KYATEC STUDY CLUB

Everyone has a right to having a distinction in mathematics

MOTTO: *Learn to Live*

MATHEMATICS VOL 1

Grade 10 - 12 Revision Questions and Answers

COMPILED BY: *STEPHEN MOONGA*

WITH THE HELP OF: *THE MATEMATICS DEPARTMENT*

KYAWAMA HIGH SCHOOL

1. (a) Evaluate.

(i) $3^2 \times 3^3$ (ii) $7^0 \times 7^4 \times 7^{-2}$ (iii) $5^3 \times 5^{-1} \times 8^0$

(iv) $96^2 - 16$ (v) $5^2 + 5^1$ (vi) $\left(\frac{2}{3}\right)^2 \times \left(\frac{2}{3}\right)^1 \times \left(\frac{2}{3}\right)^0$

(b) Factorize completely $5 - 20x^2$

(c) Solve for x, $2^x = 8$

SOLUTION

(a) (i) $3^2 \times 3^3$
$= 3^{2+3}$
$= 3^5$
$= \underline{243}$

(ii) $7^0 \times 7^4 \times 7^{-2}$
$= 7^{0+4-2}$
$= 7^2$
$= \underline{49}$

(iii) $5^3 \times 5^{-1} \times 8^0$
$= 5^{3-1} \times 8^0$
$= 5^2 \times 1$
$= \underline{25}$

(iv) $96^2 - 16$
$= 9216 - 16$
$= \underline{9200}$

(v) $5^2 + 5^1$
$= 5^{2+1}$
$= 5^3 = \underline{125}$

(vi) $\left(\frac{2}{3}\right)^2 \times \left(\frac{2}{3}\right)^1 \times \left(\frac{2}{3}\right)^0$
$= \left(\frac{2}{3}\right)^{2+1+0}$
$= \left(\frac{2}{3}\right)^3 = \underline{\frac{8}{27}}$

(b) $5 - 20x^2$
$= 5(1 - 4x^2)$
$= \underline{5(1 - 2x)(1 + 2x)}$

(c) $2^x = 8$
$2^x = 2 \times 2 \times 2$
$2^x = 2^3$
$\therefore \underline{x = 3}$

2. (a) Solve the equations,

(i) $\frac{2}{x} = \frac{5}{1}$ (ii) $4y^2 = 7y$ (iii) $4(m - 2) - 3 = -26 - m$

(iv) $\frac{2x}{3} + \frac{1}{4} = 8$ (v) $\frac{3t-1}{2} = \frac{4}{1}$ (vi) $2(2x - 5) + 2 = x + 7$

(vii) $2x - 6 = 4 - 3(x - 5)$ (viii) $\frac{2}{3} = \frac{9}{2r}$

(b) Solve the inequalities;

(i) $1 - \frac{3x}{5} < 4$ (ii) $\frac{x+6}{3} \geq \frac{3x-6}{5}$

SOLUTION

(a) (i) $\frac{2}{x} = \frac{5}{1}$ (cross multiply)
$5x = 2$
$\frac{5x}{5} = \frac{2}{5}$
$x = \underline{\frac{2}{5}}$

(ii) $4y^2 = 7y$
$4y^2 - 7y = 0$
$y(4y - 7) = 0$
$y = 0$ or $4y = 7$
$y = 0$ or $y = \underline{\frac{7}{4}}$

(iii) $4(m - 2) - 3 = -26 - m$

$4m - 8 - 3 = -26 - m$

$4m - 11 = -26 - m$

$4m + m = -26 + 11$

$5m = -15$

$\underline{m = -3}$

(iv) $\frac{2x}{3} + \frac{1}{4} = 8$

$\frac{8x+3}{12} = \frac{8}{1}$

$8x + 3 = 12 \times 8$

$8x + 3 = 96$

$8x = 96 - 3$

$8x = 93$

$x = \underline{\frac{93}{8}}$

(v) $\frac{3t-1}{2} = \frac{4}{1}$

$3t - 1 = 2 \times 4$

$3t - 1 = 8$

$3t = 8 + 1$

$3t = 9$

$\underline{t = 3}$

(vi) $2(2x - 5) + 2 = x + 7$

$4x - 10 + 2 = x + 7$

$4x - 8 = x + 7$

$4x - x = 7 + 8$

$3x = 15$

$\underline{x = 5}$

(vii) $2x - 6 = 4 - 3(x - 5)$

$2x - 6 = 4 - 3x + 15$

$2x + 3x = 4 + 15 + 6$

$5x = 25$

$\underline{x = 5}$

(viii) $\frac{2}{3} = \frac{9}{2r}$

$2 \times 2r = 3 \times 9$

$4r = 27$

$r = \underline{\frac{27}{4}}$

(b) (i) $1 - \frac{3x}{5} < 4$

$\frac{1}{1} - \frac{3x}{5} < \frac{4}{1}$

$\frac{5-3x}{5} < \frac{4}{1}$

$5 \times \left(\frac{5-3x}{5}\right) < \frac{4}{1} \times 5$

$5 - 3x < 20$

$-3x < 20 - 5$

$-3x < 15$

$\underline{x < -5}$

(ii) $\frac{x+6}{3} \geq \frac{3x-6}{5}$

$\frac{x+6}{3} \geq \frac{3x-6}{5}$

$5(x + 6) \geq 3(3x - 6)$

$5x + 30 \geq 9x - 18$

$5x - 9x \geq -18 - 30$

$-4x \geq -48$

$\underline{x \geq 12}$

3. Express each of the following as a fraction in its lowest term.
   (a) 0.586 (b) 0.06 (c) 0.05

<u>SOLUTION</u>

(a) 0.586

$= \frac{586}{1000}$

$= \underline{\frac{243}{500}}$

(b) 0.06

$= \frac{6}{100}$

$= \underline{\frac{3}{50}}$

(c) 0.05

$= \frac{5}{100}$

$= \underline{\frac{1}{20}}$

4. Express each of the following as a percentage; (a) $\frac{15}{20}$ (b) $\frac{9}{25}$

SOLUTION

(a) $= \frac{15}{20}$ (b) $= \frac{9}{25}$

$= \frac{15}{20} \times 100\%$ $= \frac{9}{25} \times 100\%$

$= 75\%$ $= 36\%$

5. Express each of the following as a single fraction.

(a) $\frac{x+5}{3} - \frac{x+2}{4}$ (b) $\frac{x}{4} - \frac{x}{7} + \frac{x}{14}$ (c) $\frac{7}{p-2} - \frac{5}{p-1}$ (d) $\frac{5}{2x-3} - \frac{1}{x+5}$

(e) $\frac{3}{2} - \frac{1-2x}{4x}$ (f) $\frac{3}{p-1} - \frac{2}{1-p}$ (g) $\frac{5}{2x-1} - \frac{7}{3x-2}$ (h) $\frac{1}{2-x} - \frac{2}{x-4}$

(i) $\frac{x-1}{3} - \frac{2x-3}{5}$

SOLUTION

(a) $\frac{x+5}{3} - \frac{x+2}{4}$

$= \frac{4(x+5)-3(x-2)}{12}$

$= \frac{4x+20-3x+6}{12}$

$= \frac{4x-3x+20+6}{12}$

$= \frac{x+26}{12}$

(b) $\frac{x}{4} - \frac{x}{7} + \frac{x}{14}$

$= \frac{7x-4x+2x}{28}$

$= \frac{3x+2x}{28}$

$= \frac{5x}{28}$

(c) $\frac{7}{p-2} + \frac{5}{p-1}$

$= \frac{7(p-1)+5(p-2)}{(p-2)(p-1}$

$= \frac{7p-7+5p-10}{(p-2)(p-1)}$

$= \frac{7p+5p-10-7}{(p-2)(p-1)}$

$= \frac{12p-17}{(p-2)(p-1)}$

(d) $\frac{5}{2x-3} - \frac{1}{x+5}$

$= \frac{5(x+5)-1(2x-3)}{(2x-3)(x+5)}$

$= \frac{5x+25-2x+3}{(2x-3)(x+5)}$

$= \frac{5x-2x+3+25}{(2x-3)(x+5)}$

$= \frac{3x+28}{(2x-3)(x+5)}$

(e) $\frac{3}{2} - \frac{1-2x}{4x}$

$= \frac{3(2x)-1(1-2x)}{4x}$

$= \frac{6x-1+2x}{4x}$

$= \frac{6x+2x-1}{4x}$

$= \frac{8x-1}{4x}$

(f) $\frac{3}{p-1} - \frac{2}{1-p}$

$= \frac{3(1-p)-2(p-1)}{(p-1)(1-p)}$

$= \frac{3-3p-2p+2}{(p-1)(1-p)}$

$= \frac{-3p-2p+2+3}{(p-1)(1-p)}$

$= \frac{-5p+5}{(p-1)(1-p)}$

(g) $\dfrac{5}{2x-1} - \dfrac{7}{3x-2}$

$= \dfrac{5(3x-2)-7(2x-1)}{(2x-1)(3x-2)}$

$= \dfrac{15x-10-14x+7}{(2x-1)(3x-2)}$

$= \dfrac{15x-14x+7-10}{(2x-1)(3x-2)}$

$= \dfrac{x-3}{(2x-1)(3x-2)}$

(h) $\dfrac{1}{2-x} - \dfrac{2}{x-4}$

$= \dfrac{1(x-4)-2(2-x)}{(2-x)(x-4)}$

$= \dfrac{x-4-4+2x}{(2-x)(x-4)}$

$= \dfrac{x+2x-4-4}{(2-x)(x-4)}$

$= \dfrac{3x-8}{(2-x)(x-4)}$

(i) $\dfrac{x-1}{3} - \dfrac{2x-3}{5}$

$= \dfrac{5(x-1)-3(2x+3)}{15}$

$= \dfrac{5x-5-6x-9}{15}$

$= \dfrac{5x-6x-9-5}{15}$

$= \underline{\dfrac{-x-11}{15}}$

6. Evaluate; (a) 15 - 8.33 (b) $0.24 \div 0.008$ (c) $(0.5)^3$ (d) $2.16 \div 0.03$
(e) $1.5 + 3\frac{1}{4}$ (f) $1\frac{2}{3} - \frac{3}{4}$ (g) $2\frac{4}{5} \div \frac{2}{5}$ (h) $7 - (^-4.5)$ (i) $1\frac{5}{6} - 1\frac{2}{3} + \frac{1}{5}$

SOLUTION

(a) 15 - 8.33

$$\begin{array}{r} 15.00 \\ -\ \ 8.33 \\ \hline 6.67 \\ \hline \end{array}$$

(b) $0.24 \div 0.008$

$= \dfrac{24}{100} \div \dfrac{8}{1000}$

$= \dfrac{24}{100} \times \dfrac{1000}{8}$

$= \underline{30}$

(c) $(0.5)^3$

$= 0.5 \times 0.5 \times 0.5$

$= \underline{0.125}$

(d) $2.16 \div 0.03$

$= \dfrac{216}{3}$

$= \underline{72}$

(e) $1.5 + 3\frac{1}{4}$

$= 1.5 + 3.25$

$= \underline{4.75}$

(f) $1\frac{2}{3} - \frac{3}{4}$

$= \frac{5}{3} - \frac{3}{4}$

$= \frac{20-9}{12}$

$= \underline{\frac{11}{12}}$

(g) $2\frac{4}{5} \div \frac{2}{5}$

$= \frac{14}{5} \div \frac{2}{5}$

$= \frac{14}{5} \times \frac{5}{2}$

$= 7 \times 1$

$= \underline{7}$

(h) $7 - (^-4.5)$

$= 7 + 4.5$

$= \underline{11.5}$

(i) $1\frac{5}{6} - 1\frac{2}{3} + \frac{1}{5}$

$= \frac{11}{6} - \frac{5}{3} + \frac{1}{5}$

$= \frac{55-50+6}{30}$

$= \underline{\frac{11}{30}}$

7. Solve the simultaneous equations

(a) $2x + 3y = 5$
$x - 4y = 8$

(b) $4x + 3y = 12$
$2x - 5y = 12.5$

(c) $2x - 0.3y = 0.4$
$0.4x + 0.2y = -0.8$

SOLUTION

(a) $2x + 3y = 5 \times 1$
$\underline{x - 4y = 8}\ \times 2$
$2x + 3y = 5$
$\underline{2x - 8y = 16}$ (Subtract)
$11y = -11$
$y = -1$

when y = -1 replacing in $2x + 3y = 5$
we get $2x + 3 \times (-1) = 5$
ie $2x - 3 = 5$
$2x = 5 + 3$
$2x = 8$ ie $x = 4$
$\underline{\therefore \text{Solution set} = (4, -1)}$

(b) $4x + 3y = 12 \quad \times 1$
$\underline{2x - 5y = 12.5} \times 2$
$4x + 3y = 12$
$\underline{4x - 10y = 25}$ (Subtract)
$13y = -13$
$y = -1$

when $y = -1$ replacing in $4x + 3y = 12$
we get $4x + 3\times(-1) = 12$
ie $4x - 3 = 12$
$4x = 12 + 3$
$4x = 15$ ie $x = 3.75$
$\therefore$ Solution set = (3.75, -1)

(c) $2x - 0.3y = 0.4 \times 0.4$
$\underline{0.4x + 0.2y = -0.8} \times 0.2$
$0.8x - 0.12y = 0.16$
$\underline{0.8x + 0.4y = -0.16}$(Subtract)
$-0.16y = 0.32$
$y = -2$

when $y = -2$ replacing in $2x – 0.3y = 0.4$
we get $2x – 0.3\times(-2) = 0.4$
ie $2x + 0.6 = 0.4$
$2x = 0.4 - 0.6$
$2x = -0.2$ ie $x = -0.1$
$\therefore$ Solution set = (-0.1, -2)

8. Factorize completely; a) $mn - km - hn + hk$ (b) $6x - 8x^2$ (c) $3x - 12x^3$ (d) $5 - 20x^2$ (e) $3w^2 - 12$

SOLUTION

(a) $mn - km - hn + hk$
$= m(n – k) - h(n – k)$
$= \underline{(m – h)(n - k)}$

(b) $6x - 8x^2$
$= \underline{2x(3 - 4x)}$

(c) $3x - 12x^3$
$= 3x(1 - 4x^2)$
$= \underline{3x(1 – 2x)(1 + 2x)}$

(d) $5 - 20x^2$
$= \underline{5(1 - 2x)(1 + 2x)}$

(e) $3w^2 - 12$
$= 3(w^2 - 4)$
$= \underline{3(w – 2)(w + 2)}$

9. (a) Find the exact values of $1.71 \div 0.03$
(b) Evaluate $\frac{4a-(b+c)}{c}$, when $a = 3, b = -4$ and $c = 2$
(c) Given that $a – b = 10$ and $2(a^2 – b^2) = 64$, find the value of $a + b$.
(d) Find the value of $(p + q)^2$, given that $p^2 + q^2 = 36$ and $pq = 7$.
(e) The operation $*$ is defined over R by $x*y = (x – y)^2$. Find the value of $2*3$.
(f) Subtract 0.5625 from 0.6875. (g) Find 30% of 25

SOLUTION

(a) $\frac{1.71}{0.03} = \frac{171}{3} = 57$ (b) $\frac{4a-(b+c)}{c} = \frac{4\times3-(-4+2)}{2} = \frac{12-(-2)}{2} = \frac{12+2}{2} = 7$

(c) $2(a^2 – b^2) = 64$ $2(a – b)(a + b) = 64$ but $a – b = 10$
$\Rightarrow 2 \times 10(a + b) = 64$ ie $20(a + b) = 64$ $\frac{20(a+b)}{20} = \frac{64}{20}$ $\therefore a + b = 3.2$

(d) $(p + q)^2 = (p + q)(p + q) = p^2 + q^2 + 2pq$, but $p^2 + q^2 = 36$ and $pq = 7$
$\Rightarrow 36 + 2 \times 7 = 36 + 14 = 50.$

(e) Since $x*y = (x-y)^2 \Rightarrow 2*3 = (2-3)^2 = (-1)^2 = 1$
(f) Subtract 0.5625 from 0.6875 ie 0.6875 - 0.5625 = 0.125
(g) 30% of \$25 $\Rightarrow \frac{30}{100} \times \$25 = \frac{3}{10} \times \$25 = \$7.50$

10. (a) Given that $\frac{cb-a}{c} = 1$, express c in terms of a and b

(b) Given that $p = \frac{q-t}{3t}$, where $t \neq 0$.

(i) Calculate the value of p When $q = \frac{1}{4}$ and $t = {}^{-1}/_{2}$.
(ii) Express t in terms of p and q.

(c) Given that a = 3, b = -5 and c = 10, find the exact value of

(i) $a + b - c$ (ii) $-\frac{3}{5}(c - 2ab)$
(iii) Given that 6x = 15y, state the ratio x: y
(iv) Given that $\frac{1}{m} = 0.0125$, find the value of m

SOLUTION

(a) $\frac{cb-a}{c} = 1$

$cb - a = c$

$cb - c = a$

$\frac{c(b-1)}{(b-1)} = \frac{a}{(b-1)}$

$\underline{c = \frac{a}{(b-1)}}$

(b) (i) $p = \frac{q-t}{3t}$

$p = \frac{\frac{1}{4}-(-\frac{1}{2})}{3(-\frac{1}{2})}$

$p = \frac{\frac{1}{4}+\frac{1}{2}}{(-\frac{3}{2})}$

$p = \frac{3}{4} \div (-\frac{1}{2})$

$p = \frac{3}{4} \times (-\frac{2}{1})$

$\underline{p = -\frac{3}{2}}$

(ii) $p = \frac{q-t}{3t}$

$3pt = q - t$

$3pt + t = q$

$\frac{t(3p+1)}{(3p+1)} = \frac{q}{(3p+1)}$

$\underline{t = \frac{q}{(3p+1)}}$

11. Given that $f(x) = \frac{3-5x}{2x}$, for $x \neq 0$, find

(a) f(3) (b) x when f(x) = -4 (c) an expression for $f^{-1}(x)$

SOLUTION

(a) $f(x) = \frac{3-5x}{2x}$

$f(3) = \frac{3-5(3)}{2(3)}$

$f(3) = \frac{3-15}{6}$

(b) $-\frac{4}{1} = \frac{3-5x}{2x}$

$3 - 5x = -8x$

$8x - 5x = -3$

(c) $f(x) = \frac{3-5x}{2x}$, Let y = f(x)

$\Rightarrow y = \frac{3-5x}{2x}$

$\Rightarrow 2xy = 3 - 5x$

$f(3) = \frac{-12}{6}$ 3x = -3 $\Rightarrow$ 2xy + 5x = 3 , Make x the subject

$\underline{f(x) = -2}$ $\underline{x = -1}$ x(2y + 5) = 3, Dividing by 2y + 5

$x = \frac{3}{2y+5}$, Replace x by $f^{-1}(x)$ and y by x

$\therefore f^{-1}(x) = \underline{\frac{3}{2x+5}}$

12. (a) Given that p = 7 and q = -3, find the value of $p^2 - q^3$.

(b) Simplify $\frac{a-2}{a^2-4}$ (c) Solve the equation 3(m – 5) = 7 –2(m – 3)

(d) Given that x = 8 and y = -2, find $x - y^2$

(e) Given that $a = \frac{3a+b}{b}$. (i) Express a in terms of b. (ii)Find a when b=2.

<u>SOLUTION</u>

(a) $p^2 - q^3$

$= 7^2 - (-3)^3$

$= 49 - 27$

$\underline{= 22}$

(b) $\frac{a-2}{a^2-4}$

$= \frac{a-2}{(a-2)(a+2)}$

$= \underline{\frac{1}{(a+2)}}$

(c) 3(m – 5) = 7 –2(m – 3)

3m + 2m = 7 + 15 + 6

5m = 28

$\underline{m = 5.6}$

(d) $x - y^2$

$= 8 - (-2)^2$

$= 8 - 4$

$\underline{= 4}$

(e) (i) $a = \frac{3a+b}{b}$

ab = 3a + b

ab - 3a = b

a(b – 3) = b

$a = \underline{\frac{b}{b-3}}$

(ii) $a = \frac{3a+b}{b}$.

$a = \frac{b}{b-3}$.

$a = \frac{2}{2-3}$

$\underline{a = -2}$

13. (a) V varies directly as the square of x and inversely as y. Given that V = 9 when X = 3 and y = 4. Find V when x = 5 and y = 2.

(b) Given that y+2 is inversely proportional to x and y = 7 when $x = \frac{1}{3}$.

Find the value of y when $x = \frac{1}{2}$.

(c) Given that y varies directly as the square root of x and y = 15.

(i) Write down an equation connecting x and y.

(ii) Find y when $x=\frac{1}{4}$.

(iii) Find x when y=80.

SOLUTION

(a) V $\alpha \frac{x^2}{y}$ $\Rightarrow$V = $\frac{kx^2}{y}$ k = $\frac{Vy}{x^2}$ $\Rightarrow$ k = $\frac{9\times4}{3\times3}$ k = 4

V = $\frac{4\times5^2}{2}$ $\Rightarrow$V = 2$\times$25 $\therefore$V = 50

(b) y+2 $\propto \frac{1}{x}$ $\Rightarrow$ y+2 = $\frac{k}{x}$ $\Rightarrow$ 7+2 = $\frac{k}{\frac{1}{3}}$ $\Rightarrow$k=9$\times\frac{1}{3}$ $\therefore$ k=3

y+2 = $\frac{3}{\frac{1}{2}}$ $\Rightarrow$ y+2 =3$\div\frac{1}{2}$ $\Rightarrow$y+2 =3$\times$2 $\Rightarrow$y+2 =6 $\therefore$ y=4.

(c) (i) y$\propto \sqrt{x}$ $\Rightarrow$ y=k$\sqrt{x}$ $\therefore$ y=5$\sqrt{x}$

(ii) y=k$\sqrt{x}$ $\Rightarrow$ y=5$\sqrt{\frac{1}{4}}$ $\Rightarrow$y=5$\times\frac{1}{2}$ $\therefore$ y=2$\frac{1}{2}$

(iii) y=k$\sqrt{x}$ $\Rightarrow$ 80=5$\sqrt{x}$ $\Rightarrow$ $\frac{5}{5}\sqrt{x}=\frac{80}{5}$ $\Rightarrow$ $\sqrt{x}$=16 $\Rightarrow$($\sqrt{x}$)^2=16^2 $\therefore$ x=256.

14. (a) After a long negotiation a shopkeeper reduced the price of the radio by 23%. The customer quickly paid K520,000 for the new price. What was the original price?

(b) Find 3$\frac{1}{2}$% of K50,720

SOLUTION

(a) Let the original price be x Kwacha

A reduction of 23% means the new price will be (100% -23%) of x Kwacha ie 77% of x Kwacha = K520,000

$\frac{77}{100}x$=520, 000 $\Rightarrow$77x=520, 000 x 100 $\Rightarrow$ $x=\frac{520,000\times100}{77}$ x=675, 324.68

Therefore, the original price was approximately K675, 324. 68

(b) 3$\frac{1}{2}$% of K50,720

(c) Ie $\frac{7}{200}$ x 50,720 $\Rightarrow$ $\frac{355040}{200}$ $\Rightarrow$1775.20 $\therefore$3$\frac{1}{2}$% of K50,720=K1775.20

15 A set P has 16 subsets. Find n(P)

SOLUTION

2^n = 16

2x2x2x2 = 16

2^4 = 16

$\therefore$n= 4

16. Use as much of the given information as is necessary to find the value of $\sqrt{0.013}$. ($\sqrt{13}$=3.61; $\sqrt{1.3}$=1.14)

SOLUTION

$\sqrt{0.013}=\sqrt{\frac{1.3}{100}}=\frac{1.14}{10}$ = 0.114

17. Find the integer n such that n + 3 < 11< n + 5

SOLUTION

$n + 3 < 11$ and $11 < n + 5$

$n < 11 - 3$ $\quad 11 - 5 < n$ or $n > 11 - 5$

$n < 8$ $\quad n > 6$

We write this as $6 < n < 8$

This means that the required integer lies between 6 and 8 and

Hence n = 7

18 (a) Simplify $\frac{3y^2-5y-12}{y^2-9}$

(b) Given the a = 2, b = -3 and c = 0. Find

(i) a – b (ii) $(b – a)^2$ (iii) $a^2 – abc$

(c) Evaluate $3^3 \times 27^{-1}$

(d) Given that $x * y = \frac{x}{y} + \frac{1}{2}$, Find the value of $\frac{3}{4} * 2$

SOLUTION

(a) $\frac{3y^2-5y-12}{y^2-9} \Rightarrow \frac{3y^2-9y+4y-12}{(y-3)(y+3)} \Rightarrow \frac{3y(y-3)+4(y-3)}{(y-3)(y+3)} \Rightarrow \frac{(3y+4)(y-3)}{(y-3)(y+3)} \therefore \frac{3y+4}{y+3}$

(b) (i) a – b $\quad$ (ii) $(b – a)^2$ $\quad$ (iii) $a^2 – abc$

=2-(-3) $\quad$ $=(-3-2)^2$ $\quad$ $= 2^2 - 2\times(-3)\times0$

= 2 + 3 $\quad$ $= (-5)^2$ $\quad$ = 4 - 0

= 5 $\quad$ = 25 $\quad$ = 4

(c) $3^3 \times 27^{-1} = 3^3 \times (3^3)^{-1} = 3^3 \times 3^{-3} = 3^{3+(-3)} = 3^0 = 1$

(d) $x * y = \frac{x}{y} + \frac{1}{2} = \frac{\frac{3}{4}}{2} + \frac{1}{2} = \frac{3}{4} \div \frac{2}{1} + \frac{1}{2} = \frac{3}{4} \times \frac{1}{2} + \frac{1}{2} = \frac{3}{8} + \frac{1}{2} = \frac{3+4}{8} = \frac{7}{8}$

19. Express 50cm to 2m as a ratio in its lowest terms.

SOLUTION

Expressing in cm we have 50cm to 200cm

Since ratio is a fraction (or number) we do not include units

ie 50: 200 = 1: 4

20. The number of orphans and vulnerable children in one province of Zambia is 599 900.

(a) Write down this number in standard form

(b) Express this number correct to 1 significant figure

SOLUTION

(a) In standard form $\quad$ (b) To one significant figure

599 900 $\quad$ 599 900

$= 5.999 \times 10^5$ $\quad$ = 600 000

21. A motorist left Kabompo at 22:00 hours and arrived in Mufumbwe after 2hours 30 minutes.
(a) Find the time at which he arrive in Mufumbwe
(b) If the motorist's average speed was 120Km/h. What is the distance between Kabompo and Mufumbwe?

SOLUTION
(a)

Departed	22: 00
	2: 30
Arrived	24: 30
Arrival time is	00: 30 hours

(b) S = $\frac{D}{T}$ $\Rightarrow$ D = S x T $\Rightarrow$ D = 120 x 2$\frac{1}{2}$ km $\Rightarrow$ D = 120 x $\frac{5}{2}$ km $\therefore$ D =300km

22. Given that h(x) = $\frac{3}{2x+1}$, find the value of
(a) x for which h is not a function
(b) h($^-$5) (c) x for which h(x) = $\frac{1}{x+1}$

SOLUTION
(a) When the denominator is zero h(x) is undefined ie does not exist
$\therefore$ h(x) is not a function when 2x+1= 0
2x+1= 0 $\Rightarrow$ 2x = -1 $\therefore$ x = -$\frac{1}{2}$
(b) h(x) = $\frac{3}{2x+1}$ $\Rightarrow$h($^-$5) = $\frac{3}{2(-5)+1}$ $\Rightarrow$h($^-$5) = $\frac{3}{-10+1}$ $\Rightarrow$h($^-$5) = $\frac{3}{-9}$ $\therefore$ h($^-$5) = $\frac{1}{-3}$
(c) If h(x) = $\frac{1}{x+1}$ then h(x) = $\frac{3}{2x+1}$ = $\frac{1}{x+1}$
$\Rightarrow$ $\frac{3}{2x+1}$ = $\frac{1}{x+1}$ $\Rightarrow$3(x+1) = 2x+1 $\Rightarrow$3x+3 = 2x+1 $\Rightarrow$3x-2x= 1-3 $\therefore$ x= -2

23. The results in Mathematics test were as follows;
2, 9, 3, 2, 4, 1, 1, 2, 8, 7, 10
Find (a) The mode of distribution (b) The median of the distribution

SOLUTION
First arrange the marks in ascending order
ie 1, 1, 2, 2, 2, 3, 4, 7, 8, 9, 10
(a) Mode is the most frequent(or common) mark

In this case, mode = 2

(b) Median is the middle mark after arranging in order
From 1, 1, 2, 2, 2, 3, 4, 7, 8, 9, 10
$\therefore$ Median = 3

24. In the year 2001 Thukuta Co-operative Union made a profit of K720 million from sales of its own grown Maize, poultry and piggery units. These contributed profit in the ratio of 2: 3: 4 respectively.
(a) Calculate how much profit came from the piggery unit
(b) If the total profit in 2001 was 25% more than in 2000, calculate the total profit in the year 2000.
(c) In 2002, the co-operative decided to expand and go into irrigation farming as well. It needed to spend K75 million for a bore hole, irrigation pipes and overheads. Given that it borrowed this sum from the Bank, at a simple interest of 40% for 9 months, calculate the total amount the co-operative paid to the Bank.
d) In 2003, the co-operative grew corn and green peas worth \$72, 000 on export market. Given that it paid Zambia Revenue Authority $17^1/_2$% VAT and the exchange rate was \$1 = K4 760. Calculate how much the co-operative earned in Zambian Kwacha.

<u>SOLUTION</u>

(a) The piggery unit produced $\frac{4}{9}$ of the total profits

ie $\frac{4}{9}$ X 720, 000, 000
= 4 X 80, 000, 000
= 320, 000, 000
$\therefore$ The piggery unit produced K320, 000, 000

(b) The profit generated in 2001 was 25% more than in 2000
Let the profit generated in 2000 be x million kwacha
$\Rightarrow$ the profit produced in 2001 was 125% of x million kwacha
ie x + 25%of x = 720, 000, 000

$\Rightarrow \frac{125x}{100} = 720,000,000 \Rightarrow x = \frac{720,000,000 \times 100}{125} \quad \therefore x = 576,000,000$

Therefore, the profit generated in 2000 was K 576, 000, 000

(c) Interest = $\frac{\text{Principal} \times Rate \times Time}{100}$

$\Rightarrow$ Interest = $\frac{75,000,000 \times 40 \times 9}{100 \times 12}$

$\therefore$ Interest = 22, 500, 000

Total Amount = Principal + Interest

Total Amount = 75, 000, 000 +22, 500, 000 = 97, 500, 000

ie the total amount required by the bank is K97, 500, 000

(d) The VAT was; | Income earned = Total sales - VAT

$17^1/_2\%$ x \$72 000 | $\Rightarrow$ Income earned = \$72, 000 - \$12, 600

$= \frac{17\frac{1}{2}}{100}$ x \$72 000 | $\Rightarrow$ Income earned = \$59, 400

$= \frac{35}{200}$ x \$72 000 | $\Rightarrow$ Income earned = K59, 400X4, 760

= \$12, 600 | $\therefore$ Income earned = K 282, 744, 000

25. (a) The scores in Mathematics results of pupils at Kyawama High School are as follows; 3, 7, 3, 2, 1, x, 2, 8, 5, 6. If the average was 4, find the mark represented by x.

(b) On a map of scale 1:10,000, a road is represented by 3.2cm.

(i) What is the actual length of road on the ground?

(ii) What will be the area on the map a field 17.8Km^2

(c) Write down the biggest and the smallest fractions from the Following; $\frac{5}{8}$, $\frac{7}{12}$ and $\frac{11}{16}$

SOLUTION

(a) $\frac{1+2+2+3+3+5+6+7+8+x}{10} = 4$

$\frac{37+x}{10} = 4$

$37 + x = 40$

$x = 40 - 37$

$\therefore x = 3$

(b) (i) 1cm $\rightarrow$ 10, 000cm

Distance on the map	Distance on the ground
1cm	10, 000cm = 100m = 0.1km
3.2cm	3.2x10, 000cm = 32, 000cm = 320m = 0.32km

(ii) Area: $(1\text{cm})^2 \rightarrow (10,000\text{cm})^2$

Area on the map	Distance on the ground
1cm^2	100, 000, 000cm^2 = 10, 000m^2 = 0.01km^2
1, 780 cm^2	178, 000, 000, 000cm^2 = 17, 800, 000m^2= 17.8km^2

(c) When expressed in decimal we get

$\frac{5}{8} = 0.625$ $\frac{7}{12} = 0.583$ $\frac{11}{16} = 0.6875$

$\therefore$ Biggest fraction = $\frac{11}{16}$ and smallest fraction = $\frac{7}{12}$

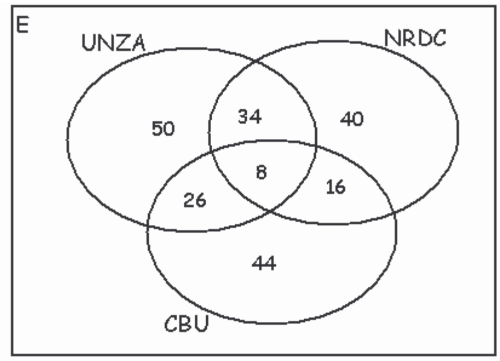
26. A survey carried out among school leavers in a certain town, involving three institutions, showed that 118 applied to the University of Zambia (UNZA), 98 applied to the Copperbelt University (CBU) and 94 applied to the National Resources Development College (NRDC). To increase the chances of selection, 42 applied to UNZA and CBU, 24 applied to CBU and NRDC, 34 applied to UNZA and NRDC and 8 applied to all the three institutions

(i)How this information on a Venn diagram.

(ii)Calculate the total number of school leavers who took part in the

SOLUTION

(i)



(ii)Total number of school leavers = 50 + 44 + 40 + 26 + 16 + 8 + 34

= 218

27. (a)In the year, 2002, Kyawama High School had 84% pass rate at Grade 12 level.

(i)Calculate the number of pupils that passed if 150 pupils sat for the examinations.

(ii)The ratio of the boys and girls that passed was 3:4 respectively. Calculate the percentage of the girls that passed the examinations in the year 2002 if 72% of the boys passed.

(b)Every year, the school enrolls the same number of pupils. Calculate the percentage pass if 45 pupils failed the examinations in 2003.

(c) In 2004, 9 pupils were accepted the University of Zambia. Given that this number is 10% of those that passed the examinations, how many pupils passed?

SOLUTION

(a) (i) Number passed = $\frac{84}{100}$ x 150 = 126

(ii) Number of boys passed = $\frac{3}{7}$ x 126 =54

Number of girls passed = $\frac{4}{7}$ x 126 =72

If 72% of the boys passed we first establish how many boys wrote the exam

Let x be the number of boys that wrote the exam

$\Rightarrow$ 72% of x = 54 ie $\frac{72x}{100}$ = 54 $\Rightarrow$ 72x = 5, 400 $\therefore$ x = 75.

75 boys wrote the exam and therefore 75 girls also wrote the exam

Percentage of the girls that passed = $\frac{72}{75} \times 100$ = 96%.

(b) The number of pupils that wrote the exam in 2003 was 150

If 45 failed then 105 passed

$\Rightarrow$ Percentage pass = $\frac{105}{150}$ x 100%

= 70%

(c) $\frac{10}{100}x = 9 \Rightarrow 10x = 900 \Rightarrow x = 90$ $\therefore$ 90 pupils passed in 2004

28. (a) Subtract $2x^2 + 3y^2 - z^2$ from $3x^2 - 7y^2 - 6z^2$

(b) The are 38 pupils in class. Each pupil had one vote (including the candidates) to choose a class monitor from the three candidates Trevor, Gerald and Sharon. Trevor gained twice as many votes as Gerald. Sharon got 5 votes. Find the number of votes Trevor received.

SOLUTION

(a) Subtracting $2x^2 + 3y^2 - z^2$ from $3x^2 - 7y^2 - 6z^2$

$3x^2 - 7y^2 - 6z^2$

$\underline{2x^2 - 3y^2 - z^2}$ (subtract)

$x^2 - 4y^2 - 5z^2$

(b) Sharon received 5 votes the remaining 33 votes had to be shared Between Trevor and Gerald

Remaining votes =38 - 5 = 33

Let x be the number of votes Gerald got

Number of votes Gerald got = x

Trevor got twice as many votes as Gerald = 2x

ie 2x + x = 33

3x =33 $\Rightarrow$ x = 11

ie Gerald received = 11 votes

$\therefore$ Trevor received = 11x 2 = 22 votes

29. (a) State the smallest integer X for which 3x > 28

(b) Given that X is an integer such that $45 < 5x - 11 \leq 54$

SOLUTION

(a) $3x > 28 \Rightarrow \frac{3x}{3} > \frac{28}{3}$ $\therefore x > 9.3$ The smallest integer is 9

(b) Given $45 < 5x - 11 \leq 54$

$5x-11 > 45 \Rightarrow 5x > 45 + 11 \Rightarrow 5x > 56$ ie $x > 11.2$

Also $5x - 11 \leq 54 \Rightarrow 5x - 11 \leq 54 + 11 \Rightarrow 5x \leq 65$ ie $x \leq 13$

$\Rightarrow 11.2 < x \leq 13$ The integer x is 12

30. Find the value of x given that $\begin{pmatrix} x & 2 \end{pmatrix}\begin{pmatrix} 3 \\ -5 \end{pmatrix} = 8$

SOLUTION

$\begin{pmatrix} x & 2 \end{pmatrix}\begin{pmatrix} 3 \\ -5 \end{pmatrix} = 8 \Rightarrow 3x - 10 = 8 \Rightarrow 3x = 8 + 10 \Rightarrow 3x = 18$ $\therefore x = 6$

31. P is the point (-4, 3) and 0 is the origin on the co-ordinates plane. Find the co-ordinates of the mid point of OP.

SOLUTION

Coordinates of the midpoint of OP $= \left(\frac{x_2+x_1}{2}, \frac{y_2+y_1}{2}\right)$

$= \left(\frac{-4+0}{2}, \frac{3+0}{2}\right)$

$= (-2, 1.5)$

32. (a) Express as a single matrix $\begin{pmatrix} 2 \\ -1 \end{pmatrix}\begin{pmatrix} 3 & 2 \end{pmatrix}$

(b) The determinant of the matrix $\begin{pmatrix} 2k & -4 \\ 3k & 1 \end{pmatrix}$ is 2. Find the value of K

SOLUTION

(a) $\begin{pmatrix} 2 \\ -1 \end{pmatrix}\begin{pmatrix} 3 & 2 \end{pmatrix} = \begin{pmatrix} 2(3) & 2(0) \\ -1(3) & -1(0) \end{pmatrix} = \begin{pmatrix} 6 & 0 \\ -3 & 0 \end{pmatrix}$

(b) $\det\begin{pmatrix} 2k & -4 \\ 3k & 1 \end{pmatrix} = 2 \Rightarrow 2k(1) - 3k(-4) = 2 \Rightarrow 2k + 12k = 2 \Rightarrow 14k = 2$ $\therefore k = \frac{1}{7}$

33. Chitanyika Simon and Chindi Brown of grade 12A class share an amount of K12, 700, 000 left by their deceased mother in the ratio 3:2. Find how much each one of them will get.

SOLUTION

SIMON	**BROWN**
$\frac{3}{5}$ X K12, 700, 000	$\frac{2}{5}$ X K12, 700, 000
=K $\frac{38,100,000}{5}$	=K $\frac{25,400,000}{5}$
= K7,620, 000	= K5, 080, 000

34. (a)Find the equation of the line of L passing through a point (0,5) whose gradient is 3.

(b) If B is a point (6, $^{-}3$) and C is ($^{-}2$, 1), find

(i) the gradient of line BC

(ii) the equation of line BC

SOLUTION

(a) $y - y_1 = m(x - x_1)$

$y - 5 = 2(x - 0)$

$y - 5 = 2x - 0$

$y = 2x + 5$

(b) If B is a point (6, $^{-}3$) and C is ($^{-}2$, 1), find

(i) the gradient of line BC

$m = \frac{y_2 - y_1}{x_2 - x_1} \Rightarrow m = \frac{1-(-3)}{-2-6} \Rightarrow m = \frac{4}{-8} \quad \therefore\ m = -\frac{1}{2}$

(ii) The equation of line BC

$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1} \Rightarrow \frac{y-(-3)}{x-6} = \frac{1-(-3)}{-2-6} \Rightarrow \frac{y+3}{x-6} = -\frac{1}{2} \quad \therefore y = -\frac{1}{2}x$

35. Simplify $\frac{3y^2 - 48}{y^2 - y - 20}$

SOLUTION

$\frac{3y^2 - 48}{y^2 - y - 20} = \frac{3(y^2 - 16)}{(y+4)(y-5)} = \frac{3(y+4)(y-4)}{(y+4)(y-5)} = \frac{3(y-4)}{(y-5)} = \frac{3y - 12}{y - 5}$

36. In a Mathematic test Moonga scores 36 marks out of 60.

(a) What percentage of the marks is Moonga's score?

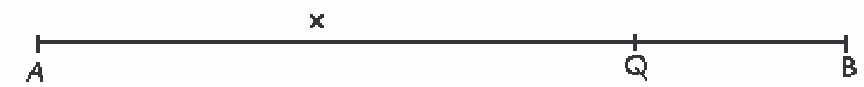
(b) If the percentage needed to pass the test is 45%, what is the pass mark.

SOLUTION

(a) $\frac{36}{60} \times 100\% = \frac{3600}{60}\% = 60\%$

(b) $\frac{45}{100} \times 60 = \frac{2,700}{100} = 27 \quad \therefore$ the pass mark = 27

37. In the diagram below, AB = 7cm and AQ = Xcm



(a) Express the length of QB in terms of x

(b) Find x if AQ = 3QB

SOLUTION

(a) $QB = (7 - x)cm$

(a) $AQ = 3QB$

$AQ = 3(7 - x)$

$x = 21 - 3x \Rightarrow x + 3x = 21 \Rightarrow 4x = 21 \therefore x = 5\frac{1}{4}$

38. A prize fund of K660 000 is divided as follows:
the first prize is half the fund
the second prize is two-third the first prize
the third prize is what remains
Calculate the actual prizes

SOLUTION

First prize

$\frac{1}{2} \times K660,000 = K330,000$

Second prize

$\frac{2}{3} \times K330,000 = K220,000$

Third prize

$K330,000 + K220,000 = K550,000 \therefore K660,000 - K550,000 = K110,000$

39. (a)For the sequence 7, 9, 11, 13, ____. Write down
(i)The eleventh term. (ii) the n^{th} term
(b) For the sequence 1, 4, 7, . . . Find the 20^{th} term
NOTE: a is the first term.

SOLUTION

(a) (i) $a_n = a + (n-1)d$

$a_{11} = 7 + (11-1)2$

$a_{11} = 7 + (10)2$

$a_{11} = 7 + 20$

$a_{11} = 27$

(ii) $a_n = 7 + (n - 1)2$

$a_n = 7 + 2n - 2$

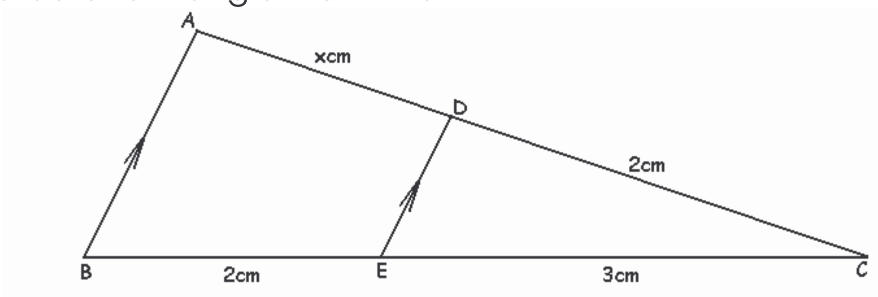
$a_n = 2n + 7 - 2$

$a_n = 2n - 5$

(b) $a=1, n=20, d=3$

$a_{20} = a + (n-1)d \Rightarrow a_{20} = 1+(20-1)3 \Rightarrow a_{20} = 1 + 19 \times 3 \therefore a_{20} = 58$

40. AB is parallel to DE, CD = 2cm, EC = 3cm and BE = 2cm, AD = x cm and area of a triangle DCE = $2cm^2$.



Calculate; (a) the ratio CD to DA. (b)the area of triangle ABC.

SOLUTION

(a) CE: CB = CD: CA

$\frac{3}{5} = \frac{2}{x+2}$ $\Rightarrow 3(x+2) = 10 \Rightarrow 3x+6 = 10 \Rightarrow 3x = 10 - 6 \Rightarrow 3x = 4$ $\therefore x = 1\frac{1}{3}$

(b) area ΔCDE: area ΔABC = 3^2: 5^2

$\frac{\text{area } \Delta CDE}{\text{area } \Delta ABC} = \frac{9}{25}$

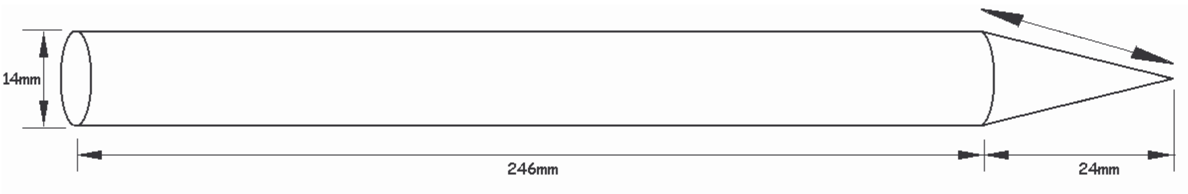
$\frac{2}{\text{area } \Delta ABC} = \frac{9}{25}$

9 area ΔABC= 50

area ΔABC= $\frac{50}{9}$

$\therefore$ area ΔABC=$5\frac{5}{9}$ cm^2.

41. The diagram below shows a carpenter's pencil with diameter 14mm and a length of 270mm. The sharpened end is in form of a cone of height 24mm (Take $\pi = \frac{22}{7}$).



(a) Calculate

(i) the slant length (ii) of the conical part.

(iii)The total surface area of the pencil.

(b) Given that $1mm^3$ weighs 3mg (assuming that density of wood and lead are the same), find

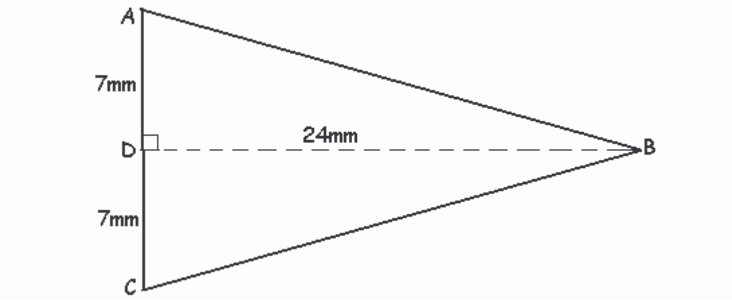
(i) The volume of the pencil before it was sharpened

(ii) The mass of the pencil

(d) The mass of the pencil that was removed after the pencil was sharpened.

SOLUTION

(a) (i) Using the cross-section of the cone yields an isosceles triangle of height 24mm



Let AB be the slant height and using Pythagoras Theorem

$AB^2 = AD^2 + BD^2 \Rightarrow AB^2 = 7^2 + 24^2 \Rightarrow AB^2 = 49 + 576 \Rightarrow AB^2 = 625 \therefore AB = 25$

$\therefore$ the slant height = 25mm.

(ii)The total surface area of the pencil

(Area of curved surface of cone = π r l)

$A = \pi r^2 + 2\pi r h + \pi r l$

$A = \frac{22}{7}\times 7^2 + 2\times\frac{22}{7}\times 7\times 246 + \frac{22}{7}\times 7\times 25$

$A = \frac{22}{7}\times 49 + 2\times\frac{22}{7}\times 7\times 246 + \frac{22}{7}\times 7\times 25$

$A = 154mm^2 + 10,824mm^2 + 550mm^2$

$A = 11,528mm^2$

(b) (i) Volume of the pencil before it was sharpened is;

$V = \pi r^2 h$

$V = \frac{22}{7}\times 7^2 \times 270mm^3$

$V = 154\times 270mm^3$

$V = 41,580mm^3$

(ii)$1mm^3$ weighs 3mg

$\Rightarrow 41,580mm^3$ weighs $3\times 41,580mg = 124,740mg$

$\therefore$ Mass of the pencil before it was sharpened = 124, 740mg

(c) The mass of the pencil that was removed after it was sharpened

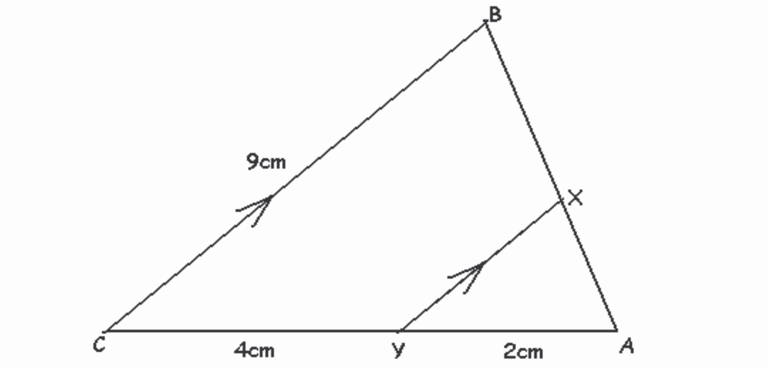
$= 3\times(\pi r^2 h - \frac{1}{3}\pi r^2 h) = 3\times(\frac{22}{7}\times 7^2 \times 24 - \frac{1}{3}\times\frac{22}{7}\times 7^2 \times 24)mg.$

$= 3\times(3,696 - 1,232)mg$

$= 3\times 2,462mg$

$= 7,392mg$

42. In the diagram, CB is parallel to YX, AY= 2cm, YC=4cm and BC = 9cm.



(i) Write the angle which correspond to AXY.
(ii)Find the length of xy
(iii)Given that the area of quadrilateral XYCD is $48cm^2$,
find the area of triangle AXY.

<u>SOLUTION</u>

(i) The angle which correspond to AXY = ABC

(ii) XY: BC = AY: AC $\Rightarrow \frac{XY}{9} = \frac{2}{6} \Rightarrow$ 6XY = 18 $\Rightarrow$ XY = $\frac{18}{6}$ $\therefore$ XY = 3cm

(iii) Ratio of sides = 1: 3
ratio of areas = $1^2 : 3^2$ = 1: 9
area ΔAXY: area ΔABC = 1: 9
area ΔAXY: area ΔAXY + 48 = 1: 9

$$\frac{\text{area AXY}}{\text{area AXY} + 48} = \frac{1}{9}$$

$\Rightarrow$ 9 area ΔAXY = area ΔAXY+ 48
$\Rightarrow$ 9 area ΔAXY - area ΔAXY= 48
$\Rightarrow$ 8 area ΔAXY = 48
$\Rightarrow$ area ΔAXY = $\frac{48}{8}$
$\therefore$ area ΔAXY = $6cm^2$

43. A map is drawn to a scale of 1: 50 000
(a) On the map, the distance of a road is represented by 9cm. Calculate the actual distance.

(b) The actual area of a game reserve is 30Km2. Calculate the area on the map reserve in square centimetres.

SOLUTION

(a) 1cm $\rightarrow$ 50, 000cm

Distance on the map	Distance on the ground
1cm	50, 000cm = 500m = 0.5km
9cm	450, 000cm =4, 500m = 4.5km

(b) Area: $(1cm)^2 \rightarrow (10, 000cm)^2$

Area on the map	Distance on the ground
$1cm^2$	$2, 500, 000, 000cm^2 = 250, 000m^2 = 0.25km^2$
$120\ cm^2$	$300, 000, 000, 000cm^2 = 30, 000, 000m^2 = 30km^2$

44. Given that P= $^-2$, q = 8 and r = 12, find; (a) r – q (b) q^2 - pr (c) $\sqrt[3]{q}$

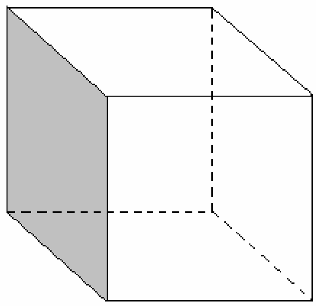
SOLUTION

(a) r – q
= 12 - 8
= 4

(b) q^2 - pr
= $8^2 - (^-2)\times 12$
= 64 + 24
= 88

(c) $\sqrt[3]{q}$
= $\sqrt[3]{8}$
= 2

45. The diagram below shows a solid metal cube of volume $125cm^3$. Given that $1cm^3$ of the metal has mass of 9 grams.



Calculate:-

(a) the length of the edge of the cube.

(b) the mass of the cube giving your answer in kilograms.

(c) the total surface area of the cube.

SOLUTION

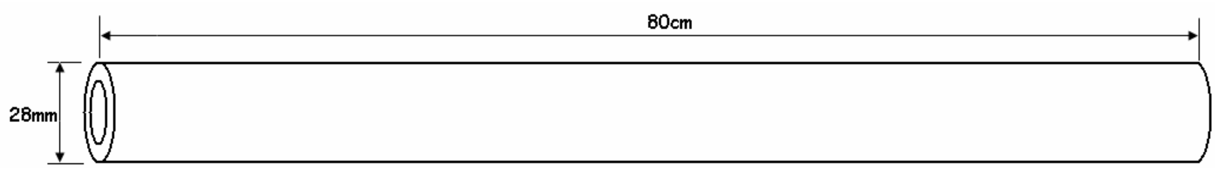
(a) $V = L^3 \Rightarrow 125 = L^3 \Rightarrow \sqrt[3]{L^3} = \sqrt[3]{125} \Rightarrow L = 5$

(b) $1cm^3 \rightarrow 9g$

$125cm^3$ has a mass of $9\times125g$ = 1, 125g = 1.25kg

(c) Total surface area
= Area of one face$\times$6
= $5\times5\times6cm^2$
= $150cm^2$

46. A cylindrical pipe is 80cm long. The external diameter is 28mm. Taking π to be $\frac{22}{7}$.



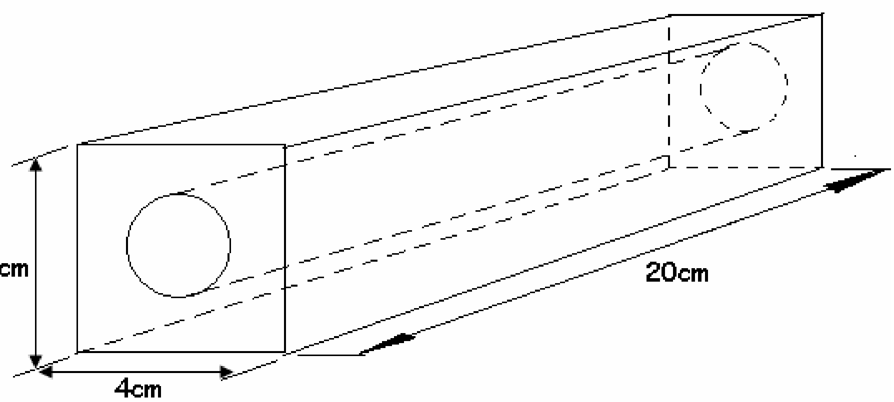
Find (a) The area of the external curved surface area of the pipe.
(b) The volume of the metal used in making a pipe given that the internal diameter is 14mm.

SOLUTION

(a) Area
$= 2\pi r h$
$= 2\times\frac{22}{7}\times 1.4 \times 80cm^2$
$= 704\ cm^2$

(b) Volume = $\pi(R^2 - r^2)h$
$= \frac{22}{7}(1.4^2 - 0.7^2)\times 80\ cm^3$
$= \frac{22}{7}(1.96 - 0.49)\times 80\ cm^3$
$= \frac{22}{7}\times 1.47\times 80\ cm^3 = 369.6\ cm^3$

47. A metal work student bored cylindrical hole from one end through a rectangular bar as shown in the figure below.



(a) Find the volume of the metal bar before the whole was made.
(b) Given that $1cm^3$ of the metal has a mass of 7.85g, find the mass of the metal bar before the hole was made.
(c)If the mass of the hollow metal bar is 2, 315g, calculate the volume of the metal removed giving your answer correct to two decimal places.
(d) Find the radius of the hole, correct to 3 significant figures.

SOLUTION

(a) $V = L^2 \times H \Rightarrow V = 4^2 x\ 20cm^3 \Rightarrow V = 16x\ 20cm^3 \therefore V = 320cm^3$

(b) $1cm^3 \rightarrow 7.85g$
$320cm^3 \rightarrow 320 \times 7.85g = 2,512g$

(c) $1cm^3 \rightarrow 7.85g$

$x \rightarrow 2,315g$

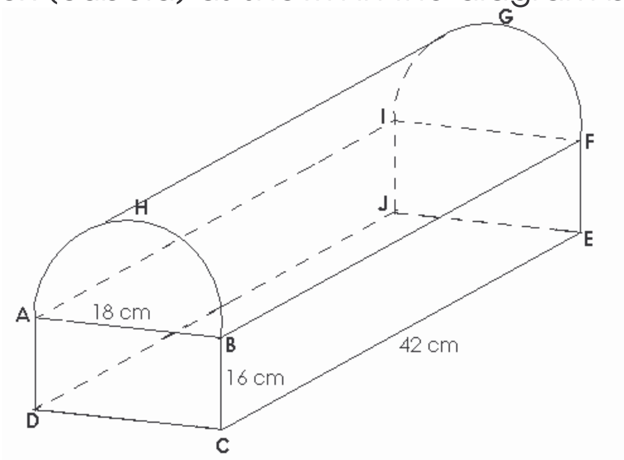
$1 : x = 7.85 : 2,315$

$\frac{1}{x} = \frac{7.85}{2,315} \Rightarrow 7.85x = 2,315 \Rightarrow x = \frac{2,315}{7.85} \quad \therefore x = 294.90cm^3$

(d) $V = \pi r^2 h \quad 294.9 = 3.142 \times r^2 \times 20$

$62.84r^2 = 294.9 \Rightarrow r^2 = \frac{294.9}{62.84} \Rightarrow r^2 = 4.692 \Rightarrow r = \sqrt{4.692} \quad \therefore r = 2.17$ cm

48. The cover of a sewing machine is made up of half a cylinder (curved part) and a box (cuboid) as shown in the diagram below.



Given that AHB is a semi-circle. AB = 18cm, BC = 16cm and CE = 42cm and π = 3.142. Calculate.

(a) The curved surface area of the cover.

(b) The surface area of the cover.

(c) The volume of the cover of the sewing machine.

SOLUTION

(a) $A = \frac{1}{2}\pi rh = \frac{1}{2} \times 3.142 \times 9 \times 42 = \frac{1}{2} \times 3.142 \times 9 \times 42 = 593.8cm^2$

(b) $A = \frac{1}{2}\pi rh + 2x\frac{1}{2} x \pi r^2 + 2(LxH) + 2(BxH)$

$A = \frac{1}{2} x\ 3.142x9x42 + 2x\frac{1}{2} x\ 3.142\ x9^2 + 2(42x16) + 2(18\ x16)$

$A = 593.8cm^2 + 254.5cm^2 + 1,344cm^2 + 576\ cm^2 \quad \therefore A = 2768.3\ cm^2$

(c) V = base area x height

$V = (\frac{1}{2} x \pi r^2 + Bxh) \times H \Rightarrow V = (127.25 + 288) \times 42cm^3 \therefore V = 1,7440.5cm^3$

GOOD LUCK MAY GOD BLESS YOU AS YOU STUDY THIS MATERIAL AND LOOK OUT FOR VOLUME 3.

MENSURATION FORMULAS ONLY:

1. Rectangle: Area = length x breadth, $(A = l \times b)$
2. Square: Area = length x length, $(A = l^2)$
3. Triangle: Area = $\frac{1}{2}$ base x height, $(A = \frac{1}{2}bh)$
4. Parallelogram: Area = base x perpendicular height, $(A = bh)$
5. Trapezium: $A = \frac{1}{2}h(a + b)$
6. Circle: Circumference $= 2\pi r$ $(C = 2\pi r)$
7. Circle: Area $= \pi r^2$ $(A = \pi r^2)$
8. Area of a sector: $A = \frac{x}{360}\pi r^2$ $(A = \frac{x}{360}\pi r^2)$
9. Area of a segment = Area of a sector - Area of a triangle
   i.e $(A = \frac{x}{360}\pi r^2 - \frac{1}{2}bh)$

PRISMS

10. Area of a cube = 6(L x L) (A = 6L2)
11. Volume of a cube: V = L x L x L $(V = L^3)$
12. Area of a cuboids:
    A = 2(length x breadth) + 2(length x height) + 2(breadth x height)
    A = 2lb + 2lh + 2bh
13. Volume of a cuboids: V = length x breadth x height, $(V = lbh)$

TRIANGULAR PRISM.

14. Volume of a triangular prism = $\frac{1}{2}$ base area x height, $(V = \frac{1}{2}bhH)$

CYLINDER

15. Curved surface area = $2\pi rh$ $(A = 2\pi rh)$
16. Volume cylinder = πr^2h, $(V = \pi r^2h)$

PYRAMIND

17. Volume of pyramid: V = $\frac{1}{3}$ base area x height, $(V = \frac{1}{3}Ah)$

CONE

18. Curved surface area = πrl $(A = \pi r l)$
19. Volume of a sphere = $\frac{1}{3}$ base area x height, $(V = \frac{1}{3}Ah)$

SPHERE

20. Surface area of a sphere = $4\pi r^2$ $(V = 4\pi r^2)$
21. Volume of a sphere: $V = {}^4/_3 \pi r^2$

FRUSTUMS:

22. Volume of a frustums = Volume of a full cone - volume of a small cone
    $(V = \frac{1}{3}\pi R^2H - \frac{1}{3}\pi r^2h)$
    Or $V = \frac{1}{3}\pi(R^2H - r^2h)$