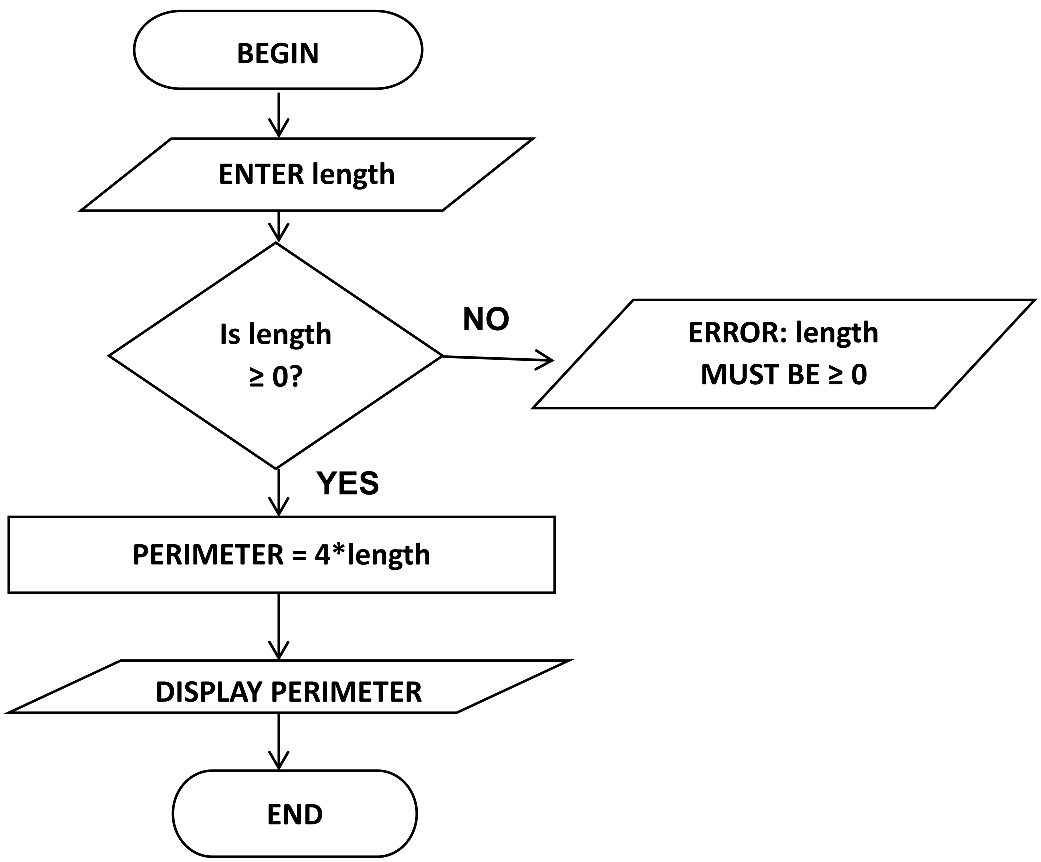
Flow Charts

The underlying factors of any flow chart are the use of the correct symbols for each step and the correct operation symbols.

This is a derivative of a flow chart and its underlying factors are the correct extraction of statements inside a flow chart symbol, listing them vertically and preservation of the logical steps also known as dentation.

Example

- Construct a **flow chart** program to calculate the **perimeter** of a square, given its **length**.
- Since the formula is:
  Perimeter = 4l
- Data needed for input is the **length**



PSEUDO CODE

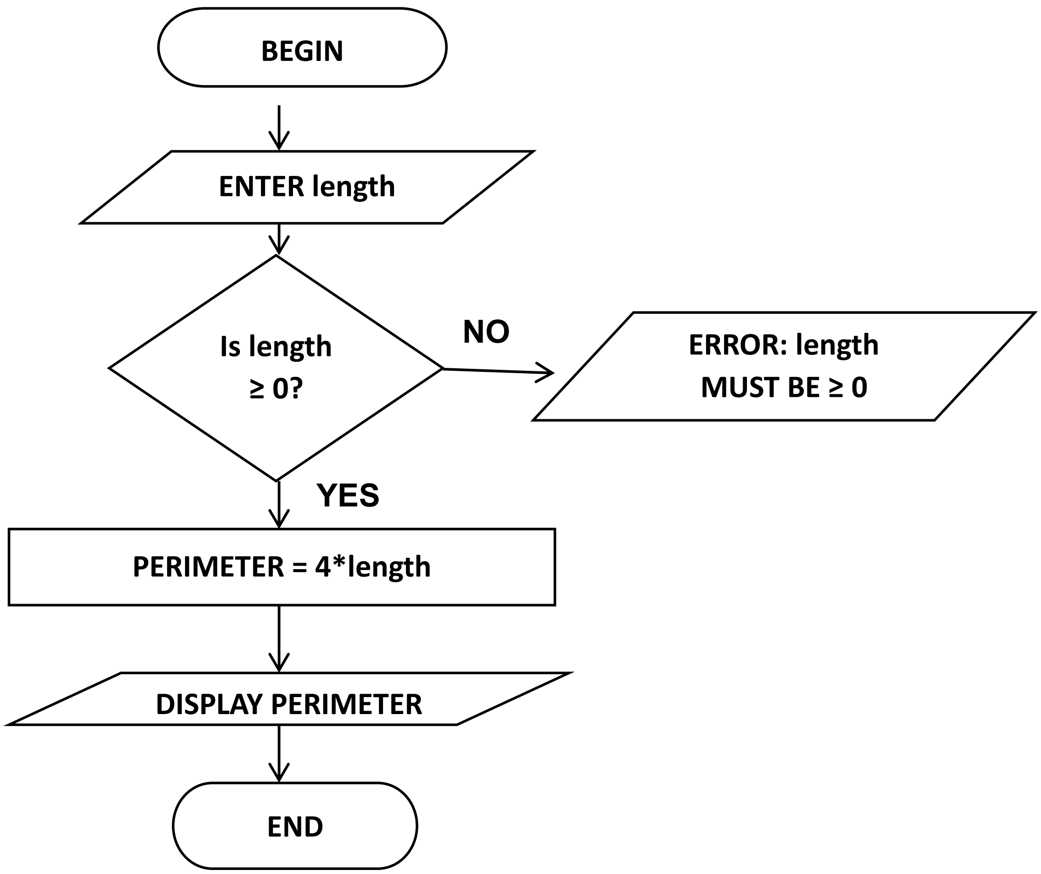
A pseudo code is a program design language that is made up of statements that are written in natural language. The design language describes the steps for the algorithms of a program exactly.

A pseudo code is the same as a flow chart except in a pseudo code shapes and not symbols are not used instead line by line statements are used.

Topic 15: Computers

Example on writing a pseudo code from a flow chart

Study the flow chart below.



Write a pseudo code corresponding to the flow chart above. [5]

SOLUTION

Start

Enter length,

If length < 0

Then display "Error message" and re-enter positive length,

Else enter perimeter = 4*length

End if

Display perimeter

Stop

1. The program below is given in the form of a pseudo code. Draw a corresponding flow chart. **[2016 P2]**

Start

Enter radius,

If radius < 0

Then display "Error message" and re-enter positive radius

Else enter height

If height < 0

Then display "Error message" and re-enter positive height

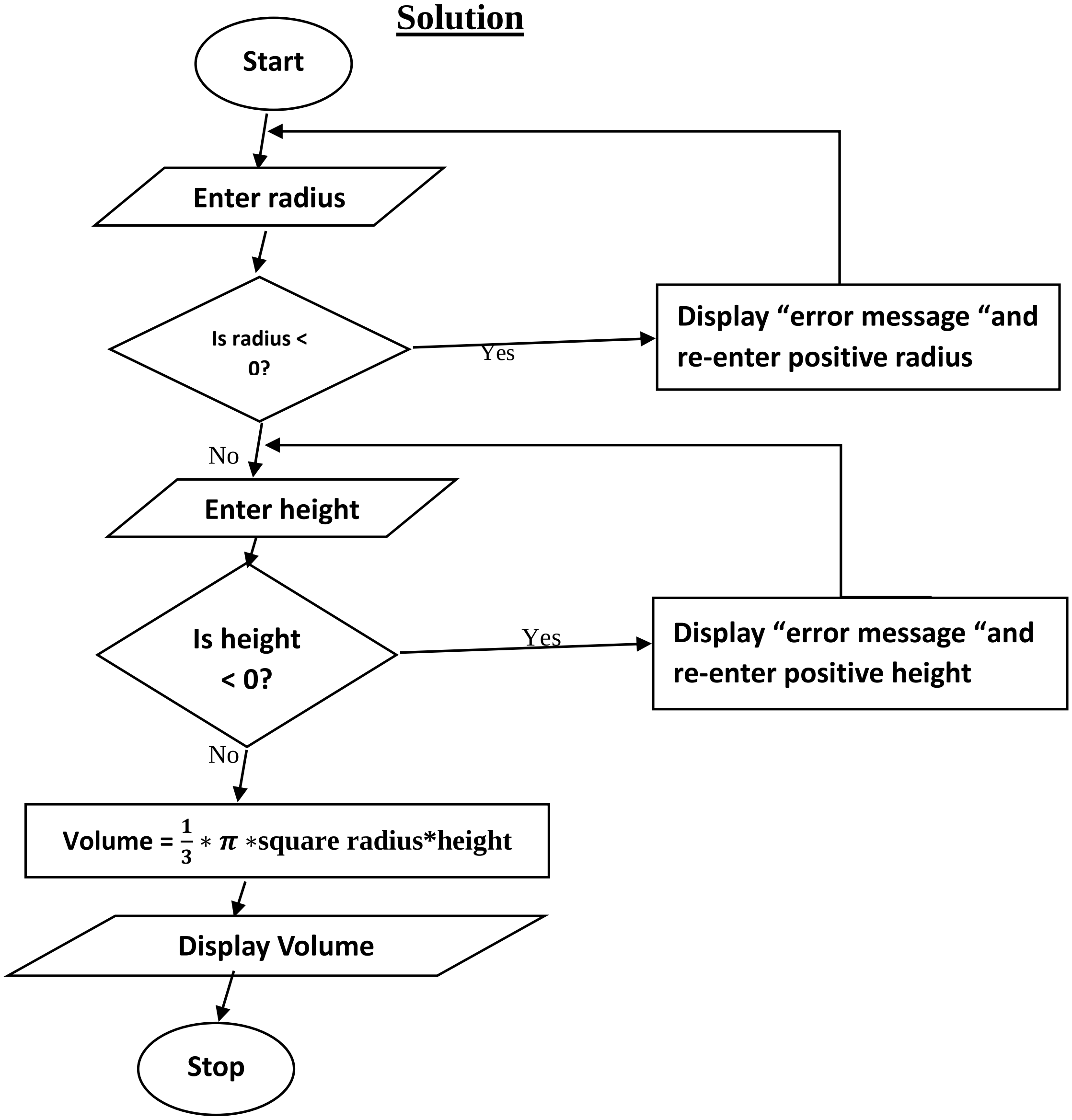
Else Volume = $\frac{1}{3} * \pi *$**square radius*height**

End if

Display Volume

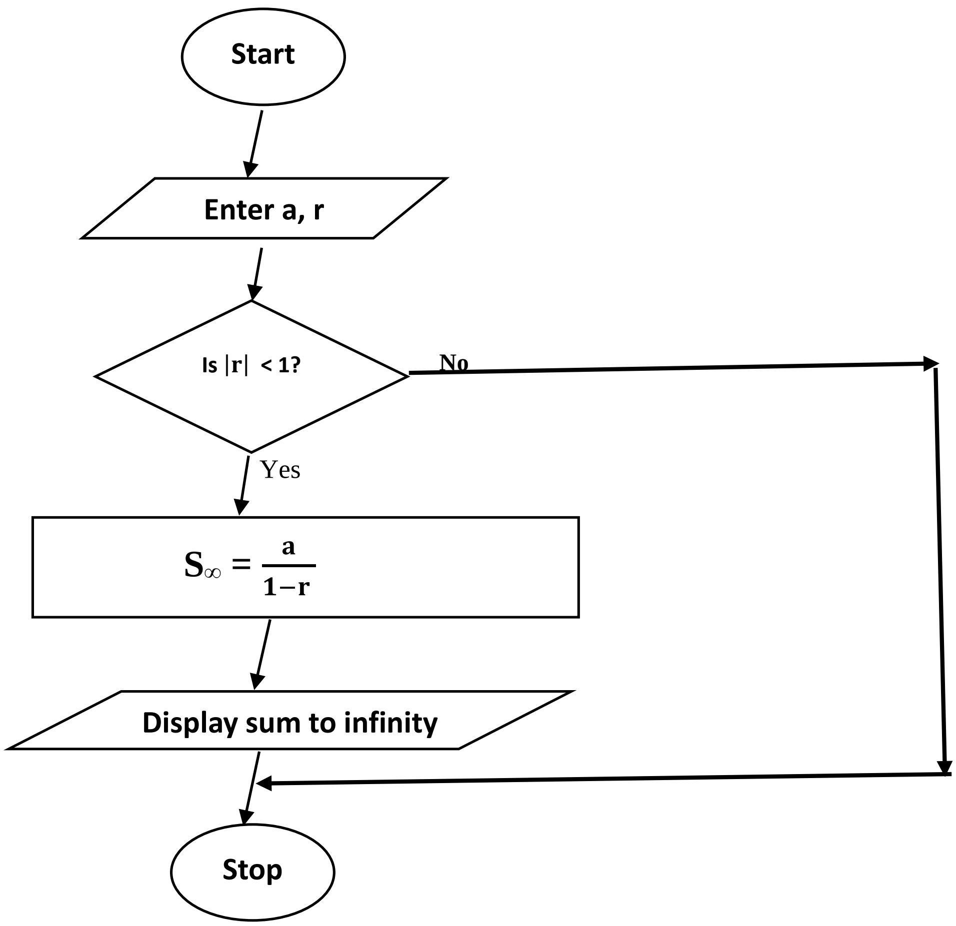
Stop

Solution



2017 GCE P2

2. Write a pseudo code corresponding to the flow chart below. [5]



Solution

Start

Enter a, r,

If |r| < 1

Then display "Print no real solutions" and re-enter |r| > 1

Else $S_\infty = \frac{a}{1-r}$

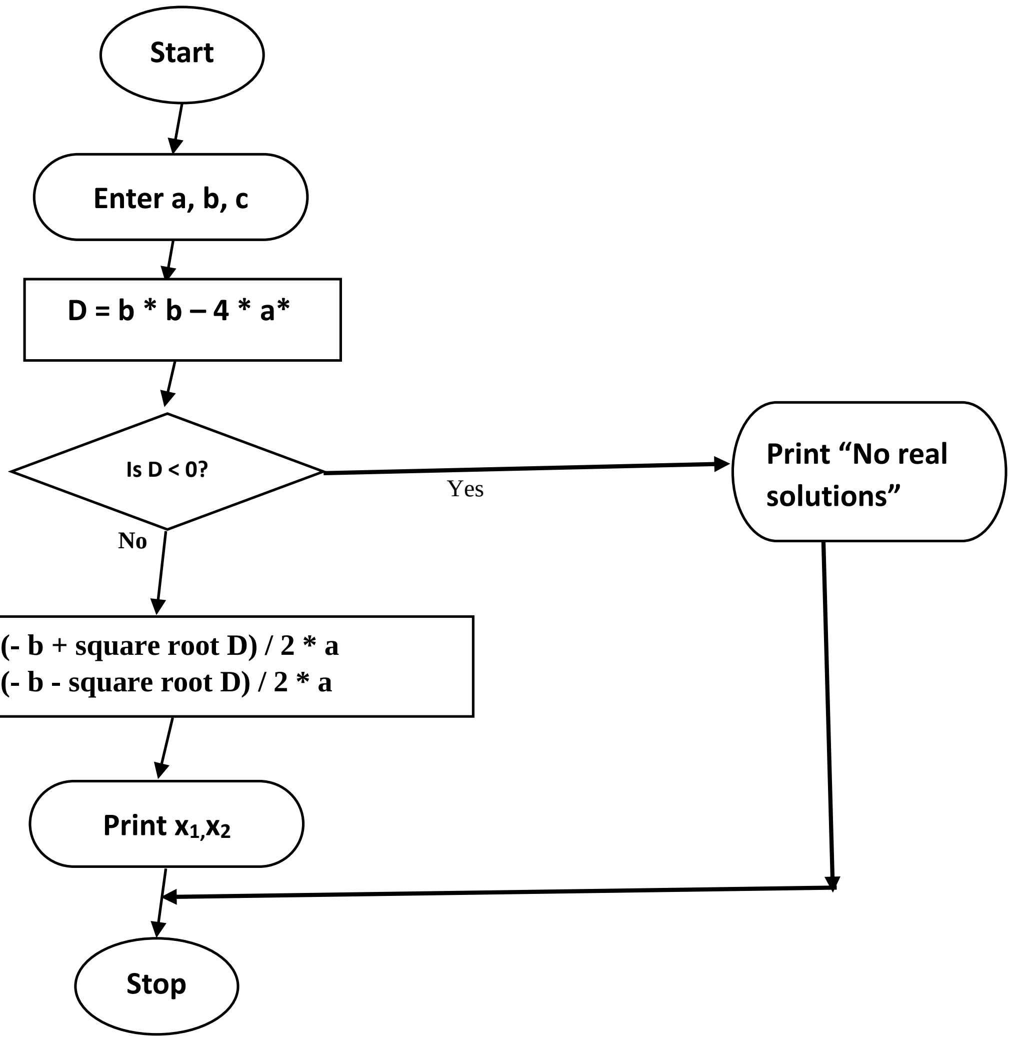
End if

Display Sum to infinity

Stop

2016 SPE P2

3. Write a pseudo code corresponding to the flow chart below. [5]



Solution

Start

Enter a, b, c

Else enter D = b * b – 4 * a*

If D < 0

Then “Print no real solutions”

Else x_1 = (- b + square root D) / 2 * a and x_2 = (- b - square root D) / 2 * a

End if

Print x_1, x_2

Stop

TOPIC 16: INTRODRODUCTION TO CALCULUS

Calculus is a branch of mathematics which was developed by Newton (1642-1727) and Leibnitz (1646-1716) to deal with changing quantities.

DIFFERENTIATION

1. DIFFERENTIATING FUNCTIONS FROM FIRST PRINCIPLES

EXAMPLES

1. If $f\left(x\right)=2x-5,\ f^{'}(x)$ from first principle.
2. **Find** $\frac{dy}{dx}$ from first principle for the function $y=2x^2$.

Solution to one

$$f'(x)=\lim_{h\to o}\frac{f(x+h)-f(x)}{h}$$

$$f(x)=2x-5$$

$$f(x+h)=2(x+h)-5$$

$$f'(x)=\lim_{h\to o}\frac{f(x+h)-f(x)}{h}$$

$$f'(x)=\lim_{h\to o}\frac{2(x+h)-5-(2x-5)}{h}$$

$$=\lim_{h\to o}\frac{2x+2h-5-2x+5}{h}$$

$$=\lim_{h\to o}\frac{2h}{h}$$

$$=\lim_{h\to o}2$$

$$f'(x)=2$$

Solution to two

$$f(x)=2x^2$$

$$f(x+h)=2(x+h)^2$$

$$\frac{dy}{dx}=\lim_{h\to o}\frac{f(x+h)-f(x)}{h}$$

$$\frac{dy}{dx}=\lim_{h\to o}\frac{2(x+h)^2-2x^2}{h}$$

$$=\lim_{h\to o}+\frac{2(x^2+2xh+h^2)-2x^2}{h}$$

$$=\lim_{h\to o}\frac{2x^2+4xh+2h^2-2x^2}{h}$$

$$=\lim_{h\to o}\frac{4xh+2h^2}{h}$$

$$=\lim_{h\to o}4x+2h$$

$$\frac{dy}{dx}=4x\ \ (note\,that\,as\,h\to o,2h=0)$$

EXERCISE

1. Find $f'(x)$ for each of the following functions by first principle.

$(a)\ f(x)=5x+4$

$(b)\ f(x)=x^2-1$

$(c)\ f(x)=20x^2-6x+7$

Expected Answers:

$(a)\ f'(x)=5$

$(b)\ f'(x)=2x$

$(c)\ f'(x)=40x-6$

2. DIIFFERENTIATING FUNCTIONS USING THE FORMULA:

A. The Derivative of $\mathbf{ax^n}$

Given a function $y = f(x) = ax^n$ **then it follows that** $\frac{dy}{dx} = f'(x) = anx^{n-1}$

1. Given that $y = 7$, find $\frac{dy}{dx}$.
2. Given that $y = 5x$, find $\frac{dy}{dx}$.
3. Find the derived function of $y = 2x^4 + 5x^3 - x^2 + 2$

Solution to one

$y = 7 \; is\, the\, same\, as \; y = 7x^0 \; since \; x^0 = 1$

$y = 7x^0$

$\frac{dy}{dx} = (0)7x^{0-1}$

$\frac{dy}{dx} = 0$

The derivative of any constant is Zero (0)

Solution to two

$y = 5x \; is\, the\, same\, as \; y = 5x^1$

$y = 5x^1$

$\frac{dy}{dx} = 1(5)x^{1-1}$

$\frac{dy}{dx} = 1(5)x^0$

$\frac{dy}{dx} = 5$

Solution to three

$y = 2x^4 + 5x^3 - x^2 + 2$

$\frac{dy}{dx} = 4(2)x^{4-1} + 3(5)x^{3-1} - 2x^{2-1} + 0$

$\frac{dy}{dx} = 8x^3 + 15x^2 - 2x^1$

$\frac{dy}{dx} = 8x^3 + 15x^2 - 2x$

B. the Derivative of $(\mathbf{ax + b})^{\mathbf{n}}$ (Chain rule)

The derivate of the function $y = (ax+b)^n$ is given by the formula $\frac{dy}{dx} = n(ax+b)^{n-1} \times a$

Example

4. If $y = (3x+5)^4$, *find* $\frac{dy}{dx}$.

Solution to four

$y = (3x+5)^4$

$\frac{dy}{dx} = 4(3x+5)^{4-1} \times 3$

$\frac{dy}{dx} = 12(3x+5)^3$

C. The Derivative of a product. (Product Rule)

If $y=(ax+b)^n(cx+d)^m$ we can let $u=(ax+b)\ and\ v=(cx+d)$. From it follows that, the derivative of a product is given by the formula

$$\frac{dy}{dx}=v\frac{du}{dx}+u\frac{dv}{dx}$$

D: The Derivative of a quotient (Quotient Rule)

If $y=f(x)$ is a ratio of functions $u\ and\ v$ where $u\ and\ v$ are also functions of x, the derivative of the function y with respect to x is given by the formula

$$\frac{dy}{dx}=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}=\frac{vu'-uv'}{v^2}$$

Examples

1. Given that $y=(3x+1)^2(2x-5)^3$, find $\frac{dy}{dx}$.

2. Differentiate $\frac{(x-3)^2}{(x+2)^2}$.

Solution to one

$y=(3x+1)^2(2x-5)^3$

$u=(3x+1)^2\ and\ v=(2x-1)^3$

$\frac{du}{dx}=2(3x+1)\times 3\ and\ \frac{dv}{dx}=3(2x-1)^2\times 2$

$\frac{du}{dx}=6(3x+1)\qquad \frac{dv}{dx}=6(2x-1)^2$

$\frac{dy}{dx}=v\frac{du}{dx}+u\frac{dv}{dx}$

$\frac{dy}{dx}=(2x-1)^3 6(3x+1)+(3x+1)^2 6(2x-1)^2$

$=6(2x-1)^3(3x+1)+6(3x+1)^2(2x-1)^2$

$=6(2x-1)^2(3x+1)[2x-1+3x+1]$

$=6(2x-1)^2(3x+1)(5x)$

$=30x(2x-1)^2(3x+1)$

Solution to two

We let $u=(x-3)^2$ and $v=(x+2)^2$

$u'=2(x-3)\ \ and\ \ v'=2(x+2)$

$\frac{dy}{dx}=\frac{vu'-uv'}{v^2}$

$\frac{dy}{dx}=\frac{(x+2)^2 2(x-3)-(x-3)^2 2(x+2)}{\left[(x+2)^2\right]^2}$

$=\frac{2(x+2)^2(x-3)-2(x-3)^2(x+2)}{(x+2)^4}$

$=\frac{2(x+2)(x-3)[(x+2)-(x-3)]}{(x+2)^4}$

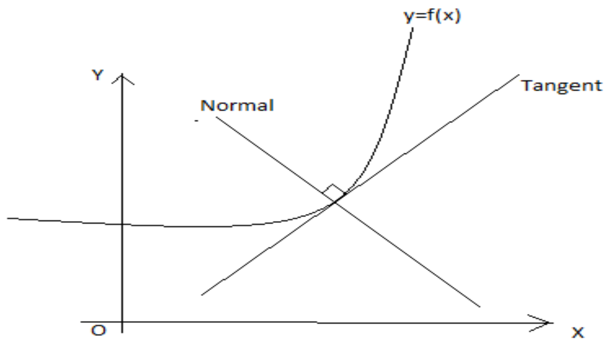
$=\frac{2(x-3)(x+2-x+3)}{(x+2)^3}$

$=\frac{2(x-3)(5)}{(x+2)^3}$

$\frac{dy}{dx}=\frac{10(x-3)}{(x+2)^3}$

TANGENTS AND NORMALS

If $y = f(x)$ is a curve, we can find the gradient at any point on the curve. This gradient is equal to the gradient of the tangent to the curve at that point.



If the gradient of the tangent is m_1 and that of the normal line is m_2 it follows that $m_1 \times m_2 = -1$

The tangent and the normal are perpendicular to each other at the point of contact.

Example

1. Given that the equation of a curve is $y = 3x^2 + 4x + 1$. Find the gradient of
   (i) The tangent at the point where $x = 4$
   (ii) The normal at the point where $x = 4$

Solution to one

$y = 3x^2 + 4x + 1$

The gradient of the tangent is $\frac{dy}{dx}$

$m_1 = 6x + 4$ and since the value of $x = 4$

$m_1 = 6(4) + 4$
$m_1 = 24 + 4$
$m_1 = 28$

Solution to two

To find the gradient of the normal, recall that

$m_1 \times m_2 = -1$
$28 \times m_2 = -1$
$m_2 = \frac{-1}{28}$

$\therefore$ The gradient of the tangent, $m_1 = 28$ and the gradient of the normal, $m_2 = \frac{-1}{28}$

FINDING THE EQUATION OF THE TANGENT AND THE NORMAL

Example

1. Find the equation of the tangent and the normal to the curve $y = 3x^2 + 4x + 1$ at the point where $x = 4$.

Solution

We have the value for x. So let's find the corresponding value for y.

$$y = 3x^2 + 4x + 1, \quad x = 4$$
$$y = 3(4)^2 + 4(4) + 1$$
$$y = 3(16) + 16 + 1$$
$$y = 48 + 16 + 1$$
$$y = 65$$

The point is $(4, 65)$

The gradient of the tangent is $m_1 = 28$ and that of the normal is $m_2 = \frac{-1}{28}$ at the point $(4, 65)$

Equation of the tangent

$$y = m_1 x + c$$
$$65 = 28(4) + c$$
$$65 = 112 + c$$
$$c = -47$$
$$y = 28x - 47$$

Equation of the normal

$$y = m_2 x + c$$
$$65 = \frac{-1}{28}(4) + c$$
$$65 = \frac{-1}{7} + c$$
$$65 + \frac{1}{7} = c$$
$$c = \frac{456}{7}$$
$$y = \frac{-1}{28}x + \frac{456}{7}$$

Exercise

1. Find the equation of the tangent and the normal to the curve $x^2 = 4y$ at the point $(6, 9)$.

Expected Answers: $y = 3x - 9$ for the tangent and $y = \frac{-1}{3}x + 11$ for the normal.

TOPIC 17: INTEGRATION

The inverse of differentiation or the reverse of differentiation is called integration.

Since integration is the reverse of differentiation the following steps must be taken:-

1. Increase the power of the variable by 1.
2. Divide the term (variable term) by the new power.
3. Then finally add the arbitrary term C

In General if $\frac{dy}{dx} = ax^n$ then it follows that $y = \frac{ax}{n+1}^{n+1} + c$

A.INDEFINITE INTEGRALS

An indefinite integral must contain an arbitrary constant (C). An integral of the form $\int f(x)\,dx$ is called an indefinite integral.

1. Integrate the following gradient functions

(a) $\frac{dy}{dx} = 3x$

(b) $f'(x) = 6x^3 + 2x^2 - x$

Solution

Solution to (a)

$\frac{dy}{dx} = 3x^1$

$\int 3x^1$

$= \frac{3x^{1+1}}{2} + c$

$= \frac{3x^2}{2} + c$

Solution to (b)

$$f'(x) = 6x^3 + 2x^2 - x$$
$$f'(x) = 6x^3 + 2x^2 - x^1$$
$$f(x) = \frac{6x^{3+1}}{4} + \frac{2x^{2+1}}{3} - \frac{x^{1+1}}{2} + c$$
$$f(x) = \frac{6x^4}{4} + \frac{2x^3}{3} - \frac{x^2}{2} + c$$
$$f(x) = \frac{3}{2}x^4 + \frac{2}{3}x^3 - \frac{1}{2}x^2 + c$$

Exercise

1. Integrate the following gradient functions

(a) $5x^2 - x + 1$ (b) $x^6 - 3x^4 + 2x^2 + 1$

B. DEFINITE INTEGRALS

A definite integral is an integral performed between the limits. Thus $A = \int_a^b f(x)\,dx$ is an integral performed between the limiting values a and b for x.

Example

1. Evaluate the definite integral of $4x^3 - 1$ between $x = 1$ *and* $x = 3$

$$f(x) = 4x^3 - 1$$

$$\int_1^3 f(x)\,dx = \int_1^3 (4x^3 - 1)\,dx$$

$$= [\frac{4}{4}x^4 - x]_1^3$$

$$= [x^4 - x]_1^3$$

$$= (3^4 - 3) - (1^4 - 1)$$

$$= (81 - 3) - (1 - 1)$$

$$= 78 - 0$$

$$= 78$$

Applications of Integration

1. Find y given that $\frac{dy}{dx} = 2x - 3$ and that $y = -4$ when $x = 1$.

Solution

If $\frac{dy}{dx} = 2x - 3$,

$y = \int (2x - 3)\,dx$

$y = x^2 - 3x + c$

when $x = 1$, $y = 1 - 3 + c = -4$ so $c = -2$

Hence $y = x^2 - 3x - 2$

2. The gradient of the tangent at a point on a curve is given by $x^2 + x - 2$. Find the equation of the curve if it passes through (2, 1).

Solution

Gradient $= \frac{dy}{dx} = x^2 + x - 2$

$y = \int (x^2 + x - 2)\,dx = \frac{x^3}{3} + \frac{x^2}{2} - 2x + c$

when $x = 2$, $y = \frac{8}{3} + \frac{4}{2} - 4 + c = 1$ Hence $c = \frac{1}{3}$.

The equation of the curve is $y = \frac{x^3}{3} + \frac{x^2}{2} - 2x + \frac{1}{3}$

$6y = 2x^3 + 3x^2 - 12 + 2.$

TOPIC 17: CALCULUS

Exercise

2016 SPE P2

1. (a) Find the equation of the tangent to the Curve $\mathbf{y = x^2 - 2x - 3}$ at the point (3, 0).

(b) A curve is such that $\frac{dy}{dx} = 5x^2 - 12x$.

Given that it passes through (1, 3), find its Equation.

2016 SPE P1

2. Given that y = $\mathbf{2x^3 - \frac{3}{2x^2}}$, find $\frac{dy}{dx}$.

2016 P2

3. The equation of a curve is $\mathbf{y = x^3 - \frac{3}{2}x^2}$. Find,

(a) The equation of the normal where x = 2.

(b) The coordinates of the stationary points.

2017 GCE P2

4. Evaluate $\int_2^5 (\mathbf{3x^2 + 2})\ dx$