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Maths
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"If education is not an emergency in your life then hope is dead"

LESSON 1 QUESTION 1 P1 2016 PAGE 1

Evaluate $8^{\frac{2}{3}}$

SOLUTIONS

Evaluate means find the value of

Then $8^{\frac{2}{3}}$ means 8 to the power $\frac{2}{3}$

So this is coming from indices, Then we can write 8 in index form and it will be 2^3 because $2^3 = 2 \times 2 \times 2 = 8$,

So where there is 8 we write 2^3

So we have

$$8^{\frac{2}{3}} = (2^3)^{\frac{2}{3}}$$

When we multiply the power inside and the power outside.

3 in side will cancel with 3 outside so we have 2^2

$$(2^3)^{\frac{2}{3}} = 2^2 = 2 \times 2 = 4$$

And the answer is 4

LESSON 1 QUESTION 2 P1 2016 (page 2)

Simplify $3(a + 5) - a(a - 2)$

SOLUTIONS

HINT; to simplify is to reduce; in this case we have to expand first by opening the brackets by multiplying the numbers and letters outside by numbers and letters inside brackets so we have

$$(3 \times a) + (3 \times 5) + (-a \times a) + (-a \times -2)$$

We have

$3a + 15 + \ -a^2 + 2a$ remember $+ \times - = -$ so we have

$$3a + 15 - a^2 + 2a$$

Collecting like terms we have

$$3a + 15 + \ -a^2 + 2a$$

$$3a + 2a + 15 - a^2$$

There are only two like terms $3a + 2a$ so we add them

And the answer is $5a + 15 - a^2$

LESSON 1 GERNERAL QUESTION page 3

A car travels 320km on 20litres of petro, how far can it go on 15litres of petrol.

SOLUTION

$$320km = 20litres$$

$$xkm = 15litres$$

So we have

$$320 = 20$$

$$x = 15$$

We can now cross multiply,

$$320 = 20$$

$$x = 15$$

We have $320 \ \times \ 15 = 20x$

Dividing both sides by 20 we have

$$4800 = 20x$$

$$\frac{4800}{20}=\frac{20x}{20} \therefore x = 240$$

And the answer is 240km

LESSON 1 QUESTION P1 2016 page 4

Express $\begin{bmatrix} 1 & 2 & -3 \end{bmatrix}\begin{bmatrix} 5 & -2 \\ 4 & 2 \\ 3 & 1 \end{bmatrix}$ as a single matrix

SOLUTION

The first matrix is a row matrix while the second matrix is a 3 x 2 matrix. The row has to multiply the first column to get the first term in the answer, then it will multiply the second column to get the second term in the answer as follows;

$$\begin{bmatrix} 1 & 2 & -3 \end{bmatrix}\begin{bmatrix} 5 & -2 \\ 4 & 2 \\ 3 & 1 \end{bmatrix}$$

$$=[(1 \times 5) + (2 \times 4) + (-3 \times 3) \qquad (1 \times -2) + (2 \times 2) + (-3 \times 1)]$$

Multiplying what is inside brackets we have;

$$[5 + 8 - 9 \qquad -2 + 4 - 3]$$

Adding the numbers we have the answer

$$\begin{bmatrix} 4 & -1 \end{bmatrix}$$

LESSON 2 QUESTION P1 2017 page 1

The first and second terms of an arithmetic progression are 100 and 95, respectively. Find; a. Tenth term

b. Sum of the first tern terms, $[S_n=\frac{n}{2}(2a + (n-1)d)]$

SOLUTIONS a page 2

This question is coming from series and sequences.

a. To answer a. we use the formula $T_n = a + (n-1)d$ where

a is the first term, n is the number of terms and d is the common difference or difference between the second and the first term.

So $a = 100$, $n = 10$ lets find d, we said d is the difference between 95 and 100 so $d = 95 - 100 = -5$

Then we can substitute in the formula $T_n = a + (n-1)d$

We have $T_{10} = 100 + (10-1)(-5)$

Subtracting 1 from 10 we have

$$T_{10} = 100 + (9)(-5)$$

Multiplying $9\ and - 5$ we have $T_{10} = 100 + -45$

Remember positive times negative is negative so we have

$$T_{10} = 100 - 45 = 55$$

$$\therefore T_{10} = 55$$

SOLUTIONS b page 3

For b, we just substitute in the formula $[S_n = \frac{n}{2}(2a + (n-1)d)]$

$$S_{10} = \frac{10}{2}[2(100) + (10-1)(-5)]$$

Divide 10 by 2 outside, multiply 2 by 100 inside, subtract 1 from 10 we have

$$S_{10} = 5[200 + (9)(-5)]$$

Multiplying 9 and -5 we have

$$S_{10} = 5[200 - 45]$$

Subtracting inside brackets we have

$$S_{10} = 5[155]$$

$\therefore S_{10}=775$

LESSON 2 Question p1 2016 page 4

There are 45 green and red marbles in a bag. Given that the probability of choosing a green marble is $\frac{2}{5}$, calculate the number of green marbles in the bag.

SOLUTIONS page 5

The question is coming from probability its simple just multiply the probability by the total number of marbles as follows;

Total number of marbles $\times$ probability of picking a green marble

Total number of marbles is 45 and the probability of choosing a green marble is $\frac{2}{5}$

So we have

$$\frac{2}{5} \times 45 = 2 \times 9$$

$$\therefore \textit{number of green marbles} = 18$$

LESSON 2 QUESTION P1 2016 page 6

The base areas of two containers that are geometrically similar are $80cm^2$ and $180cm^2$, respectively. If the capacity of the larger container is 54litres. Calculate the capacity of the smaller on.

SOLUTIONS

This comes from similar triangles but therefore we will use proportion and ratio method as follows;

$180cm^2$ = 54litres

$80cm^2$ = x

Then we have

180 = 54

80 = x

We can now cross multiply

180 = 54

80 = x

Then we have

$$180x = 4320$$ **then divide both sides by 180**

$$\frac{180x}{180} = \frac{4320}{180}$$

$$\therefore x = 24litres$$

LESSON 3 QUESTION P1 2016 page 1

The length of a wire is 5.2cm, correct to one decimal place. What is the maximum possible length?

SOLUTION

The question in under approximations from the new syllabus, they are just asking us to round off to the nearest tenth. Since the number after the point is less than five (5) then we will just ignore 2. If the number after the point is 5 or more then we round off by adding one (1) to 5.

$$\therefore \textit{the maximum length is } 5cm$$

$$\therefore x = 24litres$$

LESSON 3 QUESTION P1 2016 page 2

Find the gradient of the straight line whose equation is $3y - x = 5$

SOLUTION

This is from coordinate geometry; they are asking us to make y the subject of the formula so that we obtain an equation of the form;

$$y = mx + c$$

Then the coefficient of$x$ corresponding to m is the gradient.

$$3y - x = 5$$

$-x$ Will be crossed over the equal sign and it becomes$+x$

$$3y = 5 + x$$

Divide throughout by 3 to remain with y as the subject of the formula

$$\frac{3y}{3} = \frac{5}{3} + \frac{1}{3}x$$

$$\therefore y = \frac{1}{3}x + \frac{5}{3}$$

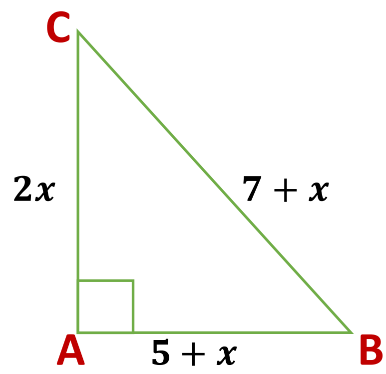
Gradient is the coefficient of x when y is the subject of the formula

$$\therefore gradient = \frac{1}{3}$$

$$\therefore x = 24litres$$

LESSON 3 QUESTION 9 P2 1990 page 3

In the right angled-triangle ABC below find the value of x in cm.



HINT

This question is coming from a grade ten topic called Pythagoras theorem then it is integrated into quadratic equation evaluation. We can use Pythagoras theorem, a formula which states that; the square of the longest side is equal to the sum of the squares of the two shortest sides of a right angled triangle. BC is the longest side then AB and AC are the two shortest sides, therefore the formula is as follows;

SOLUTION

$$BC^2 = AB^2 + AC^2$$

Then we just have to substitute with what is written on sides in the triangle

$$(7 + x)^2 = (5 + x)^2 + (2x)^2$$

Then we have to expand, the have

$$(7 + x)(7 + x) = (5 + x)(5 + x) + (2x)(2x)$$

Then opening the brackets by multiplying we have

$$7 \times 7 + 7 \times x + x \times 7 + x \times x = 5 \times 5 + 5 \times x + x \times 5 + x \times x + 4x^2$$

$$49 + 7x + 7x + x^2 = 25 + 5x + 5x + x^2 + 4x^2$$

Adding like terms on each side we have

$$49 + 14x + x^2 = 25 + 10x + 5x^2$$

Collecting like terms we have

$$49 - 25 + 14x - 10x + x^2 - 5x^2 = 0$$

Adding and subtracting

$$24 + 4x - 4x^2 = 0$$

Rewriting the equation we have

$$4x^2 - 4x - 24 = 0$$

HINT

This is now a quadratic equation. We can find the value of x by using the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Page 4

A quadratic equation is an equation of the form $ax^2 + bx + c = 0$

In the quadratic equation the number in front of x^2 is equal to a, the number in front of x is equal to b and the number without a letter is equal to c.

$$4x^2 - 4x - 24 = 0$$

$$ax^2 + bx + c = 0$$

Then we can substitute into the formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

We have

$$x = \frac{-(-4) \pm \sqrt{(-4^2) - 4(4)(-24)}}{2(4)}$$

$$x = \frac{4 \pm \sqrt{16 + 384}}{8}$$

HINT

First find the square root

$$x = \frac{4 \pm \sqrt{400}}{8}$$

HINT

We have two mathematical operations + and - at the same point hence we have to add and also subtract

$$x = \frac{4 \pm 20}{8}$$

Subtraction

Addition

$$x = \frac{4 + 20}{8} \ and \ x = \frac{4 - 20}{8}$$

$$x = \frac{24}{8} \text{ and } x = \frac{-16}{8}$$

$$\therefore x = 3cm \text{ and } x = -2cm$$

However the most correct answer must be a positive answer because length cannot be negative $\therefore x = 3cm$

LESSON 4 QUESTIONP2 2008 page 1

Solve the equation $5x^2 - 2x - 1 = 0$ giving the answer correct to two decimal places

Hint; this is a quadratic equation and we have to use the quadratic formula in order to find the values of x the formula is

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Hint; The arrows show that;

$a = 5,$

$b = -2$

$c = -1$

SOLUTION

$$ax^2 + bx + c = 0$$

$$5x^2 - 2x - 1 = 0$$

We can now substitute

Into the formula

$- \times - = +$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(5)(-1)}}{2(5)}$$

HINT

First find the square root

$$x = \frac{2 \pm \sqrt{4 + 20}}{10}$$

$$x = \frac{2 \pm \sqrt{24}}{10}$$

HINT; we have two mathematical operations $+$ and $-$ at the same point hence we have to add and also subtract

$$x = \frac{2 \pm 4.899}{10}$$

$$x = \frac{2 + 4.899}{10} \text{ and } x = \frac{2 - 4.899}{10}$$

$$x = \frac{6.899}{10} \text{ and } x = \frac{-2.899}{10}$$

$$\therefore x = 0.69 \text{ and } x = -0.29$$

LESSON 4 QUESTIONP1 2003 page 2

a. Given that $6x = 15y$, state the ratio $x : y$

b. Given that $\frac{1}{m} = 0.0125$, find the value of m

SOLUTIONS

HINT: dividing both sides by 3 to reduce the numbers because 3 can go into both 6 and 15

$$6x = 15y$$

$$\frac{\cancel{6}x}{\cancel{3}} = \frac{\cancel{15}y}{\cancel{3}}$$

HINT: make x the subject of the formula by dividing both sides by 2, then get $2x = 5y$ and make y the subject of the formula by dividing by 5.

$$2x = 5y$$

$$\frac{\cancel{2}x}{\cancel{2}} = \frac{5y}{2}$$

$$\therefore x = \frac{5y}{2} \text{ (Formula for } x\text{)}$$

$$2x = 5y$$

$$\frac{2x}{5} = \frac{\cancel{5}y}{\cancel{5}}$$

$$\therefore y = \frac{2x}{5} \text{ (Formula for } y\text{)}$$

The ratio is $x : y$ so where there is x we write the formula for x and where there is y we write the formula for y as follows;

$$\therefore x : y = \frac{5y}{2} : \frac{2x}{5}$$

LESSON 4 QUESTION, P1, 2003 page 3

c. Given that $\frac{1}{m} = 0.0125$, find the value of m

HINT: we have to first to make m the subject of the formula by cross multiplication. Introduce 1 as a denominator of 0.0125

SOLUTIONS

$$\frac{1}{m} = 0.0125$$

$$\frac{1}{m} = \frac{0.0125}{1}$$

$$0.0125m = 1$$

Dividing both sides by 0.0125 we have

$$\frac{0.0125m}{0.0125} = \frac{1}{0.0125}$$

$$\therefore m = 80$$

LESSON NUMBER 5 QUESTION, P1, 2003 page 1

Simplify $\frac{9a-15}{9a^2-25}$

HINT: we can write the numerator as $3(3a-5)$ by factorizing 3.

We can also write the denominator $(3a)^2 - 5^2$ let's substitute into the expression.

SOLUTIONS

$$\frac{9a-15}{9a^2-25}$$

$$\frac{9a-15}{9a^2-25} = \frac{3(3a-5)}{(3a)^2-5^2}$$

$(3a)^2 - 5^2$ Is a difference of two squares hence can be factorized as;

$$(3a-5)(3a+5)$$

$$= \frac{3\cancel{(3a-5)}}{\cancel{(3a-5)}(3a+5)}$$

We can cancel $(3a-5)$ in the numerator and $(3a-5)$ in the denominator

$\therefore$Answer = $\frac{3}{(3a+5)}$

LESSON 5 QUESTION, P1, 2003 page 2

Find the value of $(p+q)^2$ given that $p^2+q^2 = 36$ and $pq = 7$

HINTS;

1. First we expand $(p+q)^2$
2. We can now rearrange $p^2+2pq+q^2$ by putting the squared together
3. We know that $p^2+q^2 = 36$ and $pq = 7$ according to the question so we substitute.

SOLUTIONS

$$(p+q)^2 = (p+q)(p+q)$$
$$= (p \times p) + (p \times q) + (q \times p) + (q \times q)$$
$$= p^2 + pq + qp + q^2$$
$$\therefore (p+q)2 = p^2 + 2pq + q^2$$
$$= p^2 + q^2 + 2pq$$
$$p^2 + q^2 + 2pq = 36 + 2(7)$$
$$= 36 + 14$$
$$\therefore (p+q)^2 = 50$$

LESSON 5 QUESTION, P$_2$, 2013 page 3

Express $\frac{2a}{x-1} - \frac{a}{x-2}$ as a fraction in its simplest form

HINTS;

1. First find the common denominator by multiplying the two denominators. The denominator is $(x-1)(x-2)$
2. Divide $(x-1)$ into $(x-1)(x-2)$ the answer is $(x-2)$ then multiply this answer by $2a$.
3. Divide $(x-2)$ into $(x-1)(x-2)$ the answer is $(x-1)$ then multiply this answer by a.
4. Open the brackets by multiplying by the terms outside brackets in the numerator
5. Collecting like terms in the numerator we have
6. Subtracting and adding in the numerator we have
7. Factorizing a in the numerator we have

SOLUTIONS

$$\frac{\square}{(x-1)(x-2)}$$
$$\frac{2a(x-2)-\dots}{(x-1)(x-2)}$$
$$\frac{2a(x-2) - a(x-1)}{(x-1)(x-2)}$$
$$\frac{2ax - 4a - ax + a)}{(x-1)(x-2)}$$
$$\frac{2ax - ax - 4a + a)}{(x-1)(x-2)}$$
$$\frac{ax - 3a}{(x-1)(x-2)}$$
$$\therefore answer = \frac{a(x-3)}{(x-1)(x-2)}$$

LESSON NUMBER 5 QUESTION, P$_2$, 2013 PAGE 4

Evaluate $2\frac{5}{8} - 2\frac{1}{6} \div 1\frac{1}{12}$

HINTS;

1. Change all mixed fractions to improper fraction by multiplying the denominator by the whole number then add the numerator to get each numerator for the improper fraction e.g. in the fraction in $2\frac{5}{8}$ we multiply 8×2 then add 5 to get 21 the numerator for the improper fraction so we have $\frac{21}{8}$
2. We divide first then subtract, so the $\div$ changes to $\times$ and the fraction $\frac{13}{12}$ is inversed, the numerator becomes the denominator while the denominator becomes the numerator we have
3. Find the common denominator and it is **8** then divide **8** into **8** and multiply the answer by **21**, do the same for the other fraction and then subtract

SOLUTIONS

$$= \frac{21}{8} - \frac{13}{6} \div \frac{13}{12}$$

$$\frac{21}{8} - \left(\frac{13}{6} \div \frac{13}{12}\right) = \frac{21}{8} - \left(\frac{13}{6} \times \frac{12}{13}\right)$$

$$= \frac{21}{8} - \left(\frac{\cancel{13}}{\cancel{6}} \times \frac{\cancel{12}}{\cancel{13}}\right)$$

$$= \frac{21}{8} - \left(\frac{1}{1} \times \frac{2}{1}\right)$$

$$= \frac{21}{8} - \frac{2}{1}$$

$$= \frac{21 - 16}{8}$$

$$\therefore answer = \frac{5}{8}$$

LESSON NUMBER 6 QUESTION, P_2, 2013 (p1)

Express 0.041 as a percentage

HINTS;

1. First change 0.041 to a fraction
2. Number of decimal places in 0.041 gives us 1000
3. Percentage means out of 100 multiplying the fraction $\frac{41}{1000}$ by 100

SOLUTIONS

$$0.041 = 0 + \frac{41}{1000}$$

$$= \frac{41}{1000}$$

$$\text{Percentage} = \frac{41}{1000} \times 100$$

$$= \frac{41}{10}$$

$$\therefore \mathit{answer} = 4.1\%$$

LESSON NUMBER 6 QUESTION, P_2, 2013 (p2)

Factorise completely $3y^2 - 12$

HINTS;

1. First of all 3 can go into $3y^2$ and -12 without leaving a remainder so we factorize 3
2. The number 4 can be written in index form as 2^2 so we write that in place of 4
3. Inside the brackets we have a difference of two squares so we factorize

SOLUTIONS

$$3y^2 - 12 = 3(y^2 - 4)$$

$$= 3(y^2 - 2^2)$$

$$\therefore 3y^2 - 12 = 3[(y-2)(y+2)]$$

LESSON 6 QUESTION, P_2, 2013 (p3)

Mr. Chagwa sold a pair of shoes at K220.00, he made a profit of 10%. Calculate the cost of the shoes.

HINTS;

1. **Cost is the money spent on buying shoes before selling them**
2. **First calculate profit or 10% of the sales we know that 10% means 10 over 100**
3. **Sales = profit + cost = K220**
4. **To calculate the cost we subtract profit from the sales**

SOLUTIONS

$$10\% = \frac{10}{100}$$

$$profit = 10\%\ of\ the\ sales$$

$$= 10\% \times 220$$

$$= \frac{10}{100} \times 220 = \frac{10}{10} \times 22$$

$$= 1 \times 22$$

$$\therefore profit = K22$$

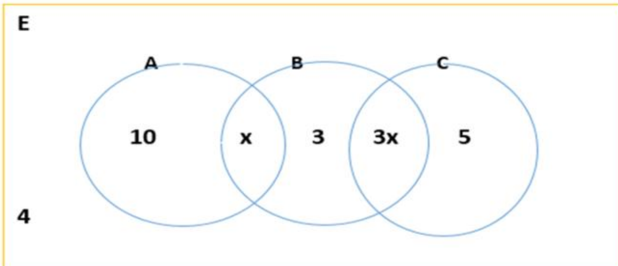
$$Cost = Sales - profit$$

$$Cost = K220 - K22$$

$$\therefore Cost = K198$$

LESSON 7 QUESTION, P_2, 2013 (page 1)

The diagram below shows three A, B and C. Given that (A∪B∪C)=50 find



i. the value of x
ii. n(A∪B)
iii. n(B∪C)'
iv. n(A'∩C')

HINTS;

1. **(A∪B∪C)** means elements in set A plus elements in set B plus elements in set C. So we add everything in the three sets and equate to 50 then we make x the subject of the formula.
2. **n(A∪B)** reads as A union B meaning elements in set A plus elements in set B.
3. **n(B∪C)'** reads as number of elements in B union C complement, meaning elements that are not in union B and C those are 10 and 4 so we add them
4. **n(A'∩C')** reads as A complement intersection C complement meaning number of elements that are not in both sets these are **3** and **4.**

SOLUTIONS(page 2)

i. $(A \cup B \cup C) = 50$

$$10 + x + 3 + 3x + 5 = 50$$

Collecting like terms we have;

$$10 + 3 + 5 + x + 3x = 50$$
$$18 + 4x = 50$$
$$4x = 50 - 18$$
$$4x = 50 - 18$$
$$4x = 32$$
$$\frac{4x}{4} = \frac{32}{4}$$
$$\therefore x = 8$$

ii. $n(A \cup B) = 10 + x + 3 + 3x$

But $x = 8$ so we substitute

$$n(A \cup B) = 10 + 8 + 3 + 3(8)$$
$$n(A \cup B) = 21 + 24$$
$$\therefore n(A \cup B) = 45$$

iii. **n(B∪C)'**$= 10 + 4$

∴n(B∪C)'$= 14$

iv. **n(A'∩C')**$= 3 + 4$

∴n(A'∩C')$= 7$

LESSON 8 QUESTION, P_1, 2016 (page 1)

Given that $f(x)=\frac{5x+4}{5}$ and $g(x)=x-1$. Find; (a) $f^{-1}(x)$

(b) $f^{-1}(2)$

(c)$fg(x)$in its simplest form

SOLUTIONS(page 2)

In (a) $f^{-1}(x)$ they are asking us to find the inverse function of $f(x)$, to do that we equate $f(x)$ *to* y such that, where there is $f(x)$ we write y

$$f(x)=\frac{5x+4}{5} \qquad \Rightarrow \qquad y=\frac{5x+4}{5}$$

Any number is over 1 so we write 1 as a denominator of y

$$\frac{y}{1}=\frac{5x+4}{5}$$

Then we cross multiply and get;

$$5x+4=5y$$

Then making x the subject of the formula we have;

$$5x=5y-4 \qquad \Rightarrow \qquad \frac{5x}{5}=\frac{5y-4}{5}$$

Dividing both sides by 5 we have;

$$\therefore x=\frac{5y-4}{5}$$

SOLUTIONS(page 3)

After we make x the subject of the formula, we write $f^{-1}(x)$ where there is x and change y to x so we have the answer;

$$\therefore f^{-1}(x) = \frac{5x-4}{5}$$

SOLUTIONS (page 4)

(b) $f^{-1}(2)$

This is the same as **(a)** $f^{-1}(x)$ it's just that where there is x they have put 2, so we have to also put **2** in $f^{-1}(x) = \frac{5x-4}{5}$ wherever there is x so we have

$$f^{-1}(2) = \frac{5(2)-4}{5} \quad \Rightarrow \quad f^{-1}(2) = \frac{10-4}{5}$$

Subtracting 4 from 10 we have the answer $f^{-1}(2) = \frac{6}{5}$

$$\therefore f^{-1}(2) = 1\frac{1}{5}$$

SOLUTIONS (page 5)

(c) $fg(x)$ **in its simplest form**

We know that; $f(x) = \frac{5x+4}{5}$ and $g(x) = x - 1$

$fg(x)$ Means we have to substitute $g(x)$ into $f(x)$ such that where there is x in $f(x)$ we write $g(x)$ so we have;

$$fg(x) = \frac{5(x-1)+4}{5}$$

Opening brackets in the numerator we have;

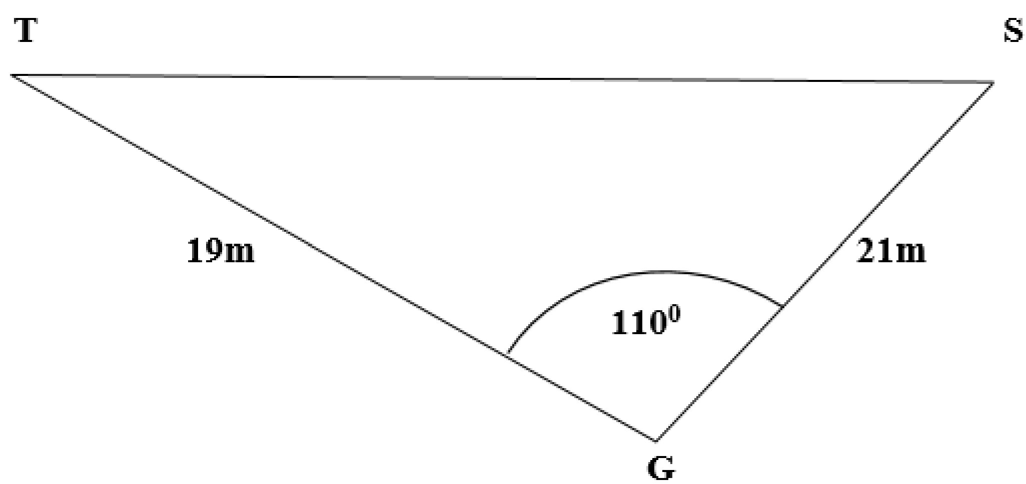
$$fg(x) = \frac{5x-5+4}{5}$$

-5 and 4 in the numerator are like terms so we add them, we have the answer;

$$\therefore fg(x) = \frac{5x-1}{5}$$

LESSON 9 QUESTION$_{10}$ P$_2$ 2008 (Page 1)

During a soccer training session, the goal keeper **'G'** was standing at the centre of the goal post, a shooting player **'S'** was standing **21m** away from the goal keeper's position, the trainer **"T"** was **19m** from the goal keeper hence angle **TGS=110^0** as shown in the diagram bellow.



a. Calculate the area of **GST** to the nearest square metre.
b. The trainer **'T'** rolls the ball along **TS** for a shooting player **'S'** to kick to the mouth of the ball goal. Calculate the distance **TS.**
c. The goal keeper **'G'** is free to intercept the ball at any point along **TS** before it reaches the Player at **'S'**. Find the shortest distance which the goal keeper could run in order to intercept the ball.

SOLUTIONS (Page 2)

HINTS; this is trigonometry, so let us come up with a formula;

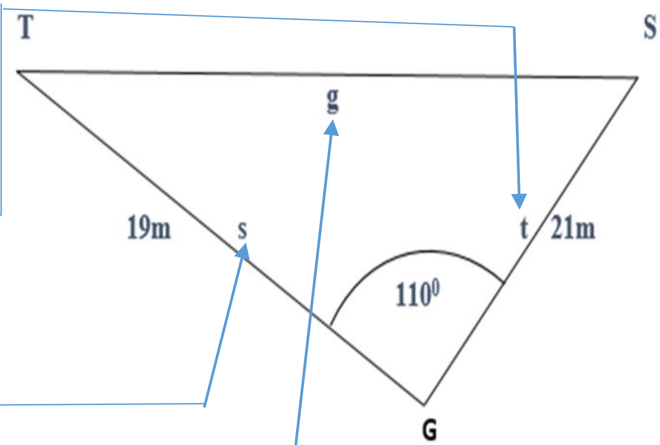
T stands for **trainer**, **S** stands for **striker** and **G** stands for **goal keeper**.

If the **trainer (T)** is standing facing inside the triangle then he will be facing opposite Line **GS** so we write small letter **t** on line **GS**

If the **striker (S)** is standing facing inside the triangle then he will be facing opposite Line **GT** so we write small letter **s** on line **GT**

If the **goal keeper (G)** is standing facing inside the triangle then he will be facing opposite Line **TS** so we write small letter **g** on line **TS**

There is an angle **TGS** formed between line **GT** and line **GS** then the side we are looking for is opposite to that angle and so the formula is **cosine rule**.



$$g^2 = s^2 + t^2 - 2stcos\theta$$

SOLUTIONS (Page 3)

Answering (a) we just use the sine formula for area;

$A = \frac{1}{2}absin\theta$ Where $a\ and\ b$ are the two sides touching the angle and $\theta = 110^0$. The sides touching the angle are **GT** and **GS** where **GT=19m** and **GS=21m** so where there is $a \times b$ we put 19×21 hence we have;

$$A = \frac{1}{2} \times 19 \times 21 \times sin110^0$$

Multiplying and finding $sin110^0$

$$A = \frac{1}{2} \times 399 \times 0.997$$

Multiplying we have;

$$A = \frac{1}{2} \times 374.9$$

Multiplying by $\frac{1}{2}$ or dividing by 2 we have the answer;

$$\therefore A = 187.47m^2$$

SOLUTIONS (Page 4)

Answering (b) we just substitute the numbers into the cosine rule from the triangle

$$g^2 = 19^2 + 21^2 - (2 \times 19 \times 21)cos110^O$$

Evaluating indices and multiplying inside brackets we have;

$$g^2 = 361 + 441 - 798cos110^0$$

Adding the first two numbers and finding $cos110^0$ we have;

$$g^2 = 802 - 798(-0.342)$$

Multiplying negative and negative we have;

$$g^2 = 802 + 798(0.342)$$

Opening brackets we have;

$$g^2 = 802 + 272.932$$

Adding we have

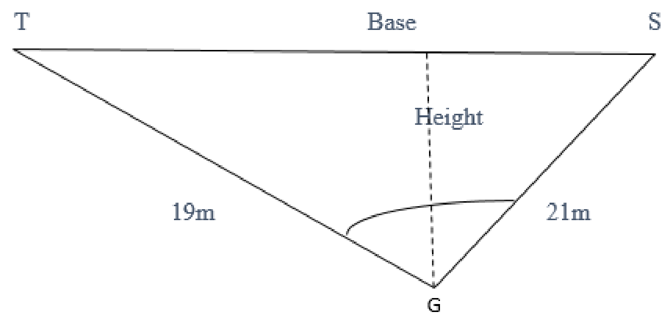
$$g^2 = 1074.932$$

Finding the square roots on both sides we have the answer;

$$\therefore g = 32.79m$$

SOLUTIONS (Page 5)

Answering **(c)** they are asking us to calculate the shortest distance from point **G** to the line **TS** hence it will be a straight line, that line will be perpendicular to the side TS and it will be the height of the triangle then the side **TS** will be the base if the triangle



So we have two formula of triangle for area that is;

$$A = \frac{1}{2} \times a \times b \times sin\theta \text{ and } A = \frac{1}{2} \times base \times height$$

So we can equate the two formula

$$\frac{1}{2} \times a \times b \times sin\theta = \frac{1}{2} \times base \times height$$

SOLUTIONS (c cont. P6)

We already calculated that area= 187.47using the formula of the left so we write this number on the left and replace for base also.

$$187.47 = \frac{1}{2} \times 32.79 \times height$$

Multiplying 32.79 by $\frac{1}{2}$ or dividing 32.79 by 2 we have

$$187.47 = 16.395height$$

Dividing both sides by16.395 we have

$$\frac{187.47}{16.395} = \frac{16.395height}{16.395}$$

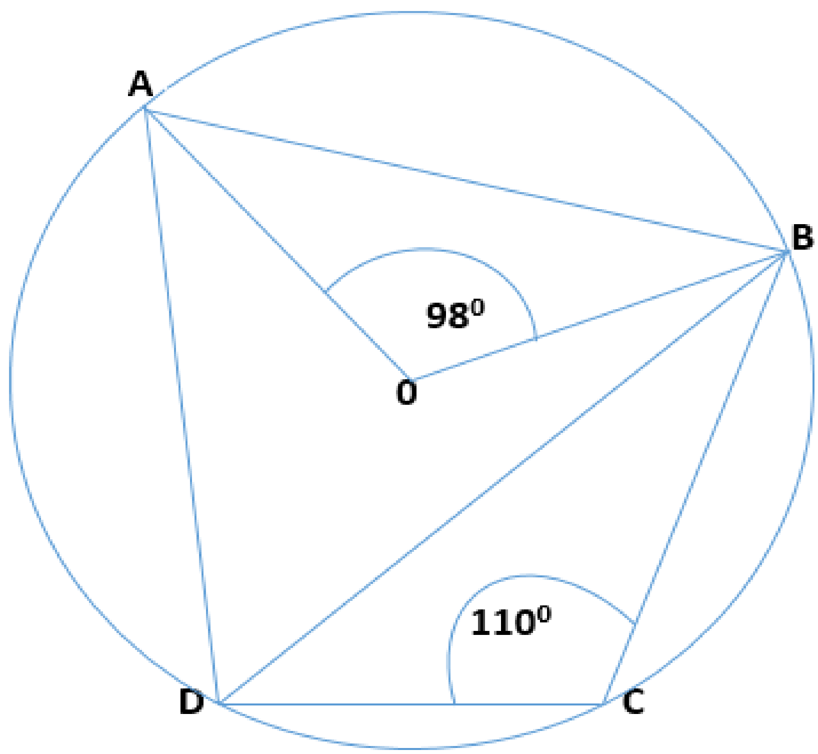
$$\therefore height = 11.43m$$

LESSON 10 ANGLES AND CIRCLE PROPERTIES (pg 1)

In the diagram, A, B, C and D are points on a circle, centre 0.

$< A\hat{O}B = 98^0 \ and \ < D\hat{C}B = 110^0$ **Find;**

$(a) A\hat{D}B \quad (b) A\hat{B}O \ (c) D\hat{A}O$



SOLUTION a (page 2)

$(a) < ADB$

$A\hat{D}B$ Is an angle at the circumference because it if found at the edge of the circle and $A\hat{O}B$ is an angle at the centre of the circle. These two angles are related because; "the angle at the centre is twice the angle at the circumference". We can write this formula as follows;

$$2A\hat{D}B = A\hat{O}B$$

We already know the value of $A\hat{O}B$, its 98^0 so we substitute for of $A\hat{O}B$ we have;

$$2A\hat{D}B = 98^0$$

Marking $A\hat{D}B$ the subject of the formula by dividing by 2

$$\frac{2A\hat{D}B}{2} = \frac{98^0}{2}$$

We have the answer;

$$\therefore A\hat{D}B = 49^0$$

SOLUTION b (page 3)

$(b)A\hat{B}O$

We have three angles in the same isosceles triangle, that is $O\hat{A}B, A\hat{B}O$ and $A\hat{O}B$.

Now that the triangle is isosceles, then; $O\hat{A}B = A\hat{B}O$. So we can equate them to x because they are the same hence we have $O\hat{A}B = A\hat{B}O = x$

The other hint is that three interior angle of a triangle add up to 180^0. This means that;

$$O\hat{A}B + A\hat{B}O + A\hat{O}B = 180^0$$

Substituting the values into the equation we have

$$x + x + 98^0 = 180^0$$

Adding the like terms we have

$$2x + 98^0 = 180^0$$

Collecting like terms we have

$$2x = 180^0 - 98^0 \quad \Rightarrow \quad 2x = 82^0$$

We then divide both sides by 2 to find the value of x

$$\frac{2x}{2} = \frac{82^0}{2}$$

We have $\therefore x = 41^0$ and the answer $\therefore A\hat{B}O = 41^0$

SOLUTION b (page 4)

$(c)D\hat{A}O$

The polygon **ABCD** is a cyclic quadrilateral. $D\hat{A}B$ is opposite to **B$\hat{C}$D** in a cyclic quadrilateral hence the two angles are **supplementary.** We also know that **supplementary** angles add up to **180^0**. This means that;

$$D\hat{A}B + B\hat{C}D = 180^0$$

We can now substitute for $B\hat{C}D = 110^0$

$$D\hat{A}B + 110^0 = 180^0$$

Collecting like terms we have;

$$D\hat{A}B = 180^0 - 110^0$$

$$\therefore\ D\hat{A}B = 70^0$$

To find $D\hat{A}O$ we subtract $O\hat{A}B$ from $D\hat{A}B$

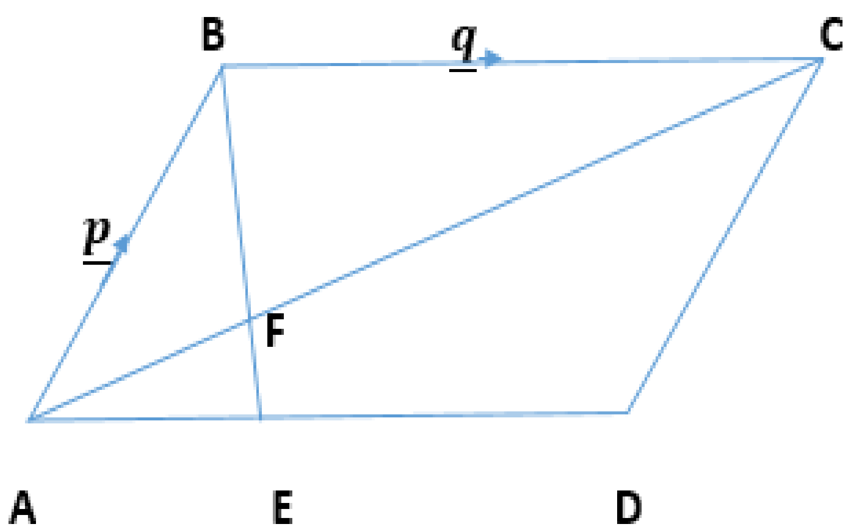
$$D\hat{A}O = D\hat{A}B - O\hat{A}B$$

$$D\hat{A}O = 70^0 - 41^0$$

$$\therefore D\hat{A}O = 29^0$$

QUESTION 4 1986 (Page 1)

The diagram below shows a parallelogram where $\overrightarrow{AB} = \underline{p}$ and $\overrightarrow{BC} = \underline{q}$, the point E on AD is such that $AE = \frac{1}{4}AD$



i. **Express in terms of $\underline{p}$ and/or $\underline{q}$ vectors (a) $\overrightarrow{AC}$ (b) $\overrightarrow{AE}$ (c) $\overrightarrow{BE}$**
ii. **AC and BE intersect at F. Given that $BF = kBE$ Express BF in terms of $\underline{p}$, $\underline{q}$ and k.**
iii. **Hence show that $\overrightarrow{AF} = (1-k)\underline{p} + \frac{1}{4}k\underline{q}$**

SOLUTIONS (i a Page 2)

The dash below $\underline{p}$ and $\underline{q}$ must be drawn to show that p and q are vectors. Do not leave them out. The arrow above $\overrightarrow{AC}$ shows the direction you are going, do not leave them out

(a) $\overrightarrow{AC}$ We first come up with the formula, what we want is to find a route that can take use to point C from A by not using a short cut. One of the routes is; we move from A to B then from B to C, meaning we have covered AC. In formula this will be;

$$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$$

Substituting for $\overrightarrow{AB}$ and $\overrightarrow{BC}$ we have the answer;

$$\therefore \overrightarrow{AC} = \underline{p} + \underline{q}$$

SOLUTIONS (i b Page 3)

(b) $\overrightarrow{AE}$ Look at the first statement in the question, there is already a formula for AE which says $AE = \frac{1}{4}AD$ so we just have to substitute into it. This being a parallelogram then $AD = BC$ but we have seen in the question that $BC = \underline{q}$ so we write $\underline{q}$ where there is AD

We have

$$AE = \frac{1}{4}AD$$

Substituting for AD we have the answer

$$\therefore AE = \frac{1}{4}\underline{q}$$

SOLUTIONS (i c Page 4)

(c) $\overrightarrow{BE}$ For BE we need to come up with the formula. We can move from point B to pint A then from point A to point E and we have reached where we want. Hence we have the formula

$$\overrightarrow{BE}=\overrightarrow{BA}+\overrightarrow{AE}$$

Take note that; when you move from **B** to **A** in the diagram, the arrow on than line is pointing upwards and you are moving in the opposite direction of the arrow, so you change $\underline{p}$ to $-\underline{p}$. We have also found **AE** in **(b)** then we can substitute

$$\overrightarrow{BE}=-\underline{p}+\frac{1}{4}\underline{q}$$

We then write a positive vector first and the negative vector last

$$\therefore \overrightarrow{BE}=\frac{1}{4}\underline{q}-\underline{p}$$

SOLUTIONS (ii Page 5)

They are asking us express BF in terms of $\underline{p}$, $\underline{q}$ and k. And they have already given us the formula $BF = kBE$, we have already found BE in **(c)** so we just substitute;

$$BF = kBE$$

Substituting for BE we have;

$$BF = k(\frac{1}{4}\underline{q}-\underline{p})$$

Opening brackets we have the answer;

$$\therefore BF = \frac{1}{4}k\underline{q}-k\underline{p}$$

SOLUTIONS (iii Page 6)

They are asking us to prove that $\overrightarrow{AF} = (1-k)\underline{p} + \frac{1}{4}k\underline{q}$ to do that we first find the formula for $\overrightarrow{AF}$ from the diagram, meaning that we want to move from point **A** to point **F**, so we can move from point **A** to point **B** then from point **B** to point **F**.

This gives us the formula

$$\overrightarrow{AF} = \overrightarrow{AB} + \overrightarrow{BF}$$

We already have $\overrightarrow{AB} = \underline{p}$ and $\overrightarrow{BF} = \frac{1}{4}k\underline{q} - k\underline{p}$ hence we can just substitute

$$\overrightarrow{AF} = \underline{p} + \frac{1}{4}k\underline{q} - k\underline{p}$$

We now collect like terms by considering vectors **p** and **q**, the first and last term are like terms so we have;

$$\overrightarrow{AF} = \underline{p} - k\underline{p} + \frac{1}{4}k\underline{q}$$

We now factorize **p** in the first and second term, so we have;

$$\therefore \overrightarrow{AF} = (1-k)\underline{p} + \frac{1}{4}k\underline{q}$$

Hence shown

LESSON NUMBER 11 INDICES (page 1)

Indices are mathematical expressions of the form a^n where a is the base and n is the power or index.

LESSON NUMBER 11 LAWS OF INDICES (page 2)

LAW NUMBER ONE; (multiplication of indices of the same base);

For $a^n \times a^m$ the bases are the same, so we just raise the base to the sum of the powers meaning; $a^n \times a^m = a^{n+m}$.

Examples; Simplify each of the following leaving the answer in index form;

(a) $3^5 \times 3^3$ (b) $7^4 \times 7^{-2}$ (c) $x^2 \times x^5$ (d) $xy^3 \times xy^3$

SOLUTIONS (Page 2)

Hint; look at the bases, for all the questions bases are the same so we just add the powers and leave answers in index form.

In **(a)** the base is 3 for both terms so we raise 3 to the power $5+3$

$$\text{a. } 3^5 \times 3^3 = 3^{5+3} = 3^8$$

In **(b)** the base is 7 for both terms so we raise 7 to the power $4+(-2)$ we also know that negative $\times$ positive$=$negative

$$\text{b. } 7^4 \times 7^{-2} = 7^{4+(-2)} = 7^{4-2} = 7^2$$

In **(c)** the base is x for both terms so we raise x to the power $2+5$

$$\text{c. } x^2 \times x^5 = x^{2+5} = x^7$$

SOLUTIONS (Page 3)

Hint; look at the bases, for the question, bases are the same so we just add the powers and leave answers in index form.

In (d) the base is xy for both terms however x has no power meaning it's raised to power 1 so we have $xy^3 = x^1y^3$replacing we have;

$$\text{a. } xy^3 \times xy^3 = x^1y^3 \times x^1y^3$$

We also know that $x^1y^3 = x^1 \times y^3$ so we have

$$= x^1 \times y^3 \times x^1 \times y^3$$

We now collect like terms (terms with the same base) we have

$$= x^1 \times x^1 \times y^3 \times y^3$$

Raising bases to the sum of the powers we have;

$$= x^{1+1} \times y^{3+3}$$

Adding powers we have

$$= x^2 \times y^6$$

We now multiply to find the answer

$$\therefore answer = x^2 y^6$$

LAW NUMBER TWO (page 4)

(Division of indices of the same base);

For $a^n \div a^m$ the bases are the same, so we just raise the base to the difference (minus) of the powers meaning;

$$a^n \div a^m = a^{n-m}.$$

Examples; Simplify each of the following leaving the answer in index form;

(a) $4^7 \div 4^5$ (b) $9^6 \div 9^{-2}$ (c) $x^5 \div x^2$ (d) $x^3 \div y^4$

SOLUTIONS (Page 5)

Hint; look at the bases, for all the questions, if bases are the same we just subtract the powers and leave answers in index form.

In (a) the base is 4 for both terms so we raise 4 to the power $7-5$

(a) $4^7 \div 4^5 = 4^{7-5} = 4^2$

In (b) the base is 9 for both terms so we raise 9 to the power $6-(-2)$

we also know that negative $\times$ negative$=$positive

(b) $9^6 \div 9^{-2} = 9^{6-(-2)} = 9^{6+2} = 9^8$

In (c) the base is x for both terms so we raise x to the power $5-2$

$$\text{(c) } x^5 \div x^2 = x^{5-2} = x^3$$

In (d) the base is x for the first term while the base is y for the second term, therefore we cannot divide unlike terms so the question remains the answer

$$\text{(d) } x^3 \div y^4 = x^3 \div y^4$$

LESSON NUMBER 12 CALCULUS (DIFFERENTIATION) (page 1)

When you are given an equation such as $y = x^n$ then differentiation of $y = x^n$ will be denoted by $\frac{dy}{dx}$ meaning differentiating y with respect x.

This will give us the formula for differentiation. The n (power of x) will be multiplied to x, then we have to subtract 1 from n (power of x) as follows;

$$y = x^n$$

$$\frac{dy}{dx} = n \times x^{n-1}$$

$$\therefore we\ have\ \frac{dy}{dx} = nx^{n-1}$$

Examples; differentiate each of the following with respect to x;

a. $y = 3x^3 - x^2$ b. $y = 6x^2 - 2x + 8$ c. $y = 2x^2$ d. $y = 9x^6 + 4$

e. $y = 3x^2 + 2x + 4x - x^2 + 2 + 10$

SOLUTIONS (page 2)

HINT; we need to just follow the formula $\frac{dy}{dx} = nx^{n-1}$. To answer a. we have $3x^3$ for this term $n = 3$, we aslo have x^2 for this term $n = 2$. We answer it as follows;

$$y \quad = \quad 3x^3 \quad - \quad x^2$$

Here we have used colours to show you what we are obtaining from each term, so follow the colours closely;

$$\frac{dy}{dx} = (3 \times 3x^{3-1}) - (2 \times x^{2-1})$$

When we multiply coefficients and subtract 1 from each power we have;

$$\frac{dy}{dx} = 9x^2 - 2x^1$$

We can ignore the power 1 in the last term we have the answer;

$$\therefore \frac{dy}{dx} = 9x^2 - 2x$$

Examples; differentiate each of the following with respect to x;

a. $y = 3x^3 - x^2$ b. $y = 6x^2 - 2x + 8$ c. $y = 2x^2$ d. $y = 9x^6 + 4$

e. $y = 3x^2 + 2x + 4x - x^2 + 2 + 10$

SOLUTIONS (page 3)

HINT; we need to just follow the formula $\frac{dy}{dx} = nx^{n-1}$. To answer b. we have $6x^2$ for this term $n = 2$, we aslo have $2x$ for this term $n = 1$ the last term is a constant 8, this term has no x so we just ignore it. Here we go;

$$y = 6x^2 - 2x + 8$$

Here we have used colours to show you what we are obtaining from each term, so follow the colours closely;

$$\frac{dy}{dx} = (2 \times 6x^{2-1}) - (1 \times 2x^{1-1}) + 0$$

When we multiply coefficients and subtract 1 from each power we have;

$$\frac{dy}{dx} = 12x^1 - 2x^0$$

We can ignore the power 1 on $12x^1$, we also know that any number to the power zero is equal to 1 so $x^0 = 1$ then we have;

$$\frac{dy}{dx} = 12x^1 - 2(1)$$

$$\therefore \frac{dy}{dx} = 12x - 2$$

SOLUTIONS (page 4)

HINT; we need to just follow the formula $\frac{dy}{dx} = nx^{n-1}$. To answer **c.** we have $2x^2$ for this term $n = 2$, and that's the only term. Here we go;

$$y = 2x^2$$

Here we have used colours to show you what we are obtaining from each term, so follow the colours closely;

$$\frac{dy}{dx} = (2 \times 2x^{2-1})$$

When we multiply coefficients and subtract 1 the power we have;

$$\frac{dy}{dx} = (4x^1)$$

We can ignore the power 1 on $4x^1$, and then we have;

$$\therefore \frac{dy}{dx} = 4x$$

SOLUTIONS (page 5)

HINT; we need to just follow the formula $\frac{dy}{dx} = nx^{n-1}$. To answer **d.** we have $9x^6$ for this term $n = 6$, the other term 4 is a constant so we ignore it

$$y = 9x^6 + 4$$

Here we have used colours to show you what we are obtaining from each term, so follow the colours closely;

$$\frac{dy}{dx} = (6 \times 9x^{6-1}) + 0$$

When we multiply coefficients and subtract 1 from the power we have;

$$\therefore \frac{dy}{dx} = 54x^5$$

SOLUTIONS (page 7)

HINT; we need to just follow the formula $\frac{dy}{dx} = nx^{n-1}$. To answer **e.**

$$y = 3x^2 + 2x + 4x - x^2 + 2 + 10$$

The terms in the same colour are like terms so we can add or subtract them, so we collect like term

$$y = 3x^2 - x^2 + 2x + 4x + 2 + 10$$

Working out the like terms we have;

$$y = 2x^2 + 6x + 12$$

We have $2x^2$ for this term $n = 2$, the other term is $6x$ for this term $n = 1$, we then have 12 a constant so we ignore it because it has no x.

$$\frac{dy}{dx} = (2 \times 2x^{2-1}) + (1 \times 6x^{1-1}) + 0$$

When we multiply coefficients and subtract 1 from the power we have;

$$\frac{dy}{dx} = 4x^1 + (6x^0)$$

Ignoring the power 1 and knowing that any number to the power zero is equal to one we have;

$$\frac{dy}{dx} = 4x + 6(1)$$

$$\therefore \frac{dy}{dx} = 4x + 6$$

LESSON NUMBER 13 QUESTION 16 p1 2012 (page1)

Given that $f(x) = \frac{3x-5}{2}$ and $g(x) = \frac{x-4}{6}$, find

a. $f(-9)$
b. $f^{-1}(x)$
c. The value of x for which $f(x) = 3g(x)$

SOLUTIONS (page 2)

a. To answer (a) $f(-9)$ this is just the same as $f(x)$, the only difference is that where there is x they have put -9 so we just have to put -9 where there is x.

$$f(x) = \frac{3x-5}{2}$$

Substituting for x we have

$$f(-9) = \frac{3(-9)-5}{2}$$

Multiplying in the numerator we have;

$$f(-9) = \frac{-27-5}{2}$$

Adding negatively in the numerator we have;

$$f(-9) = \frac{-32}{2}$$

We have the answer

$$\therefore \ f(-9) = -16$$

SOLUTIONS (page 3)

$$b. f^{-1}(x)$$

To answer this question our focus is to make x the subject of the formula, so we say let $f(x)$ be equal to y so that where there if $f(x)$ in $f(x) = \frac{3x-5}{2}$ we put just one letter y, then make x the subject of the formula so we have

$$y = \frac{3x-5}{2}$$

We then cross multiply to get

$$3x - 5 = 2y$$

Moving -5 across we have;

$$3x = 2y + 5$$

Dividing both sides by 3

$$3x = 2y + 5$$

Dividing both sides by $3y$ we have;

$$\frac{3yx}{3y} = \frac{2+5y}{3y} \quad \Rightarrow \quad x = \frac{2+5y}{3y}$$

And the answer is;

$$\therefore f^{-1}(x) = \frac{2+5x}{3x}$$

SOLUTIONS (page 4)

To answer (c) we know that $f(x) = \frac{3x-5}{2}$ and $g(x) = \frac{x-4}{6}$

So we substitute into

$f(x) = 3g(x)$ **We have** $\frac{3x-5}{2} = 3\left(\frac{x-4}{6}\right)$

Cross multiplying we have;

$$2(3x-5)=2(x-4) \qquad 6x-10=2x-8$$

Collecting like terms;

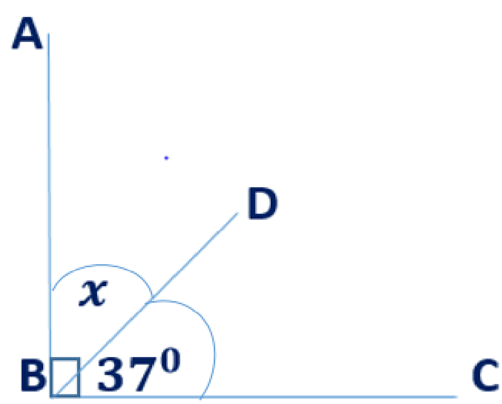
$$6x-2x=10-8 \quad \textbf{Subtract} \quad 4x=2$$

Dividing both sides by 5

$$\frac{4x}{4}=\frac{2}{4} \qquad \therefore x=\frac{1}{2}$$

LESSON NUMBER 14 ANGLES AND POLYGONS (page 1)

1. **Complementary angles; these are two acute angles that add up to 90^0. An angle is acute is it is more than 0^0 but less than 90^0.**
   Example; in the figure below $A\hat{B}C$ is a right angle, $A\hat{B}D=x$ and $D\hat{B}C$=37^0 are acute angles and they are complementary. Calculate x.



SOLUTIONS (page 2)

The two angles $A\hat{B}D$ and $A\hat{B}D$ are complementary meaning they add up to 90^0. We can then come up with the formula

$$A\hat{B}D + D\hat{B}C = A\hat{B}C$$

We know that $A\hat{B}D=x$ and $D\hat{B}C$=37^0 while $A\hat{B}C=90^0$

We can then substitute, we have;

$$x + 37^0 = 90^0$$

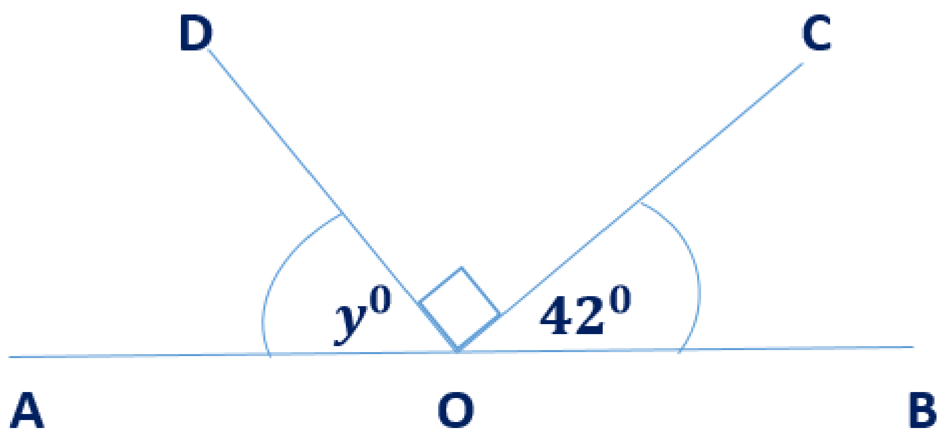
Collecting like terms we have;

$$x = 90^0 - 37^0$$

$$\therefore x = 53^0$$

LESSON NUMBER 15 ANGLES AND POLYGONS (page 3)

2. Supplementary angles; these are two or more angles that add up to 180^0. Example; in the figure below $A\hat{O}B$ is a straight line with angles, $A\hat{O}D = y^0$, $C\hat{O}D = 90^0$ and $C\hat{O}B = 42^0$. Calculate y^0.



SOLUTION (page 4)

The three angles are on the same straight line, we know that a straight line makes an angle of 180^0. This means that the sum of $A\hat{O}D$, $C\hat{O}D$ and $C\hat{O}B$ is 180^0

So we can come up with the formula;

$$A\hat{O}D + C\hat{O}D + C\hat{O}B = A\hat{O}B$$

We also know that

$A\hat{O}D = y^0, C\hat{O}D = 90^0$ and $C\hat{O}B = 42^0$

So we have to substitute into the formula, we have;

$$y^0 + 90^0 + 42^0 = 180^0$$

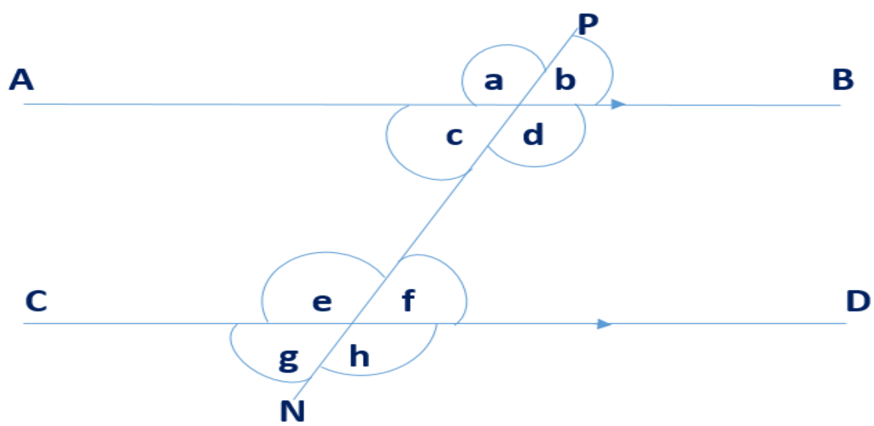
We can now collect the like terms, we have;

$$y^0 = 180^0 - 132^0$$

$$\therefore\ y^0 = 48^0$$

LESSON NUMBER 16 ANGLES AND POLYGONS (page 1)

3. **Angles associated with parallel lines**; parallel lines are straight lines that can never meet. In the figure below AB and CD are parallel lines. NP is a transversal



a. **Corresponding angles;** Corresponding angles are equal, in the figure observe closely how we equate them.

$$< g =< c,\quad < e =< a,\quad < b =< f,\quad < h =< d,$$

b. **Alternate angles;** alternate angles are also equal, observe closely how we pair these also.

$$< e =< d,\quad < c =< f,\quad < a =< h,\quad < g =< b,$$

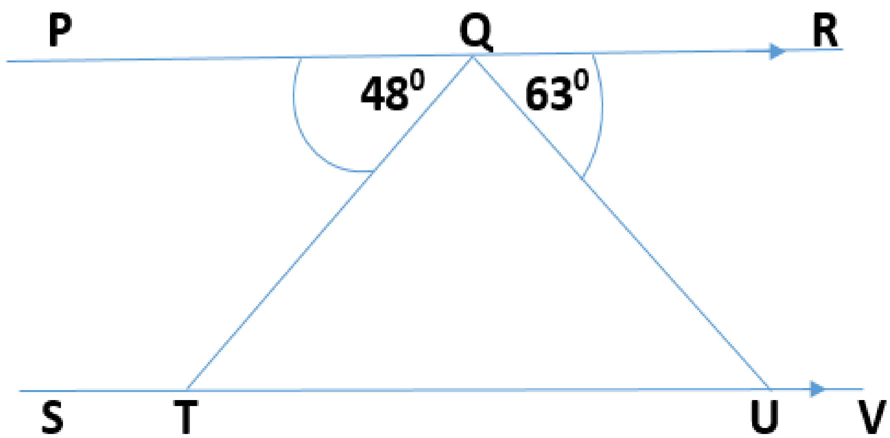
c. **Allied angles;** they are supplementary angles which lie on the same side of the transversal within two parallel lines. In this case we have only two pairs $< f \ and\ < d$ then $< e\ and\ < c$

Meaning $< f + < d = 180^0$ then $< e + < c = 180^0$

QUESTIONS (PAGE 2)

Angles associated with parallel lines (questions);

a. PR and SV are parallel lines. $P\hat{Q}T = 48^0$ and $R\hat{Q}U = 63^0$. Find: (a) $Q\hat{U}V$ (b) $Q\hat{T}V$



SOLUTIONS (page 3)

To answer (a) we can see that $Q\hat{U}V$ and $R\hat{Q}U$ are allied angles. We also know that allied angles are supplementary, hence they add up to 180^0. So we come up with the formula

$$Q\hat{U}V + R\hat{Q}U = 180^0$$

We also know that $R\hat{Q}U = 63^0$ so we substitute and have;

$$Q\hat{U}V + 63^0 = 180^0$$

Collecting like terms we have;

$$Q\hat{U}V = 180^0 - 63^0$$

Done

$$\therefore Q\hat{U}V = 117^0$$

SOLUTIONS (page 4)

To answer **(b)** we have to consider the positions of $Q\hat{T}V$ and that of $P\hat{Q}T$ the two are alternate angles. We know that alternate angles are equal this means that;

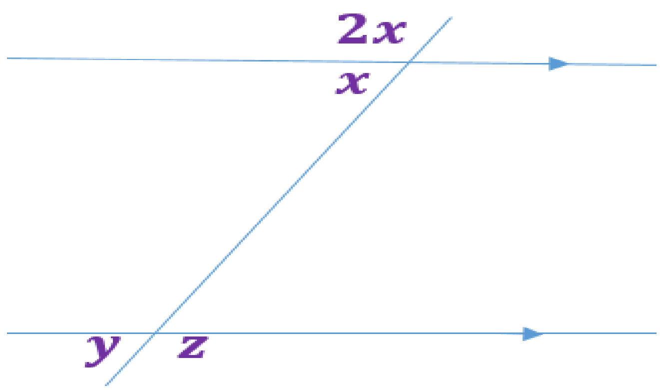
$$Q\hat{T}V = P\hat{Q}T$$

$$\therefore Q\hat{T}V = 48^0$$

LESSON NUMBER 17 QUESTION (page 5)

From the diagram bellow find,

$(a)\ x$ $(b)\ y$ $(c)\ z$



SOLUTIONS (page 6)

To answer $(a)\ x$ we have to consider the relationship between **x** and **2x** by looking at their respective positions. These two are on the same transversal line cutting parallel lines. When two angles are on the same straight line like that, then they are **supplementary** angles. We know that supplementary angles add up to 180^0. So we come up with the formula

$$2x + x = 180^0$$

We can now add the like terms

$$3x = 180^0$$

We then divide both sides by 3

$$\frac{3x}{3} = \frac{180^0}{3}$$

$$\therefore x = 60^0$$

SOLUTIONS (page 7)

To answer $(b)\ y$ we have to consider the relationship between **x** and **y** by looking at their respective positions. These two are corresponding angles. We know that **corresponding** angles are equal. So we come up with the formula

$$y = x$$

We have already calculated for x in (a)

$$\therefore y = 60^0$$

SOLUTIONS (page 8)

To answer $(c)\ z$ we have to consider the relationship between **y** and **z** by looking at their respective positions. These two are on the same straight parallel line. When two angles are on the same straight line like that, then they are **supplementary** angles. We know that **supplementary** angles add up to 180^0. So we come up with the formula

$$y + z = 180^0$$

We have already calculated y in (b) so we substitute;

$$60^0 + z = 180^0$$

Collecting like terms we have;

$$z = 180^0 - 60^0$$

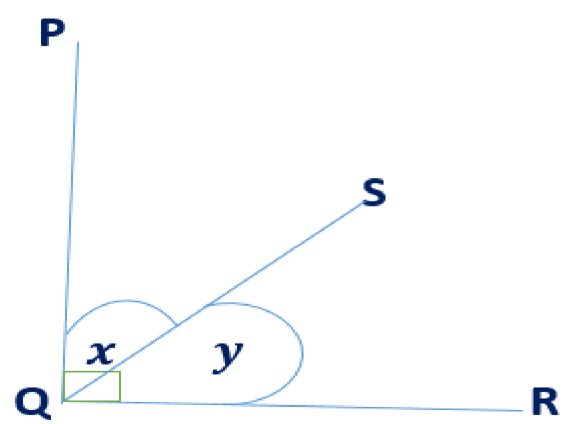
Then subtracting we have the answer;

$$\therefore z = 120^0$$

LESSON NUMBER 18 QUESTION (page 1)

In the diagram $P\hat{Q}R$ is a right angle find

(a) $If\ x = 19^0, find\ y$
(b) $If\ y = 87^0\ find\ x$



SOLUTION (page 2)

(a) $If\ x = 19^0, find\ y$

To answer these questions we need to just understand the meaning of right angle, it means an angle equal to 90^0. Both x and y are found within the same right angle. Meaning that;

$$x + y = 90^0$$

For this question $x = 19^0$ so we have;

$$19^0 + y = 90^0$$

Collecting like terms we get;

$$y = 90^0 - 19^0$$

$$\therefore y = 71^0$$

SOLUTIONS (page 3)

(a) $If\ y = 87^0, find\ x$

To answer this question we need to just understand the meaning of right angle, it means an angle equal to 90^0. Both x and y are found within the same right angle. Meaning that;

$$x + y = 90^0$$

For this question $y = 87^0$ so we have;

$$+x = 90^0$$

Collecting like terms we get;

$$x = 90^0 - 87^0$$

$$\therefore x = 03^0$$

LESSON NUMBER 19 QUESTIONS (page 1)

1. Factorise each of the following completely
   a. $x^2 - y^2$
   b. $4 - 16x^2$
2. Find the exact values of

a. $2.025 + 42.1$

b. $\frac{2}{3} \div \frac{4}{5}$

3. Express 50cm as a percentage of 2m

SOLUTION 1a (page 2)

Difference means subtraction, square means power 2. Difference of two squares means subtraction of two squared numbers or two squared terms.

To factorize $x^2 - y^2$ we open two pairs of closed brackets

$$(\quad)(\quad)$$

Write $x - y$ in the first and $x + y$ in the second brackets

$$\therefore x^2 - y^2 = (x - y)(x + y)$$

SOLUTION 1b (page 3)

To factorise $4 - 16x^2$ we first identify a number which can go into 4 and into 16 that number is 4

Then divide $\frac{4}{4} = 1$ so where there is 4 we write 1

Also divide $\frac{16x^2}{4} = 4x^2$ so where there is $16x^2$ we write $4x^2$

With the 4 outside brackets we have

$$4 - 16x^2 = 4(1 - 4x^2)$$

The 4 inside brackets can be written in index form as 2^2

$$4(1 - 4x^2) = 4(1 - 2^2x^2)$$

This 2^2x^2 is also the same as $(2x)^2$

$$4(1 - 2^2x^2) = 4(1 - (2x)^2)$$

This is now a difference of two squares and the factorization will be as follows

$$\therefore 4 - 16x^2 = 4[(1 - 2x)(1 + 2x)]$$

SOLUTIONS 2a (page 4)

To answer $(a)\ 2.025 + 42.1$ we have to consider rewriting the two numbers to be added in vertical form making sure that points are in the same vertical line as follows;

$$\begin{array}{r} 2.025 \\ \underline{+\,42.1} \end{array}$$

We the add the number up to the number down, however there are no numbers below **2** and **5** on the right, so below those we write zeroes as follows;

$$\begin{array}{r} 2.025 \\ \underline{+\,42.100} \end{array}$$

There is also no number above **4** so we write zero above **4**, we have.

$$\begin{array}{r} 02.025 \\ \underline{+\,42.100} \end{array}$$

We now have numbers in all positions hence we add and have the answer

$$\begin{array}{r} 02.025 \\ \underline{+\,42.100} \\ \underline{44.125} \end{array}$$

SOLUTIONS 2b (page 5)

To answer $(b)\frac{2}{3} \div \frac{4}{5}$ we have to change $\div$ to $\times$ then we change the second fraction to its inverse, meaning $\frac{4}{5}$ becomes $\frac{5}{4}$. After taking those steps we have;

$$\frac{2}{3} \div \frac{4}{5} = \frac{2}{3} \times \frac{5}{4}$$

We can then multiply 2 by 5 and 3 by 4, we have;

$$\frac{2}{3} \times \frac{5}{4} = \frac{10}{12}$$

The two numbers 10 and 12 have a common factor, or a number with can divide into them without leaving a remainder and that is 2. So we divide each by 2.

$$\therefore \frac{2}{3} \div \frac{4}{5} = \frac{10}{12} = \frac{5}{6}$$

SOLUTION 3 (page 6)

HINT; for this question we have to convert both measurements into same units. So let us convert 2m to cm; we know that;

$$100cm = 1m$$

$$xcm = 2m$$

This can be written as

$$100 = 1$$

$$x = 2$$

From there we can now cross multiply as follows;

$$100 = 1$$

$$x = 2$$

This will now give us

$$x = 200cm$$

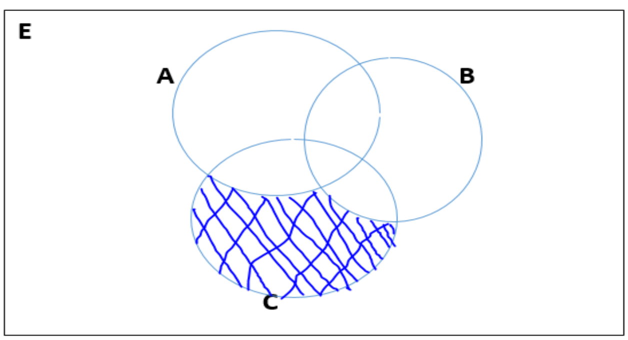
This means that 2m has been converted to cm and it is now 200cm. so to express 50cm as a percentage of 200cm, we say;

$(50cm\ over\ 200cm)$ **times 100 as follows;**

$$\frac{50}{200} \times 100 = \frac{50}{2} = 25\%$$

LESSON NUMBER 20 QUESTIONS (page 1)

1. **Express 0.002304 in standard form, correct to one decimal place**
2. **Evaluate** $(a) 42 - 8 \div 2 + 5$ $(b) \left(\frac{3}{4}\right)^{-2}$
3. **Use set notation to describe the shaded region in the diagram below.**



SOLUTION (PAGE 2)

HINT; we have to move the decimal point to the right of the first non-zero digit, in this case the first non-zero digit is 2. So the point will be between 2 and 3. So we have;

$$\mathbf{0.002304\ \mathit{becomes}\ 0002.304}$$

Then we count the number of digits the point has jumped over. There are two zeroes and a two. So it has jumped over 3 digits. To show this we first remove the zeroes in front of 2 we have;

$$\mathbf{2.304}$$

We then multiply this by 10 to the power number of digits the point jumped over, we have

$$\mathbf{2.304\ \times 10^{3}}$$

Now that the point is moving from left to right, the power will be negative

$$\mathbf{2.304\ \times 10^{-3}}$$

The question says one decimal place, meaning we remain with only on digit after the point so we have the answer;

$$\mathbf{2.3 \times 10^{-3}}$$

SOLUTION (page 3)

To answer **(a)** we have to apply **BODMAS**, this stands for **Brackets on/off**, **Division**, **Multiplication**, **Addition** and **Subtraction**. This means you deal with what is in **Brackets** first before you **Divide**, **Divide** before you **Multiply**, **Multiply** before you **Add** and **Add** before you **Subtract**.

$$(a) 42 - 8 \div 2 + 5$$

There are no brackets we have to divide $8 \div 2$ first, we have;

$$42 - 4 + 5$$

Then we have to add $-4 + 5 = 1$ we have

$$42 + 1$$

We have to subtract and find the answer;

$$43$$

SOLUTION (page 4)

To answer $(b)\left(\frac{3}{4}\right)^{-2}$ we have to use indices. The first thing we have to do is to express this with a positive index. To do that we make $\left(\frac{3}{4}\right)^{-2}$ as the denominator of 1 and we remove the negative from the power so we have;

$$\left(\frac{3}{4}\right)^{-2} = \frac{1}{\left(\frac{3}{4}\right)^{2}}$$

$\frac{1}{\left(\frac{3}{4}\right)^{2}}$ Is the same as $1 \div \left(\frac{3}{4}\right)^{2}$ then $\left(\frac{3}{4}\right)^{2}$ is the same as $\frac{3^2}{4^2}$ so we have;

$$1 \div \left(\frac{3}{4}\right)^2 = 1 \div \frac{3^2}{4^2}$$

We can now change $\div$ to $\times$ and also change $\frac{3^2}{4^2}$ to $\frac{4^2}{3^2}$ so we have;

$$1 \div \frac{3^2}{4^2} = 1 \times \frac{4^2}{3^2}$$

$$1 \times \frac{4^2}{3^2} = \frac{4^2}{3^2}$$

Multiplying each number twice by itself because of the power we have;

$$\frac{4^2}{3^2} = \frac{4 \times 4}{3 \times 3} = \frac{16}{9}$$

$\therefore$The answer is = $1\frac{7}{9}$

SOLUTION (page 5)

The shaded region does not cater for elements in set A, meaning it caters for A complement or A' and does not cater for elements in set B, meaning it caters for B complement or B' just here we have A union B complement written as $(A \cup B)'$. It caters for elements that only belong to set C. we can now connect this as;

$$C \cup (A \cup B)'.$$

LESSON NUMBER 21 VARIATIONS (page 1)

There are two types of variations, namely inverse and direct variation.

1. When y varies direct as x then we can write the formula as follows;

$$y = k \times x \qquad \therefore y = kx$$

2. When y varies inversely as x, the we write the formula for this variation as follows;

$$y = k \times \frac{1}{x} \qquad \therefore y = \frac{k}{x}$$

3. When y varies inversely as x and directly as z then we will have the formula;

$$y = k \times \frac{1}{x} \times z \quad \therefore y = \frac{kz}{x}$$

Take note that the k is just a constant of variation hence it does not come from the question.

LESSON NUMBER 21 QUESTION 17 P2 2012 (page 2)

y varies directly as x and z. Given that $y = 9$ when $x = 6$ and $z = \frac{1}{2}$, Find;

a. k(the constant of variation)
b. The value of y when $x = 4$ and $z = 3$,
c. The value of x when $y = 4\frac{1}{2}$ and $z = 5$

SOLUTIONS a (page 3)

To answer a, b, and c we first have to come up with the formula, if y varies directly as x and z then $y = k \times x \times z$. This is the same as;

$$y = kxz$$

To answer (a) we have to make k the subject of the formula by dividing both sides by xz so we have;

$$\frac{y}{xz} = \frac{kxz}{xz}$$

Then we have;

$$\therefore k = \frac{y}{xz}$$

We substitute for $y = 9$ when $x = 6$ and $z = \frac{1}{2}$ in $k = \frac{y}{xz}$

$$k = \frac{9}{6 \times \frac{1}{2}} = \frac{9}{\frac{6}{2}} \text{ this is the same as } k = 9 \div \frac{6}{2}$$

We work out $\frac{6}{2}$ because 2 can go into 6 we have

$$k = 9 \div 3 \qquad \therefore k = 3$$

SOLUTIONS b (page 4)

$$y = kxz$$

To answer (b); Finding y when $x = 4$ *and* $z = 3$, we use the same formula $y = kxz$. Substituting for $x = 4$, and $z = 3$ we have;

$$y = k \times 4 \times 3$$

But we already found $k = 3$ so we substitute for k also

$$y = 3 \times 4 \times 3 = 12 \times 3$$

Multiplying we have the answer;

$$\therefore y = 36$$

SOLUTIONS c (page 5)

$$y = kxz$$

To answer (c); Finding x when $y = 4\frac{1}{2}$ and $z = 5$ we use the same formula $y = kxz$. But first of all we make x the subject of the formula by dividing both sides by kz as follows;

$$\frac{y}{kz} = \frac{kxz}{kz} \quad \Rightarrow \quad x = \frac{y}{kz}$$

We can now substitute for **y, z** and **k** because we already have these values

$x = \frac{4\frac{1}{2}}{3 \times 5}$ this is the same as $4\frac{1}{2} \div 15$

We can now change $4\frac{1}{2}$ to an improper fraction we have $\frac{9}{2}$ then we substitute and we have;

$\frac{9}{2} \div 15$ this is the same as $\frac{9}{2} \div \frac{15}{1}$

When dividing two fractions, division changes to multiplication and the fraction on the right is inversed (numerator becomes denominator while denominator becomes numerator) so we have;

$$\frac{9}{2} \div \frac{15}{1} = \frac{9}{2} \times \frac{1}{15}$$

We then multiply and get

$$\frac{9 \times 1}{2 \times 15} = \frac{9}{30} \quad \therefore answer = \frac{3}{10}$$

LESSON NUMBER 22 QUESTION 6 p1 1983 (page 1)

Given that y varies directly as x and inversely as 2m-1, and that y=5 when x=7 and m=4. Calculate

a. The constant of variation, k
b. The value of y when x=2 and m=3
c. The value of m when y=2 and x=4

SOLUTIONS a (page 2)

We have to first of all come up with the formula y varies directly as x can be written as

$$y = kx$$

y varies inversely as 2m-1 can also be written as;

$$y = \frac{k}{2m - 1}$$

We now combine them as follows;

$$y = \left(\frac{1}{2m - 1} \times x\right) k$$

$$\therefore y = \frac{kx}{2m - 1}$$

SOLUTIONS a (page 3)

The constant of variation, k. to find the constant of variation k, we have to first make k the subject of the formula from the same formula we found

$$y = \frac{kx}{2m-1} \quad \Rightarrow \quad \frac{y}{1} = \frac{kx}{2m-1}$$

We cross multiply and have;

$$y(2m-1) = kx$$

We open the brackets and divide both sides by x and have

$$k = \frac{2ym-y}{x}$$

We can now substitute for m, x and y

$$k = \frac{(2\times5\times4)-5}{7}$$

Multiply first

$$k = \frac{40-5}{7}$$

Subtracting in the numerator

$$k = \frac{35}{7} \qquad \therefore k = 5$$

SOLUTIONS b (page 4)

The value of y when x=2 and m=3. We substitute into the formula $y = \frac{kx}{2m-1}$ for x, k and m

$$y = \frac{kx}{2m-1} \quad \Rightarrow \quad y = \frac{5\times2}{2\times3-1}$$

We first multiply before we subtract

$$y = \frac{10}{6-1}$$

We then subtract

$$y = \frac{10}{5} \qquad \therefore y = 2$$

SOLUTIONS c (page 5)

Value of m when y=2 and x=4. Use the formula $y = \frac{kx}{2m-1}$ to make m the subject of the formula

$$y = \frac{kx}{2m-1} \quad \Rightarrow \quad \frac{y}{1} = \frac{kx}{2m-1}$$

We cross multiply before we subtract

$$y(2m-1) = kx$$

We then open the brackets and obtain

$$2ym - y = kx$$

Move $-y$ to the right across the equal sign to have $+y$;

$$2ym = kx + y$$

We now divide both sides by $2y$ to remain with just m on the right side.

$$\frac{2ym}{2y} = \frac{kx+y}{2y}$$

Cancelling 2y and 2y on the left we have;

$$m = \frac{kx+y}{2y}$$

We can now substitute for k, x and y in order to find m

$$m = \frac{5\times4+2}{2\times2}$$

$$we\ first\ multiply\ we\ have$$

$$m = \frac{20+2}{4}$$

$$we\ add\ and\ get$$

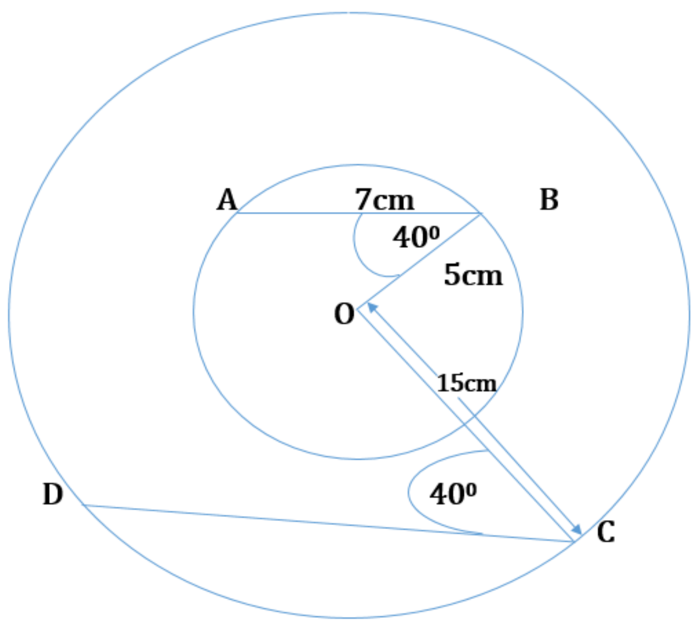
$$m = \frac{22}{4} \quad \Rightarrow \quad m = \frac{11}{2}$$

$$\therefore m = 5\frac{1}{2}$$

LESSON NUMBER 23 QUESTION 11 P2 2012 (page 1)

a) Given that $\underline{a} = \begin{pmatrix} -8 \\ 6 \end{pmatrix}$, find $|\ \underline{a}\ |$

b) The diagram shows two circles with the same centre O. The radius of the smaller circle is $5cm$ and that of the larger is $15cm$. Chord AB is $7cm$ and $A\hat{B}O = D\hat{C}O = 40^0$. Calculate the length of the chord CD.



SOLUTION a (page 2)

Given that; $\underline{a} = \begin{pmatrix} -8 \\ 6 \end{pmatrix}$, find $|\ \underline{a}\ |$

HINT this question is coming from vectors, they are asking us to find the magnitude of vector $\underline{a}$. Using the formula;

$$|\ \underline{a}\ | = \sqrt{x^2 + y^2}$$

In $\begin{pmatrix} -8 \\ 6 \end{pmatrix}$ $x = -8$ *and* $y = 6$ we substitute into the formula

$$|\ \underline{a}\ | = \sqrt{(-8)^2 + 6^2} \quad \Rightarrow \quad |\ \underline{a}\ | = \sqrt{(-8 \times -8) + (6 \times 6)}$$

Opening the brackets we have;

$$|\underline{a}| = \sqrt{64+36} \quad \Rightarrow \quad |\underline{a}| = \sqrt{100}$$

$$we\ now\ find\ the\ square\ root\ of\ 100, please\ do\ not\ use\ a\ calculator, this\ is\ paper\ 1$$

$$\therefore |\underline{a}| = 10\ units$$

SOLUTION b (page 3)

b. calculate the length of the chord CD

HINT; this is from similarity and congruence, the chord CD is for the bigger circle, there is another chord AB for the smaller circle. We also have radius OC for the bigger circle and radius OB for the smaller circle. We can now make fractions out of these by using smaller chord and smaller radius as numerator of bigger chord and radius respectively as follows;

Chords we have; $\frac{AB}{CD}$ radii we have; $\frac{OB}{OC}$

Equate the two fractions to come up with the formula;

$$\frac{AB}{CD} = \frac{OB}{OC}$$

Substitute using values from the diagram;

$\frac{7}{CD} = \frac{5}{15}$ Then cross multiply $5 \times CD = (7 \times 15)$

$$5 \times CD = (7 \times 15) \quad \Rightarrow \quad 5CD = 105$$

Divide by 5 on both sides

$$\frac{5CD}{5} = \frac{105}{5} \quad \Rightarrow \quad \therefore DC = 21cm$$

LESSON NUMBER 24 QUESTION 12 P_1 2012 (page 1)

Given that $E = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$ list the following subsets;

a. $A = \{x: x\ is\ a\ factor\ of\ 55\ \}$

b. $B = \left\{x: \frac{1}{2}x \geq 5\frac{1}{2}\right\}$

SOLUTION a (page 2)

$$A = \{x: x\ is\ a\ factor\ of\ 55\ \}$$

HINTS; we first have to understand what the word factor mean, it means a number which can go into a given number without leaving a remainder. In this case the given number is 55, so let us list all the numbers that can go into 55 without leaving a remainder;

Factors of 55 are $\{1, 5, 11, 55\}$

Now which of these numbers are also found in set E?

$$\therefore A = \{1, 5, 11\}$$

SOLUTION b (page 3)

b. $B = \left\{x: \frac{1}{2}x \geq 5\frac{1}{2}\right\}$

HINTS; we first have to work out $\frac{1}{2}x \geq 5\frac{1}{2}$, we have to change $5\frac{1}{2}$ to an improper fraction, we have $\frac{11}{2}$ substituting into $\frac{1}{2}x \geq 5\frac{1}{2}$ we have;

$$\frac{1}{2}x \geq 5\frac{1}{2} \quad \Rightarrow \quad \frac{x}{2} \geq \frac{11}{2}$$

Cross multiply

$$\frac{x}{2} \geq \frac{11}{2} \qquad \Rightarrow \qquad 2x \geq 22$$

We now divide both sides by 2

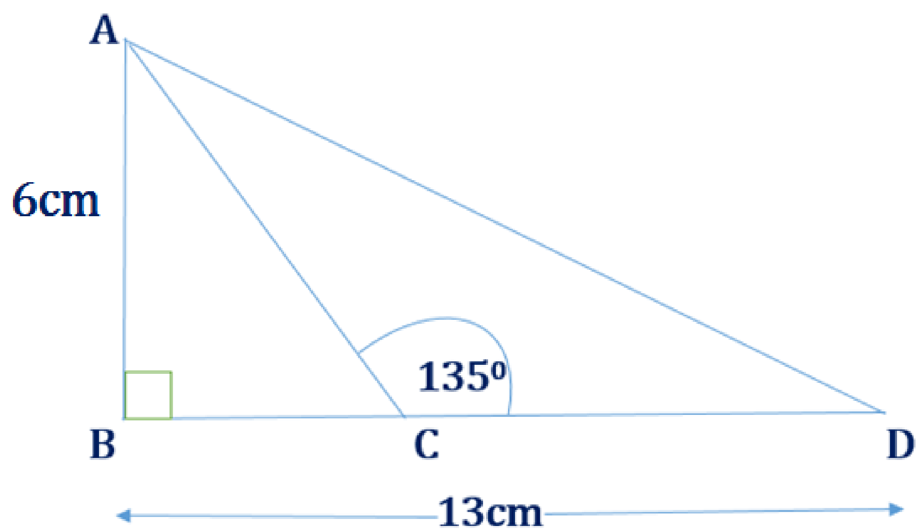
$$\frac{2x}{2} \geq \frac{22}{2} \text{ We have } x \geq 11$$

The $x \geq 11$ reads as x is greater or equal to 11, in this case we need to only list numbers equal to 11 and those greater than 11 from set E. We know that 11 is equal to 11 and its in set E, the only number greater than 11 in set E is 12 so we have ;

$$\therefore B = \{11, 12\}$$

LESSON NUMBER 25 QUESTION 13 P1 2012 (page1)

In the diagram below, BCD is a straight line, AB=6cm, BD=13cm $A\hat{C}D = 135^0$ and $A\hat{B}C = 90^0$



a. Calculate the length of CD
b. Calculate the area of triangle ACB

SOLUTION a (page 2)

For us to calculate the length of CD we must know BC so that we subtract BC from BD. Let us first find BC, to find BC we consider the angles ACB and ACD they are on the same straight line BD meaning we can find $A\hat{C}B$ by subtracting 135^0 from 180^0 hence angle $A\hat{C}B = 45^0$, We know that ABC is a triangle hence the sum of its interior angles is 180^0, we now have two known angle being 45^0 and 90^0, therefore the other angle BAC will be found as follows;

$$B\hat{A}C + A\hat{B}C + B\hat{C}A = 180^0 \Rightarrow B\hat{A}C + 90^0 + 45^0 = 180^0$$

Collecting like terms we have;

$$B\hat{A}C = 180^0 - 135^0 \qquad \Rightarrow \qquad B\hat{A}C = 45^0$$

We can see that $A\hat{C}B = B\hat{A}C = 45^0$. This means that the side opposite to $A\hat{C}B$ is equal to the side opposite to $B\hat{A}C$ therefore AB = BC = 6cm

Then we can calculate CD as follows;

$$BC + CD = BD$$

We can now replace for BC and BD

$$6 + CD = 13 \quad \Rightarrow \quad CD = 13 - 6 \qquad \therefore CD = 7cm$$

SOLUTION b (page 3)

To answer b. we have to calculate total area of the whole bigger triangle ABDA, then calculate area of smaller triangle ABCA. To find area of triangle ACDA we subtract area of triangle ABCA from area of the bigger triangle ABDA.

The formula for area of a right angled triangle is

$$A = \frac{1}{2} \times base \times height$$

We start with the bigger triangle

$$A = \frac{1}{2} \times 13 \times 6 \quad \Rightarrow \quad A = 13 \times 3$$

$\therefore A_{ABDA} = 39cm^2$

We can now calculate that of the smaller triangle ABCA

$$A = \frac{1}{2} \times base \times height$$

$$\therefore A = \frac{1}{2} \times 6 \times 6 \Rightarrow A = 3 \times 6$$

$\therefore A_{ABCA} = 18cm$

Now we have to subtract A_{ABCA} from A_{ABDA} to get A_{ACDA}

$A_{ACDA} = A_{ABDA} - A_{ABCA}$

We can now substitute

$A_{ACDA} = 39 - 18$

$\therefore A_{ACDA} = 21cm^2$

LESSON NUMBER 26 QUESTION 14 P1 2012 (page 1)

a. Solve the inequality $2y - 1 < 5$
b. The week 'Favor' went to South Africa, the exchange rate between the Zambian kwacha (K) and the US Dollar ($) was $K4,500$ to $\$1$. Given that she changed $K1,800,000$ to US$, how many Dollars did she receive?
c. Simplify $4(x - 6) - x(2x - 3)$

SOLUTION a (page 2)

Solve the inequality $2y - 1 < 5$

We have to treat $<$ just like we treat $=$ in this case so we first collect like terms

$$2y - 1 < 5 \quad \Rightarrow \quad 2y < 5 + 1$$

We then add

$$2y < 6$$

We then divide both sides by 2

$$\frac{2y}{2} < \frac{6}{2}$$

$$\therefore y < 3$$

SOLUTION b (page 3)

Here we use proportion and ratio, because $K4,500$ will give her $\$1$, but she had $K1,800,000$ we do not know how many Dollars are in $K1,800,000$

Kwacha to Dollar

$$4,500 = 1$$

$$1,800,000 = x$$

We then cross multiply to have

$$4,500\,x = 1,800,000$$

We now divide both sides by $4,500$

$$\frac{4,500\,x}{4,500} = \frac{1,800,000}{4,500}$$

$$\therefore x = \$400$$

SOLUTION c (page 4)

$$4(x - 6) - x(2x - 3)$$

Opening brackets

$$4x - 24 - 2x^2 + 3x$$

Collecting like terms

$$4x + 3x - 24 - 2x^2$$

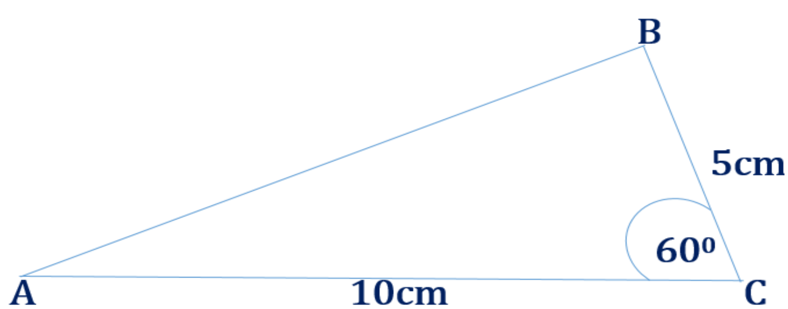
And the answer is $7x - 24 - 2x^2$

LESSON 28 QUESTION 15 P1 2012 (page 1)

a. A pack of eleven identical cards, are labeled 1 to 11, the cards are shuffled and placed upside down. If a card is picked at random from the pack, what is the probability that it is a prime numbered card?

b. In the diagram below AC =10cm, BC = 5cm and ACB = 60^0.

Given that $\sin 60^0$ = 0.866, $\cos 60^0$= 0.5 and $\tan 60^0$ = 1.73. Calculate the value of AB^2



OLUTIONS (Page 2)

To answer a. we have to understand the meaning of prime numbers and list the prime numbers from the 1 to 11 identical cards. Prime numbers are numbers with only two factors one and itself. A factor is a number which goes into a given number without leaving a remainder. So the set of prime numbers from the 11 cards will be

$$P = \{2, 3, 5, 7, 11\}$$

This shows that we have only 5 prime numbered cards

Probability of a prime numbered card = $\frac{prime\ numbered\ cards\ in\ the\ pack}{total\ number\ of\ cards\ in\ the\ pack}$

We know that there are 5 prime numbered cards and the total number of cards in the pack is 11 so we substitute

$\therefore$P (prime numbered card) = $\frac{5}{11}$

SOLUTIONS (Page 3)

To answer b. when you have two sides touching the angle and you know the lengths of the two sides, then you can use the two sides and the angle to calculate the length of the side opposite to the angle. In this case the side AB is opposite to the angle, and we have two sides touching that angle BC=5cm and AC=10cm. Use the cosine rule as follows;

$$AB^2 = AC^2+BC^2 - 2(AC)(BC)cos\theta$$

We can now substitute and have;

$$AB^2 = 10^2+5^2 - 2(10)(5)cos60^0$$

Evaluating the squared, multiplying and finding cos we have

$$AB^2 = 100 + 25 - (100 \times 0.5)$$

Adding and multiplying/opening the brackets

$$AB^2 = 125 - 50$$

$$\therefore AB^2 = 75cm^2$$

LESSON NUMBER 29 QUESTION 18 P1 2012 (page 1)

A map of a game park is drawn to a scale of 1: 50, 000.

a. **Two game camps, A and B are 7cm apart on the map. Find the actual distance between the two camps, in kilometres.**
b. **The actual area of the game park is 25km^2. Calculate the area of the game park on the map in square centimetres.**

SOLUTION (Page 2)

HINTS; The distance and area on the map uses cm and cm^2 respectively because a map is drawn on the paper hence you cannot measure using km on paper. The distance and area on the ground uses km and km^2 respectively because you cannot measure using cm on the ground.

SOLUTIONS (page 3)

To answer a. we have to use the formula of ratios and it goes as follows;

$$\frac{1}{n} = \frac{distance\ on\ the\ map\ (cm)}{distance\ on\ the\ ground\ (km)}$$

In this case n = 50,000, distance on the map is 7cm while distance on the ground is what we are looking for, so we substitute to have;

$\frac{1}{50{,}000} = \frac{7}{x}$ **Cross multiplying we have** $x = 350{,}000cm$

But distance on the ground is not supposed to be in cm so we have to convert 350, 000cm to kilometres

$$100,000 = 1$$

$$350,000 = x$$

We can now cross multiply to have

$$100,000x = 350,000$$

We now divide both sides by 100, 000

$$\frac{100,000x}{100,000} = \frac{350,000}{100,000} = 3.5$$

$$\therefore Distance\ on\ the\ ground = 3.5km$$

SOLUTIONS (Page 4)

To answer **b.** we have to use the same formula of ratio but this time for area and it goes as follows;

$$\frac{1}{n^2} = \frac{area\ on\ the\ map\ (cm^2)}{area\ on\ the\ ground\ (km^2)}$$

According to this question **n=50, 000**, area on the ground is **25km^2**, we are looking for area on the map so we can substitute as follows;

$$\frac{1}{50,000^2} = \frac{x}{25}$$

But the **25** is in square kilometres, but we want the area on the map and it must be in square centimetres so we convert **25km^2** to **cm^2**. To do so we have to multiply **25** by **100, 000^2**. So we have

$$\frac{1}{50\ 000^2} = \frac{x}{25 \times 100\ 000^2}$$

SOLUTIONS b cont. (Page 5)

We work out the denominators

$$\frac{1}{50\,000 \times 50\,000} = \frac{x}{25 \times 100\,000 \times 100\,000}$$

$$\frac{1}{2\,500\,000\,000} = \frac{x}{250\,000\,000\,000}$$

Cross multiplying we have;

$$2\,500\,000\,000x = 250\,000\,000\,000$$

Dividing both sides by 2 500 000 000

$$\frac{2\,500\,000\,000x}{2\,500\,000\,000} = \frac{250\,000\,000\,000}{2\,500\,000\,000}$$

$$\therefore area\ on\ the\ map = 100cm^2$$

LESSON NUMBER 30 QUESTION 9 PAPER1 2012 (page 1)

Solve the simultaneous equations $2x + 5y = 16$ and $3x - 2y = 5$

SOLUTIONS

HINTS; There are many methods of solving simultaneous equations but the easiest is the substitution method. In this method you have to make x or y the subject of the formula using one of the equations, then substitute into the other equation.

$$2x + 5y = 16 \text{ ………. i}$$

$$3x - 2y = 5 \text{………… ii}$$

Let us make x the subject of the formula using equation i by moving 5y across the equal sign and dividing both sides by 2.

$$2x + 5y = 16$$

$$\frac{2x}{2} = \frac{16-5y}{2} \qquad \therefore x = \frac{16-5y}{2} \text{ ....iii}$$

SOLUTIONS cont. (page 2)

We used equation i to make x the subject of the formula, so we will substitute for x in equation ii not i. So where there is x in ii we write $\frac{16-5y}{2}$

$$3x - 2y = 5 \quad \Rightarrow \quad 3\left(\frac{16-5y}{2}\right) - 2y = 5$$

We can now open the brackets by multiplying 3 by $(16 - 5y)$ we have;

$$\frac{48-15y}{2} - 2y = 5$$

We now multiply both sides by 2 so that we remove the fraction

$$\left(\frac{48-15y}{2}\right)(2) - (2y)(2) = (5)(2)$$

We have;

$$48 - 15y - 4y = 10$$

Adding and collecting like terms we have

$$-19y = 10 - 48 \quad \Rightarrow \quad -19y = -38$$

Dividing both sides by -19 we have;

$$\frac{-19y}{-19} = \frac{-38}{-19} \qquad \therefore y = 2$$

SOLUTIONS cont. (page 3)

Now that we have found the value of y, we can use any one of the two equations, to substitute for y, let's use equation i to find x.

$$2x + 5y = 16$$

$$y = 2$$

$$2x + 5(2) = 16$$

Multiplying 5 by 2

$$2x + 10 = 16$$

Collecting like terms

$$2x = 16 - 10 \quad \Rightarrow \quad 2x = 6$$

Dividing both sides by 2

$$\frac{2x}{2} = \frac{6}{2}$$

$$\therefore x = 3$$

LESSON NUMBER 31 QUESTION4 P2 2004 (page 1)

In the year 2000, workers in BM Company were awarded a pay rise of 25% based on their 1999 annual salaries. In 2001, they again received another increment of 12% on their 2000 salaries.

a. In 1999 Mr. Mulonga earned a salary of K3 000 per annum. Calculate his annual salary in the year

i. 2000

ii. 2001

b. In 2000 Mrs. Mulonga received an increment of K600 on her 1999 salary. What was her

i. Salary in 1999?

ii. Salary increment for the year 2001?

c. Mrs. Bwalya was wrongly missed in the award of increments in both years. However, after a thorough assessment, she received a total amount of K1 240 in arrears at the end of the year 2001, what was her salary in 1999?

SOLUTION a i (page 2)

To answer a. i we have to understand what **annum** means, it means a period of 12 months hence **one year**. If this person earned a salary of **K3000** per annum in **1999** then he will have a **higher** salary in **2000** because there was an increment of **25%** in that year calculated from **1999** annual salary. So we first calculate **25%** of **K3000** the salary for **1999**.

$$25\%\ of\ K3000 = \frac{25}{100} \times 3000 = 25 \times 30 = K750$$

That **K750** is the increment for the whole year **2000**, so we add it to the salary he was getting in the previous year **1999** and that was **3000**.

$$3000 + 750 = 3750$$

$$\therefore Annual\ salary\ for\ 2000 = K3\,750$$

SOLUTION a ii (page 3)

To answer **a. ii** we have to calculate his annual salary for the year **2001**. Meaning the salary to be increased is that of the year **2000** so that he gets a new annual salary in the year **2001**. The increment was **12%** of year **2000** salary

$$12\%\ of\ year\ 2000 = \frac{12}{100} \times 3\,750 = \frac{12}{10} \times 375 = \frac{12}{2} \times 75 = 6 \times 75 = 450$$

The 450 is the money to be added to the 2000 annual year salary in order to get the new salary for the year 2001 so we add

$$3\,750 + 450 = 4200$$

$$\therefore Annual\ salary\ for\ 2001 = K4200$$

SOLUTION b i (page 4)

To answer **b. i. if she received K600 increment in 2000, then the K600 was a 25% of the 1999 salary. We do not know how much she was earning in 1999 but K600 was a 25% of the 1999 salary, so we can say 1999** $salary = x$ **this can be written as;**

$$25\%\ of\ the\ 1999\ annual\ salary = K600 \Rightarrow \frac{25}{100} \times x = 600$$

We now multiply and write everything in fraction form

$$\frac{25x}{100} = \frac{600}{1}$$

$$we\ can\ now\ cross\ multiply\ to\ have;$$

$$25x = 60\ 000$$

We can now divide both sides by 25 to have;

$$\frac{25x}{25} = \frac{60\ 000}{25} \qquad \Rightarrow \qquad x = 2\ 400$$

$$\therefore Annual\ salary\ for\ 1999 = K2\ 400$$

SOLUTIONS b ii (page 5)

The annual salary for the year 2001 will be found by increasing the 2000 annual salary by its 12%. But we do not know the salary for 2000, however the annual salary is just found by increasing the 1999 by its 25%. So let's find the annual salary for the year 2000.

$$25\%\ of\ 1999\ annual\ salary\ = \frac{25}{100} \times 2\ 400 = 25 \times 24$$
$$= 600$$

$$We\ now\ add\ 1999\ annual\ salary\ + 600 = 2\ 400 + 600$$
$$= K3000$$

So we find 12% of the 2000 annual salary

$$12\%\ of\ the\ 2000\ annual\ salary = \frac{12}{100} \times 3000 = 12 \times 30$$
$$= 360$$

The 360 is the money to be added to the 2000 annual salary in order to get the 2001 annual salary so we add;

$$3000 + 360 = 3360 \quad \therefore Annual\ salary\ for\ 2001 = K3360$$

SOLUTION c (page 6)

To answer **c.** we have to understand that the K1 240 is the total amount of increments for 2000 and 2001. Now they are asking us to calculate how much she was earning in 1999. So let's take x to be the **annual salary** for the year **1999**. We know that **25% of 1999** annual salary is the increment for the year 2000, $25\%\ of\ x + x$ is going to be the salary for the year **2000**. Meaning that the increment for **2001** is **12% of the 2000 annual salary**, follow the colours in the following

If Annual salary for $1999 = x$ then increment for $2000 = 25\%\ of\ x$

If Annual salary for $2000 = 25\%\ of\ x + x$

$$increment\ for\ 2001 = 12\%\ of\ 2000\ annual\ salary$$
$$= 12\%(25\%\ of\ x + x)$$

Total increment in arrears = $increment\ for\ 2000 + increment\ for\ 2001$

$$\therefore Formula\ for\ increment\ in\ arrears = [25\%\ of\ x] + [12\%(25\%\ of\ x + x)]$$

We can now write the percentages in expanded form

$$\left[\frac{25}{100} \times x\right] + \left[\frac{12}{100} \times \left(\frac{25}{100} \times x + x\right)\right] = \left[\frac{25x}{100}\right] + \left[\frac{12}{100}\left(\frac{25x}{100} + x\right)\right]$$

SOLUTION c cont. (page 7)

We know that total arrears = 1 240 se we equate

$$\left[\frac{25x}{100}\right] + \left[\frac{12}{100}\left(\frac{25x}{100} + x\right)\right] = 1\,240$$

We now work out what is inside the brackets

$$\frac{25x}{100} + \left[\frac{12}{100}\left(\frac{25x+100x}{100}\right)\right] = 1\,240 \quad \Rightarrow \quad \frac{25x}{100} + \left[\frac{12}{100}\left(\frac{125x}{100}\right)\right] = 1\,240$$

$$\frac{25x}{100} + \frac{1500x}{10\,000} = 1\,240 \quad \Rightarrow \quad \frac{2500x+1500x}{10\,000} = 1\,240$$

Adding, changing to fraction for and cross multiplying we have

$$\frac{4000x}{10\,000} = 1\,240 \quad \Rightarrow \quad \frac{4000x}{10\,000} = \frac{1\,240}{1}$$

We now cross multiply

$$4000x = 12\,400\,000$$

Dividing both sides by 4000 to find x

$$\frac{4000x}{4000} = \frac{12\,400\,000}{4000} \therefore x = 3100$$

$$\therefore \textit{Annual salary for } 1999 = K3100$$

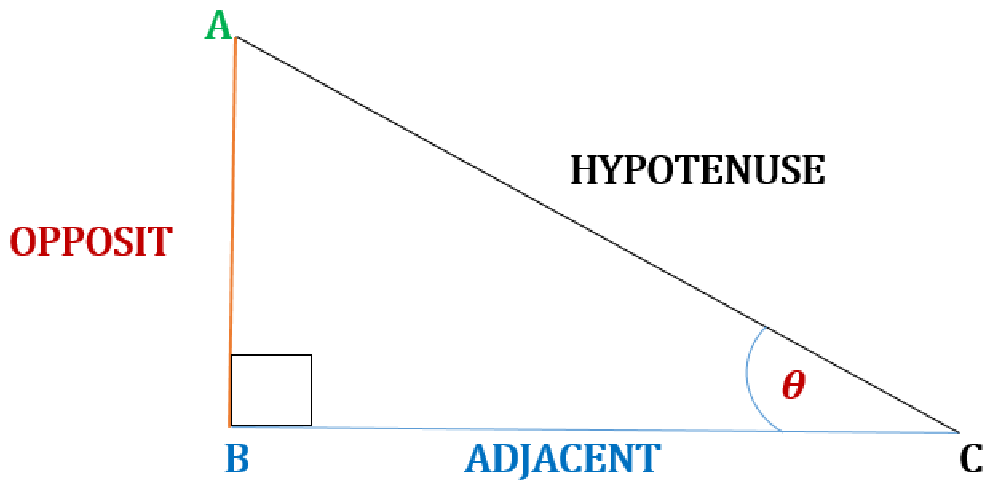
LESSON 32 TRIGOMENTRY (page 1)

Trigonometry deals with relations between sides and angles of a given triangle. In trigonometry we use the three trig-ratios, sine and cosine rule to find angles and sides of a given triangle; we also find area using the sine formula and also plot on XOY plane.

THREE TRIG-RATIOS

The three trig-ratios are modelled into a short form called SOH,CAH,TOA

Consider the right angled-triangle given below;



EXPLAINATION (PAGE 2)

Hypotenuse AC is the longest side touching the angle θ, adjacent BC is the shortest side touching the angle θ and opposite AB is the side facing angle θ

SOH,CAH,TOA

This model has three parts

SOH stands for Sinθ is equal to OPPOSITE over HYPOTENUSE

$$Sin\ \theta = \frac{\text{OPPOSITE}}{\text{HYPOTENUSE}}$$

CAH stands for Cosθ is equal to ADJACENT over HYPOTENUSE

$$\text{Cos}\theta = \frac{ADJACENT}{HYPOTENUSE}$$

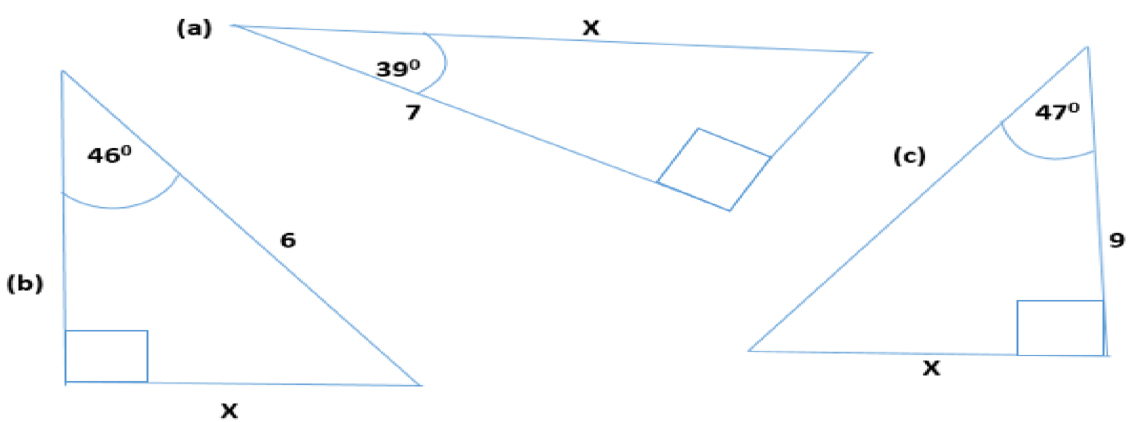
TOA stands for Tanθ is equal to OPPOSITE over ADJACENT

$$\text{Tan}\theta = \frac{OPPOSITE}{ADJACENT}$$

EXAMPLES (PAGE 3)

1. Finding sides using SOHCAHTOA

1. In each of the following right-angled triangles calculate the distance marked by **x** in cm giving your answers correct to one decimal place.



SOLUTIONS (page 4)

To answer (a) we have to first identify the type of trig-ratio we need to use, so let us collect the data we have in the question

DATA

$$\theta = 39^0$$

$$Adjacent = 7$$

$$Hypotenuse = x$$

So we are looking for hypotenuse, there are only two trig-rations we can use SOH and CAH, when looking for hypotenuse we cannot use SOH because we do not have opposite in the data so we use

$\text{Cos}\boldsymbol{\theta} = \frac{ADJACENT}{HYPOTENUSE}$ substituting we have $\text{Cos}39^0 = \frac{7}{x}$ finding $\text{Cos}39^0$ we have;

$0.777 = \frac{7}{x} \quad \Rightarrow \quad \frac{0.777}{1} = \frac{7}{x}$ Cross multiplying we have; $0.777x = 7$

Dividing both sides by 0.777

$\frac{0.777x}{0.777} = \frac{7}{0.777} \qquad \therefore x = 9.01$

SOLUTIONS (page 5)

To answer (b) we have to first identify the type of trig-ratio we need to use, so let us collect the data we have in the question

DATA

$\theta = 46^0$

$Opposite = X$

$Hypotenuse = 6$

So we are looking for opposite, there are only two trig-rations we can use SOH and TOA, when looking for opposite we cannot use TOA because we do not have adjacent in the data so we use

$Sin\ \theta = \frac{\text{OPPOSITE}}{\text{HYPOTENUSE}}$ Substituting we have $Sin46^0 = \frac{X}{6}$ finding $\text{Sin}46^0$ we have;

$$0.719 = \frac{X}{6} \quad \Rightarrow \quad \frac{0.719}{1} = \frac{X}{6}$$

Cross multiplying we have;

$$\therefore x = 4.32$$

SOLUTIONS (page 6)

To answer (c) we have to first identify the type of trig-ratio we need to use, so let us collect the data we have in the question

DATA

$$\theta = 47^0$$

$$Opposite = X$$

$$Adjacent = 9$$

So we are looking for opposite, there are only two trig-rations we can use SOH and TOA, when looking for opposite we cannot use SOH because we do not have hypotenuse in the data so we use

$\mathrm{Tan}\theta = \frac{OPPOSITE}{ADJACENT}$ **Substituting we have** $\mathrm{Tan}47^0 = \frac{x}{9}$ **finding** $\mathrm{Tan}47^0$ **we have;**

$$1.072 = \frac{X}{9} \quad \Rightarrow \quad \frac{1.072}{1} = \frac{X}{9}$$

Cross multiplying we have;

$$\therefore x = 9.65$$

(page 7)

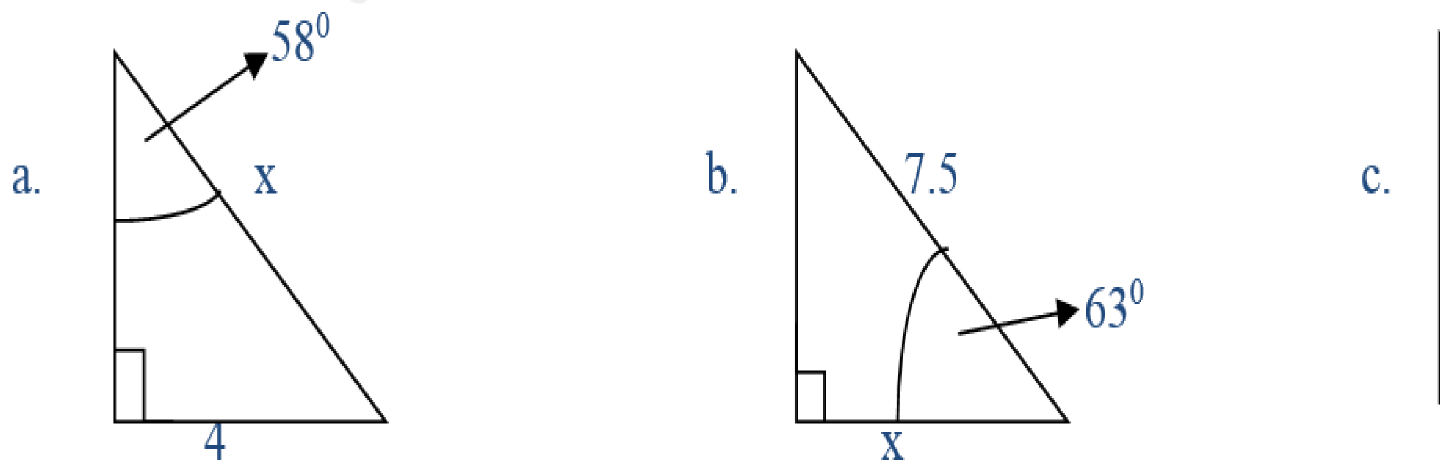
TASK; Copy the following questions, you will answer them then post your correct answers tomorrow from 19:30 to 20:00 hours. You will not be allowed to post your answer after 20 hours. Do not post answers in picture form just say;

$$(a)\ x = \ldots (b) x = \ \ldots\ (c) x = \cdots$$

Follow the rules or else you will be punished to sweep the class on Monday

EXERCISE 1;

In each of the following calculate the side marked by x in cm, giving your answ

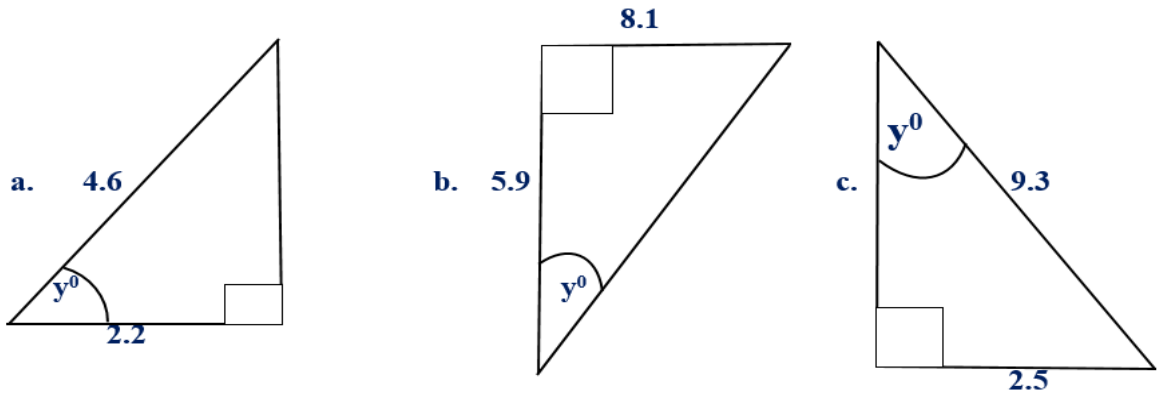


LESSON 33 TRIGOMENTRY (page 1)

1. FINDING ANGLES

Examples 2

In each of the following calculate the angle marked by y^0 giving your answers correct to 1dp.



SOLUTIONS (page 2)

To answer (a) we have to first identify the type of trig-ratio we need to use, so let us collect the data we have in the question. (Put your calculator in degrees)

DATA

$$\theta = y^0$$

$$Hypotenuse = 4.6$$

$$Adjacent = 2.2$$

According to what we have, the only trig ratio that can connect the three parts of the triangle is cosine so we use

CAH

$$\text{Cos}\theta = \frac{ADJACENT}{HYPOTENUSE} \quad \Rightarrow \quad \text{Cos}y^0 = \frac{2.2}{4.6}$$

Dividing we have;

$\text{Cos}y^0 = 0.47826$ **To find** y^0

Press Shift or 2nd f, 0.47826 when it shows; Cos^{-1} 0.47826 then press =

$$\therefore y^0 = 61.4^0$$

SOLUTIONS (page 3)

To answer (b) we have to first identify the type of trig-ratio we need to use, so let us collect the data we have in the question. (Put your calculator in degrees)

DATA

$$\theta = y^0$$

$$opposite = 8.1$$

$$Adjacent = 5.9$$

According to what we have, the only trig ratio that can connect the three parts of the triangle is tangent so we use **TOA**

$$\text{Tan}\theta = \frac{OPPOSITE}{ADJACENT} \quad \Rightarrow \quad \text{Tan}y^0 = \frac{8.1}{5.9}$$

Dividing we have;

$\text{Tan}y^0 = 1.37288$ To find y^0

Press Shift or 2ndf, 1.37288 when it shows; $\tan^{-1}$ 1.37288 then press =

$$\therefore y^0 = 53.9^0$$

SOLUTIONS (page 4)

To answer (c) we have to first identify the type of trig-ratio we need to use, so let us collect the data we have in the question. (Put your calculator in degrees)

DATA

$$\theta = y^0$$

$$opposite = 2.5$$

$$Hypotenuse = 9.3$$

According to what we have, the only trig ratio that can connect the three parts of the triangle is sine so we use SOH

$$\text{Sin}\theta = \frac{OPPOSITE}{HYPOTENUSE} \quad \Rightarrow \quad \text{Sin}y^0 = \frac{2.5}{9.3}$$

$$\text{Sin}y^0 = 0.2688$$

Press Shift or 2^nd^f, 1.37288 when it shows; tan^{-1} 1.37288 then press =

$$\therefore y^0 = 15.6^0$$

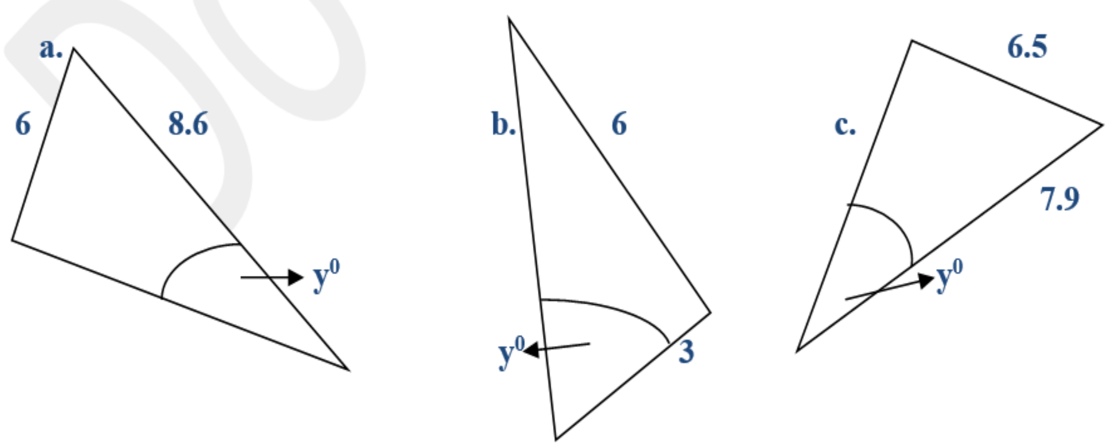
PAGE 5

TASK; Copy the following questions, you will answer them then post your correct answers on monday from 19:30 to 20:00 hours. You will not be allowed to post your answer after 20 hours. Do not post answers in picture form just say;

$$(a)\ y = \ldots (b) y = \ \ldots \ (c) y = \cdots$$

EXERCISE 2

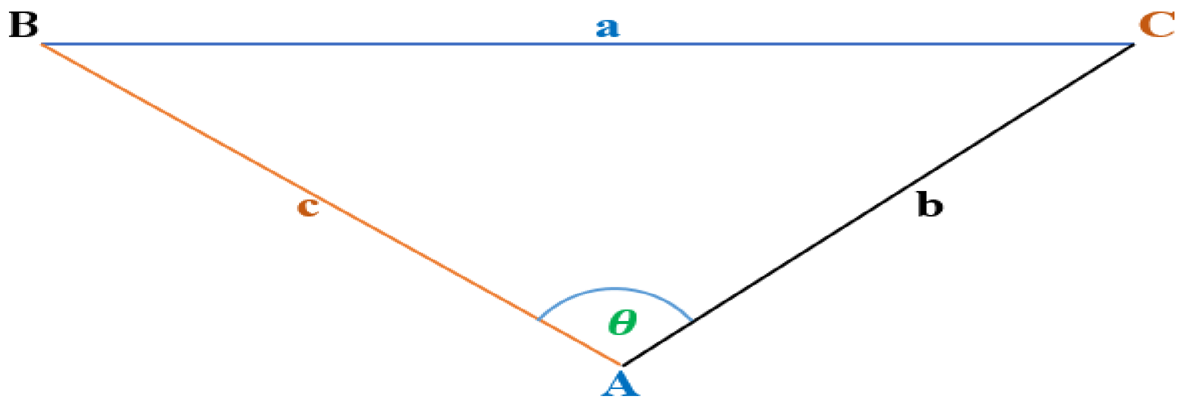
In each of the following right-angled triangles find the angle marked by y^0 correct to 2dp.



LESSON 34 COSINE RULE (page 1)

For any none-right angled triangle, any two of the adjacent sides forms an angle ϴ which is always opposite to the other side also adjacent to both sides forming up that angle.

Consider the diagram below;



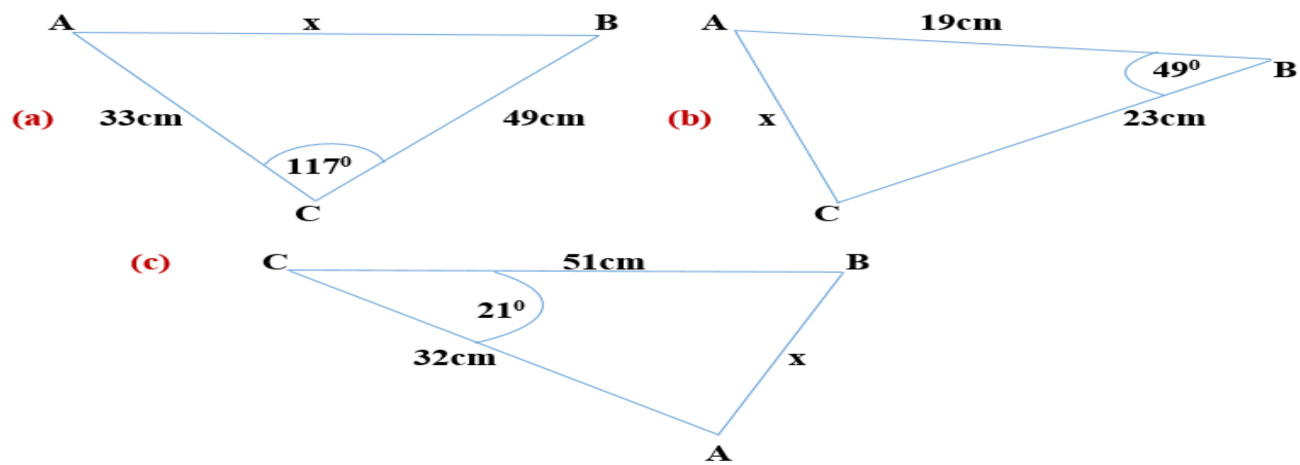
In the diagram above AC is opposite to the joint of sides at 'B' therefore we mark AC by small letter 'b', AB is opposite to the joint of sides at 'C' therefore we mark AB by small letter 'c' and BC is opposite to the joint of sides at 'A' therefore we mark BC by small letter 'a'. In the same diagram angle or $C\hat{A}B = \theta$ is opposite to BC=a. This information can be connected by the following mathematical model called Cosine Rule if we have to calculate 'a'.

$$a^2 = b^2 + c^2 - [2bc \times \cos\theta]$$

EXAMPLES: Page 2

In each of the following calculate the length of the side marked by 'x' in cm correct to 1dp.

Examples: In each of the following calculate the length of the side marked by 'x' in cm correct to 1dp.



SOLUTIONS PAGE 3

HINT; To answer (a) we have to write small letter a on BC, b on AC and c on AB. Then the formula will be $AB^2 = AC^2 + BC^2 - [2 \times AC \times BC \times \cos A\hat{B}C]$ substituting the small opposite side representative letters we have $c^2 = b^2 + a^2 - [2ba \times \cos \theta]$. We can now use the numbers.

$$c^2 = b^2 + a^2 - [2ba \times \cos \theta]$$

$$x^2 = 33^2 + 49^2 - [2 \times 33 \times 49 \times \cos 117^0]$$

Evaluating the squared, multiplying inside brackets and finding $\cos 117^0$ we have

$$x^2 = 1089 + 2401 - [3234 \times (-0.453990499)]$$

Adding the first 2 number and multiplying inside brackets we have

$$x^2 = 3490 - [-1468.205276]$$

We know that $- \times - = +$ so we have;

$$x^2 = 3490 + 1468.205274 \Rightarrow x^2 = 4957.205276$$

To find x we find the square root of $\mathbf{4957.205276}$by introducing √ on both sides

$$\sqrt{x^2} = \sqrt{4957.205276} \qquad \therefore x = 70.4\text{cm}$$

SOLUTIONS page 4

HINT; To answer (b) we have to write small letter a on BC, b on AC and c on AB. Then the formula will be $AC^2 = AB^2 + BC^2 - [2 \times AB \times BC \times \cos A\hat{B}C]$ substituting the small opposite side representative letters we have $b^2 = a^2 + c^2 - [2ac \times \cos \theta]$

We can now use the numbers.

$$b^2 = a^2 + c^2 - [2ac \times \cos \theta]$$

$$x^2 = 23^2 + 19^2 - [2 \times 23 \times 19 \times \cos 49^0]$$

Evaluating the squared, multiplying inside brackets and finding $\cos 49^0$ we have

$$x^2 = 529 + 361 - [874 \times 0.656059029]$$

Adding the first 2 number and multiplying inside brackets we have

$$x^2 = 890 - 573.3955913 \quad \Rightarrow \quad x^2 = 316.6044$$

To find x we find the square root of $\mathbf{2916.604409}$ by introducing √ on both sides

$$\sqrt{x^2} = \sqrt{316.6044} \qquad \therefore x = 17.8\text{cm}$$

SOLUTIONS PAGE 5

HINT; To answer (c) we have to write small letter a on BC, b on AC and c on AB. Then the formula will be $AB^2 = AC^2 + BC^2 - [2 \times AC \times BC \times \cos A\widehat{C}B]$ substituting the small opposite side representative letters we have $c^2 = a^2 + b^2 - [2ab \times \cos \theta]$

We can now use the numbers.

$$b^2 = a^2 + c^2 - [2ac \times \cos \theta]$$

$$x^2 = 32^2 + 51^2 - [2 \times 32 \times 51 \times \cos 21^0]$$

Evaluating the squared, multiplying inside brackets and finding $\cos 21^0$ we have

$$x^2 = 1024 + 2601 - [3264 \times 0.933580426]$$

Adding the first 2 number and multiplying inside brackets we have

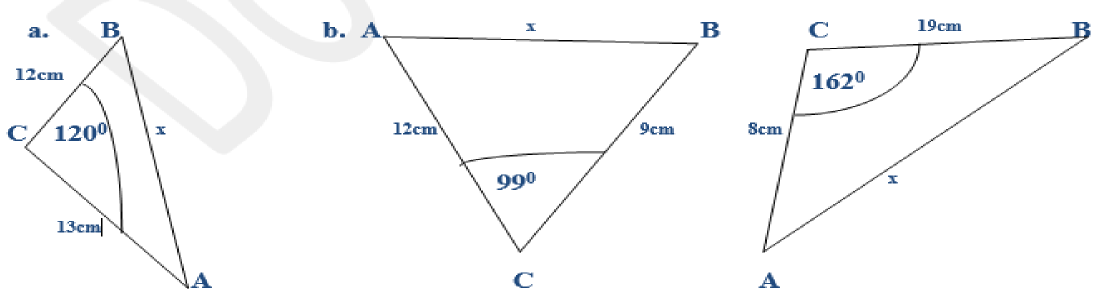
$$x^2 = 3625 - 3047.206512 \quad \Rightarrow \quad x^2 = 577.793488$$

To find x we find the square root of 577.793488 by introducing $\sqrt{}$ on both sides

$$\sqrt{x^2} = \sqrt{577.793488} \qquad \therefore x = 24.0\text{cm}$$

EXERCISE 3

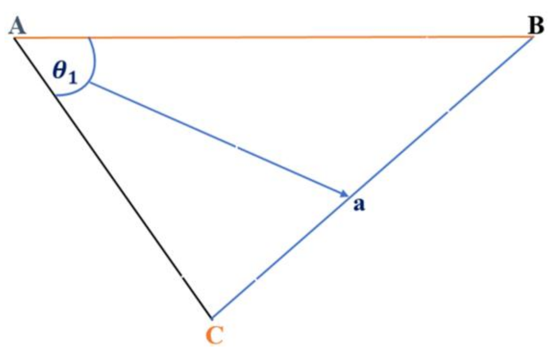
In each of the following shapes, calculate the length of the side marked by x in cm correct to 1dp.

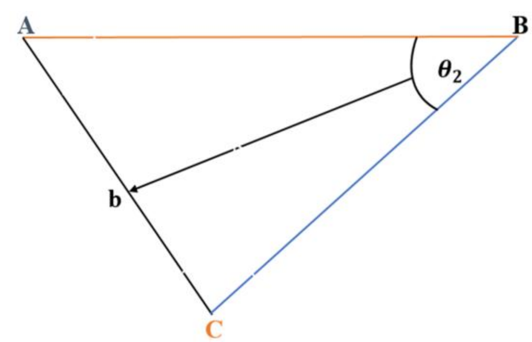


LESSON 35 SINE RULE PAGE 1

The sine rule is used when calculating a side provided that two interior angles and a side are given or calculating an angle when two sides and an interior angle are given for a triangle.

Consider the following shapes and follow up on the mathematical model (sine rule)

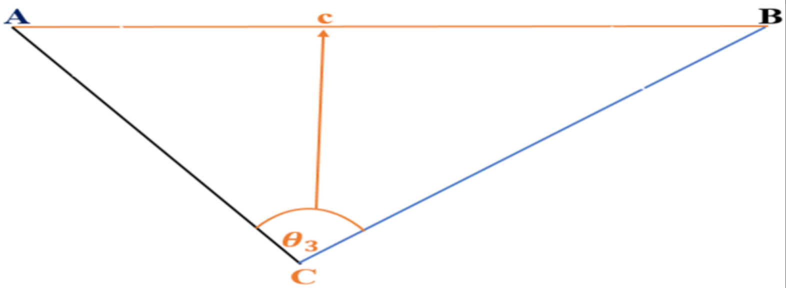
<table>
<tr>
<td>In the triangle below the point A and θ_1 are opposite to the side BC so we write small a on BC


This can be written in fraction form as;
$$\frac{a}{sin\theta_1}$$</td>
<td>In the triangle below the point B and θ_2 are opposite to the side AC so we write small b on AC


This can be written in fraction form as;
$$\frac{b}{sin\theta_2}$$</td>
</tr>
</table>

PAGE 2

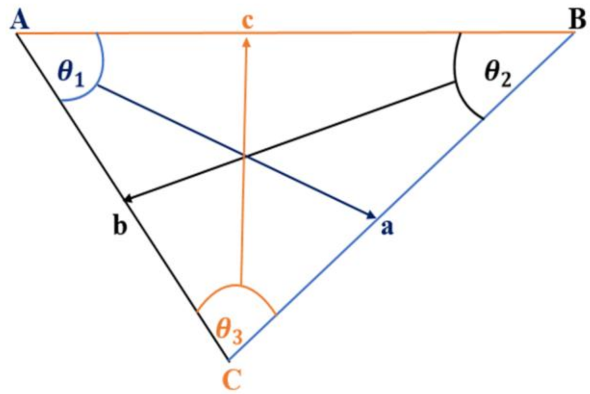
In the triangle below the point C and θ_3 are opposite to the side AB so we write small c on AB



This can be written in fraction form as;

$$\frac{c}{sin\theta_3}$$

In the triangle below the points discussed in the 3 triangles are combined in 1 sharp. Then we have to equate the three sine fractions



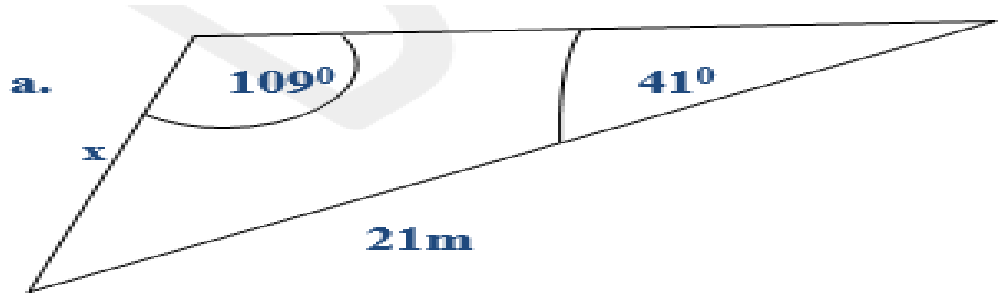
This gives us the sine rule;

$$\frac{a}{sin\theta_1} = \frac{b}{sin\theta_2} = \frac{c}{sin\theta_3}$$

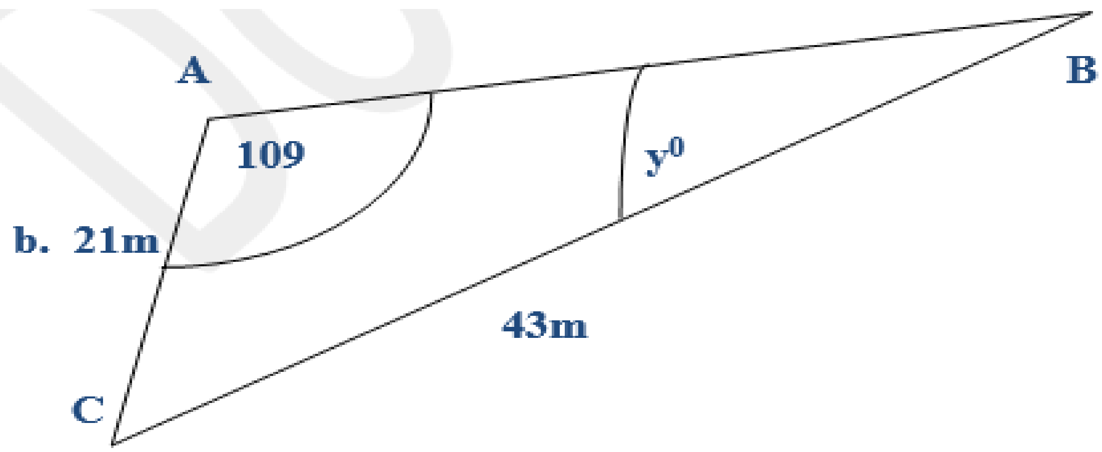
SINE RULE PAGE 3

Examples 4

a. **In the following shape calculate the length of the side marked by 'x' in m correct to 1dp**



b. **In the following diagram calculate y^0 giving the answer correct to 1dp.**



SOLUTIONS PAGE 4

HINT; when using the sine rule we only equate two fractions not three. To answer (a) the angle 41^0 is opposite to the side where x is so we can write this in fraction form as; $\frac{x}{sin41^0}$ we also have angle 109^0 opposite to the side 21m, this can be

written in fraction form as; $\frac{21}{sin109^0}$ we can now equate the two fractions as follows;

$$\frac{x}{sin41^0} = \frac{21}{sin109^0}$$

Cross multiplying we have

$$xsin109^0 = 21 \times sin41^0$$

Finding sine on both sides we have

$$0.945518575x = 21 \times 0.656059029$$

Multiplying on the right we have;

$$0.945518575x = 13.77723961$$

Dividing both sides by 0.945518575

$$\frac{0.945518575x}{0.945518575} = \frac{13.77723961}{0.945518575} \qquad \therefore x = 14.6m$$

SOLUTIONS PAGE 5

HINT; when using the sine rule we only equate two fractions not three. To answer (b) the angle 109^0 is opposite to the side where 43m is so we can write this in fraction form as; $\frac{43}{sin109^0}$ we also have angle y^0 opposite to the side 21m, this can be written in fraction form as; $\frac{21}{siny^0}$ we can now equate the two fractions as follows;

$$\frac{21}{siny^0} = \frac{43}{sin109^0}$$

Cross multiplying we have

$$43 \times siny^0 = 21 \times sin109^0$$

Finding sine on the right side we have

$$43 \times \sin y^0 = 21 \times 0.945518575$$

Multiplying on the right we have;

$$43\sin y^0 = 19.85589009$$

Dividing both sides by 43

$$\frac{43\sin y^0}{43} = \frac{19.85589009}{43} \quad \Rightarrow \quad \sin y^0 = 0.461765885$$

Press shift or 2ndf, sin 0.461765885 and press $=$

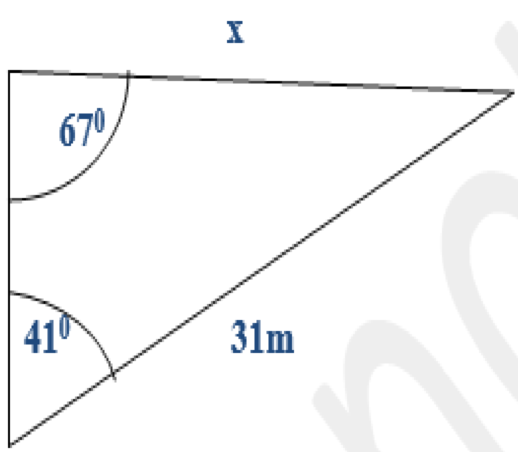
$$\therefore y = 27.5^0$$

PAGE 6

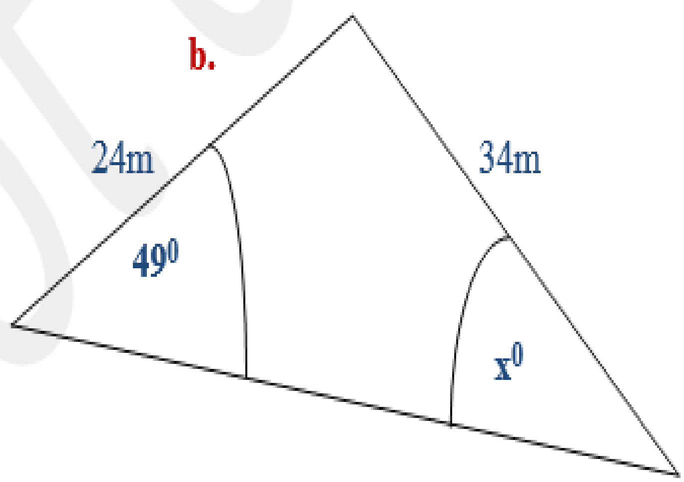
EXERCISE 4

1. In each of the following shapes calculate the length or angle marked by 'x' in m or degrees correct to 1dp.

a.

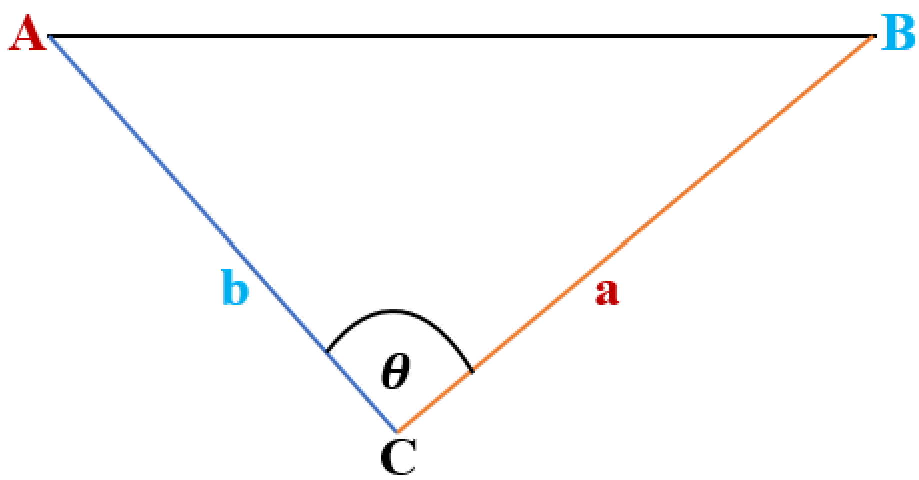


b.



LESSON 36 FINDING AREA PAGE 1

We can also use sine to calculate the area of a triangle provided that we have two known adjacent sides to a known angle. Study the figure below and derive the sine formula for Area of a triangle.



The formula for area of the triangle above is

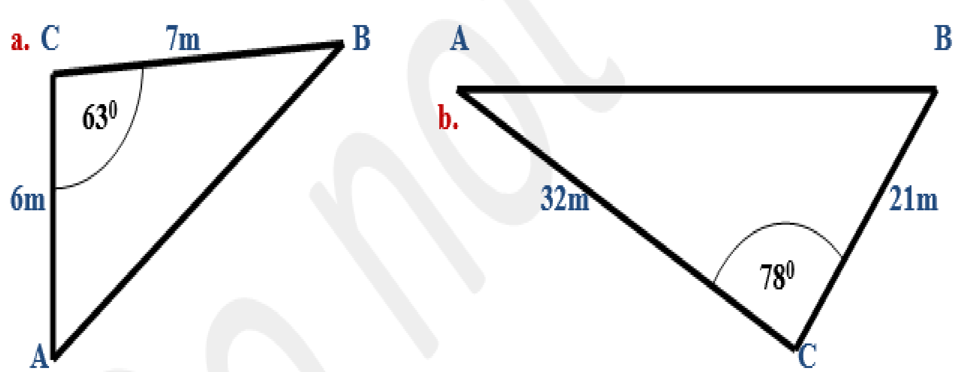
$$Area = \frac{1}{2} \times a \times b \times \sin\theta$$

Note; a and b must be adjacent to the angle

FINDING AREA PAGE 2

Examples 5

Find the area of each of the following triangles in m^2 giving the answer correct to 1dp.



SOLUTIONS PAGE 3

HINTS; to answer (a) we will use the same formula $Area = \frac{1}{2} \times a \times b \times sin\,\theta$ in this case the sides $a = 7m$ and $b = 6m$ are well known and are adjacent to the angle $\theta = 63^0$ so we can just substitute into the formula;

$$Area = \frac{1}{2} \times 7 \times 6 \times sin\,63^0$$

Dividing 2 into 6 and finding sin63^0 we have

$$Area = 1 \times 7 \times 3 \times 0.891006524$$

$$\therefore Area = 18.7m^2$$

SOLUTIONS PAGE 4

HINTS; to answer (b) we will use the same formula $Area = \frac{1}{2} \times a \times b \times sin\,\theta$ in this case the sides $a = 21m$ and $b = 32m$ are well known and are adjacent to the angle $\theta = 78^0$ so we can just substitute into the formula;

$$Area = \frac{1}{2} \times 21 \times 32 \times sin\,78^0$$

Dividing 2 into 32 and finding sin78^0 we have

$$Area = 1 \times 21 \times 16 \times 0.9781476$$

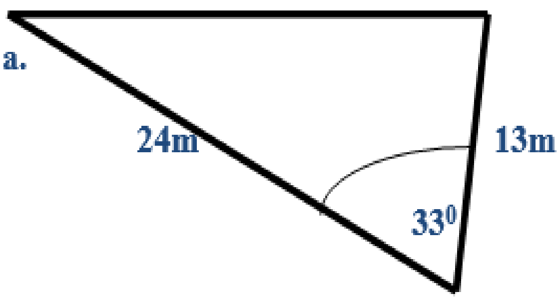
$$\therefore Area = 328.7m^2$$

PRACTICE QUESTIONS PAGE 5

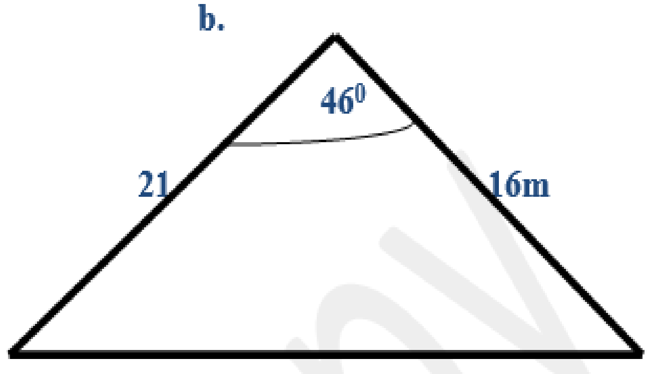
EXERCISE 5,

Calculate the area for each of the following triangle in m^2 answer correct to 1dp.

a.



b.



ANSWERS

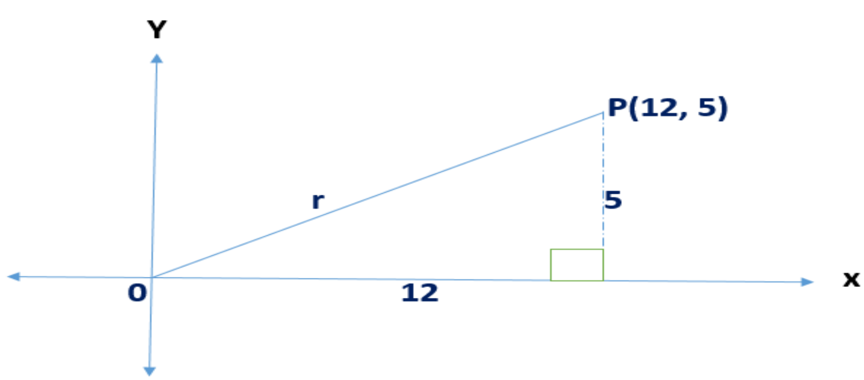
a. $\boldsymbol{A = 85.0m^2}$

b. $\boldsymbol{A = 120.8m^2}$

LESSON 37 TRIGONOMETRY ON CARTESIAN PLANE PAGE 1

HINT; this is something new in ordinary mathematics, it's from the new syllabus make sure you follow all the steps.

In the diagram below, P has coordinates (12, 5). Find the values of sin $X\hat{O}P$, cos $X\hat{O}P$ and tan $X\hat{O}P$



SOLUTIONS PAGE 2

HINT; the Cartesian plane has **two** axes, the **X** and the **Y axis**. The x-axis is the **horizontal** line while the y-axis is the **vertical** line. The point **P(12, 5)** corresponds to **P(x, y)**, this means that $x = 12$ (movement of **12** units from **0** along the **x-axis**) and $y = 5$ (movement of **5** units from **0** along the **y-axis**). This means that; to go to point **P(12, 5)**, the starting point is **0**, so you move **12** steps horizontally and then **5** steps vertically in the vertical axis. If we consider the triangle XOP we will see that; $r = hypotenuse$, $12 = adjacent$ and $5 = opposite$

SOLUTIONS sin $X\hat{O}P$ PAGE 3

To answer **sin** $X\hat{O}P$ we have to remember the **SOHCAHTOA**, this means that **sin** $X\hat{O}P$ is coming from **SOH**. We also know that **SOH** stands for $sin\theta = \frac{Opposite}{Hypotenuse}$ so we can just substitute.

$$\sin X\hat{O}P = \frac{5}{r}$$

We do not know the length of r, we can consider finding **r** first using Pythagoras theorem.

$$(hypotenuse)^2 = (opposite)^2 + (adjacent)^2$$

$$r^2 = 5^2 + 12^2 \quad \Rightarrow \quad r^2 = 25 + 144$$

$$r^2 = 169 \quad \Rightarrow \quad \sqrt{r^2} = \sqrt{169} \qquad \therefore r = 13$$

SOLUTIONS sin $X\hat{O}P$ PAGE 4

To answer sin $X\hat{O}P$ we have to remember the SOHCAHTOA, this means that sin $X\hat{O}P$ is coming from SOH. We also know that SOH stands for $sin\theta = \frac{Opposite}{Hypotenuse}$ so we can just substitute because we now have the value of r.

$$sin\, X\hat{O}P \; = \frac{5}{r} \qquad \Rightarrow \qquad sin\, X\hat{O}P \; = \frac{5}{13}$$

Dividing 5 by 13 we have

$$\therefore sin\, X\hat{O}P \; = 0.38$$

SOLUTIONS cos $X\hat{O}P$ PAGE 5

To answer cos $X\hat{O}P$ we have to remember the SOHCAHTOA, this means that cos $X\hat{O}P$ is coming from CAH. We also know that CAH stands for $cos\theta = \frac{adjacent}{Hypotenuse}$ so we can just substitute.

$$cos\, X\hat{O}P \; = \frac{12}{r} \Rightarrow cos\, X\hat{O}P \; = \frac{12}{13}$$

Dividing 12 by 13 we have;

$$\therefore cos\, X\hat{O}P \; = 0.923$$

SOLUTIONS tan $X\hat{O}P$ PAGE 6

To answer tan $X\hat{O}P$ we have to remember the SOHCAHTOA, this means that tan $X\hat{O}P$ is coming from TOA. We also know that CAH stands for $tan\theta = \frac{opposite}{adjacent}$ so we can just substitute.

$$\tan X\hat{O}P = \frac{5}{12}$$

Dividing 5 by 12 we have;

$$\therefore \tan X\hat{O}P = 0.41667$$

QUESTIONS FOR YOUR PRACTICE PAGE 7

For each of the following points, calculate the values of $sin\theta$, $cos\theta$ and $tan\theta$.

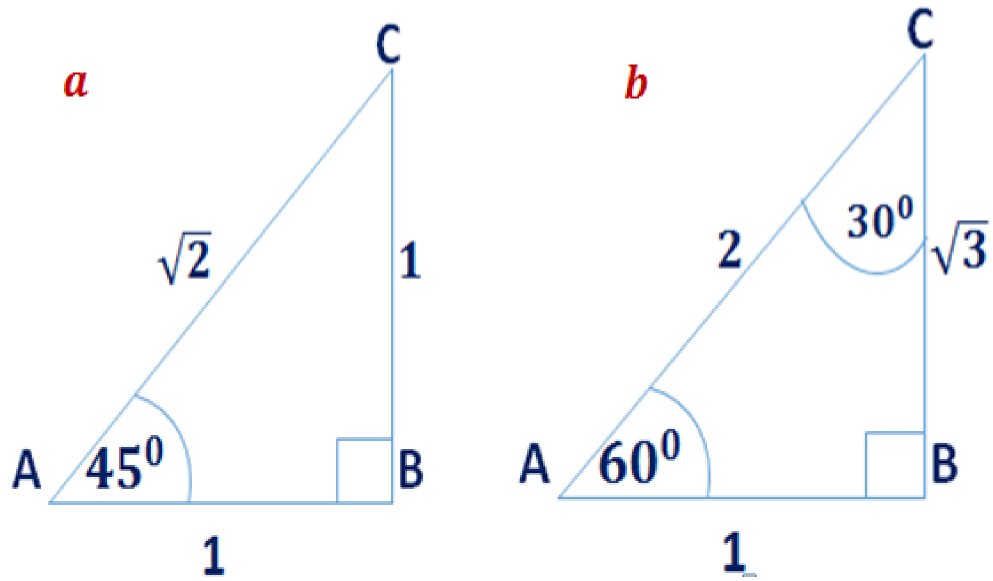
P(6, 8)

ANSWERS

$$sin\theta = 0.8 \qquad cos\theta = 0.6 \qquad tan\theta = 1.333.$$

LESSON 38 SPECIAL ANGLES PAGE 1

Special angles are angles whose ratios can be found without using a calculator. There are three special angles 45^0, 30^0 and 60^0. We work out questions on special angles by relating to the following triangles



QUESTIONS PAGE 2

Evaluate the following without using a calculator

a. sin 45^0
b. cos 45^0
c. tan 45^0
d. sin 30^0
e. cos30^0
f. tan30^0
g. sin 60^0
h. cos 60^0
i. tan 60^0

SOLUTION a PAGE 3

HINTS; to answer a, we have to relate to triangle a) on page 1 using SOHCAHTOA

$$sin\theta = \frac{Opposite}{Hypotenuse}$$

In this case $Opposite = 1, \theta = 45^0$ and $Hypotenuse = \sqrt{2}$. We now substitute to find the answer

$$\boxed{\therefore sin45^0 = \frac{1}{\sqrt{2}}}$$

SOLUTION b PAGE 4

In this case $Adjacent = 1, \theta = 45^0$ and $Hypotenuse = \sqrt{2}$.

$$cos\theta = \frac{adjacent}{Hypotenuse}$$

We now substitute to find the answer

$$\therefore cos45^0 = \frac{1}{\sqrt{2}}$$

SOLUTION c PAGE 5

In this case $Adjacent = 1, \theta = 45^0$ and $opposite = 1$.

$$tan\theta = \frac{opposite}{adjacent}$$

We now substitute to find the answer

$$tan45^0 = \frac{1}{1}$$

$$\therefore tan45^0 = 1$$

SOLUTION d PAGE 6

HINTS; to answer d, we have to relate to triangle b) on page 1 using SOHCAHTOA

In this case $Opposite = 1, \theta = 30^0$ and $Hypotenuse = 2$.

$$sin\theta = \frac{Opposite}{Hypotenuse}$$

We now substitute to find the answer

$$\therefore sin30^0 = \frac{1}{2}$$

SOLUTION e PAGE 7

In this case $Adjacent = \sqrt{3}, \theta = 30^0$ and $Hypotenuse = 2$.

$$cos\theta = \frac{adjacent}{Hypotenuse}$$

We now substitute to find the answer

$$\therefore cos30^0 = \frac{\sqrt{3}}{2}$$

SOLUTION f PAGE 8

In this case $Adjacent = \sqrt{3}, \theta = 45^0$ and $opposite = 1$.

$$tan\theta = \frac{opposite}{adjacent}$$

We now substitute to find the answer

$$\therefore tan30^0 = \frac{1}{\sqrt{3}}$$

SOLUTION g PAGE 9

HINTS; to answer d, we have to relate to triangle b) on page 1 using SOHCAHTOA

In this case $Opposite = \sqrt{3}, \theta = 60^0$ and $Hypotenuse = 2$.

$$sin\theta = \frac{Opposite}{Hypotenuse}$$

We now substitute to find the answer

$$\therefore sin60^0 = \frac{\sqrt{3}}{2}$$

SOLUTION h PAGE 10

In this case $Adjacent = 1, \theta = 60^0$ and $Hypotenuse = 2$.

$$cos\theta = \frac{adjacent}{Hypotenuse}$$

We now substitute to find the answer

$$\therefore cos60^0 = \frac{1}{2}$$

SOLUTION i PAGE 11

In this case $Adjacent = 1, \theta = 60^0$ **and** $opposite = \sqrt{3}$.

$$tan\theta = \frac{opposite}{adjacent}$$

We now substitute to find the answer

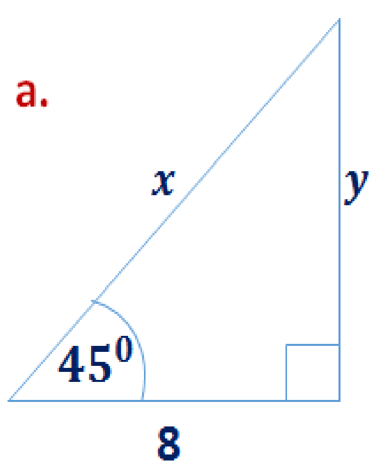
$$tan60^0 = \frac{\sqrt{3}}{1}$$

$$\therefore tan60^0 = \sqrt{3}$$

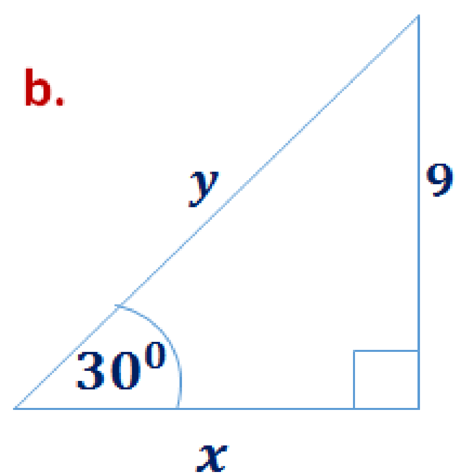
LESSON 39 APPLICATION OF SPECIAL ANGLES PAGE 1

In each of the following, calculate the lengths of the sides marked by x and y without using a calculator. Leave the answers in surd form where necessary.

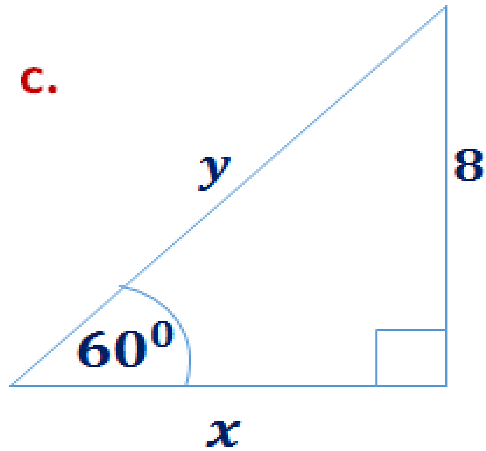
a.



b.



c.



SOLUTION a PAGE 2

In this case $Adjacent = 8, \theta = 45^0$ and $opposite = y$ and $Hypotenuse = x$. We can use $tan\theta = \frac{opposite}{adjacent}$ for finding y

$$tan45^0 = \frac{y}{8}$$

Using the previous lesson $tan45^0 = 1$ we then substitute

$$1 = \frac{y}{8} \quad \Rightarrow \quad \frac{1}{1} = \frac{y}{8}$$

Cross multiplying we have

$$\therefore y = 8$$

SOLUTION a PAGE 3

In this case $Adjacent = 8, \theta = 45^0$ and $opposite = y$ and $Hypotenuse = x$. We can use $cos\theta = \frac{adjacent}{Hypotenuse}$ for finding x

$$cos45^0 = \frac{8}{x}$$

Using the previous lesson $cos45^0 = \frac{1}{\sqrt{2}}$ we then substitute

$$\frac{1}{\sqrt{2}} = \frac{8}{x}$$

Cross multiplication gives us

$$\therefore x = 8\sqrt{2}$$

SOLUTION b PAGE 4

In this case $Adjacent = x, \theta = 30^0$ and $opposite = 9$ and $Hypotenuse = y$. We can use $tan\theta = \frac{opposite}{adjacent}$ for finding x

Substituting we have

$$tan\theta = \frac{9}{x}$$

Using the previous lesson $tan30^0 = \frac{1}{\sqrt{3}}$ we then substitute

$$\frac{1}{\sqrt{3}} = \frac{9}{x}$$

Cross multiplying we have

$$\therefore x = 9\sqrt{3}$$

SOLUTION b PAGE 5

In this case $Adjacent = x, \theta = 30^0$ and $opposite = 9$ and $Hypotenuse = y$. We can use $sine\theta = \frac{opposite}{hypotenuse}$ for finding y

Substituting we have

$$sine30^0 = \frac{9}{y}$$

Using the previous lesson $sin30^0 = \frac{1}{2}$ we then substitute

$$\frac{1}{2} = \frac{9}{y}$$

Cross multiplying we have

$$\therefore y = 18$$

SOLUTION c PAGE 6

In this case $Adjacent = x, \theta = 60^0$ and $opposite = 8$ and $Hypotenuse = y$. We can use $sine\theta = \frac{opposite}{hypotenuse}$ for finding y

Substituting we have

$$sine60^0 = \frac{8}{y}$$

Using the previous lesson $sin60^0 = \frac{\sqrt{3}}{2}$ we then substitute

$$\frac{\sqrt{3}}{2} = \frac{8}{y}$$

Cross multiplying we have

$$y\sqrt{3} = 16$$

Dividing both sides by $\sqrt{3}$ we have

$$\frac{y\sqrt{3}}{\sqrt{3}} = \frac{16}{\sqrt{3}}$$

$$\therefore y = \frac{16}{\sqrt{3}}$$

SOLUTION c PAGE 7

In this case $Adjacent = x, \theta = 60^0$ and $opposite = 8$ and $Hypotenuse = y$. We can use $tan\theta = \frac{opposite}{adjacent}$ for finding x

Substituting we have

$$tan60^0 = \frac{8}{x}$$

Using the previous lesson $tan60^0 = \sqrt{3}$ we then substitute

$$\sqrt{3} = \frac{8}{x} \qquad \Rightarrow \qquad \frac{\sqrt{3}}{1} = \frac{8}{x}$$

Cross multiplying we have

$$x\sqrt{3} = 8$$

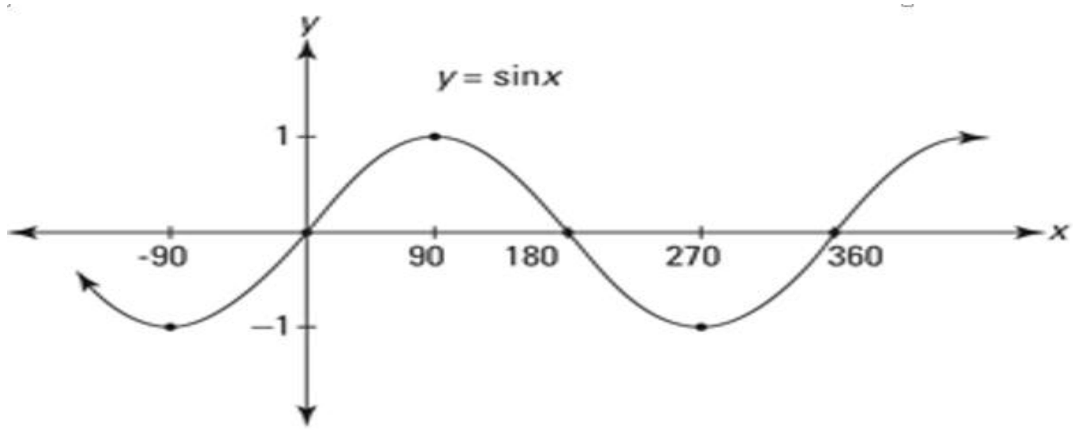
Dividing both sides by $\sqrt{3}$ we have

$$\frac{x\sqrt{3}}{\sqrt{3}} = \frac{8}{\sqrt{3}}$$

$$\therefore x = \frac{8}{\sqrt{3}}$$

LESSON 40 GRAPHS OF SINE, COSINE AND TANGENT PAGE 1

Below is the graph of $y = sinx$ for $x\epsilon[-90^0, 360^0]$ this means that x belongs to angles starting with -90^0 ending with 360^0. This graph also means that the values of x are angles in degrees written in the x $-$axis. Then the values of $sinx$ will be obtained in the $y - axis$



Examples; use the sine graph above to find the value of;

(a) $\sin -90^0$

(b) $\sin 0^0$

(c) $\sin 90^0$

(d) $\text{Sin}180^0$

SOLUTIONS PAGE 2

a. To answer (a) $\sin -90^0$ in this case the angle is $x = -90^0$, so you go to the graph and stand at -90^0 in the x-axis, then look at where the curve is, its below that point. Go to the curve straight from -90^0, when you move straight you will reach the curve where there is a thick dot (•), when you are there move straight to the right and reach the y-axis. At what number are you in the y-axis? Youre at y=-1

$\therefore \sin -90^0 = -1$ (to prove this, get your caculator and press sin-90^0=)

SOLUTIONS PAGE 3

b. To answer (b) $\sin 0^0$ in this case the angle is $x = 0^0$, so you go to the graph and stand at 0^0 in the $x - axis$, on the graphy $0^0$is not written because its position represents $y = 0$ and also $x = 0^0$ so we can not write two different values at the same point, howerver 0^0 is suppose to be between -90⁰ and 90⁰ just where the y-axis cuts the x-axis. It happens that the curve for y=sin0^0 also passes at the same point. So to find the value of $y = sin0^0$ we have to find the value of y at that point. If we have to take the coordinates of that point it will be (x, y)=(0^0, 0). This means that $y = 0$. Then we can substitute into $y = sin0^0$ so we have

0=sin0^0 $\therefore$ $sin0^0 = 0$

(to prove this, get your caculator and press sin0^0=)

SOLUTIONS PAGE 4

c. To answer (c) sin90^0 in this case the angle is $x = 90^0$, so you go to the graph and stand at 90^0 in the x-axis, then look at where the curve is, its above that point. Go to the curve straight from 90^0, when you move straight you will reach the curve where there is a thick dot (•), when you are there move straight to the left and reach the y-axis. At what number are you in the y-axis? You are at y=1

$\therefore \sin 90^0 = 1$ (to prove this, get your caculator and press sin90^0=)

SOLUTIONS PAGE 5

d. To answer (d) sin180^0 in this case the angle is $x = 180^0$, so you go to the graph and stand at 180^0 in the x-axis, then look at where the curve is, its exactly at the point where the angle 180^0 is. This means that we are just on the x-axis therefore at that point y=0

$\therefore \sin 180^0 = 0$ (to prove this, get your caculator and press sin180⁰=)

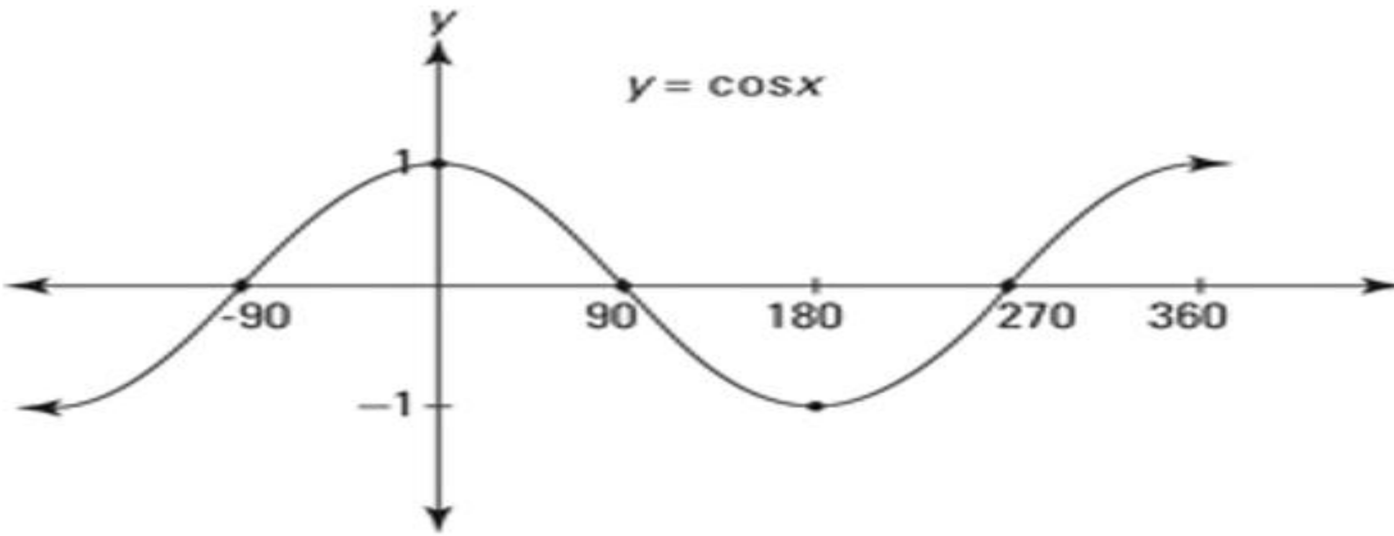
QUESTIONS FOR YOUR PRACTICE

Using what you have learnt answer

e. $\mathbf{Sin270^0}$
f. $\mathbf{Sin360^0}$

LESSON 41 GRAPHS OF SINE, COSINE AND TANGENT PAGE 1

Below is the graph of $y = cosx$ for x$\epsilon[-90^0, 360^0]$ this means that x belongs to angles starting with -90^0 ending with 360^0. This graph also means that the values of x are angles written in the x-axis. Then the values of $Cosx$ will be obtained in the y-axis



(a) $\mathbf{Cos-90^0}$
(b) $\mathbf{Cos0^0}$
(c) $\mathbf{Cos90^0}$
(d) $\mathbf{Cos180^0}$

SOLUTIONS PAGE 2

a. To answer (a) $\text{Cos} - 90^0$ in this case the angle is $x = -90^0$, so you go to the graph and stand at -90^0 in the x-axis, It happens that the curve for y=Cos-90^0 also passes at the same point. So to find the value of $y = Cos - 90^0$ we have to find the value of y at that point. If we have to take the coordinates of that point it will be (x, y)=$(-90^0, 0)$. This means that $y = 0$. Then we can substitute into $y = Cos - 90^0$ so we have

0=Cos-90^0 $\therefore\ Cos - 90^0 = 0$

(to prove this, get your caculator and press Cos-90^0=)

SOLUTIONS PAGE 3

b. To answer (b) $\text{Cos}0^0$ in this case the angle is at $x = 0^0$, so you go to the graph and stand at 0^0 in the x-axis, then look at where the curve is, its above that point. Go to the curve straight from 0^0, when you move straight you will reach the curve where there is a thick dot (•), At what number are you in the y-axis? Youre at y=1

$\therefore \text{Cos } 0^0 = 1$ (to prove this, get your caculator and press Cos0^0=)

SOLUTIONS PAGE 4

c. To answer (c) $\text{Cos}90^0$ in this case the angle is $x = 90^0$, so you go to the graph and stand at 90^0 in the x-axis, then look at where the curve is, its just at the same point so pick the value of y at that point. What is the value of y? its 0 because youre just on the x-axis

$\therefore \text{Cos}90^0 = 0$ (to prove this, get your caculator and press Cos90^0=)

SOLUTIONS PAGE 5

d. To answer (d) $\cos 180^0$ in this case the angle is $x = 180^0$, so you go to the graph and stand at 180^0 in the x-axis, then look at where the curve is, its below that point where the angle 180^0 is. Move straight downwards to where there is a thick dot. Then from that dot go straight to the y-axis on your left and pick the value of y. what is the value of y straight to that point? Its -1. This means that

$\therefore \mathbf{Cos180^0 = -1}$ (to prove this, get your caculator and press $\text{Cos}180^0=$)

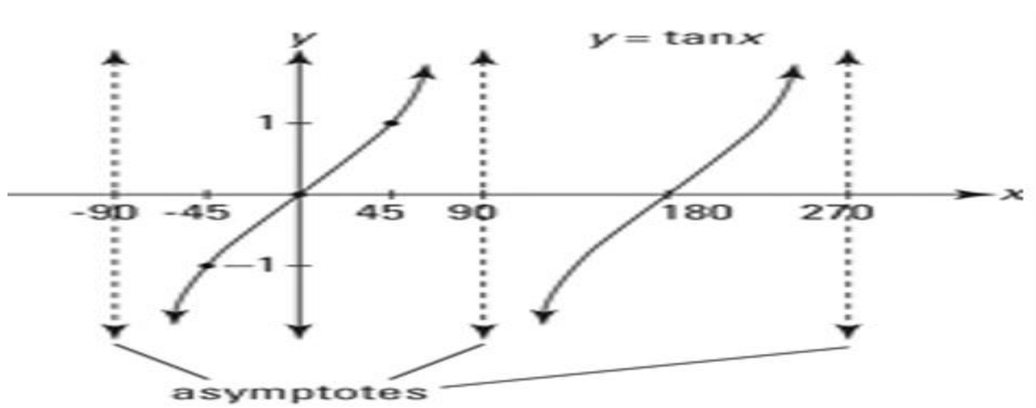
QUESTIONS FOR YOUR PRACTICE

Using what you have learnt answer

e. $\mathbf{Cos270^0}$
f. $\boldsymbol{Cos}\mathbf{360^0}$

LESSON 42 GRAPHS OF SINE, COSINE AND TANGENT PAGE 1

Below is the graph of $y = tanx$ for $x\epsilon[-90^0, 270^0]$ this means that x belongs to angles starting with -90^0 ending with 270^0. This graph also means that the values of x are angles written in the x-axis. Then the values of $tanx$ will be obtained in the y-axis



(a) $\text{Tan} - 90^0$

(b) $\text{Tan}{-45}^0$

(c) $\text{Tan}0^0$

SOLUTIONS PAGE 2

a. To answer $\text{Tan} - 90^0$, just look at the curve, it shows that the line crossing at that angle is an asymptote meaning $\text{Tan} - 90^0$ does not exist even on the calculator it will give you Maths error.

b. To answer tan -45^0 we go to the graph at -45^0 then pick the value of y on the curve just below the angle. What is the value of y there? Its -1 $\therefore$ $\tan \ -45^0 = -1$

c. To answer $tan\ 0^0$we just go the curve at 0^0 and pick the value of y at that point like we did for $sin0^0$. The answer is 0,

$$\therefore tan0^0 = 0$$

CLASS EXERCISE PAGE 3

Use the graph on page 1 to answer the following

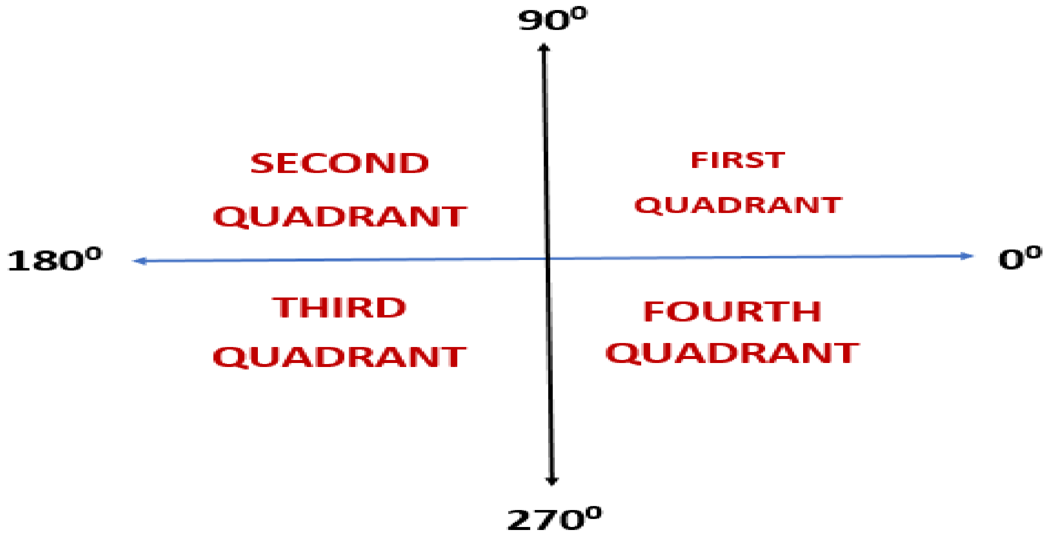
d. $\text{Tan}45^0$

e. $\text{Tan}90^0$

f. $\text{Tan } 180^0$

g. $Tan270^0$

LESSON 43 FOUR QUADRANTS OF TRIGONOMENTRY PAGE 1



In the first quadrant we have angles from 0^0 to 89^0 thus $0^0 \leq \theta \leq 89^0$

In the second quadrant we have angles from 90^0 to 180^0 thus $90^0 \leq \theta \leq 180^0$

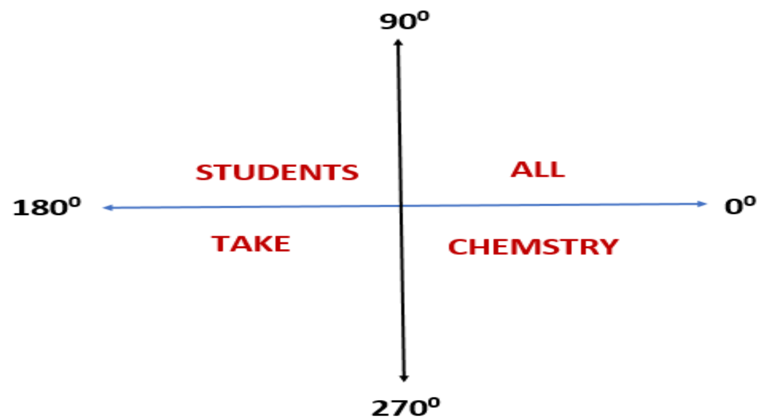
In the third quadrant we have angles from 181^0 to 269^0 thus $181^0 \leq \theta \leq 269^0$

In the fourth quadrant we have angles from 270^0 to 360^0 thus $270^0 \leq \theta \leq 360^0$

THE THREE TRIGONOMENTRIC RATIOS AND QUADRANTS PAGE 2

1. The Sine, Cosine and tangent of any angle in the first quadrant is positive. To show that the three are positive we write ALL in the first quadrant to help us remember.
2. Only the sine of angles in the second quadrant is positive, the rest are negative. To help us remember we write a word starting with letter S in the second quadrant.

3. Only Tangent of angles in the third quadrant is positive the rest are negative so we write a word starting with letter T in the third quadrant.
4. Only the Cosine of angles in the fourth quadrant is positive the rest are negative so we write a word starting with letter C in that quadrant to help us remember.



This gives us the memorizing model "ALL STUDENTS TAKE CHEMESTRY" from the first to the fourth quadrant.

SOLVING TRIGONOMETRIC EQUATIONS INVOLVING QUADRANTS PAGE 3

Examples; solve the following equations for $0^0 \leq \theta \leq 360^0$

1. $sin\theta = 0.866$
2. $cos\theta = -0.35$
3. $tan\theta = -0.81$

SOLUTIONS PAGE 4

HINTS; to solve $sin\theta = 0.866$ we have to take note that sine of the angle θ is positive, therefore the quadrants in which sine is positive is the first and second quadrant meaning will have two different answers. The formula for getting the angle in the;

a. First quadrant is $\theta_1 = 0^0 + \alpha$
b. Second quadrant is $\theta_2 = 180^0 - \alpha$

First we get α press shift or 2nd function sin 0.866 =

$$\alpha = sin^{-1}0.866 = 59.99 = 60^0$$

Getting the first answer

$$\theta_1 = 0^0 + 60^0$$

$$\therefore \theta_1 = 60^0$$

Getting the second answer

$$\theta_2 = 180^0 - \alpha$$

$$\theta_2 = 180^0 - 60^0$$

$$\therefore \theta_2 = 120^0$$

SOLUTIONS PAGE 5

HINTS; to solve $cos\theta = -0.35$ we have to take note that cosine of the angle is negative, therefore the quadrants in which cosine is negative are the second and third quadrants meaning will have two different answers. The formula for getting the angle in the;

a. Second quadrant is $\theta_2 = 180^0 - \alpha$
b. Third quadrant is $\theta_3 = 180^0 + \alpha$

First we get α press shift or 2nd function $cos0.35$= (do not include the negative)

$$\alpha = cos^{-1}0.35 = 69.51 = 70^0$$

Getting the first answer in the second quadrant

$$\theta_2 = 180^0 - 70^0$$

$$\therefore \theta_2 = 110^0$$

Getting the second answer in the third quadrant

$$\theta_3 = 180^0 + 70^0$$

$$\therefore \theta_3 = 250^0$$

SOLUTIONS PAGE 6

HINTS; to solve $tan\theta = -0.81$ we have to take note that tangent of the angle is negative, therefore the quadrants in which tangent is negative are the second and fourth quadrants meaning will have two different answers. The formula for getting the angle in the;

a. Second quadrant is $\theta_2 = 180^0 - \alpha$
b. Fourth quadrant is $\theta_4 = 270^0 + \alpha$

First we get α press shift or 2nd function $tan0.81$= (do not include the negative)

$$\alpha = tan^{-1}0.81 = 39^0$$

Getting the first answer in the second quadrant

$$\theta_2 = 180^0 - 39^0$$

$$\therefore \theta_2 = 141$$

Getting the second answer in the fourth quadrant

$$\theta_4 = 270^0 + 39^0$$

$\therefore \theta_4 = 309^0$

QUESTIONS FOR YOUR PRACTICE PAGE 7

Solve the each of the following equations for $0^0 \leq \theta \leq 360^0$

$1. sin\theta = 0.64$
$2. cos\theta = 0.42$
$3. tan\theta = 3.73$

Possible answers

1. $\theta_1 = 39.79^0 \qquad \theta_2 = 140.21^0$
2. $\theta_1 = 65.17^0 \qquad \theta_4 = 335.17^0$
3. $\theta_1 = 75.07^0 \qquad \theta_3 = 255.07^0$

LESSON 44 QUESTION 8 P1 2013 PAGE 1

a. Solve the following simultaneous equations

$b = 6 - a$ 1

$2a + 3b = 13$ 2

b. Express $3\frac{2}{5}\%$ as a decimal

c. Factorise completely $2xy + x - 10y - 5$

SOLUTIONS PAGE 2

HINT; in this case we make one of the variables the subject of the formula using one of the equations. Looking at what we have b is already the subject of the formula for equation 1 such that; $b = 6 - a$ so we substitute for b in equations 2 where there is b we put $6 - a$

Substituting for b in 2 we have

$$2a + 3b = 13 \quad \Rightarrow \quad 2a + 3(6 - a) = 13$$

We can now open the brackets by multiplying by 3

$$2a + 18 - 3a = 13$$

Collecting like terms we have;

$$2a - 3a = 13 - 18 \quad \Rightarrow \quad -a = -5$$

We are looking for a not – a so we multiply both sides by -1 to remove the –

$$(-a = -5)(-1) \qquad \therefore a = 5$$

SOLUTIONS PAGE 3

Now that we have found a we can substitute for a in equation 1 to get b

$$b = 6 - a$$

Where there is a we write 5

$$b = 6 - 5$$

$$\therefore b = 1$$

SOLUTION PAGE 4

To answer (b) $3\frac{2}{5}\%$ being expressed as a decimal, we have to change $3\frac{2}{5}\%$ from mixed to improper fraction.

$$3\frac{2}{5}\% = \frac{(5\times3)+2}{5}\% = \frac{15+2}{5}\% = \frac{17}{5}\%$$

This means that;

$$3\frac{2}{5}\% = \frac{17}{5}\%$$

We now have to divide 5 into 17 to change to decimal fraction we have;

$$3.4\%$$

However **3.4%** means $\frac{3.4}{100}$ so divide again and we have the answer

$$\therefore 3\frac{2}{5}\% = 0.034$$

SOLUTIONS PAGE 5

To Factorise $2xy + x - 10y - 5$ we have to introduce brackets, so we have

$$(2xy + x) - (10y + 5)$$

The negative for $10y$ and 5 is already outside that's why $10y$ and 5 are positive inside brackets. Then we have to consider that x is a common factor in the first brackets $(2xy + x)$ and 5 is a common factor in the second brackets $(10y + 5)$ so we Factorise them to have;

$$x(2y + 1) - 5(2y + 1)$$

We still have something common and that is $2y + 1$ so we factorise that to have;

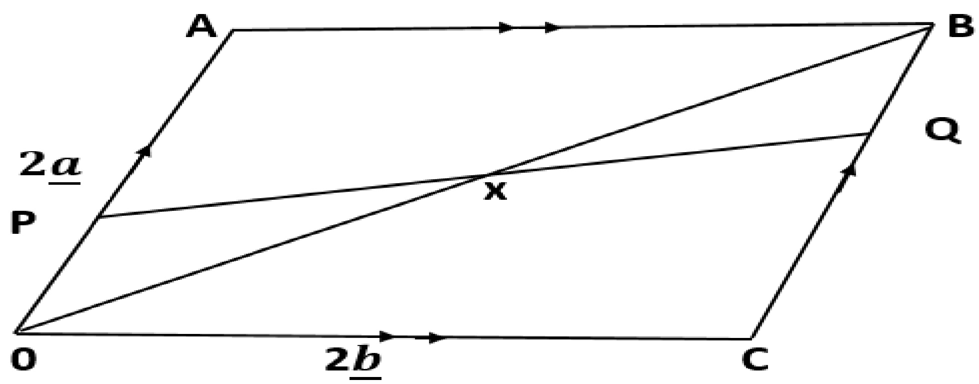
$$(2y + 1)(x - 5)$$

$$\therefore 2xy + x - 10y - 5 = (2y + 1)(x - 5)$$

LESSON 45 QUESTION 6 P2 2012 PAGE 1

a. In the diagram below OABC is a parallelogram in which $\overrightarrow{OA} = 2\underline{a}$ and $\overrightarrow{OC} = 2\underline{b}$. P is a point on OA such that $OP = \frac{1}{4}OA$ and Q is a point on CB such that;

$CQ : QB = 3 : 1$



i. Express in terms of $\underline{a}$ and/or $\underline{b}$ the vectors
   (a) $\overrightarrow{OB}$
   (b) $\overrightarrow{OP}$
   (c) $\overrightarrow{QC}$

ii. Given that $\overrightarrow{OX} = h\overrightarrow{OB}$, express $\overrightarrow{OX}$ in terms of $\underline{a}$, $\underline{b}$ and h.

b. The width of a rectangle is 3m, if the diagonal is 8.5m long calculate, the length giving your answer correct to 2 decimal places

SOLUTIONS PAGE 2

HINTS; Remember; this being a parallelogram means that the distance OA is the same as BC. And the distance AB is the same as OC.

To answer i (a) $\overrightarrow{OB}$ we assume that we can't pass direct from O to B so we need to find a route and that route is the formula. In the formula O will be the starting point while B will be the ending point. So we look at where to pass around the parallelogram. There are two formula routes in this case.

$$\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB} \qquad \text{and} \qquad \overrightarrow{OB} = \overrightarrow{OC} + \overrightarrow{CB}$$

We can now substitute and we have;

$$\overrightarrow{OB} = 2\underline{a} + 2\underline{b}$$

$$\therefore \overrightarrow{OB} = 2(\underline{a} + \underline{b})$$

SOLUTIONS PAGE 3

To answer i (b) $\overrightarrow{OP}$ we already have the formula in the question that $OP = \frac{1}{4}OA$ and we know that $\overrightarrow{OA} = 2\underline{a}$

We can now substitute and we have;

$$\overrightarrow{OP} = \frac{1}{4} \times 2\underline{a}$$

We multiply to have;

$$\overrightarrow{OP} = \frac{2\underline{a}}{4} \times$$

We now divide 2 and 4 to have the answer

$$\therefore \overrightarrow{OP} = \frac{1}{2}\underline{a}$$

SOLUTION PAGE 4

To answer i (c) $\overrightarrow{QC}$ we can see that it's on the line $\overrightarrow{CB}$ meaning its part of $\overrightarrow{CB}$ however it is a movement in the opposite direction of the arrow for $\overrightarrow{CB}$. We also have the ratio $CQ:QB = 3:1$ on the same line meaning $CQ = 3$ and $QB = 1$ so the whole line from C to B it means $CQ + QB = 3 + 1 = 4$

The formula for $\overrightarrow{QC}$ will be as follows;

$$\overrightarrow{QC} = \frac{ratio\ of\ CQ}{total\ of\ ratios} \times the\ vector\ on\ the\ line\ \overrightarrow{CB}$$

SOLUTION PAGE 4 cont.

This is the same as

$$\overrightarrow{QC} = \frac{CQ}{CQ+QB} \times \overrightarrow{CB}$$

Since $\overrightarrow{QC}$ is moving against the arrow on the line vector $\overrightarrow{CB}$ such that; it's moving in the direction of $\overrightarrow{BC}$ we introduce a negative on $\overrightarrow{CB}$ so the formula becomes

$\overrightarrow{QC} = \frac{CQ}{CQ+QB} \times -\overrightarrow{CB}$ Substituting we have $\overrightarrow{QC} = \frac{3}{3+1} \times -2\underline{a}$

$\overrightarrow{QC} = \frac{3}{4} \times -2\underline{a}$ *multiplying we have* $\overrightarrow{QC} = \frac{-6}{4}\underline{a}$

We now divide 2 into 6 and 4 to have

$\overrightarrow{QC} = \frac{-3}{2}\underline{a}$ $\therefore \overrightarrow{QC} = -1\frac{1}{2}\underline{a}$

SOLUTION PAGE 5

To answer ii Given that $\overrightarrow{OX} = h\overrightarrow{OB}$, express $\overrightarrow{OX}$ in terms of $\underline{a}, \underline{b}$ and h.

We already have the formula

$$\overrightarrow{OX} = h\overrightarrow{OB},$$

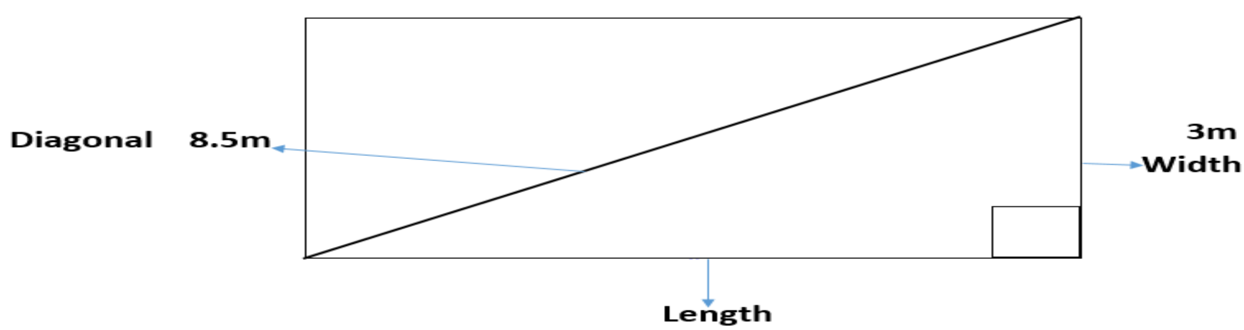
So we have to substitute into the formula and we already have $\overrightarrow{OB}$

$$\overrightarrow{OX} = h[2(\underline{a} + \underline{b})]$$

$$\therefore \overrightarrow{OX} = 2h(\underline{a} + \underline{b})$$

SOLUTIONS PAGE 6

To answer b, width of a rectangle is 3m, diagonal is 8.5m long calculate, the length. We need to know what they are talking about. So let's draw it



So to calculate length we can use Pythagoras theorem as follows;

$$(diagonal)^2 = (width)^2 + (length)^2$$

We want to find the length so we make length the subject of the formula

$$(length)^2 = (diagonal)^2 - (width)^2$$

$$(length)^2 = (8.5)^2 - (3)^2 \Rightarrow (l)^2 = (8.5 \times 8.5) - (3 \times 3)$$

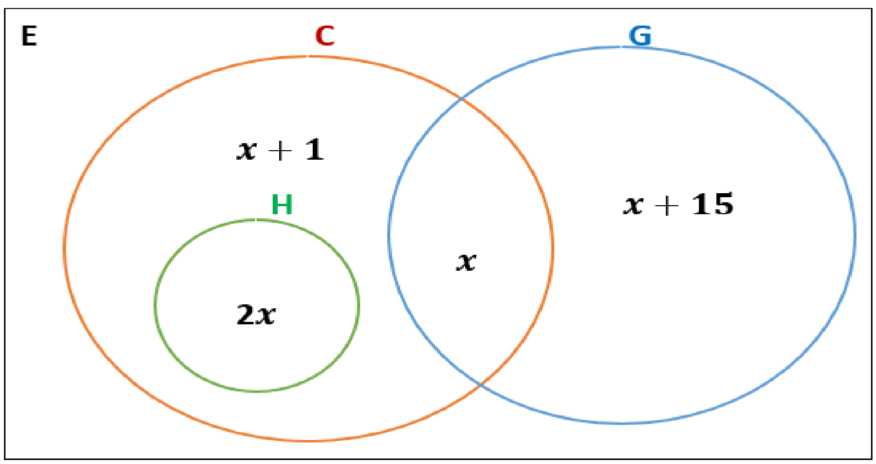
$$(l)^2 = 72.25 - 9 \qquad \Rightarrow \qquad (l)^2 = 63.25$$

Finding the square root on both sides

$$\sqrt{l^2} = \sqrt{63.25} \qquad \therefore lenght = 7.95m$$

LESSON 46 QUESTION5 P2 2012 PAGE 1

a. A class of 41 girls takes History (H), Commerce (C) and Geography (G) as optional subjects. The Venn diagram below shows their choice distribution.



i. Calculate the value of x

ii. Find

a. n(H∪G)

b. n(G'∩H')

d. The ratio of adults to children that bought tickets for a video show was 17:15 respectively. The total number of tickets sold for the video show was 4704

i. How many more adults than children attended the video show?

ii. If the tickets for adults were sold at K12 500 each and the tickets for children were sold at K8 500 each, calculate the total amount realized from the same of tickets

SOLUTIONS PAGE 2

To find x we have to add everything in the Venn diagram and equate to the total number of girls as follows;

$$x + 1 + 2x + x + x + 15 = 41$$

Collecting like terms we have;

$$x + 2x + x + x = 41 - 1 - 15$$

Adding and subtracting like terms we have

$$5x = 25$$

Dividing both sides by 5

$$\frac{5x}{5} = \frac{25}{5}$$

$$\therefore x = 5$$

SOLUTIONS PAGE 3

i. To find (a) n(H∪G) we need to understand what the questions means. It means finding the total number of elements found in the two sets H and G. counting them together. In H we have $2x$, in G we have $x + 15$ and the x in G intersection C is also part of G.

$$n(H \cup G) = 2x + x + 15 + x$$

We know that x=5 so where there is x we write 5

$$n(H \cup G) = 2(5) + 5 + 15 + 5$$

Opening brackets

$$n(H \cup G) = 10 + 5 + 15 + 5$$

$$\therefore n(H \cup G) = 35$$

SOLUTIONS PAGE 4

ii. **To find (b) $n(G' \cap H')$ we need to understand what the questions means. G' reads as G complement, meaning the elements that are not found in set G. so let's list G'**

$$G' = \{x+1, 2x\}$$

H' reads as H complement, meaning the elements that are not found in set H. so let's list H'

$$H'\{x+1, x, x+15\}$$

n(G'∩H') reads as, number of elements in G complement intersection H **complement. We have to pick the elements found in both G' and H'. The element found in both is $x+1$ so**

$$n(G' \cap H') = x+1$$

We now substitute for x=5 we have

$$n(G' \cap H') = 5+1$$

$$\therefore n(G' \cap H') = 6$$

SOLUTIONS PAGE 5

To answer (c) (i) we need to find the number of adults and also the number of children, the ratio of adults to children is $17:15$. Adults corresponds to 17 while children corresponds to 15.

Number of adults= $\frac{ratio\ for\ adults}{ratio\ for\ adults + ratio\ for\ children} \times$

$$total\ number\ of\ tickets$$

$$Number\ of\ adults = \frac{17}{17+15} \times 4704 = \frac{17}{32} \times \frac{4704}{1} = \frac{79\ 968}{32}$$

$$\therefore Number\ of\ adults = 2499$$

Number of Children= $\frac{ratio\ for\ children}{ratio\ for\ adults + ratio\ for\ children} \times$ $total\ number\ of\ tickets$

$$Number\ of\ children = \frac{15}{17+15} \times 4704 = \frac{15}{32} \times \frac{4704}{1} = \frac{70\ 560}{32}$$

$$\therefore Number\ of\ children = 2205$$

To find how many more adults than children attended we say;

$$Number\ of\ adults - Number\ of\ children$$

Substituting we have

$$2499 - 2205 = 294$$

$$\therefore more\ aduts\ than\ children = 294$$

SOLUTIONS PAGE 6

To answer (c) (ii) to find the total amount realized from the sale of tickets we multiply the amount for each adult ticket by the total number of adults plus amount for each child's ticket multiplied by the total number of children.

Total amount realized = $(12,500 \times\ adults)+(8\ 500 \times children)$

Total amount realized = $(12,500 \times\ 2499)+(8\ 500 \times 2205)$

Total amount realized = $(31237500)+(18742500)$

$$Total\ amount\ realized\ =\ K49,980,000$$

ESSON 47 CALCULUS DIFFERNTIATION PAGE 1

REVISION ON INDICES NEGATIVE TO POSITIVE POWERS

Indices are mathematical expressions in the form x^n, in this case x is the base while n is the power also called index. Some powers can be negative such as x^{-n}. Indices with a negative power can be expressed with a positive power by changing them into fractions.

Examples;

Express the following indices with a positive power

a. x^{-n}
b. 4^{-1}
c. -5^{-2}
d. $2x^{-3}$
e. $10x^{-4}$
f. 11^{-6}
g. $-7x^{-5}$

SOLUTIONS PAGE 2

HINTS; to change from negative power to positive power, the negative sign changes to numerator 1, meaning the negative sign will be removed from the power, therefore the base raised to its power now in positive form becomes the denominator.

To express x^{-n} with a positive index the negative on power n becomes 1 and it is written as the numerator of x^n

$$a.\ x^{-n} = \frac{1}{x^n}$$

To express 4^{-1} with a positive index the negative on power 1 becomes 1 and it is written as the numerator of 4^1

$$b.\ 4^{-1} = \frac{1}{4^1}$$

SOLUTIONS PAGE 3

To express -5^{-2} with a positive index the negative on power 2 becomes 1 and it is written as the numerator of -5^2 the negative on the base 5, will not be removed

$$c.\ -5^{-2} = -\frac{1}{5^2}$$

To express $2x^{-3}$ with a positive index the negative on power 3 becomes 1 and it is written as the numerator of $x^2$the 2 in front of x (coefficient) is not going to be below 1 because it doesn't have a negative power

$$d.\ 2x^{-3} = 2 \times x^{-3} = 2 \times \frac{1}{x^3} = \frac{2}{1} \times \frac{1}{x^3} = \frac{2}{x^3}$$

SOLUTIONS PAGE 4

To express $10x^{-4}$ with a positive index the negative on power 4 becomes 1 and it is written as the numerator of x^4 the 10 in front of x (coefficient) is not going to be below 1 because it doesn't have a negative power

$$e.\ 10x^{-4} = 10 \times x^{-4} = 10 \times \frac{1}{x^4} = \frac{10}{1} \times \frac{1}{x^4} = \frac{10}{x^4}$$

To express 11^{-6} with a positive index the negative on power 6 becomes 1 and it is written as the numerator of 11

$$f.\ 11^{-6} = \frac{1}{11^6}$$

To express $-7x^{-5}$ with a positive index the negative on power 5 becomes 1 and it is written as the numerator of x^5 the 7 in front of x (coefficient) is not going to be below 1 because it doesn't have a negative power

$$\text{g. } 7x^{-5} = -7 \times x^{-5} = -7 \times \frac{1}{x^5} = -\frac{7}{1} \times \frac{1}{x^5} = \boxed{-\frac{7}{x^5}}$$

LESSON 48 CALCULUS DIFFERNTIATION PAGE 1

REVISION ON INDICES POSITIVE TO NEGATIVE POWERS

Indices are mathematical expressions in the form x^n, however they can also be in fraction form such as $\frac{1}{x^n}$ in this case x is the base while n is the power also called index. In this case the power is positive, Indices with a positive power can be expressed with a negative power by changing them from fraction form.

Examples;

Express the following indices with a negative power

a. $\frac{1}{x^n}$

b. $\frac{1}{3^2}$

c. $\frac{5}{x^4}$

d. $-\frac{11}{19x^6}$

e. $\frac{7}{5x^n}$

f. $\frac{1}{x^7}$

g. $-\frac{17}{x^5}$

SOLUTIONS PAGE 2

HINTS; to change from $\frac{1}{x^n}$ we remove the numerator 1, meaning the negative has to appear on the power n

$$\text{a. } \frac{1}{x^n} \quad = \quad 1 \times x^{-n} \quad = \quad x^{-n}$$

To change from $\frac{1}{3^2}$ we remove the numerator 1, meaning the negative has to appear on the power 2

$$\text{b. } \frac{1}{3^2} \quad = \quad 1 \times 3^{-2} \quad = \quad 3^{-2}$$

SOLUTIONS PAGE 3

To change from $\frac{5}{x^4}$ we remove the numerator 5, meaning the negative has to appear on the power 4

$$\text{c. } \frac{5}{x^4} \quad = \quad 5 \times x^{-4} \quad = \quad 5x^{-4}$$

To change from $-\frac{11}{19x^6}$ we remove the coefficient fraction $-\frac{11}{19}$, meaning the negative has to appear on the power 6

$$\text{d. } -\frac{11}{19x^6} \quad = \quad -\frac{11}{19} \times x^{-6} \quad = \quad -\frac{11}{19}x^{-6}$$

SOLUTIONS PAGE 4

To change from $\frac{7}{5x^n}$ we remove the coefficient fraction $\frac{7}{5}$, meaning the negative has to appear on the power n

$$\text{e. } \frac{7}{5x^n} \quad = \quad \frac{7}{5} \times x^{-n} \quad = \quad \frac{7}{5}x^{-n}$$

To change from $\frac{1}{x^7}$ we remove the numerator 1, meaning the negative has to appear on the power 7

$$f.\ \frac{1}{x^7} = 1 \times x^{-7} = \boxed{x^{-7}}$$

To change from $-\frac{17}{x^7}$ we remove the numerator -17, meaning the negative has to appear on the power 7

$$g.\ -\frac{17}{x^7} = -17 \times x^{-7} = \boxed{-17x^{-7}}$$

LESSON 49 CALCULUS DIFFERENTIATION PAGE 1

GRADIENT FUNCTION

EXAMPLES; find the gradient function for each of the following expressions

a. $3x^4$
b. $9x^6$
c. $\frac{1}{x^2}$
d. $3x^4 - 4x^2$
e. $\frac{2}{2x^5}$
f. $-7x^3 - 3$

SOLUTIONS PAGE 2

When you are given an equation such as $y = x^n$ then differentiation of $y = x^n$ will be denoted by $\frac{dy}{dx}$ meaning differentiating y with respect x.

We can now come up with the formula for differentiation. The power of x is denoted by n. This n will be multiplied by the coefficient of x

(number in front of x in this case its 1), then we have to subtract 1 from n (power of x) as follows;

$$y = x^n$$

$$\frac{dy}{dx} = n \times 1 \times x^{n-1}$$

$$\therefore we\ have\ \frac{dy}{dx} = nx^{n-1}$$

SOLUTIONS PAGE 3

To answer a. $3x^4$ we have to identify n and the coefficient of x. $n = 4$ and the coefficient f x is 3. So we can use the formula

$$y = 3x^4$$

$$\frac{dy}{dx} = n \times 1 \times x^{n-1}$$

$$\frac{dy}{dx} = 4 \times 3x^{4-1}$$

$$\therefore \frac{dy}{dx} = 12x^3$$

SOLUTIONS PAGE 4

b. To answer b. $9x^6$ we have to identify n and the coefficient of x. $n = 6$ and the coefficient f x is 9. So we can use the formula

$$y = 9x^6$$

$$\frac{dy}{dx} = n \times 1 \times x^{n-1}$$

$$\frac{dy}{dx} = 6 \times 9x^{6-1}$$

$$\therefore \frac{dy}{dx} = 54x^5$$

SOLUTIONS PAGE 5

To answer c. $\frac{1}{x^2}$ we have to rewrite this, when we go to indices then $\frac{1}{x^2} = x^{-2}$ .We now have to identify n and the coefficient of x. $n = -2$ and the coefficient f x is 1. So we can use the formula

$$y = x^{-2}$$

$$\frac{dy}{dx} = n \times x^{n-1}$$

$$\frac{dy}{dx} = -2 \times x^{-2-1} = -2 \times x^{-3}$$

$$\frac{dy}{dx} = -2 \times \frac{1}{x^3} = -\frac{2}{1} \times \frac{1}{x^3}$$

$$\therefore \frac{dy}{dx} = -\frac{2}{x^3}$$

SOLUTIONS PAGE 6

To answer d. $3x^4 - 4x^2$. We have to identify n and the coefficient of **x**. we have two powers and two coefficients $n_1 = 4$ and $n_2 = 2$ the coefficients of x are $c_1 = 3$ and $c_2 = 4$. So we can use the formula

$$y = 3x^4 - 4x^2$$

$$\frac{dy}{dx} = n_1 \times c_1 \times x^{n_1-1} - n_2 \times c_2 \times x^{n_2-1}$$

$$\frac{dy}{dx} = (4 \times 3 \times x^{4-1}) - (2 \times 4 \times x^{2-1})$$

$$\therefore \frac{dy}{dx} = 12x^3 - 8x$$

SOLUTIONS PAGE 7

To answer d. $\frac{2}{2x^5}$ when it is in fraction form, $\frac{2}{2x^5} = x^{-5}$ we have to rewrite using indices we have one power $n = -5$. So we can use the formula

$$y = x^{-5}$$

$$\frac{dy}{dx} = n \times x^{n-1}$$

$$\frac{dy}{dx} = -5 \times x^{-5-1}$$

$$\therefore \frac{dy}{dx} = -5x^{-6}$$

$$\therefore \frac{dy}{dx} = -\frac{5}{x^6}$$

SOLUTIONS PAGE 8

To answer f. $-7x^3 - 3$ we have to identify n and the coefficient of x. $n = 3$ and the coefficient f x is -7. So we can use the formula. So we can use the formula

$$y = -7x^3 - 3$$

$$\frac{dy}{dx} = n \times 1 \times x^{n-1}$$

$$\frac{dy}{dx} = 3 \times -7 \times x^{3-1}$$

The 3 has no x it’s a constant so we leave it out

$$\therefore \frac{dy}{dx} = -21x^2$$

LESSON 50 CALCULUS FRACTION DIFFERENTIATION QUESTIONS PAGE 1

Find the gradient function for each of the following expressions

a. $\frac{5}{14x^7}$

b. $\frac{4}{x^2}$

c. $\frac{1}{4x^5}$

d. $-\frac{7x}{x^3}$

e. $\frac{4}{3x^6}$

f. $\frac{5}{2x}$

g. $-\frac{2}{2x^3}$

SOLUTIONS PAGE 2

HINT; to answer (a) $\frac{5}{14x^7}$ we have to change this to negative index form, $\frac{5}{14x^7} = \frac{5}{14} \times x^{-7}$

This means that $y = \frac{5}{14}x^{-7}$ in this equation the number in front of x^{-7} is $\frac{5}{14}$ and we can call it coefficient **c** the power of **x** is denoted by **n** and in this case its **-7**. Finding the gradient function means differentiating. The formula for differentiation is

$$\frac{dy}{dx} = n \times c \times x^{n-1}$$

So we can just substitute

$$\frac{dy}{dx} = -7 \times \frac{5}{14} \times x^{-7-1}$$

$$\frac{dy}{dx} = -\frac{5}{2} \times x^{-8}$$

We can now express x^{-8} with a positive index we have

$$\frac{dy}{dx} = -\frac{5}{2} \times \frac{1}{x^8} \qquad \frac{dy}{dx} = \frac{5}{2x^8}$$

$$\therefore \frac{dy}{dx} = -2\frac{1}{2x^8}$$

SOLUTIONS PAGE 3

HINT; to answer (b) $\frac{4}{x^2}$ we have to change this to negative index form, $\frac{4}{x^2} = 4 \times x^{-2}$

This means that $y = 4x^{-2}$ in this equation the number in front of x^{-2} is 4 and we can call it coefficient c the power of x is denoted by n and in this case its -2. Finding the gradient function means differentiate. The formula for differentiation is

$$\frac{dy}{dx} = n \times c \times x^{n-1}$$

So we can just substitute

$$\frac{dy}{dx} = -2 \times 4 \times x^{-2-1}$$

$$\frac{dy}{dx} = -8 \times x^{-3}$$

We can now express x^{-2} with a positive index we have

$$\frac{dy}{dx} = -8 \times \frac{1}{x^3} \qquad \therefore \frac{dy}{dx} = -\frac{8}{x^3}$$

SOLUTIONS PAGE 4

HINT; to answer (c) $\frac{1}{4x^5}$ we have to change this to negative index form, $\frac{1}{4x^5} = \frac{1}{4} \times x^{-5}$

This means that $y = \frac{1}{4}x^{-5}$ in this equation the number in front of x^{-5} is $\frac{1}{4}$ and we can call it coefficient **c** the power of **x** is denoted by **n** and in this case its **-5**. Finding the gradient function means differentiating. The formula for differentiation is

$$\frac{dy}{dx} = n \times c \times x^{n-1}$$

So we can just substitute

$$\frac{dy}{dx} = -5 \times \frac{1}{4} \times x^{-5-1}$$

$$\frac{dy}{dx} = -\frac{5}{4} \times x^{-6}$$

We can now express x^{-6} with a positive index we have

$$\frac{dy}{dx} = -\frac{5}{4} \times \frac{1}{x^6} \qquad \therefore \frac{dy}{dx} = -\frac{5}{4x^6}$$

SOLUTIONS PAGE 5

HINT; to answer (d) $-\frac{7x}{x^3}$ first we have to conceal x from the denominator and from the numerator, by first expanding the denominator to $-\frac{7\times x}{x\times x\times x} = -\frac{7}{x\times x} = -\frac{7}{x^2}$ we have to change this to negative index form, $-\frac{7}{x^2} = -7 \times x^{-2}$. This means that $y = -7x^{-2}$. In this equation the number in front of x^{-2} is -7

and we can call it coefficient c the power of x is denoted by n and in this case it's -2. Finding the gradient function means differentiating. The formula for differentiation is

$$\frac{dy}{dx} = n \times c \times x^{n-1}$$

So we can just substitute

$$\frac{dy}{dx} = -2 \times -7 \times x^{-2-1}$$

$$\frac{dy}{dx} = 14 \times x^{-3}$$

We can now express x^{-3} with a positive index we have

$$\frac{dy}{dx} = 14 \times \frac{1}{x^3} \qquad \therefore \frac{dy}{dx} = \frac{14}{x^3}$$

QUESTIONS

USING WHAT YOU HAVE LEARNT ANSWER THE FOLLOWING AND POST YOUR ANSWERS TOMORROW AT 20;00 HOURS

A. $\frac{4}{3x^6}$

B. $\frac{5}{2x}$

C. $-\frac{2}{2x^3}$

LESSON 51 SPECIMEN QUESTION 1 PAPER 2 2016 (PAGE 1)

a. If matrix P=$\begin{pmatrix} 4 & 2 \\ 0 & 3 \end{pmatrix}$ and Q=$\begin{pmatrix} 12 & 4 \\ -9 & m \end{pmatrix}$, find

i. The value of m for which the determinants of P and Q are equal

ii. The inverse of Q

b. If matrix A=$\begin{pmatrix} 6 & 4 \\ 2 & 5 \end{pmatrix}$ and B=$\begin{pmatrix} 5 & 6 \\ -7 & h \end{pmatrix}$, find

i. The value of h for which the determinants of A and B are equal

ii. The inverse of B

SOLUTIONS (a i page 2)

HINTS; it clearly states that determinants are equal, so the formula is that

$$determinant\ of\ P = determinant\ of\ Q$$

A two by two matrix has two diagonals, major and minor diagonal,

$$determinant = product(major - minor)diagonal$$

$$\begin{pmatrix} 4 & 2 \\ 0 & 3 \end{pmatrix}$$

Red is major diagonal while black is minor diagonal

SOLUTIONS (a i cont. page 3)

So we have

$$determinant\ of\ P = determinant\ of\ Q$$

$$((4 \times 3) - (0 \times 2)) = ((12 \times m) - (4 \times -9))$$

$$(12 - 0) = (12m - (-36))$$

Negative by negative

$$12 = 12m + 36$$

Collecting like terms and changing sign on 36

$$12 - 36 = 12m$$

Subtracting a bigger number from a small number is negative

$$-24 = 12m$$

Dividing both sides by 12 to find m

$$\frac{-24}{12} = \frac{12m}{12}$$

$$\therefore m = -2$$

SOLUTIONS (a ii page 4)

Hints; inverse of matrix Q is denoted by Q^{-1}, will be found by the formula

$$Q^{-1} = \frac{1}{determinat\ of\ Q} \times Q^{adjoind}$$

Q^T Is found by, interchanging the terms in the major diagonal and changing the signs for the terms in the minor diagonal.
For Q where there is m we write -2

$$Q = \begin{pmatrix} 12 & 4 \\ -9 & -2 \end{pmatrix}$$

Now observe the changes in movements

$$\therefore Q^{adjoind} = \begin{pmatrix} -2 & -4 \\ 9 & 12 \end{pmatrix}$$

SOLUTIONS (a ii cont. page 5)

We now have to find the determinant of Q

$$determinant = ((-2 \times 12) - (-9 \times 4))$$

$$det = (-24 - (-36))$$

$$det = (-24 + 36)$$

$$\therefore det = 12$$

We can now substitute into the formula to find the inverse

$$Q^{-1} = \frac{1}{determinat\ of\ Q} \times Q^T$$

$$Q^{-1} = \frac{1}{12} \times \begin{pmatrix} -2 & -4 \\ 9 & 12 \end{pmatrix}$$

$$\therefore Q^{-1} = \frac{1}{12}\begin{pmatrix} -2 & -4 \\ 9 & 12 \end{pmatrix}$$

SOLUTIONS (b i page 6)

Hints; the same way we found m can be used in this case

$$det\ of\ A = det\ of\ B$$

$$(6 \times 5) - (2 \times 4) = (5 \times h) - (6 \times -7)$$

$$30 - 8 = 5h - (-42)$$

$$22 = 5h + 42$$

$$22 - 42 = 5h$$

$$-20 = 5h$$

$$\frac{-20}{5} = \frac{5h}{5} \quad \therefore h = -4$$

SOLUTIONS (b ii page 7)

Hints; to get the determinant of B we use the following formula

$$B^{-1} = \frac{1}{determinat\ of\ B} \times B^{adjoind}$$

We already know $h = -4$ so we substitute

$$det\ of\ B = (5 \times -4) - (6 \times -4)$$

$$det\ of\ B = -20 + 24$$

$$\therefore det\ of\ B = 4$$

$$\therefore B^{adjoind} = \begin{pmatrix} -4 & -6 \\ 7 & 5 \end{pmatrix}$$

$$B^{-1} = \frac{1}{4} \times \begin{pmatrix} -4 & -6 \\ 7 & 5 \end{pmatrix}$$

$$\therefore B^{-1} = \frac{1}{4}\begin{pmatrix} -4 & -6 \\ 7 & 5 \end{pmatrix}$$

LESSON 52 MATRICES FURTHER QUESTIONS (PAGE 1)

QUESTION ONE

Given that A=$\begin{pmatrix} 5 & 2 \\ 1 & 0 \end{pmatrix}$ and B=$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$ find

- i. **The inverse of matrix A**
- ii. **3A – B**
- iii. **AB**

QUESTION TWO

a. **Given that P = $\begin{pmatrix} -1 & 2 \\ 0 & 1 \end{pmatrix}$ and Q = $\begin{pmatrix} 1 & 2 \\ 1 & -2 \end{pmatrix}$ find the**

i. The determinant of Q
ii. Matrix 2Q – 3P
iii. Matrix P^2

SOLUTIONS (ONE i PAGE 2)

Inverse of matrix A is denoted by A^{-1}.

$$A^{-1} = \frac{1}{determinat\ of\ A} \times A^{adjoind}$$

We can first find the determinant of $A = \begin{pmatrix} 5 & 2 \\ 1 & 0 \end{pmatrix}$

$$determinant = product(major - minor)diagonal$$

Red is major diagonal while black is minor diagonal

$$determinant\ of\ A = (5 \times 0) - (1 \times 2)$$

$$determinant\ of\ A = 0 - 2$$

$$\therefore determinant\ of\ A = -2$$

SOLUTIONS (ONE i cont. PAGE 3)

We can now find $A^{adjoind}$ by interchanging the numbers in the major diagonal, and changing signs for the numbers in the minor diagonal

$$A = \begin{pmatrix} 5 & 2 \\ 1 & 0 \end{pmatrix} \qquad \therefore A^{adjoind} = \begin{pmatrix} 0 & -2 \\ -1 & 5 \end{pmatrix}$$

We can now just substitute into the formula

$$A^{-1} = \frac{1}{determinat\ of\ A} \times A^{adjoind}$$

$$A^{-1} = \frac{1}{-2} \times \begin{pmatrix} 0 & -2 \\ -1 & 5 \end{pmatrix} \quad \therefore A^{-1} = \frac{1}{-2}\begin{pmatrix} 0 & -2 \\ -1 & 5 \end{pmatrix}$$

SOLUTIONS (ONE ii PAGE 4)

Finding $3A - B$

First of all $3A$ means $3\times A$ so the number 3 has to multiply everything in matrix A, so $3A - B$ will be

$$3A - B = 3 \times \begin{pmatrix} 5 & 2 \\ 1 & 0 \end{pmatrix} - \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

We start with multiplication

$$\begin{pmatrix} 15 & 6 \\ 3 & 0 \end{pmatrix} - \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

At this point we subtract corresponding numbers, please observe the colours

$$\begin{pmatrix} 15-(-1) & 6-0 \\ 3-0 & 0-(-1) \end{pmatrix} = \begin{pmatrix} 15+1 & 6 \\ 3 & 0+1 \end{pmatrix}$$

$$\therefore 3A - B = \begin{pmatrix} 16 & 6 \\ 3 & 1 \end{pmatrix}$$

SOLUTIONS (ONE iii PAGE 5)

Finding AB means multiplying A to B using row by column

$$AB = \begin{pmatrix} 5 & 2 \\ 1 & 0 \end{pmatrix} \times \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

Please observe the colours

$$= \begin{pmatrix} (5\times -1)+(2\times 0) & (5\times 0)+(2\times -1) \\ (1\times -1)+(0\times 0) & (1\times 0)+(0\times -1) \end{pmatrix}$$

$$= \begin{pmatrix} -5+0 & 0-2 \\ -1+0 & 0+0 \end{pmatrix}$$

$$\therefore AB = \begin{pmatrix} -5 & -2 \\ -1 & 0 \end{pmatrix}$$

SOLUTIONS (TWO i PAGE 6)

Q = $\begin{pmatrix} 1 & 2 \\ 1 & -2 \end{pmatrix}$ the determinant of Q will be found in the same way we found the determinant of A.

$$determinant = product(major - minor)diagonal$$

$$determinant\ of\ Q = (1 \times -2) - (1 \times 2)$$

$$determinant\ of\ Q = -2 - 2$$

$$\therefore determinant\ of\ Q = -4$$

SOLUTIONS (TWO ii PAGE 7)

Finding 2Q – 3P, this will be treated like ONE ii on page 4.

$$2Q - 3P = 2 \times \begin{pmatrix} 1 & 2 \\ 1 & -2 \end{pmatrix} - 3 \times \begin{pmatrix} -1 & 2 \\ 0 & 1 \end{pmatrix}$$

We multiply first

$$2Q - 3P = \begin{pmatrix} 2 & 4 \\ 2 & -4 \end{pmatrix} - \begin{pmatrix} -3 & 6 \\ 0 & 3 \end{pmatrix}$$

We now subtract according to colour and correspondence

$$\begin{pmatrix} 2 - -3 & 4 - 6 \\ 2 - \ \ 0 & -4 - 3 \end{pmatrix}$$

$$\therefore 2Q - 3P = \begin{pmatrix} 5 & -2 \\ 2 & -7 \end{pmatrix}$$

SOLUTIONS (TWO iii PAGE 8)

Finding Matri P^2 means multiplying p by itself using row by column

$$P \times P = \begin{pmatrix} -1 & 2 \\ 0 & 1 \end{pmatrix} \times \begin{pmatrix} -1 & 2 \\ 0 & 1 \end{pmatrix}$$

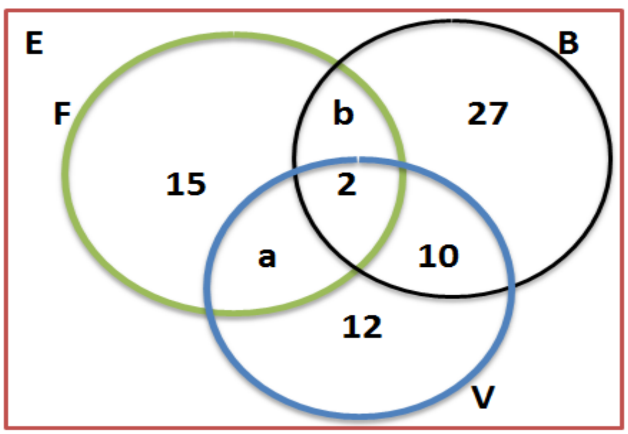
$$\begin{pmatrix} (-1 \times -1) + (2 \times 0) & (-1 \times 2) + (2 \times 1) \\ (0 \times -1) + (1 \times 0) & (0 \times 2) + (1 \times 1) \end{pmatrix}$$

$$\begin{pmatrix} 1 + 0 & -2 + 2 \\ 0 + 0 & 0 + 1 \end{pmatrix}$$

$$\therefore P^2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

LESSON 53 SETS AND PROBABILITY (PAGE 1)

A survey was carried out at Mulonga secondary school on 100 learners to determine their participation in three sports Football (F) Basketball (B) and Volleyball (V). The results are shown in the Venn diagram bellow;



Given that 42 learners play Volleyball (V) find the value of a and b

a. If a learner is selected at random, what is the probability that;

i. The learner does not play Football (F)?

ii. Plays Volleyball (V) or Football (F) but not both?

iii. A learner either participates in only one of these sports or not involved at all?

iv. A learner who plays Volleyball (V) also plays Basketball (B) but not Football (F)?

SOLUTIONS (a page 2)

To find a we have to add all numbers that are found in set V and equate to 42 the total number of learners who played volleyball. The blue circle is the set for volleyball. We have;

$$a + 2 + 10 + 12 = 42$$

Adding like terms we have

$$a + 24 = 42$$

Collecting like terms 24 becomes negative on the other side

$$a = 42 - 24$$

$$\therefore a = 18$$

SOLUTIONS (a page 3)

To find b we have to add all numbers that are found in all the sets and equate to 100 the total number of learners on which this survey was conducted. We have;

$$18 + b + 2 + 10 + 12 + 27 + 15 = 100$$

Adding like terms we have

$$b + 84 = 100$$

Collecting like terms we have

$$b = 100 - 84$$

$$\therefore b = 16$$

SOLUTIONS (b i page 4)

Leaners who do not play football are those numbers which are not found inside the green circle which is the set for football. To find the probability of learners who do not play football we add the numbers outside F and use the following formular.

$$P(not\ football) = \frac{total\ outside\ football}{total\ number\ of\ learners}$$

$$P(not\ football) = \frac{27 + 10 + 12}{100}$$

$$\therefore P(not\ football) = \frac{49}{100}$$

SOLUTIONS (b ii page 5)

To find the probability of learners who played volleyball or football but not both we add $b, 15, 12\ and\ 10$ and use the sum as a numerator in the formula over total number of learners. We will not use a and 2 because they are for both. We will not use 27 because it's for neither F nor V;

$$P(F or\ V\ but\ not\ both) = \frac{b + 15 + 12 + 10}{total\ number\ of\ learners}$$

$$P(F or\ V\ but\ not\ both) = \frac{16 + 15 + 12 + 10}{100}$$

$$\therefore P(F or\ V\ but\ not\ both) = \frac{53}{100}$$

SOLUTIONS (b iii page 6)

The numbers corresponding to only one game are 15 (only football), 27(only basketball) and 12(only volleyball). Looking at the set we do not have any number completely outside any one of the sets. We add these numbers and use the sum as the numerator of the total number of learners;

$$P(one\ game\ or\ none) = \frac{27 + 15 + 12}{total\ number\ of\ learners}$$

$$P(one\ game\ or\ none) = \frac{54}{100}$$

Dividing by 2 we have

$$\therefore P(one\ game\ or\ none) = \frac{27}{50}$$

SOLUTIONS (b iv page 7)

This one is just about understanding the language and analyzing the Venn diagram. There is only one number suiting this probability and that is 10 which is found in both V and B

$$P(V\ and\ B\ but\ not\ F) = \frac{10}{total\ number\ of\ learners}$$

$$P(V\ and\ B\ but\ not\ F) = \frac{10}{100}$$

Dividing by 10 we have

$$\therefore P(V\ and\ B\ but\ not\ F) = \frac{1}{10}$$

LESSON 54 SPECIMEN QUESTION 1 PAPER 2 2016 (page 1)

iii. At destiny secondary School, there are 100 learners in Grade 12. A survey was carried out to determine how many of these learners liked Netball, Football or Volleyball and the following were the findings;

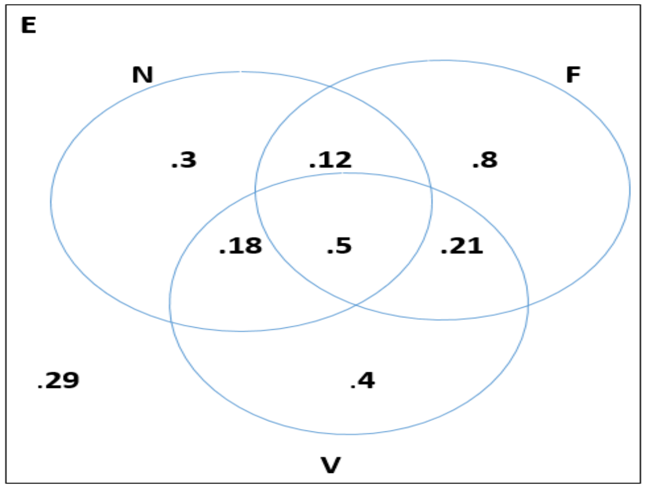
5 liked all the three games
8 liked Football only
4 liked volleyball only
3 liked Netball only
12 liked Football and Netball
21 liked Football and Volleyball
18 liked Netball and volleyball

i. **Illustrate this information on a Venn diagram**
ii. **How many learners did not like any of the three games?**
iii. **How many learners did not like volleyball?**
iv. **How many learners liked two games only?**
v. **Assuming that, the total number of Grade 12 learners is 80 and other likings remains the same except that 3 more learners liked all the games. Find the number of learners who liked neither of the 3 games**

SOLUTIONS (b i page 2)

Hints; to illustrate this information on a Venn diagram, we have to identify how many sets we are talking about. There are three (3) games in this case, hence the number of sets is also three (3). However the three sets are coming from one (1) set of 100 learners called the universal set denoted by E,

Netball set will be denoted by **N**, **Volleyball** set by **V** and **Football** set by **F**. The Venn diagram will be as follows;



SOLUTIONS (b ii page 3)

Hints; the number of learners who did not like any of the games can be denoted by x, it will be found by adding all the **likings** talked about in the question plus x and equated to the total number of learners (**100**) as follows;

$$three\ games + F + V + N + F\&N + F\&V + N\&V + x = total$$

Inserting the values we have

$$5 + 8 + 4 + 3 + 12 + 21 + 18 + x = 100$$

Adding like terms we have

$$71 + x = 100$$

Collecting like terms and changing the sign on 71 we have

$$x = 100 - 71$$

$\therefore x = 29$ **learners did not like any of the games**

SOLUTIONS (b iii page 4)

Hints; **we can denote them by x we have to add all the numbers that are part of set V and subtract the obtained sum from the total number of learners (100) as follows;**

$$\boldsymbol{three\ games + N\&V + V + V\&F + x = total}$$

Inserting numbers we have;

$$\mathbf{5 + 18 + 4 + 21 + x = 100}$$

Adding like terms we have;

$$\mathbf{48 + x = 100}$$

Collecting like terms and changing the sign on 48 we have;

$$\mathbf{x = 100 - 48}$$

$\therefore x = 52$ **Learners did not like volleyball**

SOLUTIONS (b iv page 5)

Hints; **we need only to add the numbers found in enclosures of two sets such as 12 for (N&F), 18 for (N&V) and 21 for (F&V)**

$$\boldsymbol{two\ games\ only = (N\&F) + (N\&V) + (F\&V)}$$

$$\boldsymbol{two\ games\ only = 12 + 18 + 21}$$

$$\boldsymbol{\therefore two\ games\ only = 51}$$

SOLUTIONS (b v page 6)

Hints; **in this case they have reduced the total number of learners to 80 and they have increased the number of learners who liked all the three games by 3 meaning 5+3=8 is the**

number of learners who liked all the three games. So the formula will be as follows;

$$three\ games + F + V + N + F\&N + F\&V + N\&V + x = total$$

Inserting the numbers we have;

$$8 + 8 + 4 + 3 + 12 + 21 + 18 + x = 80$$

Adding like terms we have;

$$74 + x = 80$$

Collecting like terms and changing the sign on 74 we have;

$$x = 80 - 74$$

$\therefore x = 6$ learners neither liked any of the three games

LESSON 55 SPECIMEN QUESTION 2 PAPER 2 2016 (page 1)

i. There are 6 girls and 4 boys in a drama club. Two members are chosen at random to represent the club at a meeting. What is the probability that;
   a. Both members are boys
   b. The first member is a boy or a girl
   c. One member is a girl

ii. A box contains 6 lemons, 5 oranges and 4 apples.
   a. A fruit is selected at random, what is the probability that it is a lemon?
   b. Given that 2 fruits are selected at random, what is the probability that one is an apple and the other is a lemon?
   c. If the probability of selecting an orange is $\frac{1}{3}$ what will be the probability of selecting an apple or lemon?

SOLUTIONS (i a page 2)

Hints; we first need to know the probability of choosing a boy

$$p(boy) = \frac{boys}{boys + girls}$$

$$p(b) = \frac{4}{4+6} = \frac{4}{10}$$

$$\therefore p(b) = \frac{2}{5}$$

SOLUTIONS (i a cont. page 3)

Hints; in probabilities, the word **BOTH** means **multiplication,** so both boys means

$$p(boy) \times p(boy)$$

Substituting we have

$$\frac{2}{5} \times \frac{2}{5} = \frac{4}{25}$$

$$\therefore probability\ of\ both\ boys = \frac{4}{25}$$

SOLUTIONS (i b page 4)

Hints; we have to find the probability of choosing a girl first.

$$p(girl) = \frac{girls}{boys + girls}$$

Substituting the numbers we have

$$p(g) = \frac{6}{4+6} = \frac{6}{10} \quad \therefore p(g) = \frac{3}{5}$$

SOLUTIONS (i b cont. page 5)

Hints; in probabilities, the word OR means addition boy or girl means

$$p(boy) + p(girl)$$

Substituting what we have already calculated we have

$$\frac{2}{5} + \frac{3}{5} = \frac{2+3}{5} = \frac{5}{5}$$

$$\therefore boy\ or\ a\ girl = 1$$

SOLUTIONS (i c page 6)

Hints; in this case one is a girl, meaning that the other must be a boy. So it's a boy and a girl. The word AND means multiplication in probabilities so we have;

$$p(boy) \times p(girl)$$

Substituting the already calculated probabilities we have;

$$\frac{2}{5} \times \frac{3}{5} = \frac{6}{25}$$

$$\therefore p(boy\ \&girl) = \frac{6}{25}$$

SOLUTIONS (ii a page 7)

$$p(lemons) = \frac{lemons}{oranges + apples + lemons}$$

Substituting the numbers we have

$$p(lemons) = \frac{6}{6+4+5} = \frac{6}{15} \quad \therefore p(lemons) = \frac{2}{5}$$

SOLUTIONS (ii b page 8)

Hints; they have mentioned AND so we have to multiply. We already have $p(L)$ so we have to find $p(A)$.

$$p(A) = \frac{Apple}{oranges + apples + lemons}$$

$$p(A) = \frac{4}{5+4+6} = \frac{4}{15}$$

$$\therefore p(A) = \frac{4}{15}$$

$$p(L) \times p(A) = \frac{2}{5} \times \frac{4}{15} = \frac{8}{75}$$

$$\therefore p(L) \times p(A) = \frac{8}{75}$$

SOLUTIONS (ii c page 9)

Hints; in the question it says the probability of apple or lemon when the probability of an orange is $\frac{1}{3}$. We have to know that a probability is a number from 0 to 1, therefore in this case the sum of probabilities is 1. So;

$$P(A) + P(L) + p(O) = 1$$

We know that $p(O) = \frac{1}{3}$ so we substitute

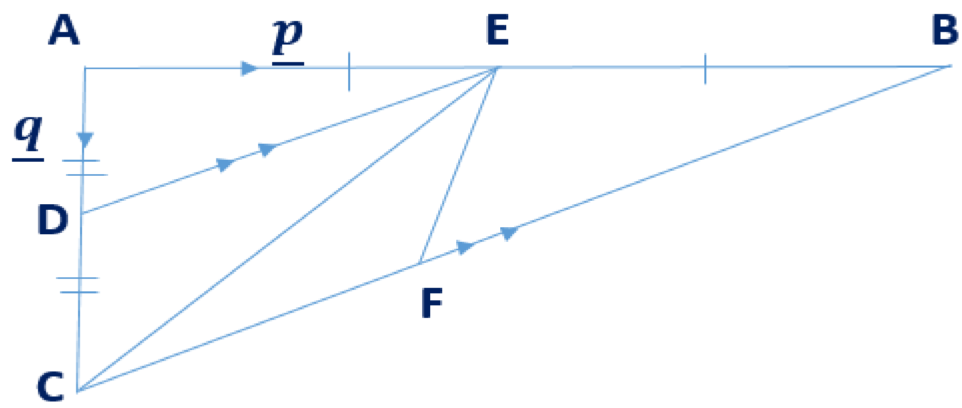
$$P(A) + P(L) + \frac{1}{3} = 1$$

Collecting like terms we have

$$P(A) + P(L) = 1 - \frac{1}{3} = \frac{3-1}{3} = \frac{2}{3} \quad \therefore P(A) + P(L) = \frac{2}{3}$$

LESSON 56 SPECIMEN QUESTION 3 PAPER 2 2016 (page 1)

In the diagram below, $\overrightarrow{AE} = \underline{p}$, $\overrightarrow{AD} = \underline{q}$, $AE = EB$, $AD = DC$, DE is parallel to CB and $CF: FB = 1: 3$.



i. Express in terms of $\underline{p}$ and/or $\underline{q}$

a. $\overrightarrow{DE}$

b. $\overrightarrow{EC}$

c. $\overrightarrow{CB}$

ii. Show that $\overrightarrow{EF} = \frac{1}{2}\left(3\underline{q} - \underline{p}\right)$.

SOLUTIONS (i a page 2)

Hints; they are asking us to find $\overrightarrow{DE}$ in vectors you don't move direct from D to E, you have to find a path to move from point D to point E. So we can move from D to A then from A to E. Hence the formula will be;

$$\overrightarrow{DE} = DA + AE$$

When you move from D to A, then you are moving opposite to the direction of the arrow hence the vector is negative so you put a negative on vector $\underline{q}$. We can now substitute into the formula

$$\overrightarrow{DE} = -\underline{q} + \underline{p}$$

$$\therefore \overrightarrow{DE} = \underline{p} - \underline{q}$$

SOLUTIONS (i b page 3)

Hints; we can use the same idea we used in solution a, the formula will be as follows;

$$\overrightarrow{EC} = EA + AC$$

EA is moving against the arrow so negative,

$\overrightarrow{AD} = \underline{q} = DC$ this means that AC is AD times 2 which will be $2AD = 2\underline{q}$. We can now substitute

$$\overrightarrow{EC} = -\underline{p} + 2\underline{q}$$

$$\therefore \overrightarrow{EC} = 2\underline{q} - \underline{p}$$

SOLUTIONS (i c page 4)

Hints; even for CB we can use the same idea, we come up with the formula,

$$\overrightarrow{CB} = CA + AB$$

We can see that CA is moving against the arrow hence negative, $AE = EB$ so AB will be AE times 2 giving us $2\underline{p}$. We now substitute

$$\overrightarrow{CB} = -2\underline{q} + 2\underline{p}$$

$$\overrightarrow{CB} = 2\underline{p} - 2\underline{q}$$

$$\therefore \overrightarrow{CB} = 2(\underline{p} - \underline{q})$$

SOLUTIONS (ii page 5)

First we come up with the formula,

$$\overrightarrow{EF} = EC + CF$$

We already found EC= $2\underline{q} - \underline{p}$, so let's think of CF, it's on the line CB and on that line there is a ratio $CF: FB = 1: 3$ in this ration CF corresponds to 1, but the total ratio is 1+3 on that line, so it will be 1 over total ratio times line CB. We already found CB= $2\underline{p} - 2\underline{q}$. The following will be the substitution

$$\overrightarrow{EF} = 2\underline{q} - \underline{p} + \frac{1}{1+3} \times CB$$

$$\therefore \overrightarrow{EF} = 2\underline{q} - \underline{p} + \frac{1}{4}(2\underline{p} - 2\underline{q})$$

SOLUTIONS (ii cont. page 6)

Opening the brackets we have;

$$\overrightarrow{EF} = 2\underline{q} - \underline{p} + \frac{2\underline{p}}{4} - \frac{2\underline{q}}{4}$$

$$\overrightarrow{EF} = 2\underline{q} - \underline{p} + \frac{\underline{p}}{2} - \frac{\underline{q}}{2}$$

Collecting like terms we have

$$\overrightarrow{EF} = 2\underline{q} - \frac{\underline{q}}{2} + \frac{\underline{p}}{2} - \underline{p}$$

Adding and subtracting

$$\overrightarrow{EF} = \frac{4\underline{q} - \underline{q}}{2} + \frac{\underline{p} - 2\underline{p}}{2}$$

$$\overrightarrow{EF} = \frac{3\underline{q}}{2} + \frac{-\underline{p}}{2}$$

Factorizing we have

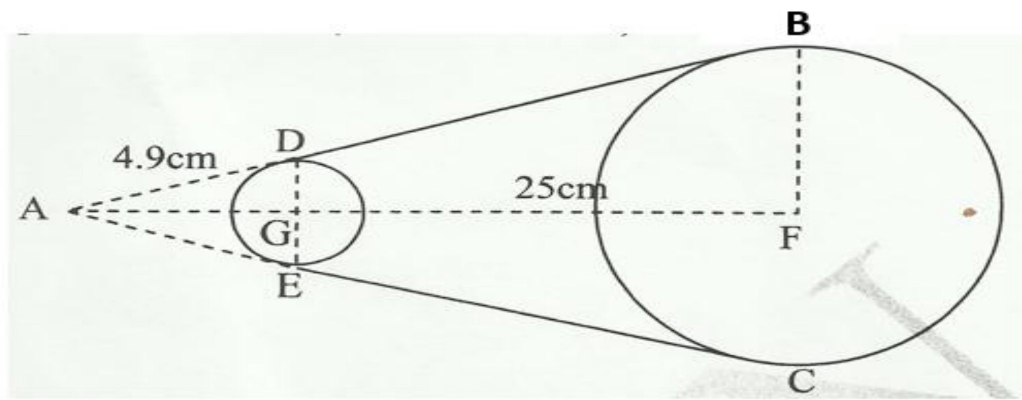
$$\therefore \overrightarrow{EF} = \frac{1}{2}(3\underline{q} - \underline{p})$$

Hence shown

LESSON 57 SPECIMEN QUESTION 4 PAPER 2 2016 (page 1)

a. Solve the equation $3x^2 + 5x + 2 = 0$, giving your answer correct to 2 decimal places.

b. The figure bellow is that of a cone ABC from which a vuvuzela DBCE was made by cutting off its sharp end as shown. Take $\pi = 3.142$



Given that $AF = 30cm, FB = 6cm, GF = 25cm\ and\ AD = 4.9cm$, calculate

i. The curved surface area of the piece that was cut off
ii. The volume of the vuvuzela

SOLUTIONS (a page 2)

To answer this question we use the formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ so we start by identifying a, b, and c from $3x^2 + 5x + 2 = 0$, the number in front of x^2 is the a, so $a = 3$, b is the number in front of x so $b = 5$, c is the number which has no x so $c = 2$

We can now substitute into the formula

$$x = \frac{-5 \pm \sqrt{5^2 - (4 \times 3 \times 2)}}{2 \times 3}$$

We multiply first

$$\therefore x = \frac{-5 \pm \sqrt{25 - 24}}{6}$$

SOLUTIONS (a continuation. page 3)

Work out what is inside the root

$$x = \frac{-5 \pm \sqrt{1}}{6} = \frac{-5 \pm 1}{6}$$

We have two signs at this point so we use them one by one

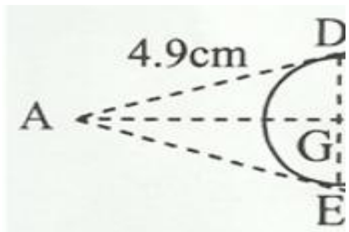
$$x = \frac{-5 + 1}{6}, \qquad x = \frac{-5 - 1}{6}$$

$$x = \frac{-4}{6}, \qquad x = \frac{-6}{6}$$

$$\therefore x = -0.67, \qquad \therefore x = -1.00$$

SOLUTIONS (b i page 4)

Area of a cone is found using the formula πrS where r is the radius, or length from $GD = 1cm$. S is the slant height $DA = 4.9cm$ we first need to calculate r using Pythagoras theorem.



$$GD^2 + AG^2 = AD^2$$

$$GD^2 = 4.9^2 - 5^2$$

$$GD^2 = 24.01 - 25$$

$$GD^2 = -0.99$$

Ignore the negative and find the square root

$$\sqrt{GD^2} = \sqrt{0.99}$$

$$\therefore r = 0.994987437cm$$

SOLUTIONS (b i cont. page 5)

To find the curved surface area we just substitute into

$$A = \pi rS$$

$$A = 3.142 \times 0.994987437 \times 4.9$$

$$\therefore A = 15.32cm^2$$

SOLUTIONS (b ii page 6)

To calculate volume of the vuvuzela, we have to calculate the total volume of the lager cone before it is cut, then we subtract volume of the small cone that was cut. Formula for volume of a bigger cone is found as;

$$\boldsymbol{Volume = \frac{1}{3} \times base\ area \times height}$$

The base area is the area of the circular part

$$\boldsymbol{Volume = \frac{1}{3} \times base\ area \times height}$$

$$\boldsymbol{Area\ of\ a\ circle = \pi r^2}$$

$$\boldsymbol{Volume = \frac{1}{3} \times \pi r^2 \times 30}$$

$$\boldsymbol{Volume = \frac{1}{3} \times (3.142 \times 6^2) \times 30}$$

$$\boldsymbol{Volume = 1 \times 3.142 \times 36 \times 10}$$

$$\boldsymbol{\therefore Volume = 1131.12cm^3}$$

SOLUTIONS (b ii cont. page 7)

We now have to find volume of the smaller cone so that we subtract from that of the bigger cone

$$\boldsymbol{Volume = \frac{1}{3} \times base\ area \times height}$$

Radius of the circle for the smaller cone is $\boldsymbol{2.1794}$, hight is 5

$$\boldsymbol{Volume = \frac{1}{3} \times \pi r^2 \times 5}$$

$$Volume = \frac{1}{3} \times 3.142 \times 2.1794^2 \times 5$$

$$Volume = \frac{1}{3} \times 3.142 \times 2.1794 \times 2.1794 \times 5$$

$$Volume = \frac{74.6191123}{3}$$

$$\therefore Volume = 24.87cm^3$$

SOLUTIONS (b ii cont. page 8)

To get the volume of the vuvuzela, we subtract volume of the small cone from volume of the bigger cone

$$V = volume(bigger - smaller)cone$$

$$V = 1131.12 - 24.87$$

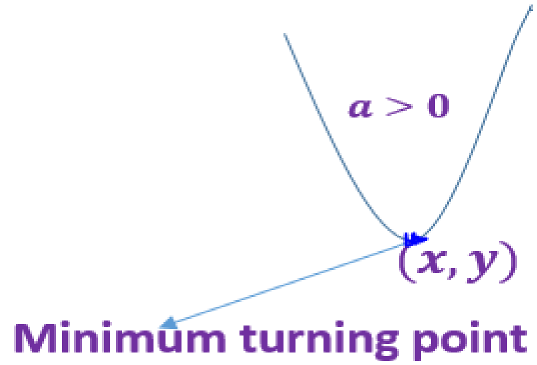
$$\therefore V = 1106.25cm^3$$

LESSON 58 QUADRATIC FUNCTIONS (page 1)

DETERMINING THE NATURE OF THE TURNING POINT & SKETCHING

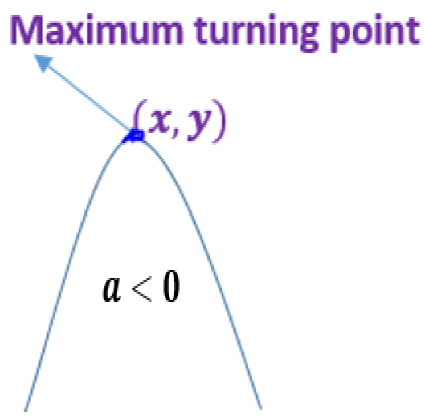
A quadratic function I of the form; $ax^2 + bx + c$

When $a > 0$ the graph will have a minimum turning point and the graph will be a follows;



NOTES PAGE 2

When $a < 0$ the graph will have a maximum turning point and the graph will be a follows;



QUESTIONS PAGE 3

Determine the nature of the turning points for each of the following functions and sketch the graph

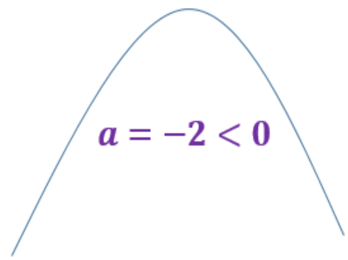
a. $f(x) = -2x^2 + 4x - 5$
b. $f(x) = x^2 - x - 15$
c. $f(x) = -2x - 4x^2$
d. $f(x) = 1 - 2x + 4x^2$
e. $f(x) = -10 + 2x - 7x^2$

SOLUTIONS (a page 4)

To determine the nature of the turning point, we have to identify a from the function, a is always the coefficient (number in front) of x^2 in the function $-2x^2 + 4x - 5$ in this case $a = -2$. If we look at the number -2 it's less than zero so $a < 0$, what nature of the turning point is it?

$$-2 < 0 \Rightarrow a < 0 \therefore we\ have\ a\ \mathbf{maximum}\ value$$

The diagram for a maximum value will be facing downwards

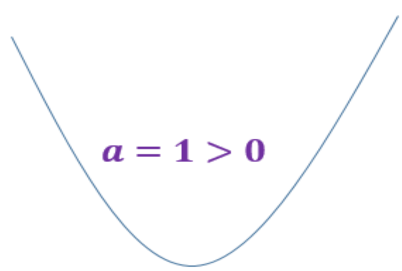


SOLUTIONS (b page 5)

To determine the nature of the turning point, we have to identify $\boldsymbol{a}$ from the function, $\boldsymbol{a}$ is always the coefficient (number in front) of x^2 in the function $f(x) = x^2 - x - 15$ in this case $\boldsymbol{a} = \mathbf{1}$. If we look at the number $\mathbf{1}$ it's greater than zero so $a > 0$, what nature of the turning point is it?

$$1 > 0 \Rightarrow a > 0 \therefore we\ have\ a\ \boldsymbol{minimum}\ value$$

The diagram for a minimum value will be facing upwards

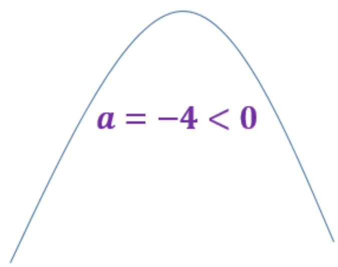


SOLUTIONS (c page 6)

To determine the nature of the turning point, we have to identify $\boldsymbol{a}$ from the function, $\boldsymbol{a}$ is always the coefficient (number in front) of x^2 in the function $f(x) = -2x - 4x^2$ in this case $\boldsymbol{a} = -4$. If we look at the number -4 it's less than zero so $a < 0$, what nature of the turning point is it?

$$-4 < 0 \Rightarrow a < 0 \therefore we\ have\ a\ maximum\ value$$

The diagram for a maximum value will be facing downwards

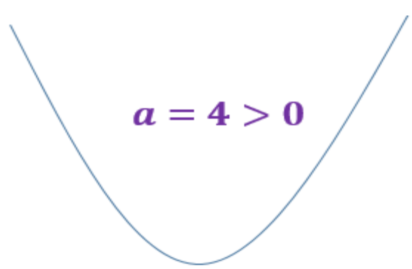


SOLUTIONS (d page 7)

To determine the nature of the turning point, we have to identify a from the function, a is always the coefficient (number in front) of x^2 in the function $f(x) = 1 - 2x + 4x^2$ in this case $a = 4$. If we look at the number 4 it's greater than zero so $a > 0$, what nature of the turning point is it?

$$4 > 0 \Rightarrow a > 0 \therefore we\ have\ a\ minimum\ value$$

The diagram for a minimum value will be facing upwards

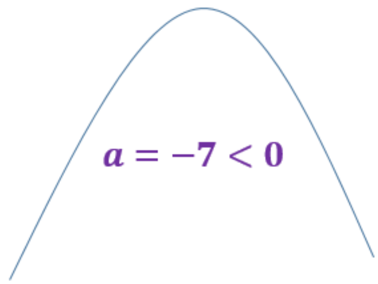


SOLUTIONS (e page 8)

To determine the nature of the turning point, we have to identify a from the function, a is always the coefficient (number in front) of x^2 in function $f(x) = -10 + 2x - 7x^2$ in this case $a = -7$. If we look at the number -7 it's less than zero so $a < 0$, what nature of the turning point is it?

$$-7 < 0 \Rightarrow a < 0 \therefore we\ have\ a\ \boldsymbol{maximum}\ value$$

The diagram for a maximum value will be facing downwards



LESSON 59 QUADRATIC FUNCTIONS (page 1)

FINDING THE COORDINATES OF THE TURNING POINT

Questions. For each of the following quadratic functions, find the coordinates of the turning point.

a. $y = 2x^2 + 4x - 5$
b. $y = 1 - 2x + 4x^2$
c. $y = 1 + x + x^2$
d. $y = 24 - 2x - x^2$

SOLUTIONS (a page 2)

To find the coordinates we have to know that we are looking for (x, y), to calculate x from a function we use the formula

$$x = \frac{-b}{2a}$$

So we first identify a and b from the function, $2x^2 + 4x - 5$. a is the number in front of x^2 while b is the number in front of x. In this case $a = 2$ and $b = 4$ so we substitute into the formula to find x

$$x = \frac{-4}{2(2)} = \frac{-4}{4} \quad \Rightarrow \quad \therefore x = -1$$

SOLUTIONS (a cont. page 3)

To find y, we substitute for x into the function

$$y = 2x^2 + 4x - 5$$

$$y = 2(-1)^2 + 4(-1) - 5$$

$$y = (2 \times 1) + (-4) - 5$$

$$y = 2 - 4 - 5$$

$$\therefore y = -7$$

We have found both x and y

$$\therefore (x, y) = (-1, -7),$$

SOLUTIONS (b page 4)

So we first identify a and b from the function $1 - 2x + 4x^2$. a is the number in front of x^2 while b is the number in front of x. In this case $a = 4$ and $b = -2$ so we substitute into the formula $x = \frac{-b}{2a}$ to find x

$$x = \frac{-(-2)}{2(4)} = \frac{2}{8} \quad \Rightarrow \quad \therefore x = \frac{1}{4}$$

To find y, substitute for x into the function $y = 1 - 2x + 4x^2$

$$y = 1 - 2\left(\frac{1}{4}\right) + 4\left(\frac{1}{4}\right)^2 \Rightarrow y = 1 - \frac{1}{2} + 4\left(\frac{1}{16}\right)$$

$$y = \frac{1}{1} - \frac{1}{2} + \frac{1}{4} = \frac{4-2+1}{4} \quad \therefore = \frac{3}{4}$$

We have found both x and y

$$\therefore (x, y) = \left(\frac{1}{4}, \frac{3}{4}\right)$$

SOLUTIONS (c page 5)

So we first identify a and b from the function $1 + x + x^2$. a is the number in front of x^2 while b is the number in front of x. In this case $a = 1$ and $b = 1$ so we substitute into the formula $x = \frac{-b}{2a}$ to find x

$$x = \frac{-1}{2(1)} = \frac{-1}{2} \quad \Rightarrow \quad \therefore x = \frac{-1}{2}$$

To find y, substitute for x into the function $y = 1 + x + x^2$

$$y = 1 + \frac{-1}{2} + \left(\frac{-1}{2}\right)^2 \Rightarrow y = 1 - \frac{1}{2} + \frac{1}{4}$$

$$y = \frac{4-2+1}{4} \quad \therefore y = \frac{3}{4}$$

We have found both x and y

$$\therefore (x, y) = \left(-\frac{1}{2}, \frac{3}{4}\right)$$

SOLUTIONS (d page 6)

So we first identify a and b from the function, $24 - 2x - x^2$. a is the number in front of x^2 while b is the number in front of x. In this case $a = -1$ and $b = -2$ so we substitute into the formula $x = \frac{-b}{2a}$ to find x

$$x = \frac{-(-2)}{2(-1)} = \frac{2}{-2} \quad \Rightarrow \quad \therefore x = -1$$

To find y, substitute for x into the function $24 - 2x - x^2$

$$y = 24 - 2(-1) - (-1)^2$$

$$y = 24 + 2 - 1 \qquad \therefore y = 25$$

We have found both x and y

$$\therefore (x, y) = (-1, 25),$$

LESSON 60 QUESTION 1 PAPER 2 2004 PAGE 1

a. Given that $a = 3, b = 2$ and $c = 4$ find the value of $\frac{a}{b} + \frac{b}{c}$

b. Solve $3(m - 5) = 7 - 2(m - 3)$

c. Factorise completely $4 - 16x^2$

SOLUTION (a page 2)

We first substitute for $a = 3, b = 2$ and $c = 4$

$$\frac{a}{b} + \frac{b}{c} = \frac{3}{2} + \frac{2}{4}$$

Finding the common denominator in this case its 4

$$\frac{..+..}{4}$$

Denominator 2 into 4 is 2, multiply 2 by numerator 3 its 6

$$\frac{6+..}{4}$$

Denominator 4 into 4 is 1, multiply 1 by numerator 2 its 2

$$\frac{6+2}{4}$$

Adding numerators we have

$$\frac{8}{4}$$

Dividing by 4 we have the answer

$$\therefore Ans = 2$$

SOLUTION (b PAGE 3)

$$3(m-5) = 7-2(m-3)$$

Open brackets on the left multiply by 3, on the right by 2

$$3 \times m - 3 \times 5 = 7 - 2 \times m - 2 \times -3$$

Negative times negative on the right will give u positive

$$3m - 15 = 7 - 2m + 6$$

Collect like terms numbers with m to the left no m right

Then – $2m$ becomes $+2m$ and – 15 becomes $+15$

$$3m + 2m = 7 + 6 + 15$$

$$5m = 13 + 15$$

$$5m = 28$$

Divide both sides by 5 to find m

$$\frac{5m}{5} = \frac{28}{5} \therefore m = 5\frac{3}{5}$$

SOLUTION (c page 4)

$$4 - 16x^2$$

4 can go into $4 = 1$ and $16 = 4$ so we factorise it

$$4(1 - 4x^2)$$

Write numbers inside brackets in index form

$$4\left(1^2 - 2^2x^2\right)$$

Know that $1^2 - 2^2x^2$ is a difference of two squares

If it was $y^2 - x^2$ factorizing will be $(y - x)(y + x)$

Factorise the difference of two squares

$$\therefore 4\left(1^2 - 2^2x^2\right) = 4[(1 - 2x)(1 + 2x)]$$

LESSON 61 QUESTION 1 PAPER 2 2015 PAGE 1

a. Evaluate $2\frac{1}{4} + 4\frac{1}{2} \div \frac{2}{9}$

b. Solve the equation $\frac{x+4}{2} = \frac{2x-1}{3}$

c. Simplify $\frac{h^2-k^2}{h+k}$

d. Express $K75.00$ as a percentage 0f $K50$

SOLUTION (a page 2)

$$2\frac{1}{4} + 4\frac{1}{2} \div \frac{2}{9}$$

We first change fractions to improper then divide

$$2\frac{1}{4} + 4\frac{1}{2} \div \frac{2}{9} = \frac{9}{4} + \frac{9}{2} \div \frac{2}{9}$$

Division changes to multiplication, and the third fraction is changed upside down

$$\frac{9}{4}+\frac{9}{2}\times\frac{9}{2}$$

We multiply first

$$\frac{9}{4}+\frac{81}{4}$$

Finding common denominator and adding

$$\frac{9}{4}+\frac{81}{4}=\frac{9+81}{4}=\frac{90}{4}=\frac{45}{2}=22\frac{1}{2}$$

SOLUTION (b page 3)

$$\frac{x+4}{2}=\frac{2x-1}{3}$$

Cross multiplication

$$\frac{x+4}{2}=\frac{2x-1}{3}$$

$$3(x+4)=2(2x-1)$$

Opening brackets

$$3x+12=4x-2$$

Collecting like terms

$$3x-4x=-2-12$$

Subtracting

$$-x=-14$$

Multiplying both sides by negative 1

$$(-x=-14)(-1)$$

$$\therefore x=14$$

SOLUTION (c page 4)

$$\frac{h^2 - k^2}{h + k}$$

The numerator is a difference of two squares so we first factorise it like we did in the previous lesson

$$\frac{(h - k)(h + k)}{h + k}$$

We can now divide to have the answer

$$h - k$$

SOLUTION (d PAGE 5)

$K75.00$ as a percentage Of $K50$

We have to make a fraction where K75 will be the numerator and K50 will be the denominator then multiply by 100

$$\frac{75}{50} \times 100 = 75 \times 2 = 150\%$$

LESON 62 SEQUENCES AND SERIES QUESTIONS (PAGE 1)

a. **Find the sum 10 terms of 1, 5, 9, 13, 17 ……**
b. **The fifth term of an AP is 10 and the tenth term is 20, find the values of a and d, hence find the 20th term.**
c. **For the sequence 14, 17, 20, 23, ……… Find the,**
   i. **9th term**
   ii. **Sum of the first 24 terms.**

SOLUTIONS (a page 2)

Sum of 10 terms for 1, 5, 9, 13, 17 ……

The formula for sum is $S_n = \frac{n}{2}[2a + (n - 1)d]$

$n = 10$ (the number of terms)

$a = 1$ (The first term)

$d = 5 - 1 = 4$ (Second- first=Common difference)

So we substitute

$$S_{10} = \frac{10}{2}[2(1) + (10 - 1)4]$$

$$S_{10} = 5[2 + (9)4]$$

$$S_{10} = 5[2 + 36]$$

$$S_{10} = 5[38]$$

$$\therefore S_{10} = 190$$

SOLUTIONS (b page 3)

The formula for term of an AP is $t_n = a + (n - 1)d$

Fifth term of an AP is 10

Meaning term number 5 is 10, so n=5, then term number 5 is 10. So we have;

$$term_5 = a + (5 - 1)d = 10$$

$$a + 4d = 10 \ldots \ldots \ldots \ldots equation\ i$$

The tenth term is 20

Meaning term number 10 is 20, so n=10, then term number 10 is 20. So we have;

$$term_{10} = a + (10 - 1)d = 20$$

$$a + 9d = 20 \ldots \ldots \ldots \ldots \ldots equation\ ii$$

SOLUTIONS (b cont. page 4)

We can now solve equation i and ii simultaneously

$$a + 4d = 10 \ldots \ldots . . i$$

$$a + 9d = 20 \ldots \ldots . . ii$$

Using equation i make a the subject of the formula we have;

$$a = 10 - 4d$$

We now substitute for a in equation ii.

$$10 - 4d + 9d = 20$$

We have like terms $-4d + 9d = 5d$ so we have

$$10 + 5d = 20$$
$$5d = 20 - 10$$
$$5d = 10$$
$$\frac{5d}{5} = \frac{10}{5} \qquad \therefore d = 2$$

SOLUTIONS (b cont. page 5)

We now want to find a

We can use one of the equations to find a

Using equation i to make a the subject of the formula we have;

$$a = 10 - 4d$$

We know the value of $d = 2$, so we substitute for d

$$a = 10 - 4(2)$$
$$a = 10 - 8$$
$$\therefore a = 2$$

SOLUTIONS (c i page 6)

The formula for finding a given term is $t_n = a + (n - 1)d$ we want to find term number 9 so we have $n = 9$ then we have to know that in 14, 17, 20, 23

$$a = 14 \text{ (The first term)}$$

$$d = 17 - 14 = 3 \text{ (Second - first) term}$$

We substitute into the formula

$$t_9 = 14 + (9 - 1)3$$

$$t_9 = 14 + (8)3$$
$$t_9 = 14 + 24$$
$$\therefore t_9 = 38$$

SOLUTIONS (c ii page 7)

Sum of the first **24** terms formula $S_n = \frac{n}{2}[2a + (n-1)d]$ so we just substitute

$$S_{24} = \frac{24}{2}[2(14) + (24-1)3]$$
$$S_{24} = 12[28 + (23)3]$$
$$S_{24} = 12[28 + 69]$$
$$S_{24} = 12[97]$$
$$S_{24} = 1164$$

LESON 63 QUESTION ON SERIES AND SEQUENCES (PAGE 1)

a. For the sequence $2, 5, 8, 11, \dots \dots$ find
   i. The sixth term
   ii. An expression for the nth term
   iii. The 30th term using the expression found in ii
   iv. The sum of the first 30 terms

SOLUTION (a. page 2)

Finding the six term in $2, 5, 8, 11, \dots \dots$ we have to first determine the common difference or the number being added to each term in order to get the next term. Common difference is found by subtracting the first term from the second term. The first term in this case is 2 and the second term is 5.

$$Common\ difference = second\ term - first\ term$$

$$Common\ difference = 5 - 2$$

$$\therefore Common\ difference = 3$$

To get the sixth term we have to find the fifth term by adding the common difference 3 to the fourth term 11 then add the common difference to the fifth term.

$$Fifth\ term = fourth\ term + common\ difference$$

$$Fifth\ term = 11 + 3 = 14$$

$$Sixth\ term = fifth\ term + common\ difference$$

$$Sixth\ term = 14 + 3$$

$$\therefore Sixth\ term = 17$$

SOLUTION (ii. page 3)

Finding the expression for the n^th term in this sequence, we have to use the formula $T_n = a + (n-1)d$ in this formula T_n stands for n^th term, a stands for first term, n stands of number of terms and d stands for common difference.

In $2, 5, 8, 11$

$$a = 2$$

$$d = 5 - 2 = 3$$

We then substitute into the formula

$$T_n = a + (n-1)d$$

$$T_n = 2 + (n-1)3$$

Opening brackets we have

$$T_n = 2 + 3n - 3$$

Collecting like terms we have

$$T_n = 3n + 2 - 3$$

$$\therefore T_n = 3n - 1$$

SOLUTION (iii. page 4)

Finding the 30th term using the expression $T_n = 3n - 1$ in this case $n = 30$. So we substitute

$$T_{30} = 3(30) - 1$$

$$T_{30} = 90 - 1$$

$$\therefore T_{30} = 89$$

SOLUTION (iv. page 5)

To find the sum of the first 30 terms we use the formula

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

We already have all the numbers so we just substitute

$$S_{30} = \frac{30}{2}[2(2) + (30 - 1)3]$$

$$S_{30} = 15[4 + (29)3]$$

$$S_{30} = 15[4 + 87]$$

$$S_{30} = 15[91]$$

$$\therefore S_{30} = 1365$$

QUESTION FOR YOUR PRACTICE (page 6)

a. For the sequence $1, 5, 9, 13, \dots \dots \dots \dots$ find
   i. The 7th term
   ii. An expression for the nth term
   iii. The 20th term using your expression

iv. The sum of the first 40 terms

LESSON 64 FUNCTIONS (page 1)

For the function $f(x) = 4x - 6$ and $g(x) = \frac{6x+2}{2x+3}$, x$\neq$ 2, find

a. $f(4)$
b. $g(2)$
c. $f^{-1}(x)$
d. $g^{-1}(x)$
e. $f^{-1}(6)$
f. $g^{-1}(1)$

SOLUTION (a page 2)

To find $f(4)$ substitute 4 where there is x in $f(x) = 4x - 6$

$$f(4) = 4(4) - 6$$

$$f(4) = 16 - 6$$

$$\therefore f(4) = 10$$

SOLUTION (b page 3)

To find $g(2)$ we substitute 2 where there is x in $g(x) = \frac{6x+2}{2x+3}$

$$g(x) = \frac{6(2) + 2}{2(2) + 3}$$

$$g(x) = \frac{12 + 2}{4 + 3}$$

$$g(x) = \frac{14}{7}$$

$$\therefore g(x) = 2$$

SOLUTION (c page 4)

To find $f^{-1}(x)$ first equate $f(x) = 4x - 6$ to y then make x the subject of the formula

$$f(x) = 4x - 6 = y$$

$$4x - 6 = y$$

$$4x = y + 6$$

Dividing both sides by 4

$$\frac{4x}{4} = \frac{y+6}{4}$$

$$x = \frac{y+6}{4}$$

Then where there is x write $f^{-1}(x)$ where there is y write x

$$\therefore f^{-1}(x) = \frac{x+6}{4}$$

SOLUTION (d page 5)

To find $g^{-1}(x)$ equate $g(x) = \frac{6x+2}{2x+3}$ to y and make x the subject of the formula.

$$g(x) = \frac{6x+2}{2x+3} = y \qquad \Rightarrow \qquad \frac{6x+2}{2x+3} = y$$

We can now introduce a denominator 1 on y

$\frac{6x+2}{2x+3} = \frac{y}{1}$ **Now cross multiply** $(6x + 2)1 = y(2x + 3)$

SOLUTION (d cont. page 6)

Opening brackets

$$6x + 2 = 2yx + 3y$$

Terms with x collected on the same side

$$6x - 2yx = 3y - 2$$

Factorize x on the left

$$x(6 - 2y) = 3y - 2$$

Divide both sides by $(6 - 2y)$ to make x the subject

$$\frac{x(6-2y)}{6-2y} = \frac{3y-2}{6-2y}$$

$$x = \frac{3y-2}{6-2y}$$

Where there is x write $g^{-1}(x)$ where there is y write x

$$\therefore g^{-1}(x) = \frac{3x-2}{6-2x}$$

SOLUTION (e page 7)

To find $f^{-1}(6)$ substitute 6 for x in $f^{-1}(x) = \frac{x+6}{4}$ where there is x so we have

$$f^{-1}(6) = \frac{6+6}{4}$$

$$f^{-1}(6) = \frac{12}{4}$$

$$\therefore f^{-1}(6) = 3$$

SOLUTION (f page 8)

To find $g^{-1}(1)$ just substitute 1 for x in $\therefore g^{-1}(x) = \frac{3x-2}{6-2x}$ where there is x so we have

$$g^{-1}(1) = \frac{3(1)-2}{6-2(1)}$$

$$g^{-1}(1) = \frac{3-2}{6-2}$$

$$\therefore g^{-1}(1) = \frac{1}{4}$$

LESSON 65 FUNCTIONS (PAGE 1)

a. Given functions $g(x) = 6x + 2$ and $f(x) = 2x + 3$ find;

i. $gf(x)$
ii. $gf(2)$
iii. $f^{-1}(x)$
iv. $f^{-1}(x)$

SOLUTION (a i page 2)

To find $gf(x)$ it means we have to write or substitute $f(x)$ into the function $g(x)$ such that where there is x in $g(x) = 6x + 2$ we write $2x + 3$. We here we go

$$g(x) = 6x + 2$$

$$gf(x) = 6(2x + 3) + 2$$

We can now open the brackets

$$gf(x) = 12x + 18 + 2$$

Adding like terms we have;

$$\therefore gf(x) = 12x + 20$$

SOLUTION (a ii page 3)

To find $gf(2)$ we have to just substitute 2 in $gf(x) = 12x + 20$ where there is x we write 2 so here we go.

$$gf(x) = 12x + 20$$

$$gf(2) = 12(2) + 20$$

$$gf(2) = 24 + 20$$

$$\therefore gf(2) = 44$$

SOLUTION (a iii page 4)

To find $f^{-1}(x)$ equate $f(x) = 2x + 3$ to y and make x the subject of the formula.

$$2x + 3 = y$$

Move 3 to the other side

$$2x = y - 3$$

Divide both sides by 2

$$\frac{2x}{2} = \frac{y-3}{2}$$

$$x = \frac{y-3}{2}$$

Where there is x write $f^{-1}(x)$ where there is y write x

$$\therefore f^{-1}(x) = \frac{x-3}{2}$$

SOLUTION (a iii page 5)

To find $f^{-1}(-5)$ just substitute -5 for x in $f^{-1}(x) = \frac{x-3}{2}$

$$f^{-1}(-5) = \frac{-5-3}{2}$$

Adding in the numerator

$$f^{-1}(-5) = \frac{-8}{2}$$

Divide 2 into numerator and denominator

$$\therefore f^{-1}(-5) = -4$$

LESSON 66 MIXED QUESTIONS (PAGE 1)

a. Factorize each of the following

- i. $x^2 - y^2$
- ii. $49x^2 - 144$
- iii. $169 - 4x^2$

b. Solve the equation $sin\theta = 0.766$ for $0^o \leq \theta \leq 180^o$

SOLUTION (a i page 2)

To factorise $x^2 - y^2$ we have to know that subtraction means finding the difference, x and y are raised to the power 2, meaning they are squares. This is called a difference of two squares. Factorization is done as follows;

$$x^2 - y^2 = (x - y)(x + y)$$

SOLUTION (a ii page 3)

To factorise $49x^2 - 144$ we have to know that 49 and 44 are perfect squared numbers such that they have whole number square-roots. So we have to create a difference of two squares. The square root of 46 is 7 and that of 144 is 12. So we have

$$49 = 7^2 \text{ and } 144 = 12^2$$

We then substitute

$$49x^2 - 144 = 7^2x^2 - 12^2$$

$$7^2x^2 - 12^2 = (7x)^2 - (12)^2$$

Now factorize the difference of two squares $(7x)^2 - (12)^2$

$$\therefore (49x^2 - 144) = (7x - 12)(7x + 12)$$

SOLUTION (a iii page 4)

To factorise $169 - 4x^2$ know that **169** and **4** are perfect squared numbers such that they have whole number square-roots. So we have to create a difference of two squares. The square root of **169** is **13** and that of **4** is **2.** So we have

$$169 = 13^2 \text{ and } 4 = 2^2$$

We then substitute

$$169 - 4x^2 = 13^2 - (2x)^2$$

Now factorize the difference of two squares $13^2 - 2^2$

$$\therefore 169 - 4x^2 = (13 - 2x)(13 + 2x)$$

SOLUTION (b page 5)

To solve the equation $sin\theta = 0.766$ for $0^o \leq \theta \leq 180^o$ we first have to interpret $0^o \leq \theta \leq 180^o$ it means the answer should not be less than 0^o and should not be more than 180^o

Get a calculator, put it in degrees and press

Shift sin 0.766 $=$

The first answer will come

$$\theta = 49.96^o$$

This question has two answers, to get the second answer we subtract the first answer from 180^o

$$\theta = 180^o - 49.96^o$$

$$\therefore \theta = 130^o$$

LESON 67 QUESTIONS (PAGE 1)

Find the gradient from each of the following equations of the lines

a. $2x - y = 5$
b. $x - 2y = 4$
c. $4x + 16y = 32$
d. $4x + 2y = 17$
e. $3y = 4 - 6x$
f. $3x = y - 3$

SOLUTION (a page 2)

When finding gradient from an equation, first make y the subject of the formula then pick the number in front of x $(coefficient\ of\ x)$ as gradient of the line

$$2x - y = 5$$

Move $2x$ across the equal sign

$$-y = -2x + 5$$

Multiplying throughout by negative one to remove the negative on y

$$(-y = -2x + 5) \times -1$$

$$y = 2x - 5 \therefore gradient = 2$$

SOLUTION (b page 3)

$$x - 2y = 4$$

Move x across $=$ it becomes $-x$

$$-2y = -x + 4$$

Multiply both sides by negative 1

$$(-2y = -x + 4) \times -1$$

$$2y = x - 4$$

Divide by 2 to make y the subject of the formula

$$\frac{2y}{2} = \frac{x}{2} - \frac{4}{2}$$

$$y = \frac{1}{2}x - 2 \qquad \therefore gradient = \frac{1}{2}$$

SOLUTION (c page 4)

$$4x + 16y = 32$$

$$16y = 32 - 4x$$

$$\frac{16y}{16} = \frac{32}{16} - \frac{4x}{16}$$

$$y = 2 - \frac{1}{4}x \quad \therefore gradient = -\frac{1}{4}$$

SOLUTION (d page 5)

$$4x + 2y = 17$$

$$2y = 17 - 4x$$

$$\frac{2y}{2} = \frac{17}{2} - \frac{4x}{2}$$

$$y = \frac{17}{2} - 2x \quad \therefore gradient = -2$$

SOLUTION (e page 6)

$$3y = 4 - 6x$$

$$\frac{3y}{3} = \frac{4}{3} - \frac{6x}{3}$$

$$y = \frac{4}{3} - 2x$$

$$\therefore gradient = -2$$

SOLUTION (f page 7)

$$3x = y - 3$$
$$3x - 3 = y$$

$$\therefore gradient = 3$$

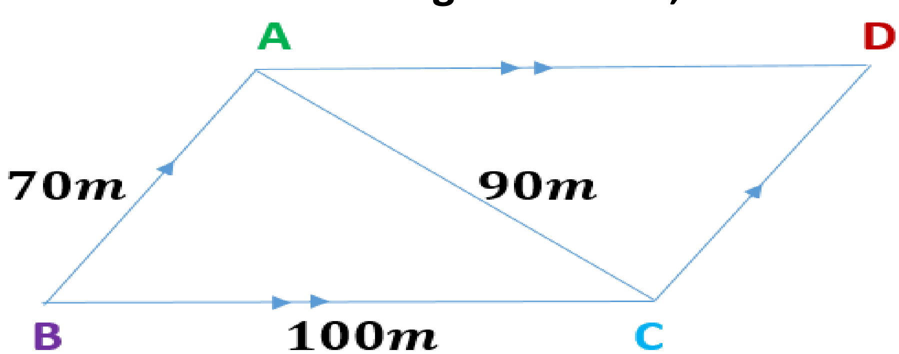
QUESTIONS FOR YOUR PRACTICE

Find the gradient from each of the following equations of lines

a. $5y - x = 2$
b. $3x - y = 4$
c. $4x + 2y = 7$

LESSON 68 SPECIMEN QUESTION 7 PAPER 2 2016 (PAGE 1)

A farm is in a shape of a parallelogram, ABCD. To accommodate two different crops, the farm is divided into two parts as shown in the diagram below;



Given that AB=70m, AC=90m and BC=100m, calculate

i. Angle BAC
ii. Area of the triangle ABC
iii. Shortest distance from D to AC

SOLUTION (a i page 2)

HINTS; to calculate the angle $BAC = \theta$ the angle is opposite to **BC** and is formed by sides **AB** and **AC** we have to use the **cosine rule** and it will be stated as follows.

$$BC^2 = AB^2 + AC^2 - 2(AB \times AC)COS\theta$$

We now substitute for lengths

$$100^2 = 70^2 + 90^2 - 2(70 \times 90)COS\theta$$

Multiplying we have

$$10,000 = 4,900 + 8,100 - 12,600COS\theta$$

Adding like terms

$$10,000 = 13,000 - 12,600COS\theta$$

Collecting like terms

$$10,000 - 13,000 = -12,600COS\theta$$

SOLUTION (a i cont. page 3)

$$10,000 - 13,000 = -12,600COS\theta$$

Subtracting on the left

$$-3,000 = -12,600COS\theta$$

Dividing both sides by $-12,600$ on the right

$$\frac{-3,000}{-12,600} = \frac{-12,600COS\theta}{-12,600}$$

$$0.238095238 = COS\theta$$

Press shift, COS $0.238095238 =$

$\theta = 76.22585301$ $\therefore angle\ BAC = 76.23^o$

SOLUTION (a ii page 4)

HINTS; to find the area of triangle ABC we have to use the angle that we have just calculated in **i** and the two sides adjacent to or touching the angle $BAC = 76.23^o$ the two sides are AC and AB, the formula is as follows;

$$A = \frac{1}{2} \times AC \times AB \times sin\theta$$

Substituting we have

$$A = \frac{1}{2} \times 90 \times 70 \times sin76.23^o$$

Dividing 2 into 90 and finding $sin76.23^o$

$$A = 45 \times 70 \times 0.971259042$$

Multiplying 45×70

$$A = 3,150 \times 0.971259042$$

$$\therefore A = 3,059.47m^2$$

SOLUTION (a iii page 5)

HINTS; to find the shortest distance from D to AC we have to consider the following;

- This being a parallelogram, $AD = BC$ and $AB = DC$
- Triangle BAC is equal to triangle DCA
- The area for triangle BAC is equal to that of DCA
- The shortest distance from D to AC is equal to the shortest distance from B to AC

There is also another formula for of a triangle

$$A = \frac{1}{2} \times base \times height$$

SOLUTION (a iii cont. page 6)

In this case the base will be AC=90m while the height will be the shortest distance we are looking for, so we substitute into the formula

$$A = \frac{1}{2} \times base \times height$$

$$A = \frac{1}{2} \times 90 \times height$$

We already calculated area= $3,059.47$ so we substitute

$$3,059.47 = \frac{1}{2} \times 90 \times h$$

Dividing 2 into 90 on the right we have

$$3,059.47 = 45h$$

Dividing both sides by 45 we have

$$\frac{3,059.47}{45} = \frac{45h}{45}$$

$$67.98822 = h$$

$$\therefore shortest\ distance = 67.99m$$

LESSON 69 GEOMETRIC PROGRESSIONS (PAGE 1)

When we have $1, 2, 4, 8, 16, \ldots \ldots \ldots ..$ the first term is 1 and the remaining terms are formed by multiplying by 2 (common ratio) each term. The formed set of these numbers forms a geometric progression (GP). The first term is denoted by a while the common ratio is denoted by r.

QUESTIONS

For each of the following sequences, say whether it is a geometric progression or not.

a. $10, 11, 12, 13, \ldots$

b. $3, 6, 12, 24, \ldots$

c. $-100, 50, -25, 12.5$

d. $\frac{9}{100}, \frac{3}{10}, 1, \frac{10}{3}$

1. For each of the following GPs find the 6th term
   a. $100, 50, 25, 12.5$
   b. $3, 6, 12, 24, \ldots$

SOLUTION (1 a page 2)

The sequence $10, 11, 12, 13, \ldots$ can only be a GP if it has a common ration r that is a number which each term is being multiplied by in order to get the next term, to get r we have to divide the second term by the first term or the third term by the second, the answer must be the same. So we try

$$r = 11 \div 10 = 1.1$$

$$r = 12 \div 11 = 1.090909091$$

$$r = 13 \div 12 = 1$$

In this case there is no common ratio hence $10, 11, 12, 13,$ is not a GP

SOLUTION (1 b page 3)

$$3, 6, 12, 24, \ldots$$

We have to find out if there is a common ratio

$$r = 6 \div 3 = 2$$

$$r = 12 \div 6 = 2$$

$$r = 24 \div 12 = 2$$

In this case there is a common ratio of 2 hence $3, 6, 12, 24, \ldots,$ is a GP

SOLUTION (1 c page 4)

$$-100, 50, -25, 12.5$$

Test for common ratio

$$r = 50 \div -100 = -0.5$$

$$r = -25 \div 50 = -0.5$$

Common ratio -0.5, hence $-100, 50, -25, 12.5$ is a GP

SOLUTION (1 d page 5)

$$\frac{9}{100}, \frac{3}{10}, 1, \frac{10}{3}$$

$$r = \frac{3}{10} \div \frac{9}{100} = \frac{3}{10} \times \frac{100}{9} = \frac{10}{3}$$

$$r = 1 \div \frac{3}{10} = 1 \times \frac{10}{3} = \frac{10}{3}$$

There is a common ration of $\frac{10}{3}$ hence $\frac{9}{100}, \frac{3}{10}, 1, \frac{10}{3}$ is a GP

SOLUTION (2 a page 6)

To find the 6th term for $100, 50, 25, 12.5$ we use the formula

$$T_n = ar^{n-1}$$

Where $n = 6, a = 100$ and $r = 0.5$

$$T_6 = (100)(0.5)^{6-1}$$

$$T_6 = (100)(0.5)^5$$

$$T_6 = (100)(0.5 \times 0.5 \times 0.5 \times 0.5 \times 0.5)$$

$$T_6 = (100)(0.03125) \therefore T_{10} = 3.125^5$$

SOLUTION (2 b page 7)

For $3, 6, 12, 24, \ldots$

$$T_n = ar^{n-1}$$

$$T_6 = (3)(2)^{6-1}$$

$$T_6 = (3)(2)^5$$

$$T_6 = 3 \times 32$$

$$\therefore T_6 = 96$$

LESSON 70 SPECIMEN P2 QUESTIONS 5 2016 (PAGE 1)

a. The 3rd and 4th terms of a geometric progression are 4 and 8 respectively. Find
   i. The common ratio, first and second term
   ii. The sum of the first 10 terms
   iii. The sum infinity of this geometric progression

b. Simplify $\frac{13k}{20a^2} \times \frac{5}{39k^2}$

SOLUTION (a i page 2)

To find the common ratio we divide 4 into 8 because they are coming one after the other;

$$\therefore r = 8 \div 4 = 2$$

Now that 4 is the third term, then we obtain the second term by dividing r into the third term.

$$Second\ term = 4 \div r = 4 \div 2 = 2$$

$$\therefore second\ term = 2$$

To get the first term we divide r into the second term

$$\therefore First\ term = 2 \div r = 2 \div 2 = 1$$

SOLUTION (a ii page 3)

Sum of the first 10 terms, we use the following formula;

$$S_n = \frac{a(1-r^n)}{(1-r)} \quad n = 10, r = 2\ and\ a = 1 \quad S_{10} = \frac{1(1-2^{10})}{(1-2)}$$

$$S_{10} = \frac{1(1-1024)}{-1}$$

$$S_{10} = \frac{1(-1023)}{-1} \Rightarrow S_{10} = \frac{-1023}{-1}$$

$$\therefore S_{10} = 1023$$

SOLUTION (a iii page 4)

To find the sum of infinity (∞) terms, it means we do not know the number of terms because they are too many to count, so we first find the value of $S_n = \frac{a(1-r^n)}{(1-r)}$ such that we do not know the real value of n as follows;

$$S_n = \frac{1(1-2^n)}{(1-2)}$$

$$subtruating\ in\ the\ denominator$$

$$S_n = \frac{1(1-2^n)}{-1}$$

Dividing 1 outside brackets by denominator -1

$$S_n = -1(1-2^n)$$

opening brackes by multiplying by -1 outside

$$S_n = 2^n - 1$$

This shows that, when n becomes too large, then the value of $S_n = 2^n - 1$ becomes too large or an infinite number too

$$\therefore S_\infty = 2^\infty - 1 = \infty$$

SOLUTION (b page 5)

$$\frac{13k}{20a^2} \times \frac{5}{39k^2}$$

$$Divide\ 13k\ and\ 39k^2\ by\ 13k, 20a^2\ and\ 5\ by\ 5$$

$$\frac{1}{4a^2} \times \frac{1}{3k}$$

$$multiply\ numerator\ by\ numerator$$

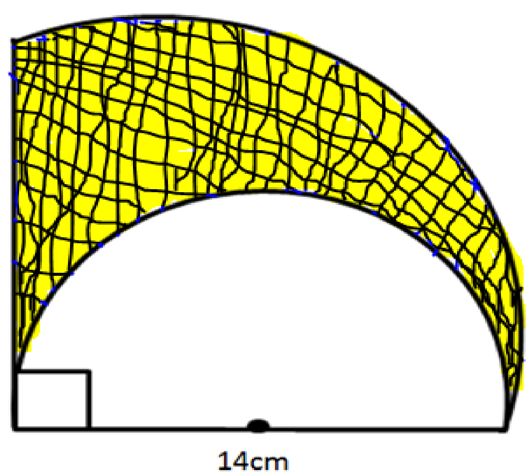
$$\frac{1}{3k \times 4a^2}$$

$$multiply\ denominator\ by\ denominator$$

$$\therefore Ans = \frac{1}{12ka^2}$$

LESSON 71 CIRCLE PAGE 1

Calculate the area of the shaded part of the sector below when the diameter of the semi-circle is 14cm



SOLUTION PAGE 2

First we calculate area of the shaded region together with that of the un-shaded region. A sector is a quarter of a circle, the formula for area of a circle is as follows

$$A_{circle} = \pi r^2$$

There are 4 sectors in a circle; to find area of a sector we divide area of a circle by 4. So the formula will be

$$A_{circle} = \frac{1}{4}\pi r^2$$

Where $r = 14$ or radius of the sector and $\pi = \frac{22}{7}$

SOLUTION cont. PAGE 3

We can now substitute into the formula

$$A_{sector} = \frac{1}{4}\pi r^2 \quad \Rightarrow \quad A_{sector} = \frac{1}{4} \times \frac{22}{7} \times 14^2$$

Expanding on 14^2 we have

$$A_{sector} = \frac{1}{4} \times \frac{22}{7} \times 14 \times 14$$

Divide 2 into 4 and 22 then 7 into 14 and 7

$$A_{sector} = \frac{1}{2} \times 11 \times 2 \times 14$$

Dividing 2 and 2

$$A_{sector} = 11 \times 14 \quad \Rightarrow \quad \therefore A_{sector} = 154cm^2$$

SOLUTION cont. PAGE 4

Now we find area of the un-shaded region

The **un-shaded** region is a **semi-circle** or **half-circle**, area of the semi-circle, is found by dividing formula for area of a circle by **2** because there are **2** semi-circles in a complete circle.

$$A_{semi-circle} = \frac{1}{2}\pi r^2$$

In this case $r = 7$ and $\pi = \frac{22}{7}$

$$A_{semi-circle} = \frac{1}{2} \times \frac{22}{7} \times 7^2$$

Expanding 7^2 we have

$$A_{semi-circle} = \frac{1}{2} \times \frac{22}{7} \times 7 \times 7$$

Dividing 2 and 22 then 7 and 7 we have

$$A_{semi-circle} = 11 \times 7$$

$$\therefore A_{semi-circle} = 77cm^2$$

SOLUTION cont. PAGE 5

To find area of the shaded region, we subtract area of the un-shaded region or semi-circle from area of the sector

$$A_{shaded\ region} = A_{sector} - A_{semi-circle}$$

Substituting we have

$$A_{shaded-region} = 154 - 77$$

$$A_{shaded\ region} = 77cm^2$$

LESSON 72 INTEGRATION page 1

When you are given to integrate $\int x^n dx$, what you have to do is add 1 to the power, meaning the power, becomes $n + 1$ get this same power and make it the denominator of x^{n+1} then add a constant c. thus $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

When you are given to integrate a constant or a number without x such as $\int ndx$ just introduce x and multiply it to n then add a constant c. thus $\int ndx = nx + c$

QUESTIONS INTEGRATE EACH OF THE FOLLOWING; PAGE 2

a. $\int x^3 dx$
b. $\int (8x^3 + 3x)dx$
c. $\int 5dx$
d. $\int (x^2 + 3x + 4)dx$

SOLUTION (a page 3)

We just have to add 1 to the power 3, then we use this power as the denominator, we also have to add a constant **c** to the final answer

$$\int x^3 dx = \frac{x^{3+1}}{3+1}$$

Adding the power and denominator

$$\frac{x^4}{4}$$

Adding a Constance

$$\therefore Answ = \frac{1}{4}x^4 + c$$

SOLUTION (b page 4)

We have the first power **3**, so we add **1** to that **3**, we then make this **sum** as the **denominator** of the **first** term. On the last term, the power is **1**, so we have **1+1** as the power and denominator of the last term

$$\int (8x^3 + 3x)dx = \frac{8x^{3+1}}{3+1} + \frac{3x^{1+1}}{1+1}$$

Adding the powers and denominators

$$\frac{8x^4}{4} + \frac{3x^2}{2}$$

Dividing the fractions and adding a constant

$$\therefore Ans = 2x^4 + 1\frac{1}{2}x^2 + c$$

SOLUTION (c page 5)

In this case there is no x on the number, we are just integrating 5 with respect to x, so we only introduce x and multiply it to 5.

$$\int 5dx = 5x + c$$

SOLUTION (d page 6)

On x squared, we add 1 to the power **2**, then we have **2+1** as the numerator. On $3x$, the power is 1, so we add 1 to 1, meaning the denominator will be 1 plus 1 also. On the last 4, there is no x so we just introduce x and multiply it to the 4.

$$\int (x^2 + 3x + 4)dx = \frac{x^{2+1}}{2+1} + \frac{3x^{1+1}}{1+1} + 4x$$

Adding powers and denominators

$$\frac{x^3}{3}+\frac{3x^2}{2}+4x$$

Dividing **2** into **3**

$$\therefore Ans=\frac{1}{3}x^3+1\frac{1}{2}x^2+4x+c$$

LESSON 73 INTEGRATION PAGE 1

Integrate each of the following

a. $\int(2x^7+x-2)dx$
b. $\int 7xdx$
c. $\int(x^2-x)dx$
d. $\int(3x^2-x^3)dx$

SOLUTION (a page 2)

On 2x power 7, we add 1 to the power 7, then we have $7+1$ as the numerator . On x, the power is 1, so we add 1 to 1, meaning the denominator will be 1 plus 1 also. On the last -2, there is no x so we just introduce x and multiply it to the 2.

$$\int(2x^7+x-2)dx=\frac{2x^{7+1}}{7+1}+\frac{x^{1+1}}{1+1}-2x$$

Adding powers and denominators

$$\frac{2x^8}{8}+\frac{x^2}{2}-2x$$

Dividing 2 into 2 and 8

$$\therefore Ans = \frac{1}{4}x^8 + \frac{1}{2}x^2 - 2x + c$$

SOLUTION (b page 3)

In this case, the power is 1, so we will add 1 to the power, this will also be the denominator of $7x$ then c will be added as a constant

$$\int 7x dx = \frac{7x^{1+1}}{1+1}$$

Adding powers and denominators

$$\frac{7x^2}{2}$$

Dividing 2 into 7 we have

$$\therefore Ans = 3\frac{1}{2}x^2 + c$$

SOLUTION (c page 4)

Just like we have been integrating other questions, so shall we integrate this one too

$$\int (x^2 - x) dx = \frac{x^{2+1}}{2+1} - \frac{x^{1+1}}{1+1}$$

Adding powers and denominators

$$\frac{x^3}{3} - \frac{x^2}{2}$$

Adding a constant c

$$\therefore Ans = \frac{1}{3}x^3 - \frac{1}{2}x^2 + c$$

SOLUTION (d page 5)

With less explanation follow the steps

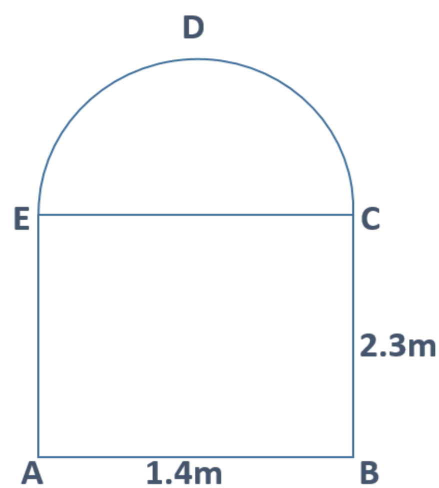
$$\int(3x^2 - x^3)dx = \frac{3x^{2+1}}{2+1} - \frac{x^{3+1}}{3+1}$$

$$\frac{3x^3}{3} - \frac{x^4}{4}$$

$$Ans = x^3 - \frac{1}{4}x^4 + c$$

LESSON 74 QUESTION 6 PAPER 2 FOR 2008 (page 1)

The diagram below represents a metallic window frame of a sports hall. It consists of a rectangular frame **ABCE** and a semi-circular arc **CDE**. The two parts are welded together at point **C** and **E**.



Given that $AB = 1.4m$ and $BC = 2.3m$, $(Take\ \pi = \frac{22}{7})$

a. Calculate

i) The length of the metal CDE

ii) The total length of the metal required to make the frame ABCDE

b. Grass panes would be required to cover this window frame, find the area of the total sheet of grass pane required.

c. Given that glass costs K42.5 pay square metre, calculate how much was spent on glass panes for the window.

SOLUTION (a i) page 2

CDE is a **semi-circle**, semi means **half**, and this means **half** of a **circle**. The formula for calculating length of a semi-circle is $\pi d \div 2$ where **d** is the **diameter** or a straight line cutting the circle into two equal halves. $diametor = EC = AB = 1.4m$.

$$CDE = (\pi \times d) \div 2$$

$$CDE = \left(\frac{22}{7} \times 1.4\right) \div 2$$

$$CDE = \left(\frac{22 \times 1.4}{7}\right) \div 2$$

$$CDE = \left(\frac{30.8}{7}\right) \div 2$$

$$CDE = 4.4 \div 2$$

$$\therefore CDE = 2.2m$$

SOLUTION (a ii) page 3

The length of the metal required is in two parts, that's the rectangle **ABCE** and the arc **CDE**. To get the length required in forming the rectangle ABCE we just find the perimeter of ABCE. The formula for perimeter of a rectangle is;

$$Perimeter = 2(length + bleadth)$$

$$P = 2(l + b)$$

To find the total length we combine the formula

$$total\ length = 2(l + b) + 2.2$$

$$total\ length = 2(2.3 + 1.4) + 2.2$$

$$total\ length = 2(3.7) + 2.2$$

$$total\ length = 7.4 + 2.2$$

$$\therefore total\ length = 9.6m$$

SOLUTION (b page 4)

To find the **total area** required, we have to add area of the rectangle, to area of a semi-circle. The formula are;

$$Area\ of\ a\ rectangle = length \times breadth = l \times b$$

$$Area\ of\ a\ semi-circle = \frac{\pi r^2}{2}$$

r is radius or half of diameter or $\frac{1.4}{2} = 0.7$

$$Total\ area = l \times b + \frac{\pi r^2}{2}$$

$$Total\ area = 2.3 \times 1.4 + \frac{\frac{22}{7} \times 0.7^2}{2}$$

SOLUTION (b cont. page 5)

$$Total\ area = 3.22 + \frac{\frac{22}{7} \times 0.49}{2}$$

$$Total\ area = 3.22 + \frac{\frac{22 \times 0.49}{7}}{2}$$

$$Total\ area = 3.22 + \frac{\frac{10.78}{7}}{2}$$

$$Total\ area = 3.22 + \frac{10.78}{7} \div 2$$

SOLUTION (b cont. page 6)

$$Total\ area = 3.22 + 1.54 \div 2$$

$$Total\ area = 3.22 + 0.77$$

$$Total\ area = 3.99$$

$$\therefore Total\ area = 3.99m^2$$

SOLUTION (c page 7)

In this case we have 3.99 square metres, but one square metre is K42.5.

$$total\ expence = 42.5 \times 3.99$$

$$\therefore total\ expence = K169.575$$

LESSON 75 QUESTION 2 PAPER 1 2016 PAGE 1

a. Simplify each of the following

i. $2m - 4n - 3(3n - m)$

ii. $3(x^2 + yx) - x(4y - x)$

b. Evaluate each of the following

i. $3^2 + 2^3 \times 2^0$

ii. $\frac{3^7 \times 3^{-3}}{9}$

SOLUTION a i PAGE 2

$$2m - 4n - 3(3n - m)$$

Open brackets by multiplying everything by -3

$$2m - 4n - 9n + 3m$$

Collecting like terms

$$2m + 3m - 4n - 9n$$

Adding and subtracting like terms

$$\therefore Ans = 5m - 13n$$

SOLUTION a ii PAGE 3

$$3\left(x^2 + yx\right) - x(4y - x)$$

Open brackets

$$3x^2 + 3yx - 4yx + x^2$$

Collecting like terms

$$3x^2 + x^2 + 3yx - 4yx$$

Adding and subtracting like terms

$$\therefore Ans = 4x^2 - yx$$

SOLUTION b i PAGE 4

$$3^2 + 2^3 \times 2^0$$

First **multiply**, know that multiplication of indices of the **same base** such as 2^3 and 2^0 requires that you just pick the base then add the powers. The base is **2**.

$$3^2 + 2^{3+0} = 3^2 + 2^3$$

Then expand according to powers

$$3 \times 3 + 2 \times 2 \times 2$$

Multiply

$$9 + 8$$

$$\therefore Ans = 17$$

SOLUTION b ii PAGE 5

$$\frac{3^7 \times 3^{-3}}{9}$$

First multiply in the numerator

$$\frac{3^{7+(-3)}}{9}$$

Positive times negative is negative

$$\frac{3^{7-3}}{9} = \frac{3^4}{9}$$

SOLUTION b ii cont. PAGE 6

Write the denominator 9 in index form

$$\frac{3^4}{3^2}$$

Division of indices of the same base requires subtraction of powers

$$\frac{3^4}{3^2} = 3^{4-2} = 3^2 = 3 \times 3$$

$$\therefore Ans = 9$$

LESSON 76 QUESTIONS PAPER 1 2016 PAGE 1

1. **Bronze** is made up of **zinc**, **tin** and **copper** in the ratio $1:4:25$ respectively. A bronze statue contains 120 grams of tin. Find the required quantities of the other metals.

SOLUTION 1 PAGE 2

Zinc, tin and copper corresponds to the ratio $1:4:25$

Meaning that; $Zinc = 1, Tin = 4$ and $Copper = 25$ in ratios

Quantity of tin is 120 grams mass of bronze unknown, write x

Find x in order to find other quantities using the formula

$$\frac{tin}{\text{Zinc+tin+copper}} \times Bronze = quantity\ of\ tin$$

Substituting into the formula

$$\frac{4}{1+4+25} \times x = 120 \quad \Rightarrow \quad \frac{4x}{30} = 120$$

SOLUTION 1 cont. PAGE 3

Introduce 1 as a denominator of 120 and cross multiply

$$\frac{4x}{30} = \frac{120}{1} \quad \Rightarrow \quad 4x = 3600$$

Divide both sides by 4 to find x

$$\frac{4x}{4} = \frac{3600}{4} \quad \Rightarrow \quad \therefore x = 900$$

SOLUTION 1 cont. PAGE 4

Find zinc using the same formula by writing zinc as numerator

$$\frac{\text{Zinc}}{\text{Zinc+tin+copper}} \times Bronze = quantity\ of\ zinc$$

Substituting into the formula

$$\frac{1}{1+4+25} \times 900 = quantity\ of\ Zinc$$

Multiplying and adding

$$\frac{900}{30} = quantity\ of\ Zinc$$

$$\therefore quantity\ of\ Zinc = 30\ grams$$

SOLUTION 1 cont. PAGE 5

Find copper using the same formula by writing copper as numerator

$$\frac{\text{copper}}{\text{Zinc} + \text{tin} + \text{copper}} \times Bronze = quantity\ of\ copper$$

Substituting into the formula

$$\frac{25}{1+4+25} \times 900 = quantity\ of\ copper$$

Multiplying and adding

$$\frac{22500}{30} = quantity\ of\ copper$$

$$\therefore quantity\ of\ copper = 750\ grams$$

SOLUTION 1 cont. PAGE 6

Another way of finding quantity of copper

$$quantity\ of\ copper = Bronze - (zinc + tin)$$

Substituting into the formula

$$quantity\ of\ copper = 900 - (30 + 120)$$

Adding in brackets

$$quantity\ of\ copper = 900 - 150$$

$$\therefore quantity\ of\ copper = 750\ grams$$

LESSON 77 QUESTIONS PAPER 1 2017 GCE PAGE 1

1. Simplify $2a - 7b - 2(a - 3b)$
2. Factorise completely $\frac{x^2}{4y^2} - \frac{1}{9}$
3. Given that $\begin{pmatrix} 2 & x \\ -5 & 1 \end{pmatrix}\begin{pmatrix} 4 \\ 3 \end{pmatrix} = \begin{pmatrix} 14 \\ -17 \end{pmatrix}$, find the value of x.

4. Two tins are geometrically similar, if the ratio of their volumes is $27:64$, find the ratio of their curved surface areas

SOLUTION 1 PAGE 2

$$2a - 7b - 2(a - 3b)$$

Open brackets by multiplying -2 to everything inside brackets

$$2a - 7b - 2a + 6b$$

Collect like terms

$$2a - 2a + 6b - 7b$$

Subtract terms

$$0 - b$$

$$\therefore Ans = -b$$

SOLUTION 2 PAGE 3

This is a difference of two squares, for example factorise

$$x^2 - y^2$$

Difference means minus and two squares means two terms raised to the power 2. To factorise this we have;

$$\therefore Ans = (x - y)(x + y)$$

SOLUTION 2 cont. PAGE 4

$$\frac{x^2}{4y^2} - \frac{1}{9}$$

First change the expression into a difference of two squares

4 can be written as 2^2, 1 as 1^2 and 9 as 3^2 so you have

$$\frac{x^2}{2^2y^2} - \frac{1^2}{3^2}$$

Everything is raised to power 2, so introduce brackets

$$\left(\frac{x}{2y}\right)^2 - \left(\frac{1}{3}\right)^2$$

This is a difference of two squares because the first and second term is raised to power 2 and there is a minus in the middle.

$$\therefore Ans = \left(\frac{x}{2y} - \frac{1}{3}\right)\left(\frac{x}{2y} + \frac{1}{3}\right)$$

SOLUTION 3 PAGE 5

$$\begin{pmatrix} 2 & x \\ -5 & 1 \end{pmatrix}\begin{pmatrix} 4 \\ 3 \end{pmatrix} = \begin{pmatrix} 14 \\ -17 \end{pmatrix}$$

Multiply on the left using row by column please observe the colour of numbers as we multiply.

$$\begin{pmatrix} 2 \times 4 + x \times 3 \\ -5 \times 4 + 1 \times 3 \end{pmatrix} = \begin{pmatrix} 14 \\ -17 \end{pmatrix}$$

Multiply

$$\begin{pmatrix} 8 + 3x \\ -20 + 3 \end{pmatrix} = \begin{pmatrix} 14 \\ -17 \end{pmatrix}$$

Equate $8 + 3x$ of the left to 14 on the right

$$8 + 3x = 14$$

Collect like terms and subtract

$$3x = 14 - 8 \quad \Rightarrow \quad 3x = 6$$

Divide both sides by 3

$$\frac{3x}{3} = \frac{6}{3} \quad \therefore x = 2$$

SOLUTION 4 PAGE 6

Volume ratio is $27:64$, the units of volume are raised to power 3 or cubic units. So find the cube-root ratio.

$$3:4$$

The units of area are raised to power 2 or squared units. Raise the volume cube-root ratio to power 2

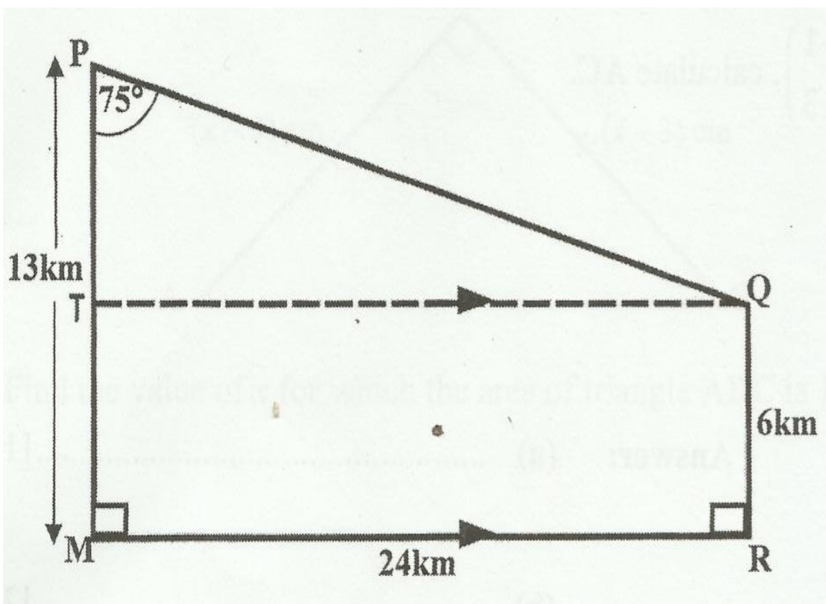
$$3^2:4^2$$

Then $3^2 = 3 \times 3 = 9$ and $4^2 = 4 \times 4 = 16$

$$\therefore Ans = 9:16$$

LESSON 78 QUESTIONS PAPER 1 2016 GCE PAGE 1

1. In the diagram below, MP=13km, QR=6km and MR=24km. MR is parallel and equal to TQ, $P\widehat{M}R = Q\widehat{R}M = 90^0$ and $T\widehat{P}Q = 75^0$



Find

a. $P\hat{Q}T$

b. The length PQ

2. The median of $2x + 3$, x and $2x + 12$ is 9, where x is a positive integer. Find the value of x.
3. What is the position of 999 in the sequence represented by the n^{th} term $n^3 - 1$?

SOLUTION 1 a PAGE 2

Sum of interior angles of a triangle is 180, PTQ is a triangle.

$$P\hat{T}Q + T\hat{Q}P + Q\hat{P}T = 180^0$$

Substitute

$$90^0 + P\hat{Q}T + 75^0 = 180^0$$

Collect like terms

$$P\hat{Q}T = 180^0 - 90^0 - 75^0$$

$$P\hat{Q}T = 90^0 - 75^0$$

$$\therefore P\hat{Q}T = 15^0$$

SOLUTION 1 b PAGE 3

Using Pythagoras theorem

$$PQ^2 = PT^2 + TQ^2$$

Substitute

$$PQ^2 = 7^2 + 24^2$$

$$PQ^2 = 49 + 576$$

$$PQ^2 = 625$$

$$\sqrt{PQ^2} = \sqrt{625} \quad \therefore PQ = 25km$$

SOLUTION 2 PAGE 4

To find the median, arrange the numbers in ascending then pick the middle number as the median

$$x,\ 2x+3,\ 2x+12$$

In this case the number in the middle is $2x+3$ so equate this number to 9 and find the value of x

$$2x+3=9$$

Collecting like terms

$$2x=9-3 \quad \Rightarrow \quad 2x=6$$

Divide both sides by 2 to find x

$$\frac{2x}{2}=\frac{6}{2}$$

$$\therefore x=3$$

SOLUTION 2 PAGE 5

To find the term number equate the n^{th} to 999 then find the value of n

$$n^3-1=999$$

Collecting like terms

$$n^3=999+1 \quad \Rightarrow \quad n^3=1000$$

To find the value of n we find the cube-root of 1000

$$\sqrt[3]{n^3}=\sqrt[3]{1000}$$

$$\therefore n=10$$

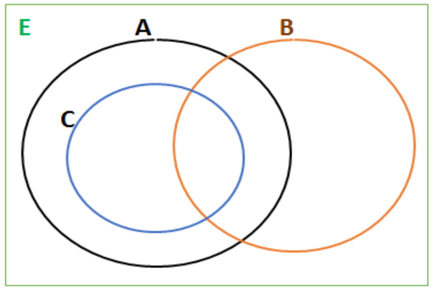
LESSON 79 QUESTIONS PAPER 1 2016 PAGE 1

a. It is given that $10^x=3$ and $10^y=9$. What is the value of 10^{y-x}?

b. A set has three elements. How many subsets does it have?

c. An ultra-modern stadium has a capacity of **43 492**. Express this number in standard form correct to **2** significant figures.

d. Using the Venn diagram in the answer space, shade the region represented by $B' \cap (A \cap C)$



SOLUTION a PAGE 2

In indices when powers are subtracting like in 10^{y-x} it means 10^{y} was being divided by 10^{x}. The rule is that "when dividing indices of the **same base** just **subtract** the power on the base". In this case 10 is the base, y and x are the powers

$$10^{y-x} = 10^{y} \div 10^{x}$$

Know that $10^{x} = 3$ and $10^{y} = 9$ the substitute

$$10^{y} \div 10^{x} = 9 \div 3$$

$$\therefore Ans = 3$$

SOLUTION b PAGE 3

The formula for finding the number of subsets is 2^{n} where n is the number of elements which is **3** in this case.

$$2^{n} = 2^{3}$$

The 2^{3}means multiplying **2 three** times by itself

$$2^{3} = 2 \times 2 \times 2 \qquad \therefore Ans = 8$$

SOLUTION c PAGE 4

Expressing a number in standard form means, moving the point and writing it between the first two numbers. In 43 492 we cannot see the point, meaning it's after the last number.

Moving the point

$$\mathbf{4.3\,492}$$

When writing in standard form we multiply by 10

$$\mathbf{4.3\,492 \times 10}$$

Count the digits the point has moved before it was written between 4 and 3 its 4, so raise 10 to power 4

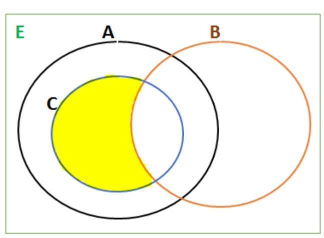
$$\mathbf{4.3\,492 \times 10^4}$$

Two significant figures will be first two digits

$$\therefore \boldsymbol{Ans = 4.3 \times 10^4}$$

SOLUTION d PAGE 5

In $B' \cap (A \cap C)$ we have B' is B-complement or elements that do not belong to set B hence no part of set B must be shaded, $\cap$ meaning intersection and $A \cup C$ meaning A union C or shading A and C but no part of set B must be shaded.



LESSON 80 QUESTIONS FROM PAPER 1 GCE 2017 PAGE 1

Question 7

a. For the sequence $11, 13, 15, 17, \ldots$, find the 13th term. [1]

b. If the arithmetic mean of 5 and c is 11, calculate c. **[2]**

Question 8

a. If $A^T = (1 \; -2 \quad 3 \; -4 \quad 5)$, write the matrix A. **[1]**

b. Find the derivative of $y = 2x^3 - 2x^2 - 3x + 1$, with respect x. **[2]**

SOLUTION 7 a PAGE 2

$11, 13, 15, 17, \ldots$, find the 13^{th} term

The formula for n^{th} term is $T_n = a + (n-1)d$ where

$$n = term\ number = 13$$

$$a = first\ term = 11$$

$$d = common\ difference = (second - first)term$$

$$d = 13 - 11 = 2$$

Substitute into the formula

$$T_{13} = 11 + (13 - 1)2$$

Subtract in brackets

$$T_{13} = 11 + (12)2$$

Open brackets

$$T_{13} = 11 + 24$$

$$\therefore T_{13} = 35$$

SOLUTION 7 b PAGE 3

5 and c is 11, calculate c

The formula for mean is $mean = \frac{sum\ of\ terms}{2}$ where

Terms are 5 and c

Substitute into the formula

$$11 = \frac{5+c}{2}$$

Introduce denominator 1 on 11

$$\frac{11}{1} = \frac{5+c}{2}$$

Cross multiply

$$1(5+c) = 2(11) \qquad \Rightarrow \qquad 5+c = 22$$

Collect like terms

$$c = 22 - 5 \qquad\qquad \therefore c = 17$$

SOLUTION 8 a PAGE 4

The matrix $A^T = (1 \ -2 \quad 3 \ -4 \quad 5)$ is a transpose role matrix because of the power T on A

To find A just write A^T in column form

$$\therefore A = \begin{pmatrix} 1 \\ -2 \\ 3 \\ -4 \\ 5 \end{pmatrix}$$

SOLUTION 8 b PAGE 5

$$y = 2x^3 - 2x^2 - 3x + 1$$

This can also be written with a power 1 on $3x$

$$y = 2x^3 - 2x^2 - 3x^1 + 1$$

Finding the derivative means differentiation in calculus denoted by $\frac{dy}{dx}$.

When differentiating, multiply the power to the number in front (coefficient) x and subtract 1 from the power.

$$\frac{dy}{dx} = 3 \times 2x^{3-1} - 2 \times 2x^{2-1} - 1 \times 3x^{1-1} + 1$$

Ignore the 1 at the end because it has no x

Continuation on the next page

SOLUTION 8 b cont. PAGE 6

$$\frac{dy}{dx} = 3 \times 2x^{3-1} - 2 \times 2x^{2-1} - 1 \times 3x^{1-1}$$

Multiply coefficients

$$\frac{dy}{dx} = 6x^{3-1} - 4x^{2-1} - 3x^{1-1}$$

Subtract powers

$$\frac{dy}{dx} = 6x^{2} - 4x^{1} - 3x^{0}$$

Any number raised to power zero is equal to 1, so$x^0 = 1$ we any number raised to power 1 is equal to that number so

$$x^1 = x$$

Substitute for x^0 and x^1

$$\frac{dy}{dx} = 6x^{2} - 4x - 3(1)$$

$$\therefore \frac{dy}{dx} = 6x^{2} - 4x - 3$$

ASSIGNMENT FOR YOUR PRACTICE PAGE 7

a. For the sequence $3, 6, 9, 13, \ldots,$ find the 10^{th} term.

b. If the arithmetic mean of 8 and c is 14, calculate c.

c. If $A^T = (-2 \quad 1 \quad 3 \quad 7)$, write the matrix A.

d. Find the derivative of $y = 3x^3 - 4x^2 - 6x + 4,$ with respect x.

LESSON 81 QUESTIONS FROM PAPER 1 GCE 2017 PAGE 1

Question 9

a. Given that $E = \{1, 2, 3, 4, 5, 6, 7, 8\}$ $A = \{1, 8\}$ and $B = \{2, 3, 4, 5, 6, 7\}$. List $(A \cup B)'$ **[1]**

b. Solve the equation $25^x = 5$. **[2]**

Question 11

a. A die and a coin are rolled and tossed, respectively. What is the probability of getting a five and a tail? **[1]**

b. Given that $x = 3.2 \times 3^4$ and $y = 4 \times 3^2$, evaluate $\frac{x}{y}$. **[2]**

SOLUTION 9 a PAGE 2

$E = \{1, 2, 3, 4, 5, 6, 7, 8\}$ $A = \{1, 8\}$ $B = \{2, 3, 4, 5, 6, 7\}$

The $(A \cup B)'$ reads as A union B complement, meaning set of elements that are not found in A union B but only in E. Union means combining elements for set A and B.

First list $(A \cup B)$ to see if there will be element remaining in E that will not be in A union B

$$(A \cup B) = \{1, 2, 3, 4, 5, 6, 7, 8\}$$

All elements in E are in $(A \cup B)$ so nothing will be in $(A \cup B)'$

$$\therefore (A \cup B)' = \{\ \} \text{ or } \emptyset$$

SOLUTION 9 b PAGE 3

$$25^x = 5$$

Write 25 and 5 in index form

$$(5^2)^x = 5^1$$

Open brackets

$$5^{2x} = 5^1$$

Bases are the same so equate the powers

$$2x = 1$$

Divide both sides by 2

$$\frac{2x}{2} = \frac{1}{2} \qquad \therefore x = \frac{1}{2}$$

SOLUTION 11 a PAGE 4

A die is a cube with 6 faces as seen below

There is only 1 side with 5 dots out of 6 sides hence the probability of 5 dots is 1 over 6 or $p(5) = \frac{1}{6}$

A coin has 2 sides, a Head and a Tail, the probability of getting a Tail is just 1 out of 2 sides $p(T) = \frac{1}{2}$

The word AND in probability means multiply

Probability of getting a five and a tail$=$ $p(5) \times p(T)$

$$p(5) \times p(T) = \frac{1}{6} \times \frac{1}{2} \qquad \therefore p(5) \times p(T) = \frac{1}{12}$$

SOLUTION 11 b PAGE 5

$x = 3.2 \times 3^4$ and $y = 4 \times 3^2$, evaluate $\frac{x}{y}$

Substitute

$$\frac{x}{y} = \frac{3.2 \times 3^4}{4 \times 3^2}$$

Expand on the indices

$$\frac{3.2 \times 3 \times 3 \times 3 \times 3}{4 \times 3 \times 3}$$

Divide the 3s

$$\frac{3.2 \times 3 \times 3}{4}$$

Multiply

$$\frac{3.2 \times 9}{4} = \frac{28.8}{4}$$

Divide 4 into 28.8

$$\therefore \frac{x}{y} = 7.2$$

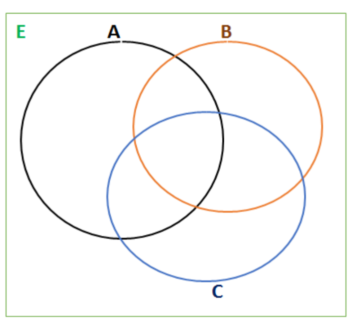
LESSON 82 QUESTIONS PAPER 1 2016 PAGE 1

2. Given that $\overrightarrow{AB} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$, find $|\overrightarrow{AB}|$

3. If $A = \begin{pmatrix} 1 & -2 \\ -1 & 4 \end{pmatrix}$ and $C = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$, calculate AC

4. Given that $\underline{a} = \begin{pmatrix} 3 \\ -4 \end{pmatrix}$, find $|\underline{a}|$

5. On the Venn diagram below, shade the region defined by $A' \cap (B \cup C)$



SOLUTION 1 PAGE 2

To find $|\overrightarrow{AB}|$ (the length of AB) we use the formula $\sqrt{x^2+y^2}$

using $\overrightarrow{AB} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$, $x = -3$ and $y = 4$

Substitute into the formula

$$\sqrt{(-3)^2+(4)^2} = \sqrt{(-3 \times -3) + (4 \times 4)} = \sqrt{9 + 16} = \sqrt{25}$$

$$\therefore Ans = 5$$

SOLUTION 2 PAGE 3

To find AC we multiply the two matrices using row by column

$$A \times C = \begin{pmatrix} 1 & -2 \\ -1 & 4 \end{pmatrix} \times \begin{pmatrix} -1 \\ 3 \end{pmatrix}$$

$$A \times C = \begin{pmatrix} 1 \times -1 + -2 \times 3 \\ -1 \times -1 + 4 \times 3 \end{pmatrix}$$

Multiplying

$$A \times C = \begin{pmatrix} -1 + -6 \\ 1 + 12 \end{pmatrix}$$

$$\therefore AC = \begin{pmatrix} -7 \\ 13 \end{pmatrix}$$

SOLUTION 2 PAGE 4

To find $|\underline{a}|$ (the length of $\underline{a}$) we use the formula $\sqrt{x^2+y^2}$

Using $\underline{a} = \begin{pmatrix} 3 \\ -4 \end{pmatrix}$, $x = 3$ and $y = -4$

Substitute into the formula

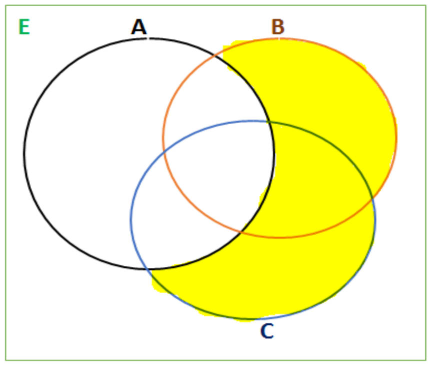
$$\sqrt{(3)^2+(-4)^2} = \sqrt{(3 \times 3) + (-4 \times -4)} = \sqrt{9 + 16} = \sqrt{25}$$

$$\therefore Ans = 5$$

SOLUTION 4 PAGE 5

In $A' \cap (B \cup C)$ we have A' is A-complement or elements that do not belong to set A hence no part of set A must be shaded, $\cap$ meaning intersection and $B \cup C$ meaning B union C or shading B and C but no part of set A must be shaded.

$\therefore A' \cap (B \cup C) \ =$



LESSON 83 QUESTION 13 GCE PAPER 1 FOR 2017 PAGE 1

The functions g and f are defined as $g: x \to \frac{x-1}{2}$ and $f: x \to 3x - 5$. Find;

a. $g^{-1}(x)$
b. x, if $f(x) = g(x)$
c. $g^{-1}f(x)$
d. $g^{-1}f(4)$
e. $g^{-1}(-2)$

SOLUTION 13 a PAGE 2

To find $g^{-1}(x)$ equate $\frac{x-1}{2}$ to y

$$y = \frac{x-1}{2}$$

Introduce denominator 1 on y

$$\frac{y}{1} = \frac{x-1}{2}$$

Cross multiply

$$x - 1 = 2y$$

Make x the subject of the formula

$$x = 2y + 1$$

Substitute $g^{-1}(x)$ for x and x for y

$$\therefore g^{-1}(x) = 2x + 1$$

SOLUTION 13 b PAGE 3

To find x, if $f(x) = g(x)$ equate the two functions

$$\frac{x-1}{2} = 3x - 5$$

Introduce denominator 1 on $3x - 5$

$$\frac{x-1}{2} = \frac{3x-5}{1}$$

Cross multiply

$$2(3x - 5) = 1(x - 1) \quad \Rightarrow \quad 6x - 10 = x - 1$$

Collect like terms add/subtract

$$6x - x = -1 + 10 \quad \Rightarrow \quad 5x = 9$$

Divide both sides by 5

$$\frac{5x}{5} = \frac{9}{5} \qquad \therefore x = 1\frac{4}{5}$$

SOLUTION 13 c PAGE 4

To find $g^{-1}f(x)$, substitute $f(x)$ into $g^{-1}(x)$ by writing $3x-5$ in $2x+1$ where there is x.

$$g^{-1}f(x)=2(3x-5)+1$$

Open brackets

$$6x-10+1$$

Add/subtract like terms

$$\therefore\ g^{-1}f(x)=6x-9$$

SOLUTION 13 d PAGE 5

To find $g^{-1}f(4)$ substitute for x in $g^{-1}f(x)$

$$g^{-1}f(x)=6x-9$$

Write 4 where there is x

$$g^{-1}f(4)=6(4)-9$$

Multiply 4 and 6

$$g^{-1}f(4)=24-9$$

$$\therefore g^{-1}f(4)=15$$

SOLUTION 13 d PAGE 6

To find $g^{-1}(-2)$ substitute for x in $g^{-1}(x)$

$$g^{-1}(x)=2x+1$$

Write -2 where there is x in $g^{-1}(x)=2x+1$

$$g^{-1}(-2)=2(-2)+1$$

$$g^{-1}(-2)=-4+1$$

$$\therefore g^{-1}(-2)=-3$$

LESSON 84 QUESTION 14 GCE PAPER 1 FOR 2017 PAGE 1

The table below shows the relationship between two variables x and y. It is given that y varies inversely as the square root of x, where x is positive.

y	2	r	$\frac{8}{9}$
x	16	1	a

a. Write an expression for y in terms of x and the constant of variation k.

b. Find the value of

i. k

ii. a

iii. ar

SOLUTION (14 a page 2)

To find the expression, follow the statement, y varies inversely as the square root of x.

Inversely means 1 over the square-root of x or $\frac{1}{\sqrt{x}}$. But you have to consider the constant of variation k in the numerator so it becomes;

$$y = \frac{1 \times k}{\sqrt{x}} \qquad \therefore y = \frac{k}{\sqrt{x}}$$

SOLUTION (14 b i page 3)

To find k, make k the subject of the formula using expression

$$y = \frac{k}{\sqrt{x}}$$

Introduce denominator $\mathbf{1}$ on $\boldsymbol{y}$

$$\frac{y}{1} = \frac{k}{\sqrt{x}}$$

Cross multiply

$$y\sqrt{x} = 1 \times k \qquad \Rightarrow \qquad k = y\sqrt{x}$$

Use the first column where $y = 2$ and $x = 16$ to substitute

$$k = 2\sqrt{16} = 2(4)$$

$$\therefore k = 8$$

SOLUTION 14 b ii page 4

To find a, use expression the expression $y = \frac{k}{\sqrt{x}}$. Use the last column where $y = \frac{8}{9}, x = a$ when $k = 8$ to substitute

$$\frac{8}{9} = \frac{8}{\sqrt{a}}$$

Cross multiply

$$8\sqrt{a} = 72$$

Divide both sides by 8

$$\frac{8\sqrt{a}}{8} = \frac{72}{8} \qquad \Rightarrow \qquad \sqrt{a} = 9$$

Introduce a square on both sides

$$\left(\sqrt{a}\right)^2 = 9^2 \qquad \Rightarrow \qquad \sqrt{a} \times \sqrt{a} = 9 \times 9$$

$$\therefore a = 81$$

SOLUTION 14 b iii page 5

To find ar first calculate r using the expression $y = \frac{k}{\sqrt{x}}$. Use the third column where $y = r, x = 1$ when $k = 8$ to substitute

$$r = \frac{8}{\sqrt{1}} \quad \Rightarrow \quad r = \frac{8}{1} \quad \Rightarrow \quad r = 8$$

Find ar now

$$ar = a \times r = 81 \times 8$$

$$\therefore ar = 648$$

LESSON 85 QUESTION 2 PAPER 2 FOR 2015 PAGE 1

a. Given that $B = \begin{pmatrix} 0 & -2 \\ 2 & -8 \end{pmatrix}$, find

i) $\frac{-3}{2}B$

ii) The determinant of B

iii) B^2

b. Solve the equation $m^2 - m - 5 = 0$

SOLUTION (1 a i page 2)

Substitute then multiply $\frac{-3}{2}$ to all the numbers in the matrix

$$\frac{-3}{2}B = \frac{-3}{2}\begin{pmatrix} 0 & -2 \\ 2 & -8 \end{pmatrix} = \begin{pmatrix} \frac{-3}{2} \times 0 & \frac{-3}{2} \times -2 \\ \frac{-3}{2} \times 2 & \frac{-3}{2} \times -8 \end{pmatrix}$$

Multiply and consider divisions inside

$$\begin{pmatrix} 0 & -3 \times -1 \\ -3 & -3 \times -4 \end{pmatrix}$$

Multiply further

$$\therefore \frac{-3}{2}B = \begin{pmatrix} 0 & 3 \\ -3 & 12 \end{pmatrix}$$

SOLUTION (1 a ii page 3)

$$B = \begin{pmatrix} 0 & -2 \\ 2 & -8 \end{pmatrix}$$

Determinant $= product\ of\ (major - minor)\ diagonal$

In major we have 0 and -8, in minor we have 2 and -2

Det of $B = (0 \times -8) - (2 \times -2)$

Multiply to open brackets

Det of $B = 0 - -4$

Negative times negative is positive

Det of $B = 0 + 4$

$\therefore$Det of $B = 4$

SOLUTION (1 a iii page 4)

B^2 means multiply B two times by itself using row by column

$$B^2 = \begin{pmatrix} 0 & -2 \\ 2 & -8 \end{pmatrix}^2 = \begin{pmatrix} 0 & -2 \\ 2 & -8 \end{pmatrix} \times \begin{pmatrix} 0 & -2 \\ 2 & -8 \end{pmatrix}$$

Observe the colours to see row by column

$$\begin{pmatrix} 0 \times 0 + -2 \times 2 & 0 \times -2 + -2 \times -8 \\ 2 \times 0 + -8 \times 2 & 2 \times -2 + -8 \times -8 \end{pmatrix}$$

Multiply, add and subtract

$$\begin{pmatrix} 0 + -4 & 0 + 16 \\ 0 + -16 & -4 + 64 \end{pmatrix} = \begin{pmatrix} 0 - 4 & 16 \\ 0 - 16 & 60 \end{pmatrix}$$

$$\therefore B^2 = \begin{pmatrix} -4 & 16 \\ -16 & 60 \end{pmatrix}$$

SOLUTION (1 b page 5)

$$m^2 - m - 5 = 0$$

Use the quadratic formula

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

In this case

$a = 1$ (Number in front of m^2)
$b = -1$ (Number in front of m)
$c = -5$ (Number without m m)

Substitute into the formula

$$m = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-5)}}{2(1)}$$

Continuation on the next page

SOLUTION (1 b cont. page 6)

Multiply in the root and denominator

$$m = \frac{1 \pm \sqrt{1 - 4(-5)}}{2(1)} \quad \Rightarrow \quad m = \frac{1 \pm \sqrt{1 + 20}}{2}$$

add in the root and find square root of 21

$$m = \frac{1 \pm \sqrt{21}}{2} \qquad m = \frac{1 \pm 4.582575695}{2}$$

Add and subtract to use $\pm$

$$m = \frac{1+4.582575695}{2} \quad \text{and} \quad m = \frac{1-4.582575695}{2}$$

$$m = \frac{5.582575695}{2} \quad \text{and} \quad m = \frac{-3.582575695}{2}$$

Divide by 2 in both

$$m = 2.79128747 \quad \text{and} \quad m = -1.791287847$$

Round off to 2 decimal places

$$\therefore m = 2.79 \quad \text{and} \quad \therefore m = -1.79$$

ASSIGNMENT FOR YOUR PRACTICE PAGE 7

a. Given that $A = \begin{pmatrix} 0 & -6 \\ -9 & -3 \end{pmatrix}$, find

iv) $\frac{-2}{3}A$

v) The determinant of A

vi) A^2

b. Solve the equation $2x^2 - x - 1 = 0$

LESSON 86 DIFFERENTIATION IN CALCULUS (PAGE 1)

Differentiate each of the following with respect to x

a. $y = 4x^5$

b. $y = x^2 + 7$

c. $y = 3x^4 + 19x - 2$

d. $y = 3x^4 - 4x^2 - 11x$

e. $y = 5x^3 - 2x^2$

SOLUTION (a page 2)

Differentiation in calculus is denoted by $\frac{dy}{dx}$. Example, given that $y = ax^n$ find $\frac{dy}{dx}$

To differentiate, you multiply the power n of x to the number a in front (coefficient) of x and subtract 1 from the power n of x.

$$\frac{dy}{dx} = n \times ax^{n-1} = nax^{n-1} \text{ (Formula)}$$

Use the formula

For $y = 4x^5$ $n = 5$ and $a = 4$

Drop n to multiply a, introduce -1 on n

$$\frac{dy}{dx} = 5 \times 4x^{5-1}$$

Multiply coefficients $5 \times 4 = 20$

$$\frac{dy}{dx} = 20x^{5-1}$$

Subtract on the power $5 - 1 = 4$

$$\therefore \frac{dy}{dx} = 20x^4$$

SOLUTION (b page 3)

For $y = x^2 + 7$ $n = 2$ and $a = 1$

Ignore 7 because it has no x

Drop n to multiply a, introduce -1 on n

$$\frac{dy}{dx} = 2 \times 1x^{2-1}$$

Multiply coefficients $2 \times 1 = 2$

$$\frac{dy}{dx} = 2x^{2-1}$$

Subtract on the power $2 - 1 = 1$

$$\frac{dy}{dx} = 2x^{1}$$

Ignore the power 1

$$\therefore \frac{dy}{dx} = 2x$$

SOLUTION (c page 4)

For $y = 3x^4 + 19x - 2$ there are two powers and two coefficients $n_1 = 4$, $a_1 = 3$, $n_2 = 1$, and $a_2 = 19$

Ignore -2 because it has no x

Drop n_1 to multiply a_1, introduce -1 on n_1

And

Drop n_2 to multiply a_2, introduce -1 on n_2

$$\frac{dy}{dx} = 4 \times 3x^{4-1} + 1 \times 19x^{1-1}$$

Multiply coefficients $4 \times 3 = 12$ and $1 \times 19 = 19$

$$\frac{dy}{dx} = 12x^{4-1} + 19x^{1-1}$$

Subtract on the power $4 - 1 = 3$ and $1 - 1 = 0$

$$\frac{dy}{dx} = 12x^3 + 19x^0$$

Any number or letter to the power zero is equal to 1 $x^0 = 1$

$$\frac{dy}{dx} = 12x^3 + 19(1) \qquad \therefore \frac{dy}{dx} = 12x^3 + 19$$

SOLUTION (d page 5)

For $y = 3x^4 - 4x^2 - 11x$ there are three powers and three coefficients. Treat them like you treated those on page 4

$$\frac{dy}{dx} = 4 \times 3x^{4-1} - 2 \times 4x^{2-1} - 1 \times 11x^{1-1}$$

Multiply coefficients and subtract powers

$$\frac{dy}{dx} = 12x^3 - 8x^1 - 11x^0$$

Any number or letter to the power zero is equal to 1 $x^0 = 1$

$$\frac{dy}{dx} = 12x^3 - 8x^1 - 11(1)$$

Ignore power 1

$$\therefore \frac{dy}{dx} = 12x^3 - 8x - 11$$

SOLUTION (e page 6)

$$y = 5x^3 - 2x^2$$

$$\frac{dy}{dx} = 3 \times 5x^{3-1} - 2 \times 2x^{2-1}$$

$$\frac{dy}{dx} = 15x^2 - 4x^1 \qquad \therefore \frac{dy}{dx} = 15x^2 - 4x$$

LESSON 87 DIFFERENTIATION IN CALCULUS (PAGE 1)

Differentiate each of the following with respect to x

1. $y = \frac{1}{10}x^5$

2. $y = \frac{1}{3}x^3 + 7x^5$

3. $y = \frac{3}{8}x^4 + 9x^3 - 6$

4. $y = \frac{3}{16}x^4 - 2x^3 - 11x^2$

5. $y = \frac{5}{9}x^3 - 2x^2$

SOLUTION (1 page 2)

Differentiation in calculus is denoted by $\frac{dy}{dx}$. Example, given that $y = ax^n$ find $\frac{dy}{dx}$

To differentiate, you multiply the power n of x to the number a in front (coefficient) of x and subtract 1 from the power n of x.

$$\frac{dy}{dx} = n \times ax^{n-1} = nax^{n-1} \quad \text{(Formula)}$$

For $y = \frac{1}{10}x^5$ $n = 5$ and $a = \frac{1}{10}$

Drop n to multiply a, introduce -1 on n

$$\frac{dy}{dx} = 5 \times \frac{1}{10}x^{5-1}$$

Multiply coefficients $5 \times \frac{1}{10} = \frac{1}{2}$

$$\frac{dy}{dx} = \frac{1}{2}x^{5-1}$$

Subtract on the power $5 - 1 = 4$

$$\therefore \frac{dy}{dx} = \frac{1}{2}x^4$$

SOLUTION (2 Page 3)

For $y = \frac{1}{3}x^3 + 7x^5$ there **two** powers and coefficients

Drop n to multiply a, introduce **-1** on n for all terms

$$\frac{dy}{dx} = 3 \times \frac{1}{3}x^{3-1} + 5 \times 7x^{5-1}$$

Multiply coefficients $3 \times \frac{1}{3} = 1$ and $5 \times 7 = 35$

$$\frac{dy}{dx} = 1x^{3-1} + 35x^{5-1}$$

Subtract the powers $3 - 1 = 2$ and $5 - 1 = 4$

$$\frac{dy}{dx} = 1x^2 + 35x^4 \qquad \therefore \frac{dy}{dx} = x^2 + 35x^4$$

SOLUTION (3 Page 4)

For $y = \frac{3}{8}x^4 + 9x^3 - 6$ there **two** powers and coefficients

Ignore -6 because it has no x

Drop n to multiply a, introduce **-1** on n for all terms

$$\frac{dy}{dx} = 4 \times \frac{3}{8}x^{4-1} + 3 \times 9x^{3-1}$$

Multiply coefficients $4 \times \frac{3}{8} = \frac{3}{2}$ and $3 \times 9 = 27$

$$\frac{dy}{dx} = \frac{3}{2}x^{4-1} + 27x^{3-1}$$

Subtract the powers $4 - 1 = 3$ and $3 - 1 = 2$

$$\frac{dy}{dx} = \frac{3}{2}x^{3} + 27x^{2}$$

Divide 2 into 3 $\frac{3}{2} = 1\frac{1}{2}$

$$\therefore \frac{dy}{dx} = 1\frac{1}{2}x^{3} + 27x^{2}$$

SOLUTION (4 Page 5)

In $y = \frac{3}{16}x^4 - 2x^3 - 11x^3$ are three powers and coefficients

Drop n to multiply a, introduce -1 on n for all terms

$$\frac{dy}{dx} = 4 \times \frac{3}{16}x^{4-1} - 3 \times 2x^{3-1} - 2 \times 11x^{2-1}$$

Multiply coefficients $4 \times \frac{3}{16} = \frac{3}{4}$, $3 \times 2 = 6$ and $2 \times 11 = 22$

$$\frac{dy}{dx} = \frac{3}{4}x^{4-1} - 6x^{3-1} - 22x^{2-1}$$

Subtract the powers $4 - 1 = 3$ and $2 - 1 = 1$

$$\frac{dy}{dx} = \frac{3}{4}x^{3} - 6x^{2} - 22x^{1}$$

Ignore the power 1 on x^1 $\therefore \frac{dy}{dx} = \frac{3}{4}x^{3} - 6x^{2} - 22x$

SOLUTION (5 Page 6)

$$y = \frac{5}{9}x^3 - 2x^2$$

Drop n to multiply a, introduce -1 on n for all terms

$$\frac{dy}{dx} = 3 \times \frac{5}{9}x^{3-1} - 2 \times 2x^{2-1}$$

Multiply coefficients $3 \times \frac{5}{9} = \frac{5}{3}$ and $2 \times 2 = 4$

$$\frac{dy}{dx} = \frac{5}{3}x^2 - 4x^1$$

$$\therefore \frac{dy}{dx} = 1\frac{2}{3}x^2 - 4x$$

LESSON 88 QUESTION 2 paper 2 for 2004 (Page 1)

a. A straight line is given by the equation $2x + 3y = 3$, find the gradient

b. Express as a single fraction, in its simplest form $\frac{3}{2} - \frac{1-2x}{4x}$

c. Solve the equation $3x^2 - x - 1 = 0$ giving your answers correct to two decimal places

SOLUTION (a page 2)

$$2x + 3y = 3$$

Here you have to make y the subject of the formula, and then pick the number in front (coefficient) of x as gradient.

Move $2x$ across $=$ it becomes $-2x$

$$3y = 3 - 2x$$

Divide throughout by 3

$$\frac{3y}{3} = \frac{3}{3} - \frac{2}{3}x \qquad \Rightarrow \qquad \therefore y = 1 - \frac{2}{3}x$$

$$\therefore gradient = -\frac{2}{3}$$

SOLUTION (b page 3)

$$\frac{3}{2} - \frac{1-2x}{4x}$$

We first find the common denominator then simplify, the common denominator is $4x$

$$\frac{\qquad\qquad}{4x}$$

Divide 2 into $4x$ you get $2x$ the multiply that to 3

$$\frac{2x(3)\ -}{4x}$$

Divide $4x$ into $4x$ you get 1 the multiply that to $1-2x$

$$\frac{2x(3) - 1(1-2x)}{4x}$$

Multiply in the numerator and collect like terms

$$\frac{6x - 1 + 2x}{4x} \quad \Rightarrow \quad \frac{6x + 2x - 1}{4x}$$

Add like terms

$$Ans = \frac{8x - 1}{4x}$$

SOLUTION (c page 4)

This is a quadratic equation in the form

$$ax^2 + bx + c = 0$$

$$3x^2 - \quad x - 1 = 0$$

In this case

$a = 3$ (Number in front of x^2)
$b = -1$ (Number in front of x)
$c = -1$ (Number without x)

Here we use the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Continuation on the next page

SOLUTION (c cont. page 5)

Substitute into the formula

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(3)(-1)}}{2(3)}$$

Multiply the negatives on $-(-1) = 1$ and $2(3) = 6$

$$x = \frac{1 \pm \sqrt{(-1)^2 - 4(3)(-1)}}{6}$$

Evaluate the square on $(-1)^2 = 1$

$$x = \frac{1 \pm \sqrt{1 - 4(3)(-1)}}{6}$$

Multiply $-4(3)(-1) = 12$ in the root and add $1 + 12 = 13$

$$x = \frac{1 \pm \sqrt{1 + 12}}{6} \quad \Rightarrow \quad x = \frac{1 \pm \sqrt{13}}{6}$$

Continuation on the next page

SOLUTION (c cont. page 6)

Find $\sqrt{13} = 3.605551275$ using a calculator

$$x = \frac{1 \pm 3.605551275}{6}$$

Because of $\pm$ we add and subtract

$$x = \frac{1 + 3.605551275}{6} \text{ or } x = \frac{1 - 3.605551275}{6}$$

Add and subtract

$$x = \frac{4.605551275}{6} \text{ or } x = \frac{-2.605551275}{6}$$

Divide by 6

$$x = 0.767591879 \text{ or } x = -0.434258545$$

Round off to 2 decimal places

$$\therefore x = 0.77 \quad \text{or} \quad \therefore x = -0.43$$

LESSON 89 QUESTION FOR 2004 (PAGE 1)

Given that y varies directly as x and inversely as $2m - 1$, and that $y = 5$ when $x = 7$ and $m = 4$. Calculate the,

i. constant of variation, k
ii. value of y when $x = 2$ and $m = 3$
iii. value of m when $y = 2$ and $x = 4$
iv. ky when $m = 3$ and $x = 10$

SOLUTION (i page 2)

$y = 5$ when $x = 7$ and $m = 4$

First write the equation of variation; y varies directly as x means $y = kx$, y varies inversely as 2m-1 means $y = \frac{k}{2m-1}$ so combine the two but k will only be written once, you have;

$$y = \frac{kx}{2m-1}$$

Substitute into the expression for y, x and m

$$5 = \frac{7k}{2\times4-1}$$

Multiply and subtract in the denominator

$$5 = \frac{7k}{8-1} \quad \Rightarrow \quad 5 = \frac{7k}{7}$$

Introduce denominator 1 on 5, cross multiply, divide both-sides by 7

$$\frac{5}{1} = \frac{7k}{7} \quad \Rightarrow \quad 7k = 35 \quad \Rightarrow \quad \frac{7k}{7} = \frac{35}{7} \quad \therefore k = 5$$

SOLUTION (ii page 3)

To find the value of y when $x = 2$ and $m = 3$ use equation of variation knowing that $k = 5$

$$y = \frac{kx}{2m-1}$$

Substitute for k, x and m

$$y = \frac{5\times2}{2\times3-1}$$

Multiply in both denominator and numerator

$$y = \frac{10}{6-1}$$

Subtract in the denominator and divide

$$y = \frac{10}{5} \qquad \therefore y = 2$$

SOLUTION (iii page 4)

To find the value of m when $y = 2$ and $x = 4$ use equation of variation.

$$y = \frac{kx}{2m-1}$$

Substitute for k, x and y

$$2 = \frac{4 \times 5}{2\mathrm{m}-1}$$

Multiply in the numerator and introduce denominator 1 on 2

$$\frac{2}{1} = \frac{20}{2\mathrm{m}-1}$$

Cross multiply and open brackets

$$2(2m-1) = 20 \quad \Rightarrow \quad 4m - 2 = 20$$

Collect like terms add and divide both sides by 4

$$4m = 20 + 2 \quad \Rightarrow \quad 4\mathrm{m} = 22 \quad \Rightarrow \quad \frac{4\mathrm{m}}{4} = \frac{22}{4}$$

$$m = \frac{11}{2} \qquad \therefore m = 5\frac{1}{2}$$

SOLUTION (iv page 5)

$k = 5, m = 3$ **and** $x = 10$

To find ky first find the value of y using the equation of variation

$$y = \frac{kx}{2m-1}$$

Substitute for k, x and m

$$y = \frac{5\times10}{2\times3-1}$$

Multiply in the numerator and denominator, subtract and divide

$$y = \frac{50}{6-1} \quad \Rightarrow \quad y = \frac{50}{5} \qquad \therefore y = 10$$

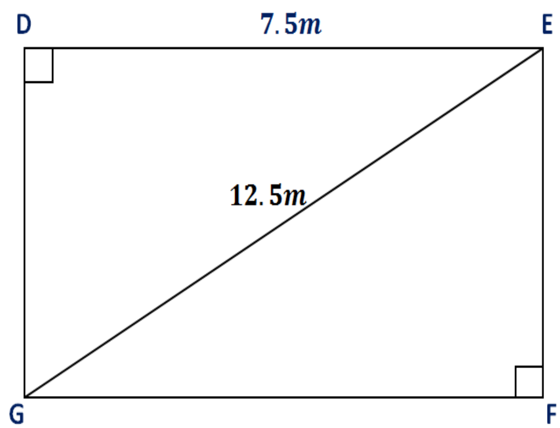
To find ky just multiply

$$k \times y = 5 \times 10$$

$$\therefore ky = 50$$

LESSON 91 QUESTION 12 PAPER 1 FOR GCE 2016 PAGE 1

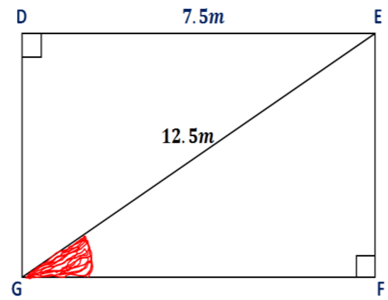
a. The rectangle $DEFG$ below is a piece of land in which $GE = 12.5m$ and $DE = 7.5m$. find the value of $\cos E\hat{G}F$



b. The cost of advertising in a local newspaper for one week is $K15.40$ per word plus a fixed charge of $K41.25$. What is the cost of advertising 15 words for one week?

SOLUTION a PAGE 2

The angle $E\hat{G}F$ is formed between the sides $GE = 12.5m$ and $GF = 7.5m$ of the right angled triangle at point G inside rectangle $DEFG$ as shaded in red below



The cosine of an angle is found as $Cos\theta = \frac{adjacent}{hypotenuse}$

Continuation on the next page

SOLUTION a cont. PAGE 3

In the formula

$$\theta = the\ angle$$

$$adjacent = shortest\ side\ touching\ the\ angle$$

$$hypotenuse = longest\ side\ touching\ the\ angle$$

Substitute into the formula

$$CosE\widehat{G}F = \frac{7.5}{12.5}$$

Multiply denominator and numerator by 10 to remove the point

$$CosE\widehat{G}F = \frac{7.5\times10}{12.5\times10} \quad \Rightarrow \quad CosE\widehat{G}F = \frac{75}{125}$$

Divide denominator and numerator by 25

$$\therefore CosE\widehat{G}F = \frac{3}{5}$$

SOLUTION b PAGE 4

One week is $K15.40$ per word plus a fixed charge of $K41.25$

You have to find the cost of 15 words for one week

First multiply $K15.40$ by 15 words

Do not use a calculator

$$\begin{array}{r} 15.40 \\ \times \quad 15 \\ \hline 1540 \\ +7700 \\ \hline 231.00 \\ \hline \end{array}$$

Add the fixed charge 41.25 to 231.00

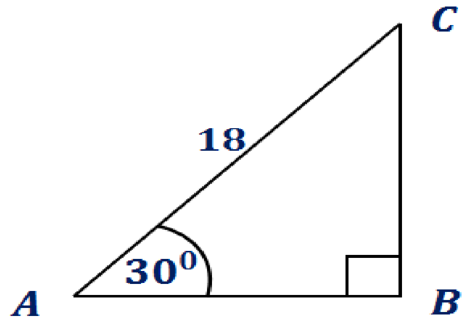
$$\begin{array}{r} 231.00 \\ +41.25 \\ \hline 272.25 \\ \hline \end{array}$$

$\therefore total\ cost = K272.25$

LESSON 92 QUESTION 15 PAPER 1 FOR GCE 2017 PAGE 1

a. A manufacturing company paid a total dividend of $K12\ 600$ at the end of 2015 on 6 000 shares. If Musonda owed 200 shares in the company, how much was paid in dividends to her.

b. The diagram below shows triangle ABC in which $AC = 18cm$, $C\hat{A}B = 30^{\circ}$ and $A\hat{B}C = 90^{\circ}$



Calculate the length of BC

SOLUTION a PAGE 2

Equate $12\ 600$ to 6 000 and 200 to x

$$6\ 000 = 12\ 600$$

$$200 = x$$

Cross multiply

$$6000x = 12\ 600 \times 200$$

Divide both sides by 6000

$$\frac{6000x}{6000} = \frac{12\ 600 \times 200}{6000}$$

Cancel 6000 on the left and some zeros on the right

Continuation on the next page

SOLUTION b cont. PAGE 3

$$x = \frac{1260 \times 2}{6}$$

Divide 6 and 2

$$x = \frac{1260 \times 1}{3}$$

Multiply in the numerator

$$x = \frac{1260}{3}$$

Divide 3 into 1260

$$\therefore x = K420$$

SOLUTION b PAGE4

Calculate BC by using the sine trigonometric ratio

$$sin\theta = \frac{opposite}{hypotenuse}$$

Where

$$\theta = the\ angle$$

$$opposite = side\ opposite\ to\ the\ angle$$

$$hypotenuse = side\ longest\ side\ touching\ the\ angle$$

BC is opposite to the angle, AC is the longest side

Continuation on the next page

SOLUTION b cont. PAGE 5

Substitute into the formula

$$sin30^{\circ} = \frac{BC}{18}$$

Referring to lesson 38, then 30° is a special angle

It also indicates that $sin30^{\circ} = \frac{1}{2}$ so substitute

$$\frac{1}{2} = \frac{BC}{18}$$

Cross multiply

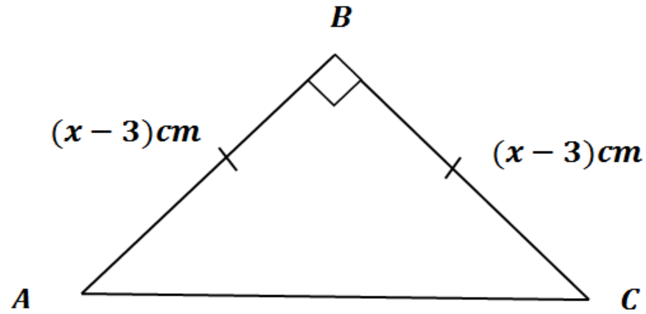
$$2BC = 18$$

Divide both sides by 2

$$\frac{2BC}{2} = \frac{18}{2} \qquad \therefore BC = 9cm$$

LESSON 93 QUESTION 13 PAPER 1 FOR GCE 2016 PAGE 1

The diagram below shows an isosceles triangle ABC where $AB = BC = (x-3)cm$ and $A\hat{B}C = 90^{\circ}$



Find the value of x for which the area of the triangle ABC is $18cm^2$

SOLUTION PAGE 2

The 90° interior angle shows that the triangle is isosceles meaning it have two equal sides.

The dashes on AB and BC shows that AB=BC

The formula for finding area of a right angled triangle is

$$Area = \frac{1}{2} \times base \times height$$

Use one of the sides as base and the other as height and substitute

$$18 = \frac{1}{2} \times AB \times BC \quad \Rightarrow \quad 18 = \frac{1}{2}(x-3)(x-3)$$

Continuation on the next page

SOLUTION cont. PAGE 3

Introduce denominator 1 on 18

$$\frac{18}{1} = \frac{(x-3)(x-3)}{2}$$

Cross multiply

$$(x-3)(x-3) = 36$$

Open brackets

$$x^2 - 3x - 3x + 9 = 36$$

Add like terms$-3x$ and $-3x$

$$x^2 - 6x + 9 = 36$$

Collect like terms

$$x^2 - 6x + 9 - 36 = 0$$

Continuation on the next page

SOLUTION cont. PAGE 4

Add like terms $+9$ and -36

$$x^2 - 6x - 27 = 0$$

This is a quadratic equation so use the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

In this case

$a = 1$ (Number in front of x^2)
$b = -6$ (Number in front of x)
$c = -27$ (Number without x)

Continuation on the next page

SOLUTION cont. PAGE 5

Substitute into the formula

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(-27)}}{2(1)}$$

Multiply the signs

$$x = \frac{6 \pm \sqrt{6^2 + 4(1)(27)}}{2(1)}$$

Multiply in the numerator and denominator

$$x = \frac{6 \pm \sqrt{36+108}}{2}$$

Add in the root and find the square-root

$$x = \frac{6 \pm \sqrt{144}}{2} \quad \Rightarrow \quad x = \frac{6 \pm 12}{2}$$

Continuation on the next page

SOLUTION cont. PAGE 6

Operation $\pm$ means you add for the first value and subtract for second value

$$x = \frac{6+12}{2} \qquad \text{And} \qquad x = \frac{6-12}{2}$$

Adding and subtracting

$$x = \frac{18}{2} \qquad \text{And} \qquad x = \frac{-6}{2}$$

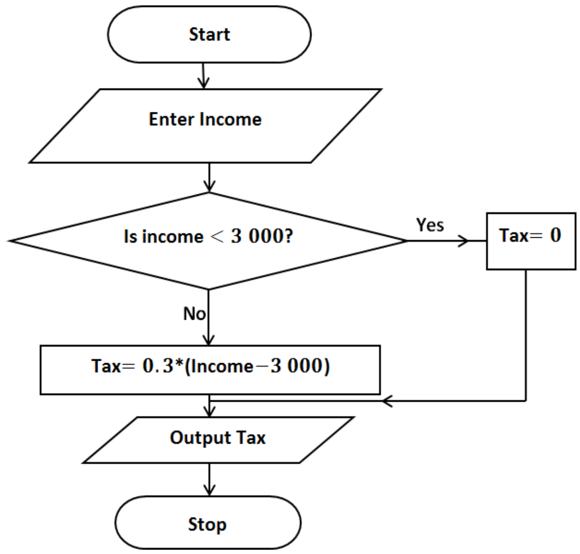
Divide by 2

$$\therefore x = 9 \qquad \text{And} \qquad \therefore x = -3$$

LESSON 94 QUESTION 16 PAPER 1 FOR GCE 2017 PAGE 1

a. The curved surface area of a cone is $88cm^2$. Given that the base radius is $4cm$, calculate the slant height of the cone using $\pi = \frac{22}{7}, A = \pi r l$.

b. The diagram below shows a flow chart for a program to calculate tax on an income.



Complete the below

Income	Tax
K2 900	
K 5 000	

SOLUTION a PAGE 2

They have already given you the formula $A = \pi r l$

The slant height is l substitute into the formula

$$88 = \frac{22}{7} \times 4 \times l$$

Multiply on the right

$$88 = \frac{88l}{7}$$

Introduce denominator 1 on 88 and cross multiply

$$\frac{88}{1} = \frac{88l}{7} \quad \Rightarrow \quad 88l = 88 \times 7$$

Divide both sides by 88

$$\frac{88l}{88} = \frac{88 \times 7}{88} \quad \Rightarrow \quad \therefore l = 7cm$$

SOLUTION b PAGE 3

In box 1 you're just starting the program

In box 2 you enter the income then the program will do the calculation of tax

In box 2 the suggestion will be made by the program,

If income is less than 3 000, the program will use the arrow where it says yes, tax will be zero and will be displayed in the Output Tax box.

Continuation on the next page

SOLUTION b cont. PAGE 4

Use the first income $2\ 900$, is it less than 3 000?

Yes so the calculation will follow the yes route and tax will be zero

Income	Tax
K2 900	0
K5 000	

SOLUTION b cont. PAGE 5

Use the second income $5\ 000$ is it less than 3 000?

No so the calculation will follow the No route and tax will be calculated as follows

$$\text{Tax} = 0.3*(\text{Income} - 3\ 000)$$

Substitute into the formula

$$\text{Tax} = 0.3*(5\ 000 - 3\ 000)$$

Subtract in brackets

$$Tax = 0.3*2\ 000$$

The star * means multiply

$$Tax = 0.3 \times 2\ 000$$

Continuation on the next page

SOLUTION b cont. PAGE 6

Change 0.3 to a fraction, it has one decimal pace so divide 3 by 10, the point becomes plus

$$0 + \frac{3}{10} = \frac{3}{10}$$

Substitute for 0.3

$$Tax = \frac{3}{10} \times 2\ 000$$

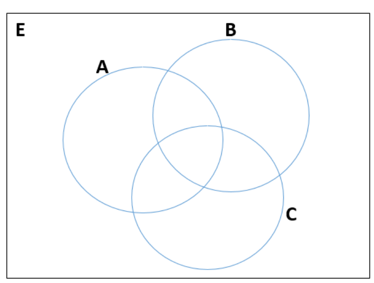
Divide 10 and 2 000

$$Tax = 3 \times 2\ 00 \quad \Rightarrow \quad Tax = 600$$

Income	∴Tax
K2 900	K0
K5 000	K600

LESSON 95 SPECIMEN QUESTIONS P1 2016 PAGE 1

a. In the Venn diagram below, shade the region $A' \cap (B \cap C)$



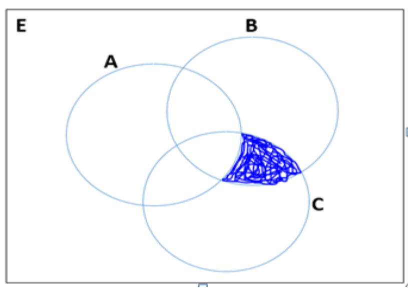
b. Given that $M = \begin{pmatrix} 4 & 1 & 3 \\ 2 & -5 & 4 \end{pmatrix}$ and $N = \begin{pmatrix} 3 & -2 \\ -1 & 4 \\ -2 & 0 \end{pmatrix}$ express MN as a single matrix.

SOLUTION a page 2

HINT; A' **Stands for A-complement or region of elements that are not found in set A, the symbol ∩ stands for intersection, it shows the overlap of elements that are found in both sets, or common elements in given sets.**

$A' \cap (B \cap C)$ **This reads as A-complement intersection B intersection C. It means you must not shade any part of set A,** $B \cap C$ **means shade the region for elements found in both set B and set C but not the region for elements found only in B or in C.**

$\therefore$ ***Answer*** $=$



SOLUTION a page 3

HINTS; expressing MN **as a single matrix means multiplying M by N. First know that M and N have rows and columns. The horizontal line where we have 4, 1 and 3 in M is a first row the vertical line where we have 4 and 2 in M is a first column. So there are 2 rows and 3 columns in M. There are 3 rows and 2 columns in N.**

$$MN = \begin{pmatrix} 4 & 1 & 3 \\ 2 & -5 & 4 \end{pmatrix} \times \begin{pmatrix} 3 & -2 \\ -1 & 4 \\ -2 & 0 \end{pmatrix}$$

Continuation on the next page

SOLUTION a page 4

Use row by column to multiply matrices, in this case the first row of M will multiply the first and second column of N. The second row of M will also multiply the first and second column of N. Observe the following multiplication patterns.

$$\boldsymbol{MN} = \begin{pmatrix} (4\times 3)+(1\times -1)+(3\times -2) & (4\times -2)+(1\times 4)+(3\times 0) \\ (2\times 3)+(-5\times -1)+(4\times -2) & (2\times -2)+(-5\times 4)+(4\times 0) \end{pmatrix}$$

Observe the colours

Green is the multiplication of the first row in M and the first column in N. row by column

Purple is the multiplication of the first row in M and the second column in N. row by column

Blue is the multiplication of the second row in M and the first column in N. row by column

Black is the multiplication of the second row in M and the second column in N. row by column

Continuation on the next page

SOLUTION a page 5

Multiply in brackets

$$\boldsymbol{MN} = \begin{pmatrix} 12+(-1)+(-6) & -8+4+0 \\ 6+5+(-8) & -4+(-20)+0 \end{pmatrix}$$

Multiply the signs

$$\boldsymbol{MN} = \begin{pmatrix} 12-1-6 & -8+4+0 \\ 6+5-8 & -4-20+0 \end{pmatrix}$$

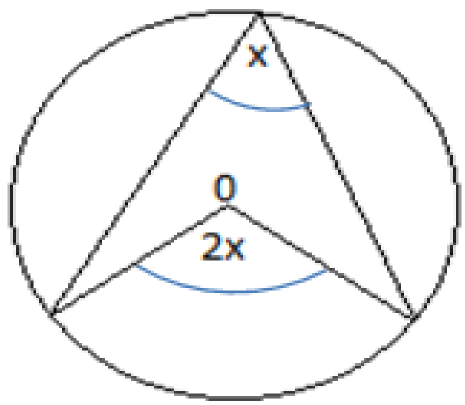
$$\therefore MN = \begin{pmatrix} 5 & -4 \\ 3 & -24 \end{pmatrix}$$

LESSON 96 CIRCLE THEOREM (page 1)

Theorems of angles in a Circle.

2. Angle at the centre theorem:

- **Angle subtended at the centre is twice the angle subtended at the circumference.**

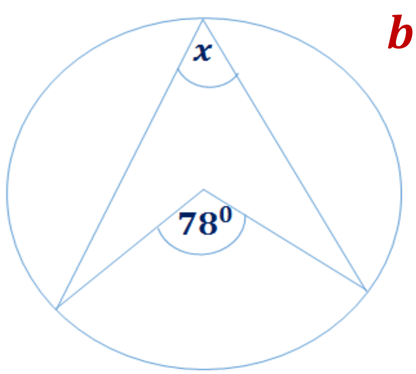


HINTS; the angle x, is at the edge of the circle hence it is said to be subtended at the circumference. To find the angle at the centre we have to multiply x by 2. This only applies when the two angles are facing the same direction like x and $2x$.

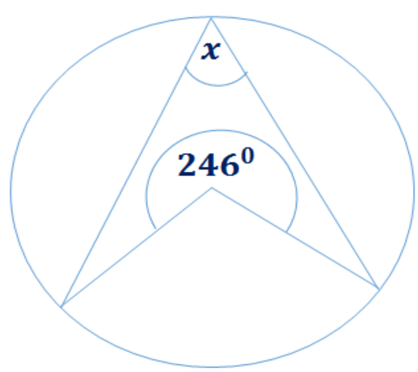
QUESTIONS (page 2)

In each of the diagrams below, find the value of x

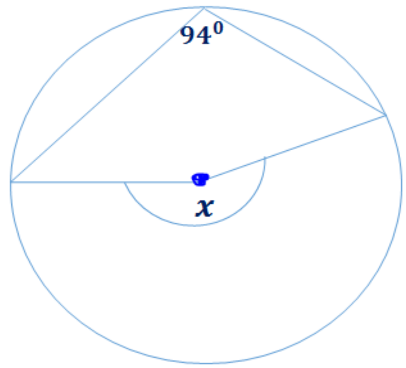
a



b



c



Copy the question and wait scroll down for answers

SOLUTION a (Page 3)

Hints; the angle at the centre 78^0 is twice the angle at the circumference x. This means that the angle at the circumference must be multiplied by 2 in order to get the angle at the centre. So we have;

$$2 \times x = 78^0 \quad \Rightarrow \quad 2x = 78^0$$

Dividing both sides by 2 to find x

$$\frac{2x}{2} = \frac{78^0}{2} \quad \Rightarrow \quad \therefore x = 39^0$$

SOLUTION b (Page 4)

In b the angles x and 246^0 are not facing in the same direction, they are facing each other, so we have to find the angle on the other side of the centre that is facing in the same direction with x. To do that we subtract 246^0 from 360^0 to have 114^0. This angle is the remaining part at the centre facing the same direction with x. Then we calculate;

$$2 \times x = 114^0 \quad \Rightarrow \quad 2x = 114^0$$

Dividing both sides by 2 we have

$$\frac{2x}{2} = \frac{114^0}{2} \quad \Rightarrow \quad \therefore x = 57^0$$

SOLUTION c (Page 5)

For c the angle at the centre is not known, we only know the angle at the circumference, to find the angle at the centre we multiply the angle at the circumference by 2.

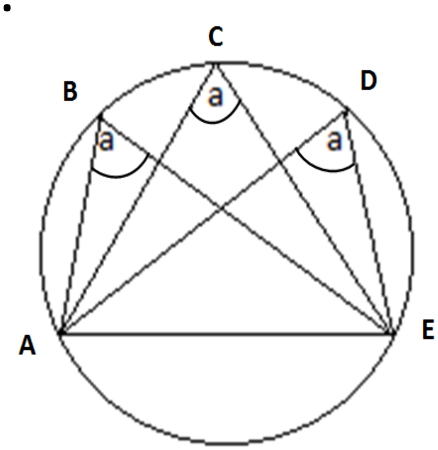
$$x = 94^0 \times 2$$

$$\therefore x = 188^0$$

LESSON 97 CIRCLE THEOREM (page 1)

3. Angle in the same chord:

- **Angles subtended in the same segment on a chord of a circle are equal.**

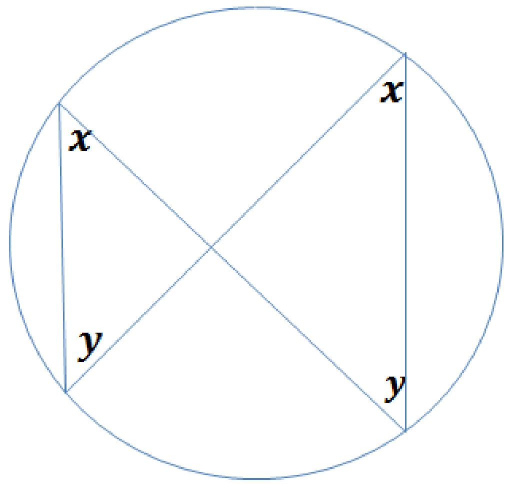


Hints; a chord is a straight line, dividing a circle into two un-equal parts, the two parts are segments, in this case AE is a chord dividing the circle into the major (larger) and minor

(smaller) segment, ABE, ACE and ADE are touching both ends of the circle hence the angles they are forming at the circumference are equal.

PAGE 2

4. Angles in the same segment and chord are equal:



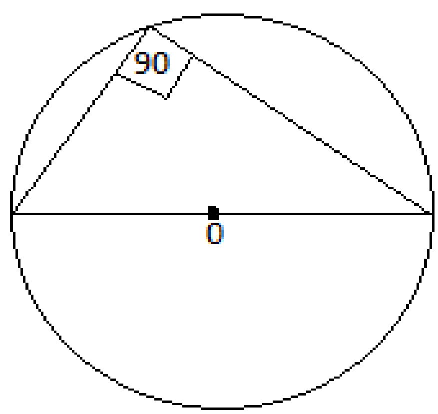
$$x = x$$

$$y = y$$

PAGE 3

5. Angle in a semi-circle:

- **Angle in a semi-circle $= 90°$.**

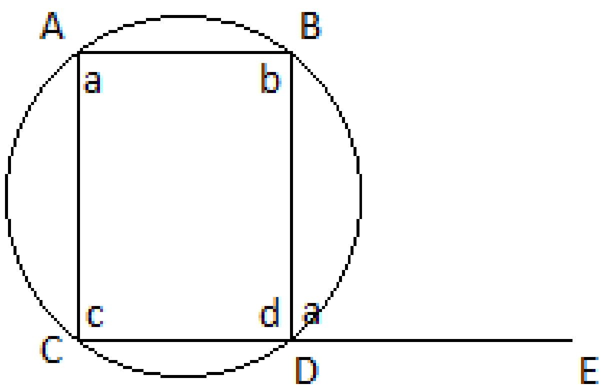


NONE; a semi-circle is formed when there is a diameter or a line passing through the centre of the circle dividing it into two equal halves.

PAGE 4

6. Angles associated with a cyclic-quadrilateral:

The opposite angles of a cyclic-quadrilateral are supplementary. The exterior angle of a cyclic-quadrilateral is equal to the opposite interior angle.



$a + d = 180$

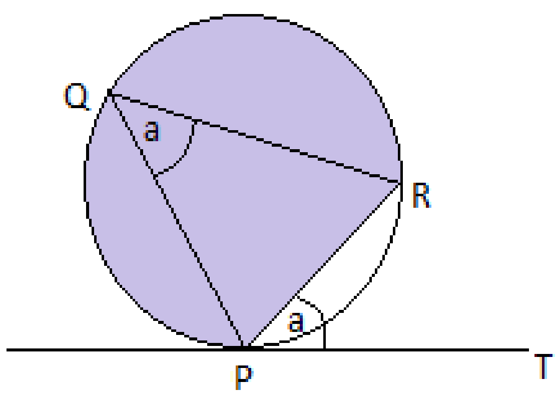
$b + c = 180$

$\angle CAB = \angle BDE$

PAGE 5

7. Alternate Segment theorem:

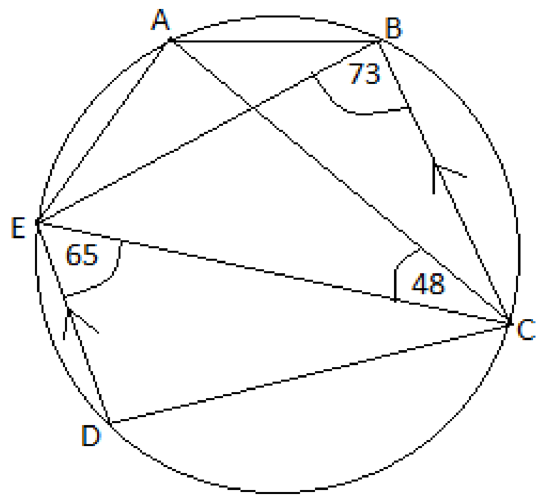
- **The angle between a chord and a tangent at the point of contact is equal to any angle in the alternate Segment.**



$\angle PQR = \angle TPR = a$

LESSON 98 CIRCLE THEOREM (page 1)

In the diagram, A, B, C, D and E lie on the circumference.



DE is parallel to CB. Angle ACE = 48°, angle CED = 65° and angle CBE = 73°. Calculate;

a) $angle\ ABE$ **b)** $angleACB$

c) $angleBEC$ **d)** $angleCDE$

SOLUTION a (page 2)

Look at the diagram, angle ABE is subtended on chord AE, angle ACE is also subtended on the same chord AE. This means that the two angles are equal according to theorem 2 of the previous lesson. Therefore;

$$< \mathbf{ABE} = < \mathbf{ACE} = \mathbf{48^0}$$

$$\therefore < \mathbf{ABE} = \mathbf{48^0}$$

SOLUTION b (page 3)

Go to the diagram, line DE and CB are parallel, this means that, when you move from D to E to C to B you form a zed (Z). This means that angle DEC and angle ECB are equal because they are alternate angles;

$$< DEC =< ECB = 65^0$$

We are finding ACB **which is part of angle** $ECB = 65^0$

In angle $ECB = 65^0$ we already have angle $ACE = 48^0$

To find angle ACB we have to subtract ACE from ECB

$$< ACB =< ECB - < ACE$$

$$< ACB = 65^0 - 48^0$$

$$\therefore < ACB = 17^0$$

SOLUTION c (page 4)

Go to the diagram, move from B to C to E and back to B, you form a triangle. The rule is that, the sum of three interior angles of a triangle is 180^0 this means that

$$< BEC + < ECB + < CBE = 180^0$$

Substituting we have

$$< BEC + 65^0 + 73^0 = 180^0$$

Adding

$$< BEC + 138^0 = 180^0$$

Collecting like terms

$$< BEC = 180^0 - 138^0$$

$$\therefore < BEC = 42^0$$

SOLUTION d (page 5)

Look at the diagram; CBE and CDE are opposite angles in a cyclic-quadrilateral. This means that angle CBE and angle CDE add up to 180^0 according to theorem 5 of the previous lesson. Thus;

$$< \text{CDE} + < \text{CBE} = 180^0$$

Substituting for $< \text{CBE} = 73^0$

$$< \text{CDE} + 73^0 = 180^0$$

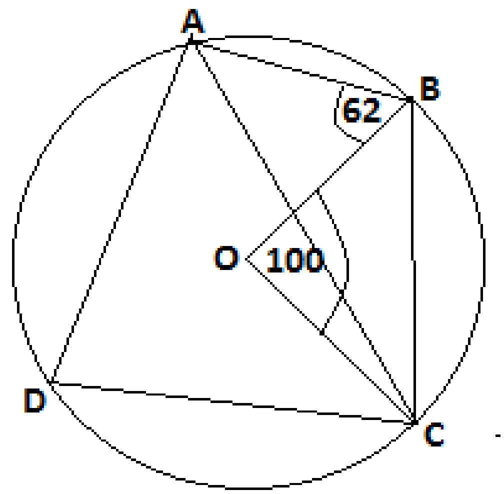
Collecting like terms

$$< \mathbf{CDE} = \mathbf{180^0} - \mathbf{73^0}$$

$$\therefore < \mathbf{CDE} = \mathbf{107^0}$$

PRACTICE QUESTION PAGE 6

(1) O is the centre of the circle through A,B,C and D. Angle BOC= 100° and angle OBA=62°.



Calculate angle;

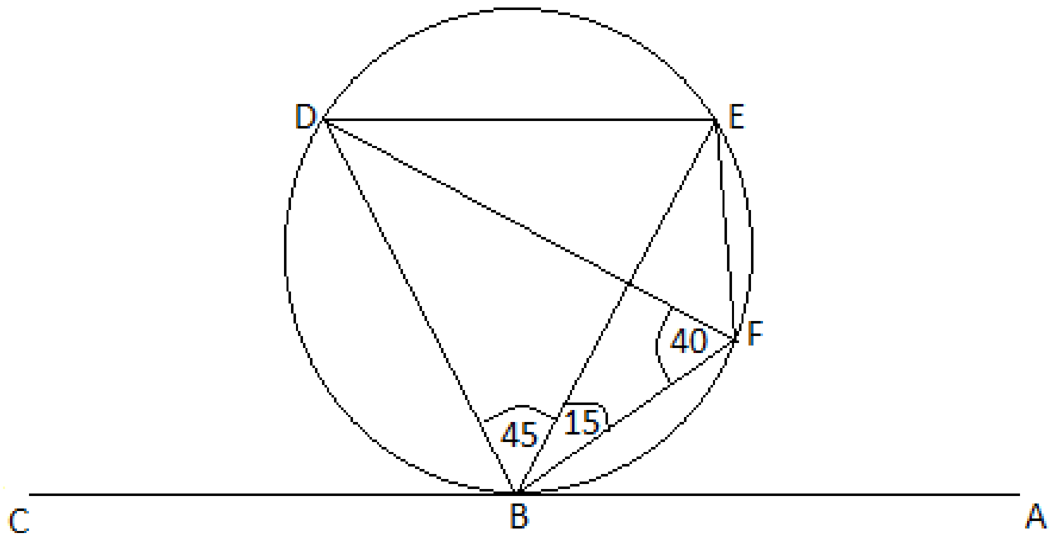
(i) **BAC,** **(ii) OCB,** **(iii) ADC.**

ANSWERS:

(i) 50°, (ii) 40°, (iii) 78°

LESSON 99 CIRCLE THEOREM (page 1)

In the diagram below, ABC is a tangent to the circle BDEF at B, angle DFB= 40°, angle EBF=15° and angle DBE= 45°.



Find (a) angle DEB,

(b) angle DEF,

(c) angle ABF.

SOLUTION a (page 2)

Angle DEB and angle DFB are subtended at the same chord BD in the same segment hence they are equal; this is according to theorem 2 of the previous lesson. Thus;

$$<\mathbf{DEB} =< \mathbf{DFB} = \ \mathbf{40^0}$$

$$\therefore < \mathbf{DEB} = \ \mathbf{40^0}$$

SOLUTION b (page 3)

Angle DEF and angle DBF are opposite angles in a cyclic-quadrilateral. This means that the sum of these angles is $\mathbf{180^0}$ according to theorem 5 of the previous lesson. Thus;

$$< \mathbf{DEF} + < \mathbf{DBF} = \ \mathbf{180^0}$$

Substituting for $\mathbf{DBF} = \mathbf{45^0 + 15^0 = 60^0}$

$$< \mathbf{DEF} + \mathbf{60^0} = \mathbf{180^0}$$

Collecting like terms

$$< \mathbf{DEF} += \mathbf{180^0} - \mathbf{60^0}$$

$$\therefore < \mathbf{DEF} = \mathbf{120^0}$$

SOLUTION c (page 4)

According to theorem 5 of the previous lesson angle ABF is equal to angle BDF. To find BDF we have to consider the three interior angles in triangle BFDB. The sum of interior angles BDF, DFB and FBD will be $\mathbf{180^0}$. Thus;

$$< \boldsymbol{BDF} + < \boldsymbol{DFB} + < \boldsymbol{FBD} = \mathbf{180^0}$$

Substituting we have;

$$< \boldsymbol{BDF} + \mathbf{40^0} + \mathbf{60^0} = \mathbf{180^0}$$

Adding

$$< \boldsymbol{BDF} + \mathbf{100^0} = \mathbf{180^0}$$

Collecting like terms

$$< \boldsymbol{BDF} = \mathbf{180^0} - \mathbf{100^0} \quad \Rightarrow \quad < \boldsymbol{BDF} = \mathbf{80^0}$$

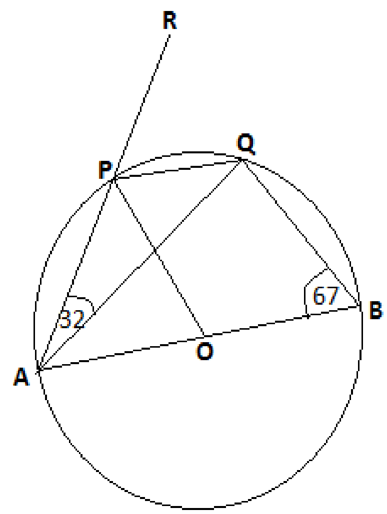
$$< \mathbf{ABF} =< \mathbf{BDF} = \mathbf{80^0}$$

$$\therefore < \mathbf{ABF} = \mathbf{80^0}$$

PRACTICE QUESTION PAGE 5

In the diagram, AB is a diameter of the circle, centre O. P and Q are two points on the circle and APR is a straight line.

Given that angle QBA = 67° and angle PAQ = 32° ,



Calculate

(a) **angle QAB,**

(b) **angle RPQ,**

(c) **angle POB.**

ANSWERS:

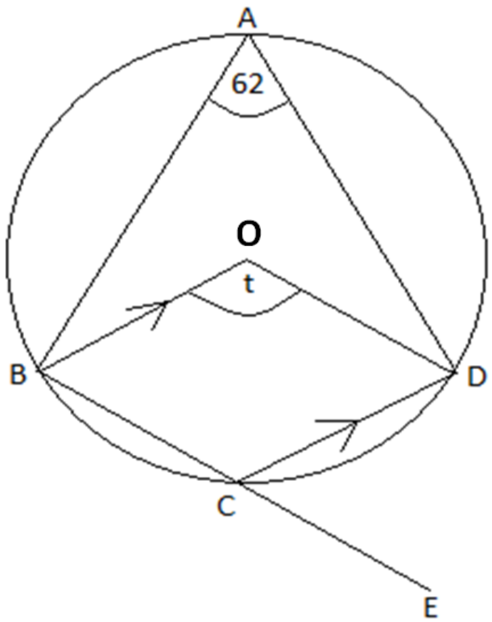
(a) 23°, (b) 67°, (b) 110°

LESSON 100 CIRCLE THEOREM (page 1)

In the diagram below, A,B,C and D lie on the circumference of the circle, center o.

BO is parallel to CD, angle

BAD=62° and BCE is a straight line.



Calculate:

(a) **angle t,**

(b) **angle BCD,**

(c) **angle OBC,**

(d) **angle DCE.**

SOLUTION a (page 2)

Angle t is at the centre, so it's twice angle BAD at the circumference

$$Angle\ t = 2 \times angleBAD$$

$$Angle\ t = 2 \times 62°$$

$$\therefore Angle\ t = 124°$$

From here connect to page 3

SOLUTION b (page 3)

Angle BCD + Angle BAD= 180° (opp. angles of a cyclic quad.)

Angle BCD+62° =180°

Collecting like terms

Angle BCD =180° − 62°

∴ Angle BCD = 118°.

From here connect to page 4

SOLUTION c (page 4)

Angle OBC and Angle BCD are allied angles associated with parallel lines hence they add up to $180°$

$$< OBC + < BCD = 180°$$

Substituting for $< BCD = 118°$

$$< OBC° + 118° = 180°$$

Collecting like terms

$$< OBC° = 180° - 118°$$

$$\therefore < OBC° = 62°$$

From here connect to page 5

SOLUTION d (page 5)

The angle DCE is an exterior angle of cyclic quadrilateral hence its equal to the opposite interior angle BAD

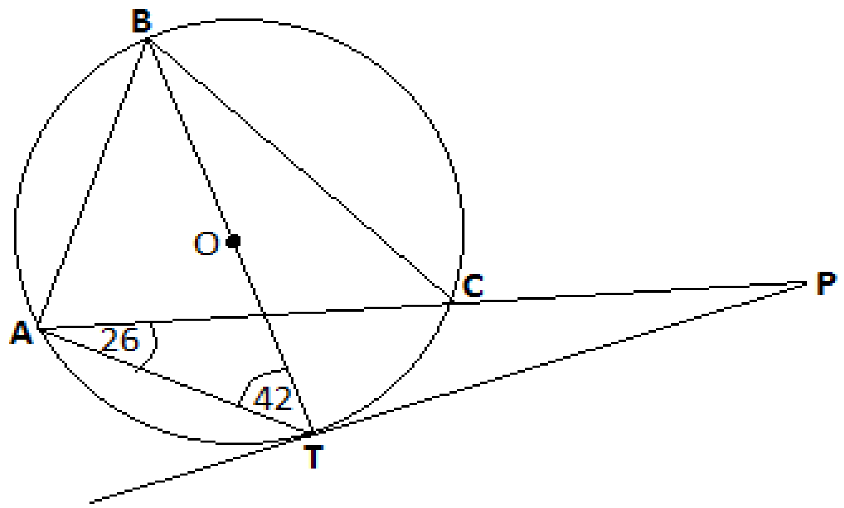
$$Angle\ DCE = AngleBAD = 62°$$

$$\therefore Angle\ DCE = 62°$$

From here connect to page 6

QUESTIONS FOR YOUR PRACTICE

BT is a diameter of a circle and A and C are points on the circumference. The tangent to the circle at the point T meets AC produced at P. Given that angle ATB = 42° and angle CAT = 26°,



Calculate

(i) angle CBT,

(ii) angle ABT,

(iii) angle APT.

ANSWERS:

(i) 26° , (ii) 48° , (iii) 22°

END OF LESSON 100

LESSON 101 SPECIMEN QUESTION 7 P1 2016 PAGE 1

For the sequence 11, 14, 17, 20,…….. Find the

a. 15th term

b. Subtract the 5th term from the 15th term

c. Sum of the first 20 terms

SOLUTION a PAGE 2

HINT; to answer a. use the formula

$$T_n = a + (n - 1)d$$

T_n Means nth term in this case it will be 15th term

a Means the first term, in this case the first number is 11

n Means the number of terms, in this case 15 terms

d Means the common difference, you get it by subtracting the first term from the second term $d = 14 - 11 = 3$

Continuation on the next page

SOLUTION a cont. PAGE 3

So you can now substitute

$$T_{15} = 11 + (15 - 1)3$$

First subtract inside brackets

$$T_{15} = 11 + (14)3$$

Multiply

$$T_{15} = 11 + 42$$

Add

$$\therefore T_{15} = 53$$

From here connect to page 4

SOLUTIONS b PAGE 4

First find the 5th term. There are 4 terms already in the question and you calculated the common difference on page 2

To get the 5th term just add the common difference 3 to the fourth term 20

$$5^{th}\ term = 20 + 3 = 23$$

15th term is 53 on page 3

$$15^{th}\ term - 5^{th}\ term = 53 - 23$$

$$\therefore Ans = 30$$

From here connect to page 5

SOLUTIONS c PAGE 5

HINT; to answer b. we have to use the formula

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

S_n Means sum of n terms in this case 20 terms

n Means number of terms, in this case 20 terms

a Means the first term in this case 11

d Means common difference in this case 3

$$S_{20} = \frac{20}{2}[2 \times 11 + (20-1)3]$$

Dividing 20 by2, multiplying 2 by 11 and subtracting 1 from 20

Continuation on the next page

SOLUTIONS b PAGE 6

$$S_{20} = 10[22 + (19)3]$$

Multiplying 3 by 19 we have

$$S_{20} = 10[22 + 57]$$

Adding inside brackets we have

$$S_{20} = 10[79]$$

$$\therefore S_{20} = 790$$

END OF LESSON 101

LESSON 102 SPECIMEN QUESTION 8 P1 2016

a. Given that $E = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $P = \{2, 3, 5, 7\}$ and $Q = \{2, 5, 8, 9\}$ list $(P \cap Q)'$

b. A plan of a house is drawn to a scale of $1:20$. If the actual area on the ground floor of the house is $80m^2$, calculate the area on the map in square centimetres.

c. Given that $f(x) = \frac{3}{2x-5}$ find $f(-2)$

SOLUTION a PAGE 2

HINT; $P \cap Q$ reads as P intersection Q, it means the set of elements common in both set P and set Q. First list this set;

$$P \cap Q = \{2, 5\}$$

See that 2 and 5 are the only elements common in both sets. Now they are asking you to list $(P \cap Q)'$ this reads as P intersection Q complement, meaning the set of elements that are not found in $P \cap Q = \{2, 5\}$ so list all elements other than 2 and 5.

$$\therefore (P \cap Q)' = \{1, 3, 4, 6, 7, 8, 9, 10\}$$

From here connect to page 3

SOLUTION b PAGE 3

HINT; remember the formula

$$\frac{1}{n^2} = \frac{area\ on\ the\ map\ in\ cm^2}{area\ on\ the\ ground\ in\ m^2}$$

With what you have

$$n = 20$$

$$area\ on\ the\ ground = \ 80m^2$$

You are looking for area on the map in squared centimeters. So convert $80m^2$ to cm^2

$$1m^2 = 10,000cm^2$$

$$80m^2 = x$$

Cross multiply

Continuation on the next page

SOLUTION b cont. PAGE 4

$$x = 80 \times 10,000 = 800,000cm^2$$

So you have converted area on the ground to cm^2.

$$\frac{1}{20^2} = \frac{x}{800,000} \quad \Rightarrow \quad \frac{1}{20 \times 20} = \frac{x}{800,000}$$

Multiply 20 by 20

$$\frac{1}{400} = \frac{x}{800,000}$$

Cross multiply

$$400x = 800,000$$

Divide both sides by 400

$$\frac{400x}{400} = \frac{800,000}{400}$$

$$\therefore x = 2000cm^2$$

From here connect to page 5

SOLUTION c PAGE 5

$f(x) = \frac{3}{2x-5}$ find $f(-2)$

Here it's just a matter of substitution, where there is x you write 2 then work out

$$f(-2) = \frac{3}{2(-2)-5}$$

Open brackets

$$f(-2) = \frac{3}{-4-5}$$

Subtract in the denominator

$$f(-2) = \frac{3}{-9}$$

Divide by 3

$$\therefore f(-2) = -\frac{1}{3}$$

END OF LESSON 102

LESSON 103 SPECIMEN QUESTION 10 P1 2016 PAGE1

a. Quiz questions are numbered from **1** to **20**. A question is selected random, what is the probability that the number of the question selected is a **perfect square** or a **prime number**?

b. The position vector of **A** is $\begin{pmatrix} -2 \\ 1 \end{pmatrix}$ and $\overrightarrow{AB} = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$, find the coordinates of the point **B.**

c. Express $\frac{x+2}{3} - \frac{2x-3}{4}$ as a single matrix in its simplest form

SOLUTION a PAGE 2

HINTS; count perfect square numbers from 1 to 20. These are numbers with a non-decimal placed square root. List them;

$$N^2 = \{1, 4, 9, 16\}$$

There are 4 perfect square numbers. The probability of selecting a perfect square number is found using the formula

$$P(N^2) = \frac{total\ perfect\ square\ numbers}{total\ number\ of\ questions}$$

$$P(N^2) = \frac{4}{20} \qquad \therefore P(N^2) = \frac{1}{5}$$

Know and count prime numbers, prime numbers are numbers with only 2 factors 1 and itself. List them;

$$P = \{2, 3, 5, 7, 11, 13, 17, 19\}$$

Continuation on the next page

SOLUTION a cont. PAGE 3

So you have 8 prime numbers. The probability of selecting a prime number is found using the formula

$$P(p) = \frac{total\ \ prime\ numbers}{total\ number\ of\ questions}$$

$$P(p) = \frac{8}{20} \qquad \therefore P(p) = \frac{2}{5}$$

They are asking you to find the probability of selecting a perfect square **OR** a prime number. Take note of the word **OR**, this word means **PLUS** in probabilities, so it means;

$$P(N^2)\ OR\ P(p) = P(N^2) + P(p)$$

Substitute

$$P(N^2) + P(p) = \frac{1}{5} + \frac{2}{5} = \frac{1+2}{5}$$

$$\therefore Ans = \frac{3}{5}$$

From here connect to page 4

SOLUTION b PAGE 4

HINTS; come up with the formula. In vectors AB comes from $A + B$.

$$A + B = AB$$

Make B the subject of the formula

$$B = AB - A$$

Substitute for A and AB

$$B = AB - A$$

Continuation on the next page

SOLUTION b cont. PAGE 5

$$B = \begin{pmatrix} -3 \\ 2 \end{pmatrix} - \begin{pmatrix} -2 \\ 1 \end{pmatrix}$$

Subtract the matrices

$$B = \begin{pmatrix} -3 - (-2) \\ 2 - 1 \end{pmatrix}$$

Know that $- \times - = +$ so you have

$$B = \begin{pmatrix} -3 + 2 \\ 2 - 1 \end{pmatrix}$$

$$\therefore B = \begin{pmatrix} -3 + 2 \\ 2 - 1 \end{pmatrix}$$

From here connect to page 6

SOLUTION c PAGE 6

$$\frac{x+2}{3} - \frac{2x-3}{4}$$

Find the common denominator

$$\frac{\qquad\qquad}{12}$$

Divide 3 into 12 and multiply the answer to $x + 2$

$$\frac{4(x+2) -}{12}$$

Divide 4 into 12 and multiply the answer to $2x - 3$

$$\frac{4(x+2) - 3(2x-3)}{12}$$

Continuation on the next page

SOLUTION c cont. PAGE 7

Open brackets

$$\frac{4x+8-6x+9}{12}$$

Collect like terms

$$\frac{9+8+4x-6x}{12}$$

Add and subtract

$$\therefore Ans = \frac{17-2x}{12}$$

END OF LESSON 103

LESSON 104 QUESTION 7 P_2 GCE 2014 PAGE 1

a. Evaluate $2\frac{3}{4} - 1\frac{5}{6} - 4$

b. Solve the inequation $1 - 2x \leq \frac{x}{2} - 9$

c. After a reduction of 20%, the price of a camera is K380.00. Calculate the original price.

SOLUTION a PAGE 2

$$2\frac{3}{4} - 1\frac{5}{6} - 4$$

Change mixed fractions into improper fractions

$$\frac{11}{4} - \frac{11}{6} - 4$$

Introduce denominator 1 on 4

$$\frac{11}{4}-\frac{11}{6}-\frac{4}{1}$$

Find the common denominator

$$\frac{\qquad}{12}$$

Divide 4 into 12 and multiply the answer to 11

$$\frac{3(11)-}{12}$$

Divide 6 into 12 and multiply the answer to 11

Continuation on the next page

SOLUTION a cont. PAGE 3

$$\frac{3(11)-2(11)}{12}$$

Divide 1 into 12 and multiply the answer to4

$$\frac{3(11)-2(11)-12(4)}{12}$$

Open brackets

$$\frac{33-22-48}{12}$$

Subtract in the numerator

$$-\frac{37}{12}$$

Divide by 12

$$\therefore Asw=-3\frac{1}{12}$$

From here connect to page 4

SOLUTION b PAGE 4

$$1 - 2x \leq \frac{x}{2} - 9$$

Collect like terms

$$1 + 9 \leq \frac{x}{2} + 2x$$

Add $1 + 9$ and introduce denominator 1 on $2x$

$$10 \leq \frac{x}{2} + \frac{2x}{1}$$

Find the common denominator on the right

$$10 \leq \frac{\quad}{2}$$

Divide 2 into 2 and multiply the answer to x

$$10 \leq \frac{1(x) +}{2}$$

Continuation on the next page

SOLUTION b cont. PAGE 5

Divide 1 into 2 and multiply the answer to $2x$

$$10 \leq \frac{1(x)+2(2x)}{2}$$

Open brackets

$$10 \leq \frac{x+4x}{2}$$

Add in the numerator on the right

$$10 \leq \frac{5x}{2}$$

Introduce denominator 1 on 10 and cross multiply

$$\frac{10}{1} \leq \frac{5x}{2} \Rightarrow 5x = 20$$

Divide both sides by 5

$$\frac{5x}{5} = \frac{20}{5} \qquad \therefore x = 4$$

From here connect to page 6

SOLUTION c PAGE 6

Come up with the formula, Let the original amount be x and, the reduction was 20% of x. The formula will be

$$Original\ price - reduction = 380$$

Substitute into the formula

$$x - (20\% \text{ of } x) = 380$$

In mathematics $\%$ out of 100 and of means multiplication

$$x - \left(\frac{20}{100} \times x\right) = 380$$

Multiply in brackets

$$x - \frac{20x}{100} = 380$$

Introduce denominator 1 on x

$$\frac{x}{1} - \frac{20x}{100} = 380$$

Continuation on the next page

SOLUTION c cont. PAGE 6

Find the common denominator on the left

$$\frac{\qquad}{100} = 380$$

Divide 1 into 100 and multiply the answer to x

$$\frac{100(x) -}{100} = 380$$

Divide 100 into 100 and multiply the answer to $20x$

$$\frac{100(x) - 1(20x)}{100} = 380$$

Open brackets and introduce denominator 1 on 380

$$\frac{100x - 20x}{100} = \frac{380}{1}$$

Subtract and cross multiply

$$\frac{80x}{100} = \frac{380}{1} \quad \Rightarrow \quad 80x = 38\,000$$

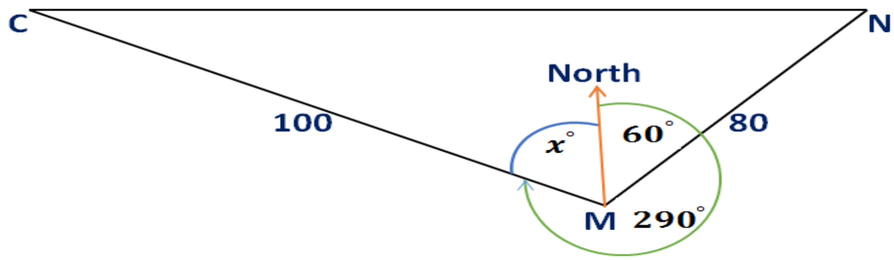
Divide both sides by 80

$$\frac{80x}{80} = \frac{38\,000}{80} \qquad \therefore x = \text{K}475$$

END OF LESSON 104

LESSON 105 SPECIMEN QUESTION 7 P2 2009 PAGE 1

Three towns Choma (C), Monze (M) and Namwala (N) are such that the distance from Monze to Choma is 100km and Monze to Namwala 80km. the bearing of Namwala from Monze is 060° and the bearing of Choma from Monze is 290°



a. Calculate the

i. Value of x

ii. Distance from Choma to Namwala correct to 2 decimal places

iii. Area covered by triangle CMN correct to 2 decimal places

b. Given that Nikoh (H) is a bus station on the Choma-Namwala route such that MH is the shortest distance from Monze to Nikoh, calculate the shortest distance MH, correct to 2 decimal places.

c. Hence, find how far Nikoh is from Namwala, giving your answer correct to 2 decimal places.

SOLUTION a. (i) PAGE 2

The angle x will be found as follows

$$x + 60 + 290 = 360$$

Add like terms on the left

$$x + 350 = 360$$

Collect like terms

$$x = 360 - 350$$

$$\therefore x = 10^\circ$$

From here connect to page 3

SOLUTION a. (ii) PAGE 3

Use the cosine rule to find CN

$$a^2 = b^2 + c^2 - 2(bc)cos\theta$$

a **is the side CN opposite to the** $C\widehat{M}N$ **inside the shape.**

b **and** c **are sides CM and MN respectively touching** $C\widehat{M}N$

θ **is the angle** $C\widehat{M}N = 60 + 10 = 70^{\circ}$

$$a^2 = 100^2 + 80^2 - 2(100 \times 80)cos70^{\circ}$$

Press these into the calculator

$$a^2 = 10927.67771$$

Find the square root on both sides

$\sqrt{a^2} = \sqrt{10927.67771}$ $\quad$ $\therefore x = 104.54km$

From here connect to page 4

SOLUTION a. (iii) PAGE 4

Use the sine formula

$$Area = \frac{1}{2}(bc)sin\theta$$

Substitute into the formula

$$Area = \frac{1}{2}(100 \times 80)sin70^{\circ}$$

Divide 2 and 100

$$Area = (50 \times 80)sin70^{\circ}$$

Press these into the calculator

$$Area = 3758.770483$$

Round off to 2 decimal places

$$\therefore Area = 3758.77km^2$$

From here connect to page 5

SOLUTION b PAGE 5

Shortest distance MH should be height from M straight to the base CN. The other formula for area of a triangle is

$$Area = \frac{1}{2} \times base \times height$$

Substitute into the formula

$$3758.77 = \frac{1}{2} \times 104.54 \times h$$

Cross multiply

$$7517.54 = 104.54h$$

Divide both sides by 104.54 multiply

$$\frac{7517.54}{104.54} = \frac{104.54h}{104.54}$$

$$\therefore h = 71.91km$$

From here connect to page 6

SOLUTION c PAGE 6

Distance HN makes a right angle with height HM therefore forming a right angled triangle with hypotenuse MN. Use Pythagoras theorem

$$MN^2 = MH^2 + HN^2$$

Make HN^2 subject of the formula

$$HN^2 = MN^2 - MH^2$$

Substitute into the formula

$$HN^2 = 80^2 - 71.91^2$$

Press these into the calculator

$$HN^2 = 1228.9519$$

Find the square root on both sides

$$\sqrt{HN^2} = \sqrt{1228.9519}$$

$$\therefore HN = 35.06km$$

END OF LESSON 105

LESSON 106 QUESTION 2 PAPER 2 2011 (Page 1)

a. Express $\frac{3x+3}{3} - \frac{2x-1}{4}$ as a single fraction in its simplest form

b. Given that matrix $A = \begin{pmatrix} 1 & x \\ -1 & 2 \end{pmatrix}$

i. Write an expression in terms of x , for determinant of A

ii. Find the value of x given that the determinant of A is 5

iii. Write A^{-1}

c. Solve the equation $\frac{12}{x+2} = \frac{3}{5}$

SOLUTION a (page 2)

HINTS; first find the common denominator for 3 and 4. It will be 12, then divide 3 into 12 you will get 4, multiply 4 to $x + 2$. Divide 4 into 12, you will get 3, multiply 3 to $2x - 3$ as follows

$$\frac{3x+3}{3} - \frac{2x-1}{4} = \frac{4(3x+3)-3(2x-1)}{12}$$

Opening brackets

$$\frac{12x+12-6x+3}{12}$$

Collecting like terms, adding and subtracting

$$\frac{12+3+12x-6x}{12}$$

$$\frac{15+6x}{12} \Rightarrow \frac{3(5+2x)}{12} \Rightarrow \therefore Ans = \frac{5+2x}{4}$$

From here connect to page 3

SOLUTION b i (page 3)

HINTS; first determine the diagonals of the matrix, 1 and 2 are in the major diagonal then x and -1 are in the minor diagonal

The formula for determinant is as follows;

$$Determinant = product(major - minor)diagonal$$

$$Determinant = (1 \times 2) - (x \times -1)$$

$$Determinant = 2 - (-x)$$

Negative times negative is positive

$$\therefore Determinant = 2 + x$$

From here connect to page 4

SOLUTION b ii (page 4)

HINTS; just equate the expression you got in b i to 5 because the question says determinant is 5

$$2 + x = 5$$

Collect like terms

$$x = 5 - 2$$

$$\therefore x = 3$$

From here connect to page 5

SOLUTION b iii (page 5)

HINTS; to find A^{-1} or inverse of the matrix, we use the following formula.

$$A^{-1} = \frac{1}{determinant} \times A_{adjoind}$$

$A_{adjoind}$ is found by interchanging positions in the major diagonal, where there is 1 write 2, where there is 2 write 1. Then change the signs for numbers in the minor diagonal. Remember $x = 3$ and $determinant = 5$

$$\therefore A^{-1} = \frac{1}{5}\begin{pmatrix} 2 & -3 \\ 1 & 1 \end{pmatrix}$$

From here connect to page 6

SOLUTION c (page 6)

HINTS; fractions are separated by an equal sign hence you have to cross multiply

$$\frac{12}{x+2} = \frac{3}{5} \quad \Rightarrow \quad 3(x+2) = 12 \times 5$$

Opening brackets/multiplying

$$3(x+2) = 12 \times 5 \quad \Rightarrow \quad 3x + 6 = 60$$

Collecting like terms, subtracting and dividing by 3

$$3x = 60 - 6 \quad \Rightarrow \frac{3x}{3} = \frac{54}{3}$$

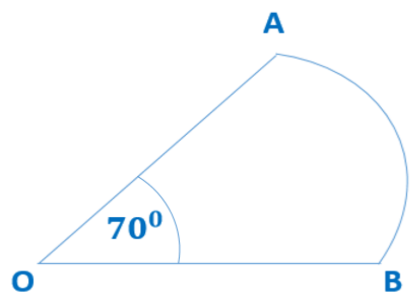
$$\therefore x = 18$$

END OF LESSON 106

LESSON 107 SPECIMEN QUESTION 11 P1 2016 PAGE 1

a. Find the transpose of the matrix $\begin{pmatrix} 1 & 3 & -1 \\ -2 & 0 & 4 \end{pmatrix}$

b. The diagram below shows a sector of a circle with centre O and $A\hat{O}B = 70^0$. Given that the area of the sector AOB is 5.5cm^2, calculate the radius of the sector, $\left[\pi = \frac{22}{7}\right]$



c. Integrate $3x^2 - 4x + 5$ with respect to x

SOLUTIONS a PAGE 2

Transposing a matrix means changing the rows to columns and columns to rows. The first row becomes the first column; the second row becomes the second column as follows

$$A = \begin{pmatrix} 1 & 3 & -1 \\ -2 & 0 & 4 \end{pmatrix}$$

$$\therefore A^T = \begin{pmatrix} 1 & -2 \\ 3 & 0 \\ -1 & 4 \end{pmatrix}$$

From here connect to page 3

SOLUTIONS a PAGE 3

You need to remember the formula for area of a sector.

$$Area = \frac{x^0}{360^0} \times \pi r^2$$

$x^0 = 70^0$ **Is the angle at the centre,** $\pi = \frac{22}{7}$**, then substitute;**

$$5.5 = \frac{70}{360} \times \frac{22}{7} \times r^2$$

Divide 70 and 7

$$5.5 = \frac{10}{360} \times \frac{22r^2}{1}$$

Divide 10 and 360

$$5.5 = \frac{1}{36} \times \frac{22r^2}{1}$$

Multiply

Continuation on the next page

SOLUTIONS a PAGE 4

$$5.5 = \frac{22r^2}{36}$$

Dividing 22 and 36 by 2

$$5.5 = \frac{11r^2}{18}$$

Cross multiplying we have

$$11r^2 = 99$$

Dividing both sides by 11

$$\frac{11r^2}{11} = \frac{99}{11} \qquad \Rightarrow \qquad r^2 = 9$$

Find the square root

$$\sqrt{r^2} = \sqrt{9} \qquad \therefore r = 3cm$$

From here connect to page 5

SOLUTIONS c PAGE 5

Symbol $\int()dx$ on $\int(3x^2 - 4x + 5)\,dx$ indicates integration. When integrating, **add 1** to the **power** of x, use the power of x as denominator, introduce x to numbers that have no x, also add a constant **C** at last.

$$\frac{3x^{2+1}}{2+1} - \frac{4x^{1+1}}{1+1} + 5x$$

Add the powers and denominators

$$\frac{3x^3}{3} - \frac{4x^2}{2} + 5x$$

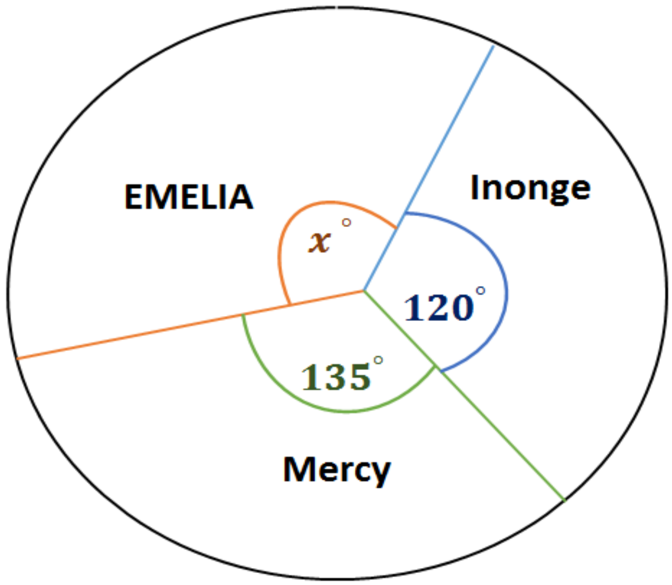
Divide the fractions and adding c

$$\therefore Ans = x^3 - 2x^2 + 5x + C$$

END OF LESSON 107

LESSON 108 QUESTION 14 PAPER 1 GCE 2016 PAGE 1

The pie chart below shows how members of a club voted for the position of chair person.



a. **What fraction of the members voted for Emelia**
b. **If 20 members voted for Inonge, how many did not vote for her?**

Copy the question above, scroll down for answers and listen to possible audios for each answer page

SOLUTION a PAGE 2

Find the value of x by considering that, the total number of degrees in a pie chart is 360°, then

$$x + 135^\circ + 120^\circ = 360^\circ$$

Add like terms

$$x + 255^\circ = 360^\circ$$

Collect like terms

$$x = 360^{\circ} - 255^{\circ}$$

$$\therefore x = 105^{\circ}$$

The fraction will be

Continuation on the next page

SOLUTION a cont. PAGE 3

$$fraction\ of\ votes\ for\ Emelia = \frac{Angle\ for\ Emelia}{Complete\ revolution}$$

Substitute into the formula

$$fraction\ of\ votes\ for\ Emelia = \frac{105^{\circ}}{360^{\circ}}$$

Divide both by 5

$$fraction\ of\ votes\ for\ Emelia = \frac{21}{72}$$

Divide both by 3

$$\therefore fraction\ of\ votes\ for\ Emelia = \frac{7}{24}$$

From here connect to page 4

SOLUTION b PAGE 4

Find the total number of voters using proportion and ratio. Let the total number of members be equal to x.

Angles voters

$$120 = 20$$

$$360 = x$$

Cross multiply

$$120x = 360 \times 20$$

Divide both sides by 120

$$\frac{120x}{120} = \frac{7200}{120} \Rightarrow \therefore x = 60$$

Continuation on the next page

SOLUTION b cont. PAGE 5

To find those who did not vote for her, you have to consider that, the total number of voters was 60, then 20 of the 60 voted for her.

$$Did\ not\ vote = total\ voters - those\ voted\ for\ her$$

Substitute into the formula

$$Did\ not\ vote = 60 - 20$$

$$\therefore Did\ not\ vote = 40$$

SOLUTION b PAGE 6

Another way finding those who did not vote for Inonge

The angle for Inonge is 120 so find the angle not for her

$$Angle\ not\ for\ her = complete\ angle - angle\ for\ her$$

Substitute into the formula

$$Angle\ not\ for\ her = 360 - 120$$

$$\therefore Angle\ for\ her = 240^{\circ}$$

Continuation on the next page

SOLUTION b cont. PAGE 7

Then use proportion and ratio

Angles		voters
120	=	20
240	=	x

Cross multiply

$$120x = 240 \times 20$$

Divide both sides by 120

$$\frac{120x}{120} = \frac{4800}{120} \qquad \therefore x = 40$$

END OF LESSON 108

LESSON 109 SPECIMEN QUESTION 12 & 13 P1 2016 PAGE 1

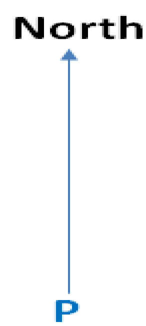
QUESTION 12

a. Solve the equation $2^{2m-3} = 8^m$
b. A translation T maps the point (1, 4) onto (4, 6). Express T as a column vector

QUESTION 13

A girl walks 30m from point P on a bearing of 060^0 to point Q. she then walks 20m due east to R.

a. Using a scale of 1cm to represent 4m, complete the diagram in the answer space below to illustrate her movement.
b. Find the bearing of R from P.



SOLUTIONS a PAGE 2

HINTS; first come up with same bases on both sides by expressing **8** on the right to index form because $8 = 2^3$ so;

$$2^{2m-3} = 2^{3m}$$

Equate the powers ignoring the bases

$$2m - 3 = 3m$$

Collect like terms

$$2m - 3m = 3$$

Subtract like terms

$$-m = 3 \qquad \therefore m = -3$$

From here connect to page 3

SOLUTIONS 12 b PAGE 3

HINTS; understand that T is a column vector which was added to the column vector of point (1, 4) in order to get (4, 6). This can be written as;

$$\begin{pmatrix}1\\4\end{pmatrix} + \mathbf{T} = \begin{pmatrix}4\\6\end{pmatrix}$$

Make T the subject of the formula

$$\mathbf{T} = \begin{pmatrix}4\\6\end{pmatrix} - \begin{pmatrix}1\\4\end{pmatrix}$$

Subtract corresponding numbers

$$= \begin{pmatrix}4-1\\6-4\end{pmatrix} \qquad \therefore \mathbf{T} = \begin{pmatrix}3\\2\end{pmatrix}$$

From here connect to page 4

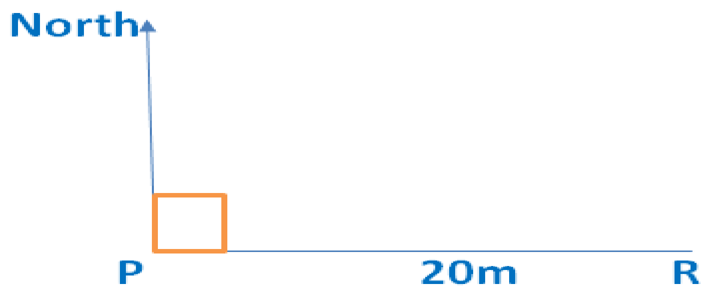
SOLUTIONS 13 a PAGE 4

On you diagram the distance of 30m will be $30 \div 4 = 7.5cm$ and 20m will be $20 \div 4 = 5cm$ but do not indicate the cm, indicate the m, however the actual measurements must be done in cm

From here connect to page 5

SOLUTIONS 13 b PAGE 5

To find the bearing of R from P, you have to know that movement eastwards will make an angle of 90^0. If we move line QR to point P we will have the following shape.



You can see that the angle formed is a 90^0 and that is the bearing, however bearings are written in 3 digits

$$\therefore \boldsymbol{bearing\ of\ R\ from\ P = 090^0}$$

END OF LESSON 109

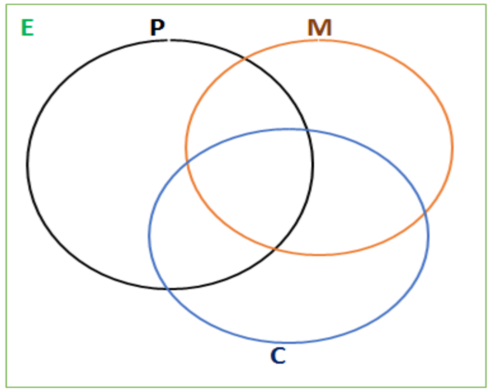
LESSON 110 SETS PAGE 1

In a class of 40 pupils, 16 like physics, 17 like maths, 24 like chemestry and the rest like other subjects not stated. 5 like all the 3 subjects. 4 like maths and physics only, 6 like physics and chemestry only, 3 like chemestry and maths only.

a. **Illustrate this information in a Venn diagram**
b. **Using your Venn diagram, find**
   i. **The number of pupils who like one subject only**
   ii. **The number of pupils who do not like any of the three subjects**
   iii. **Total number of pupils who liked two subjects**

SOLUTION a PAGE 2

The Venn diagram will be as follows before inserting values

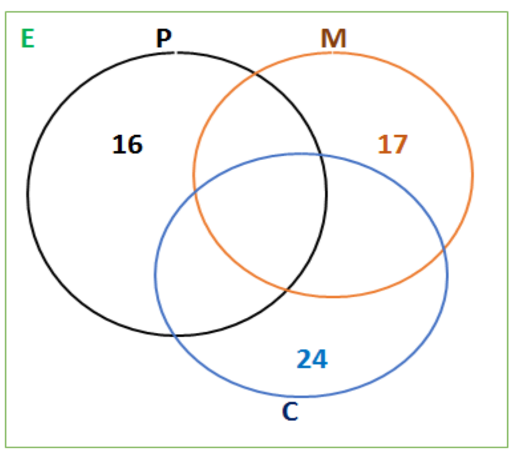


Where $E = \ Universal$ set, P=Physics, $M = Maths$ and C=Chemestry. Use a rough paper before we write the final answer. Let's insert values step by step now

Continuation on the next page

SOLUTION a PAGE 3

We insert 16 like physics, 17 like maths, 24 like chemestry



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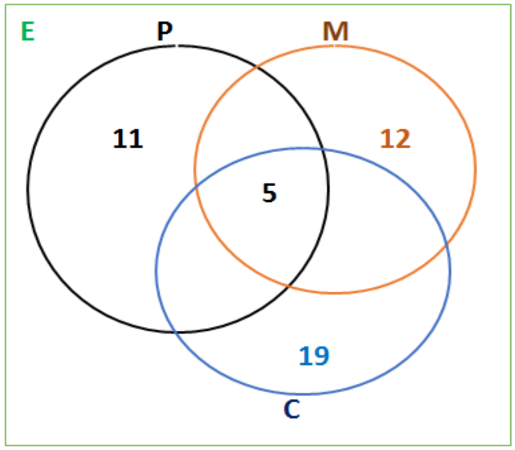
SOLUTION a PAGE 4

5 like all the 3 subjects we have to subtract 5 from each of the values entered on page 3 and insert 5 into the intersection set formed by three circles.



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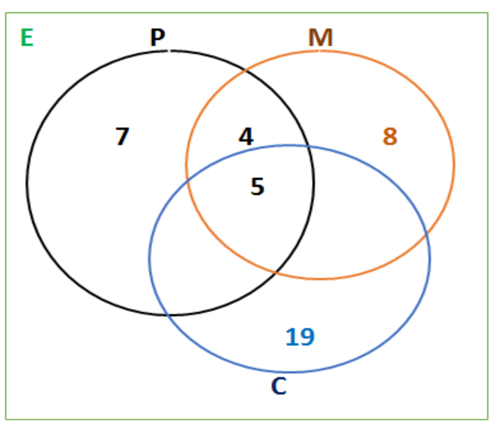
SOLUTION a PAGE 5

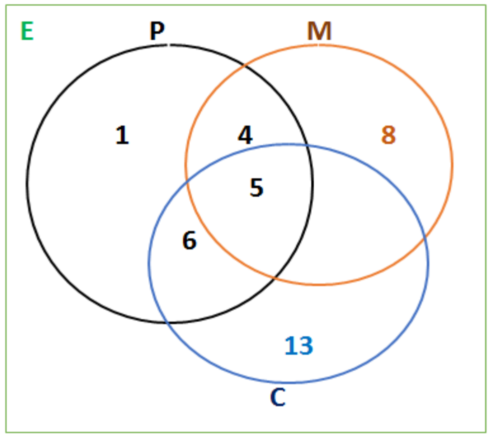
4 like maths and physics only, so we have to subtract 4 from the number in physics and maths, then write 4 in the intersection of physics and maths.



Continuation on the next page

SOLUTION a PAGE 6

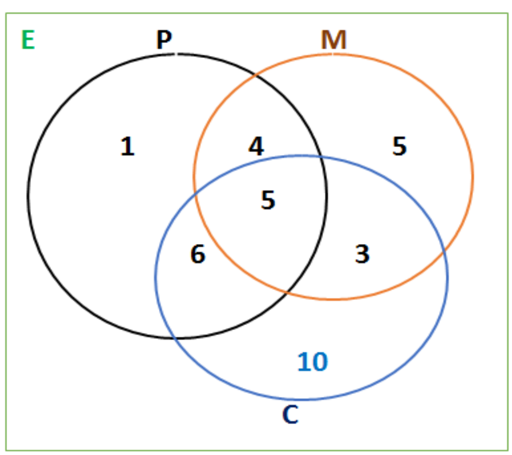
6 like physics and chemestry only, so we have to subtract 6 from the number in physics and chemestry, then write 6 in the intersection of physics and chemestry.



Continuation on the next page

SOLUTION a PAGE 7

3 like chemestry and maths only, subtract 6 from the number in maths and chemestry, then write 3 in the intersection of maths and chemestry.



Continuation on the next page

SOLUTION a PAGE 8

Add all the numbers plus x equal to 40. Then write the value of x in the universal set if it's not a zero.

$$1 + 4 + 5 + 5 + 6 + 3 + 10 + x = 40$$

$$34 + x = 40$$

Collect like terms

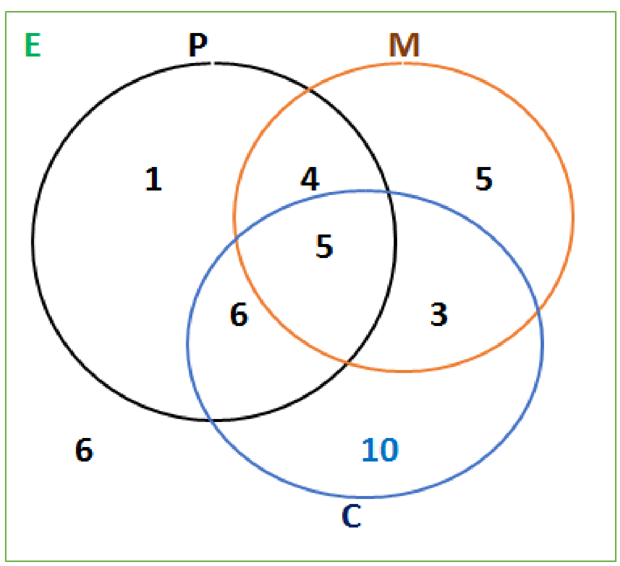
$$x = 40 - 34$$

$$\therefore x = 6$$

Continuation on the next page

SOLUTION a PAGE 9

The complete Venn diagram will have 6 in universal set



From here connect to page 10

SOLUTION b i PAGE 10

Finding the number of pupils who liked one subject only means adding numbers that are only in one set such as 1 liking physics only, 5 liking maths only and 10 liking chemestry only

$$1 + 5 + 10$$

$$\therefore Ans = 16$$

SOLUTION b ii

The number of pupils who did not like any one of the subjects is found in the universal set only. In this case we have 6

$$\therefore Ans = 6$$

From here connect to page 11

SOLUTION b iii PAGE 11

Finding the number of pupils who liked two subjects only means adding numbers that are in intersections of two sets such as 4 liking physics and maths, 6 liking physics and chemestry and 3 liking chemestry and maths

$$4 + 6 + 3$$

$$\therefore Ans = 13$$

END OF LESSON 110

LESSON 111 VARIATIONS QUESTIONS P1

1. Given that $y = \frac{kx}{z^2}$ and $x = 3$ when $y = 6$ and $z = 2$ find the value of
   i. The constant k
   ii. y when $x = 4$ and $z = 8$
   iii. z when $x = 9$ and $y = 2$
   iv. x when $y = 4$ and $z = 6$
2. $\int (3x^2 - 4x + 3)dx$

SOLUTIONS 1 i (page 2)

Just use the equation; $y = \frac{kx}{z^2}$ first make k the subject of the formula by introducing denominator 1 on y and cross multiply.

$$\frac{y}{1} = \frac{kx}{z^2}$$

Cross multiply

$$kx = yz^2$$

Dividing both sides by x

$$\frac{kx}{x} = \frac{yz^2}{x} \quad \Rightarrow \quad k = \frac{yz^2}{x}$$

Substitute and evaluate

$$k = \frac{6 \times 2^2}{3} \quad \Rightarrow \quad k = \frac{6 \times 2 \times 2}{3} \quad \Rightarrow \quad k = \frac{24}{3}$$

$$\therefore k = 8$$

From here connect to page 3

SOLUTIONS 1 ii (page 3)

Substitute in the equation $y = \frac{kx}{z^2}$, you already have k.

$$y = \frac{8 \times 4}{8^2}$$

$$the\ term\ 8^2 = 8 \times 8$$

$$y = \frac{32}{8 \times 8} \quad \Rightarrow \quad y = \frac{32}{64}$$

Divide by 32 you have

$$\therefore y = \frac{1}{2}$$

From here connect to page 4

SOLUTIONS 1 iii (page 4)

Make z the subject of the formula

$$\frac{y}{1} = \frac{kx}{z^2}$$

Cross multiply

$$yz^2 = kx$$

Divide both sides by y

$$\frac{yz^2}{y} = \frac{kx}{y} \quad \Rightarrow \quad z^2 = \frac{kx}{y}$$

To remain with z, find the square root on both sides

$$\sqrt{z^2} = \sqrt{\frac{kx}{y}} \qquad \therefore z = \sqrt{\frac{kx}{y}}$$

Continuation on the next page

SOLUTIONS 1 iii cont. (page 5)

Substitute for the numbers

$$z = \sqrt{\frac{8\times 9}{2}}$$

Multiply in the root

$$z = \sqrt{\frac{72}{2}}$$

Divide in the root and find the square root

$$z = \sqrt{36} \qquad \therefore z = 6$$

From here connect to page 6

SOLUTIONS 2 (page 6)

$$\int (3x^2 - 4x + 3)dx$$

Add 1 to each power and introducing x to the constant 3

$$3x^{2+1} - 4x^{1+1} + 3x$$

Use the powers as denominators

$$\frac{3x^{2+1}}{2+1} - \frac{4x^{1+1}}{1+1} + 3x \Rightarrow \frac{3x^3}{3} - \frac{4x^2}{2} + 3x$$

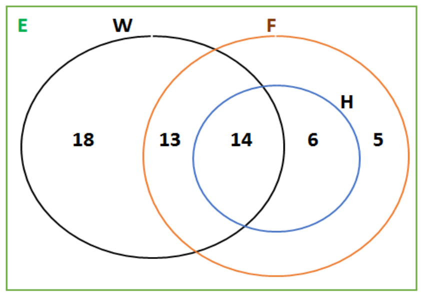
Divide

$$\therefore Ans = x^3 - 2x^2 + 3x + c$$

END OF LESSON 111

LESSON 112 QUESTION 5 P2 2016 SETS P1

a. The Venn diagram below shows the results of a survey conducted at Mulonga clinic on patients who were attended to on a particular day. Set W represents those who complained of body weakness, set H those who complained of headache and set F those who complained of fever.



i. Use the information in the Venn diagram to find the number of people who complained of;
   a. Body weakness only,
   b. Body weakness and fever only,
   c. Headache

ii. Calculate the percentage of those who complained of all the three ailments.

b. Solve the inequality $\frac{3}{2}n + 5 < 14$

c. Given that 4% of the bricks get damanged when they reach the building site, how many bricks should a builder order if $4\ 800$ bricks are needed to finish a job?

SOLUTIONS a i a (page 2)

The number of patients who complained of body weakness only should only be found in set W only,

$$\therefore n(W) = 18$$

SOLUTIONS a i b

Body weakness and fever only will be found in the intersection set of W and F

$$\therefore n(W \cap F) = 13$$

From here connect to page 3

SOLUTIONS a i c (page 3)

The patients in headache are 14 and 6, it does not say headache only so consider any number that is part of headache

$$n(H) = 14 + 6$$

$$\therefore n(H) = 20$$

From here connect to page 4

SOLUTIONS ii (page 4)

First identify the number for those who complained of all the three ailments. The number will be found in the intersection for W, H and F.

$$n(W \cap F \cap H) = 14$$

Then find the total number of patients

$$18 + 13 + 14 + 6 + 5 = 56$$

The percentage will be found as follows;

$$\frac{n(W \cap F \cap H)}{Total\ number} \times 100$$

Substitute the values and divide

$$\frac{14}{56} \times 100 \quad \Rightarrow \quad \frac{1}{4} \times 100 \qquad \therefore Ans = 25\%$$

SOLUTIONS b (page 5)

$$\frac{3}{2}n + 5 < 14$$

Collect like terms and subtract

$$\frac{3}{2}n < 14 - 5 \quad \Rightarrow \frac{3n}{2} < 9$$

Cross multiply

$$3n < 18$$

Divide both sides by 3

$$\frac{3n}{3} < \frac{18}{3} \qquad \therefore n < 6$$

From here connect to page 6

SOLUTIONS c (page 6)

First calculate 4% of 4 800 or bricks to be damaged

$$\frac{4}{100} \times 4\,800$$

Divide 100 and 4 800 by 100

$$4 \times 48$$

Multiply

$$\therefore Bricks\ to\ be\ damaged = 192$$

Continuation on the next page

SOLUTIONS c cont. (page 7)

It means that if you carry 4 800 bricks only, then 192 will be damaged on the way hence you will reach with 192 bricks less than the required number. In order to reach with the required number, you have to add 192 extra to 4 800 so that when the 192 get damaged on the way, you will reach with 4 800.

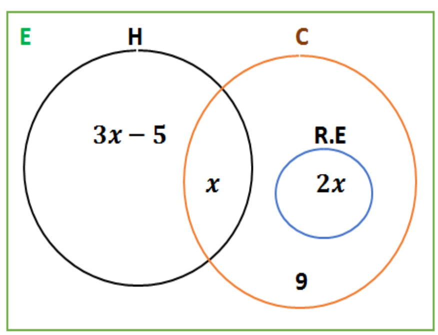
$$4\,800 + 192$$

$$\therefore Ans = 4992$$

END OF LESSON 112

LESSON 113 QUESTION 6 P2 2015 PG1

a. At Mulonga Technical Secondary School, a group of 70 pupils take optional subjects namely: History (H), Commerce (C) and Religious Education (R.E) as illustrated in the Venn diagram below.



i. Calculate the value of x

ii. Find the number of pupils who take

a) History only

b) Commerce

b. A box contains 14 identical balls, three of which are blue. Two balls are drawn at random from the box, one after the other without replacement. Calculate the probability that

i. The two balls are both blue
ii. Atleast one ball drawn is blue

SOLUTIONS a i (page 2)

To calculate x equate the sum of all the entries to 70

$$3x - 5 + x + 2x + 9 = 70$$

Collect like terms

$$3x + x + 2x = 70 + 5 - 9$$

Adding and subtracting

$$6x = 66$$

Divide both sides by 6

$$\frac{6x}{6} = \frac{66}{6}$$

$$\therefore x = 11$$

From here connect to page 3

SOLUTIONS a ii a) (page 3)

History only will be found in the set for history only not in any intersection

$$History\ only = 3x - 5$$

Substitute 11 for x

$$History\ only = 3(11) - 5$$

Multiply

$$History\ only = 33 - 5 \quad \therefore History\ only = 28$$

From here connect to page 4

SOLUTIONS a ii b) (page 4)

For commerce there is no word ONLY therefore consider adding all elements within commerce including the $2x$ in R.E

$$commerce = x + 2x + 9$$

Adding like terms

$$commerce = 3x + 9$$

Substitute 11 for x

$$commerce = 3(11) + 9$$

Multiply

$$commerce = 33 + 9$$

$$\therefore commerce = 42$$

From here connect to page 5

SOLUTIONS b i (page 5)

First you should know that, when one blue ball is picked, there was no replacement hence the number of blue balls was reducing and the total number of balls was reducing too.

$$p(first\ blue) = \frac{blue\ balls}{total\ number\ of\ balls\ in\ the\ box}$$

Substituting the values

$$\therefore p(blue) = \frac{3}{14}$$

Then find probability of picking the second blue ball

$$\therefore p(second\ blue) = \frac{2}{13}$$

Continuation on the next page

SOLUTIONS b i cont. (page 6)

Both means multiplication, so multiply the probability for both first and second ball.

$$p(first\ blue) \times P(second\ blue)$$

Substitute the values and divide

$$\frac{3}{14} \times \frac{2}{13} \quad \Rightarrow \quad \frac{3}{7} \times \frac{1}{13}$$

Multiply

$$\therefore P(both\ blue) = \frac{3}{91}$$

From here connect to page 7

SOLUTIONS b ii (page 7)

Atleast in probability means adding, also understand that, atleast one must be blue or both of them must be blue. To find the answer just add the probability for the first blue and for the second blue.

$$P(atlease\ one\ blue) = p(first\ blue) + P(second\ blue)$$

Substitute the values

$$P(atlease\ one\ blue) = \frac{3}{14} + \frac{2}{13}$$

Find the common denominator

$$\frac{\qquad}{182}$$

Continuation on the next page

SOLUTIONS b ii cont. (page 8)

Divide 14 into 182 and multiply the answer 13 to 3

$$\frac{13(3)+}{182}$$

Divide 13 into 182 and multiply the answer 14 to 2

$$\frac{13(3)+14(2)}{182}$$

Multiply in the numerator

$$\frac{39+28}{182}$$

$$\therefore P(atlease\ one\ blue) = \frac{67}{182}$$

END OF LESSON 113

LESSON 114 QUESTION 1 PAPER 1 2016 PAGE 1

a. Find the value of

i. $3 - 3 \times 3 + 3$

ii. $7 - 8 \div 2\left(\frac{3}{4} + \frac{1}{4}\right) \times 3$

b. Evaluate each of the following

i. $42 \div 0.07$

ii. $0.9 \div 0.003$

SOLUTION a i PAGE 2

We have to use $BODMAS$ when we have different mathematical operations. These letters stands for:

1. $BO = Brackets$, first deal with what is in brackets, open or introduce brackets

2. $D = \ Division$, after brackets you have to divide

3. $M = Multiplication$, after division you have to multiply

4. $A = Addition$, after multiplication you have to add

5. $S = Subtraction$, it mean you subtract after adding

Continuation on the next page

SOLUTION a i cont. PAGE 3

$$3 - 3 \times 3 + 3$$

First multiply

$$3 - 9 + 3$$

Then add

$$3 - 6 \quad \therefore Ans = -3$$

From here connect to page 4

SOLUTION a ii PAGE 4

$$7 - 8 \div 2\left(\frac{3}{4} + \frac{1}{4}\right) \times 3$$

First add in brackets with common denominator 4

$$7 - 8 \div 2\left(\frac{3+1}{4}\right) \times 3 = 7 - 8 \div 2\left(\frac{4}{4}\right) \times 3$$

Divide 4 by 4 in brackets

$$7 - 8 \div 2(1) \times 3$$

Multiply 2 by 1 to open brackets

$$7 - 8 \div 2 \times 3$$

Divide 2 into 8

$$7 - 4 \times 3$$

Multiply

$$7 - 12 \quad \therefore Ans = -5$$

From here connect to page 5

SOLUTION b i PAGE 5

$$42 \div 0.07$$

Write in fraction form

$$\frac{42}{0.07}$$

Multiply denominator and numerator by 100 because the largest number of decimal places is 2

$$\frac{42 \times 100}{0.07 \times 100} = \frac{4200}{7}$$

Dividing 7 and 4200 by 7

$$Ans = 600$$

From here connect to page 6

SOLUTION b ii PAGE 6

$$0.9 \div 0.003$$

Write in fraction form

$$\frac{0.9}{0.003}$$

Multiply denominator and numerator by 1000 because the largest number of decimal places is 3

$$\frac{0.9 \times 1000}{0.003 \times 1000} = \frac{900}{3}$$

Dividing 3 and 900 by 3

$$Ans = 300$$

END OF LESSON 114

LESSON 115 APPROXIMATIONS PAGE 1

1. ERRORS

In general when a dimension is given to the nearest n, then the error is found by

$$error = \frac{n}{2}$$

2. ABSOLUTE ERRORS

The absolute error of a dimension is the absolute difference between the true value and the recorded value. When the word absolute is used, it means that the negative is ignored.

$$Absolute\ error\ = |recorded\ value - true\ value|$$

3. RELATIVE ERRORS

The relative error of a dimension is the ratio of the absolute error to the true value.

$$Relative\ error = \frac{absolute\ error}{true\ value}$$

4. PERCENTAGE ERRORS

The percentage error states the relative error as a percentage,

$$Percentage\ error = \frac{absolute\ error}{true\ value} \times 100\%$$

QUESTIONS PAGE 2

1. Find the error in each of the following;

a. $340m$ to the nearest $10m$

b. $3600g$ to the nearest $100g$

2. The true value of the length of a rectangle is 10cm, if this is recorded as 10.2cm, find

a. The absolute error

b. The relative error

c. The percentage error

SOLUTION (1 a) PAGE 3

Hint; in this case, the measurement was not really $340m$ it could have been less or more than but it has been rounded off to the nearest ten metres so $n = 10m$ so substitute into the first formula for errors on page 1;

$$error = \frac{n}{2} = \frac{10m}{2}$$

Divide 10 and 2

$$\therefore error = 5m$$

From here connect to page 4

SOLUTION (1 b) PAGE 4

In this case $n = 100g$ so use the first formula for errors on page 1

$$error = \frac{n}{2} = \frac{100g}{2}$$

Divide 100 and 2

$$\therefore error = 50g$$

SOLUTION (2 a) PAGE 5

Use the second formula on page 1

$$Absolute\ error\ = |recorded\ value - true\ value|$$

Substitute the values

$$Absolute\ error\ = |10.2 - 10|$$

Subtract

$$\therefore Absolute\ error\ = 0.2cm$$

SOLUTION (2 b) PAGE 6

Use the third formula on page 1

$$Relative\ error = \frac{absolute\ error}{true\ value}$$

Substitute the values and remove the decimal point

$$Relative\ error = \frac{0.2cm \times 10}{10cm \times 10} \Rightarrow \quad Relative\ error = \frac{2cm}{100cm}$$

Divide 2 and 100

$$\therefore Relative\ error = 0.02$$

From here connect to page 7

SOLUTION (2 c) PAGE 7

Use the fourth formula on page 1

$$Percentage\ error = \frac{absolute\ error}{true\ value} \times 100\%$$

Substitute the values

$$Percentage\ error = \frac{0.2}{10} \times 100\%$$

Divide 10 and 100 by 10

$$Percentage\ error = 0.2 \times 10\%$$

Multiply

$$\therefore Percentage\ error = 2\%$$

QUESTIONS FOR YOUR PRACTICE PAGE 8

1. Find the error in each of the following
a. 30m to the nearest 15m
b. 38000g to the nearest 1000g
2. A mass of 24kg is recorded as 24.3kg find the;
a. Absolute error
b. Relative error
c. Percentage error

END OF LESSON 115

LESSON 116 APPROXIMATIONS PAGE 1

1. A dimension is stated as $4.3cm \pm 0.05$, find
a. The lower limit and upper limit of the dimension
2. The length and breadth of a rectangle, given to the nearest centimeter are, 15cm, and 10cm respectively. Find the;
a. Shortest possible length and the shortest possible breadth of the rectangle.
b. Longest possible length and the longest possible breadth of the rectangle.
c. Limits between which the area must lie

SOLUTIONS 1 a PAGE 2

Hint; in this case they have given us 0.05 as the absolute error. The sign $\pm$ means you have to find two limits, To get the lower limit (also called lower bound) use subtraction, to get the upper limit (also called upper bound) use addition.

$$lower\ limit = dimension - absolute\ error$$

Substitute the values and subtract

$$lower\ limit = 4.3 - 0.05 \quad \therefore lower\ limit = 4.25cm$$

Find the upper limit now

$$upper = dimension + absolute\ error$$

Substitute the values and add

$$upper\ limit = 4.3 + 0.05 \quad \therefore upper\ limit = 4.35cm$$

SOLUTIONS 2 a PAGE 3

Hint; to answer this question first find the absolute error for 15cm and 10cm. NOTE; for whole numbers, 15 (length) and 10 (breadth) the absolute error is not indicated, use 0.5 when getting the shortest possible length and shortest possible breadth, when finding the lower limit subtract the 0.5 from length and breadth respectively

$$Shortest\ possible\ length = length - absolute\ error$$

$$Shortest\ possible\ length = 15 - 0.5$$

$$\therefore Shortest\ possible\ length = 14.5cm$$

$$Shortest\ possible\ breadth = breadth - absolute\ error$$

$$Shortest\ possible\ breadth = 10 - 0.5$$

$$\therefore Shortest\ possible\ breadth = 9.5cm$$

SOLUTIONS (2b page 4)

Hint; for the longest possible length and longest possible breadth, add the 0.5 to length and breadth respectively.

$$longest\ possible\ length = length + absolute\ value$$

$$longest\ possible\ length = 15 + 0.5$$

$$\therefore longest\ possible\ length = 15.5cm$$

$$longest\ possible\ breadth = breadth + absolute\ value$$

$$longest\ possible\ breadth = 10 + 0.5$$

$$\therefore longest\ possible\ breadth = 10.5cm$$

SOLUTIONS (2c page 5)

Hint; calculate the smallest possible area and the largest possible area. $Area\ of\ a\ triangle = length \times breadth$

$$smallest\ possible\ area = Shortest\ possible\ length \times Shortest\ possible\ breadth$$

$$smallest\ possible\ area = 14.5 \times 9.5$$

$$\therefore smallest\ possible\ area = 137.75cm^2$$

$$largest\ possible\ area = longest\ possible\ length \times longest\ possible\ breadth$$

$$largest\ possible\ area = 15.5 \times 10.5$$

$$\therefore largest\ possible\ area = 162.75cm^2$$

The following is the limit they are asking you to find.

$$smallest\ possible\ area \leq Area \leq largest\ possible\ area$$

$$\therefore ans \quad 137.75cm^2 \leq A \leq 162.75cm^2$$

QUESTIONS FOR YOUR PRACTICE (page6)

1. **The length of a black board is stated as 7.5m$\pm$0.06m**
   a. **Find the lower and upper bounds**

b. Find the difference between the lower and upper bound correct to one decimal place

2. The rectangle has the following dimensions, length 23cm, and breadth 17cm. find
   a. The shortest possible length and breadth
   b. The longest possible length and breadth
   c. The limits between which area must lie

END OF LESSON 116

LESSON 117 SPECIMEN APPROXIMATIONS P1 2016

a. The length, l, of a line is measured to be $8.1cm$, correct to 1 decimal place. Complete the statement $\ldots. \leq l <. \ldots \ldots$ about l.

b. The mass of a bag of mealie meal is 25.3 kg measured correct to 1 decimal place. What is the relative error?

c. Find the percentage error.

SOLUTION (a) PAGE 2

Hints; The length, l, is measured to be 8.1cm correct to 1 decimal place, it means that the length is not exactly 8.1, it must have been between 8.04 and 8.15. All those numbers can be rounded off to 8.1 if we consider 1 decimal place so the length is actually less than 8.1cm but is greater or equal to 8.0cm so the answer will be

$$8.05 \leq l < 8.15$$

SOLUTION (b) PAGE 3

Hint; first find the absolute error, to do that, count the number of decimal places on 25.3kg, you have one decimal place, so the absolute error will be 0.05 we write a zero (0) in place of the numbers on the left of the point, each number on the right becomes zero (0) then a 5. If it was 25.334kg then the absolute error was going to be 0.0005.

SOLUTION (b) cont. PAGE 4

$$Relative\ error = \frac{absolute\ error}{recorded\ value}$$

The recorded value is 25.3kg

$$Relative\ error = \frac{0.05}{25.3}$$

Paper 1 no calculator use, just remove the decimal points

$$Relative\ error = \frac{0.05 \times 100}{25.3 \times 100} = \frac{5}{2530} = \frac{1}{506}$$

$$\therefore Relative\ error = \frac{1}{506}$$

SOLUTION (c) PAGE 5

$$percentage\ error = relative\ error \times 100$$

Substituting the values

$$percentage\ error = \frac{1}{506} \times 100 = \frac{100}{506}$$

Dividing by 2

$$\frac{100}{506} = \frac{50}{253}$$

$$\therefore percentage\ error = \frac{50}{253}$$

END OF LESSON 117

LESSON 118 APPROXIMATIONS PAGE 1

1. In each of the following, find the absolute error, relative error and percentage error.

a. $5.51cm$
b. $5.50cm$
c. $18l$
d. $18.0m$

SOLUTION (a $5.51cm$) PAGE 2

$$Absolute\ error\ = 0.005$$

$$Relative\ error = \frac{absolute\ error}{recorded\ value}$$

Substituting the values

$$Relative\ error = \frac{0.005}{5.51}$$

Multiply by 1000 to remove the point

$$\frac{0.005\times1000}{5.51\times1000} \times \frac{5}{5510}$$

Dividing by 5 we have

$$\frac{5}{5510} = \frac{1}{1102}$$

$$\therefore Relative\ error = \frac{1}{1102}$$

SOLUTION ($b\ cont.$ $5.50cm$) PAGE 3

Convert to percentage

$$percentage\ error = \frac{1}{1102} \times 100$$

Multiply numerators

$$\frac{100}{1102}$$

Divide 100 and 1102 by 2

$$\therefore percentage\ error = \frac{50}{551}\%$$

SOLUTION (b $5.50cm$) PAGE 4

$$Absolute\ error = 0.005$$

$$Relative\ error = \frac{0.005}{5.50}$$

Removing the decimal point

$$\frac{0.005 \times 1000}{5.50 \times 1000} = \frac{5}{5500}$$

Dividing 5 and 5500 by 5

$$\therefore Relative\ error = \frac{1}{1100}$$

Convert to percentage

$$percentage\ error = \frac{1}{1100} \times 100$$

$$percentage\ error = \frac{100}{1100}$$

$$\therefore percentage\ error = \frac{1}{11}\%$$

SOLUTION (c $18l$) PAGE 5

$$Absolute\ error = 0.5$$

$$Relative\ error = \frac{0.5}{18}$$

Remove the decimal point

$$\frac{0.5 \times 10}{18 \times 10} \Rightarrow \frac{5}{180}$$

Divide 5 and 180 by 5

$$\therefore Relative\ error = \frac{1}{36}$$

Convert to percentage

$$percentage\ error = \frac{1}{36} \times 100 = \frac{100}{36}$$

Divide 100 and 36 by 4

$$\frac{100}{36} = \frac{25}{9}$$

Change to mixed fraction

$$\therefore percentage\ error = 2\frac{7}{9}\%$$

SOLUTION (d 18.0m) PAGE 6

$$Absolute\ error = 0.05$$

$$Relative\ error = \frac{0.05}{18.0}$$

Removing the decimal point

$$\frac{0.05 \times 100}{18.0 \times 100} \Rightarrow \frac{5}{1800}$$

Divide 5 and 1800 by 5

$$Relative\ error = \frac{1}{360}$$

Convert to percentage

$$percentage\ error = \frac{1}{360} \times 100 = \frac{100}{360}$$

Divide 100 and 360 by 20

$$\therefore percentage\ error = \frac{5}{18}\%$$

END OF LESSON 118

LESSON 119 APPROXIMATIONS QN 17 P1 2017 QUESTIONS P1

Question 0ne

A bag of potatoes has mass $(15.4 \pm 0.05)kg$

a. **Find the tolerance of this mass.**
b. **Write down the relative error of the mass as a fraction in its simplest form.**

Question Two

Find the tolerance in each of the following

a. **The upper limit is $7m$ and the lower limit is $3m$**
b. **The measurements lie between $(7.5 \pm 1.3)cm$**

SOLUTION $(1\ a)$ PAGE 2

Tolerance is the difference between the upper limit and the lower limit.

Find the lower limit.

$$lower\ limit = 15.4 - 0.05$$

$$\therefore Lower\ limit = 15.35$$

Find the upper limit

$$Upper\ limit = 15.4 + 0.05$$

$$\therefore Upper\ limit = 15.45$$

$$Tolerance = upper\ limit - lower\ limit$$

Substitute into the formula

$$Tolerance = 15.45 - 15.35$$

$$\therefore Tolerance = 0.1$$

SOLUTION (1 b) PAGE 3

$$Relative\ error = \frac{Absolute\ error}{True\ value}$$

Substitute into the formula

$$Relative\ error = \frac{0.05}{15.4}$$

Remove the decimal point

$$Relative\ error = \frac{0.05 \times 100}{15.4 \times 100} = \frac{5}{1540}$$

Divide 5 and 1540 by 5

$$\therefore Relative\ error = \frac{1}{308}$$

SOLUTION (2 a) PAGE 4

$$Tolerance = upper\ limit - lower\ limit$$

Substitute into the formula

$$Tolerance = 7 - 3$$

$$\therefore Tolerance = 4m$$

SOLUTION (2 b) PAGE 5

Find the lower limit.

$$lower\ limit = 7.5 - 1.3$$

$$\therefore Lower\ limit = 6.2$$

Find the upper limit

$$Upper\ limit = 7.5 + 1.3$$

$$\therefore Upper\ limit = 8.8$$

$$Tolerance = upper\ limit - lower\ limit$$

Substitute into the formula

$$Tolerance = 8.8 - 6.2$$

$$\therefore Tolerance = 2.6cm$$

END OF LESSON 119

LESSON 120 SPECIMEN QUESTION 22 P1 2016 (page 1)

a. Solve the equation $(2x + 1)^2 = 9$
b. Sketch the graph of $y = x^2 - 1$
c. Find the coordinates of the turning point of $y = x^2 - 1$

SOLUTION a PAGE 2

Hints; the terms on both sides are perfect squares; just find the square roots on both sides.

$$\sqrt{(2x + 1)^2} = \sqrt{9}$$

$$2x + 1 = \pm 3$$

When 3 is positive

$$2x = 3 - 1$$

$$2x = 2$$

$$\frac{2x}{2} = \frac{2}{2}$$

$$\therefore x = 1$$

Continuation on the next page

SOLUTION a cont. PAGE 3

When 3 is negative

$$2x = -3 - 1$$

$$2x = -4$$

$$\frac{2x}{2} = \frac{-4}{2}$$

$$\therefore x = -2$$

SOLUTION b PAGE 4

Hints; first come up with the table of values by using

$-3, -2, -1, 0, 1, 2, 3$ **as values of x you can use any numbers from the number line but these are mostly used. We find the values of y by substituting for x in $y = x^2 - 1$**

x	-3	-2	-1	0	1	2	3
y							

When $x = -3$ substitute into $y = x^2 - 1$

$$y = (-3)^2 - 1 \quad \Rightarrow \quad y = (-3 \times -3) - 1$$

$$y = 9 - 1 \quad \Rightarrow \quad \therefore y = 8$$

So we write 8 in the table of values just under – 3

x	-3	-2	-1	0	1	2	3
y	8						

SOLUTION b cont. PAGE 5

When $x = -2$

$$y = (-2)^2 - 1 \ \Rightarrow \ y = (-2 \times -2) - 1$$

$$y = 4 - 1 \ \Rightarrow \ \therefore y = 3$$

Write 3 in the table just below -2

x	-3	-2	-1	0	1	2	3
y	8	3					

When $x = -1$

$$y = (-1)^2 - 1 \quad \Rightarrow \quad y = (-1 \times -1) - 1$$

$$y = 1 - 1 \quad \Rightarrow \quad y = 1 - 1 \ \therefore y = 0$$

Write 0 in the table just below -1

x	-3	-2	-1	0	1	2	3
y	8	3	0				

SOLUTION b cont. PAGE 6

When $x = 0$

$$y = (0)^2 - 1 \quad \Rightarrow \quad y = 0 - 1 \quad \therefore y = -1$$

Write -1 in the table just below 0

x	-3	-2	-1	0	1	2	3
y	8	3	0	-1			

When $x = 1$

$$y = (1)^2 - 1 \quad \Rightarrow \quad y = (1 \times 1) - 1 \ \therefore y = 0$$

Write 0 in the table just below 1

x	-3	-2	-1	0	1	2	3
y	8	3	0	-1	0		

SOLUTION b cont. PAGE 7

When $x = 2$

$$y = (2)^2 - 1 \quad \Rightarrow \quad y = 2 \times 2 - 1 \quad \Rightarrow \quad y = 4 - 1 \quad \therefore \ y = 3$$

Write 3 in the table just below 2

x	-3	-2	-1	0	1	2	3
y	8	3	0	-1	0	3	

When $x = 3$

$$y = (3)^2 - 1 \Rightarrow y = 9 - 1 \quad \therefore y = 8$$

Write 8 in the table just below 3

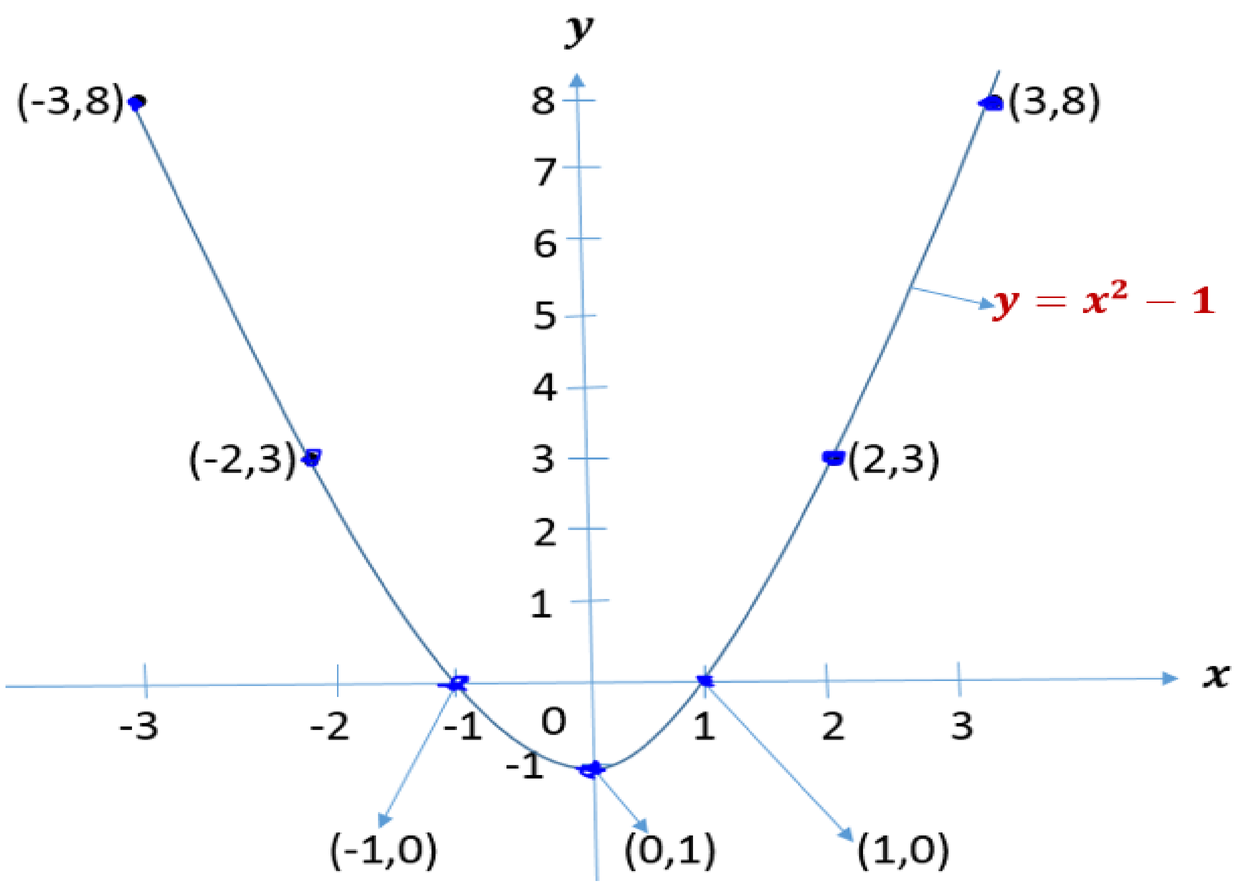
x	-3	-2	-1	0	1	2	3
y	8	3	0	-1	0	3	8

SOLUTION b cont. PAGE 8

Know that every point is in the form (x, y) so the points are;

$$(-3,8), (-2,3), (-1,0), (0,-1), (1,0), (2,3) and (3,8)$$

N0w draw the graph on the XOY plane



SOLUTION c PAGE 9

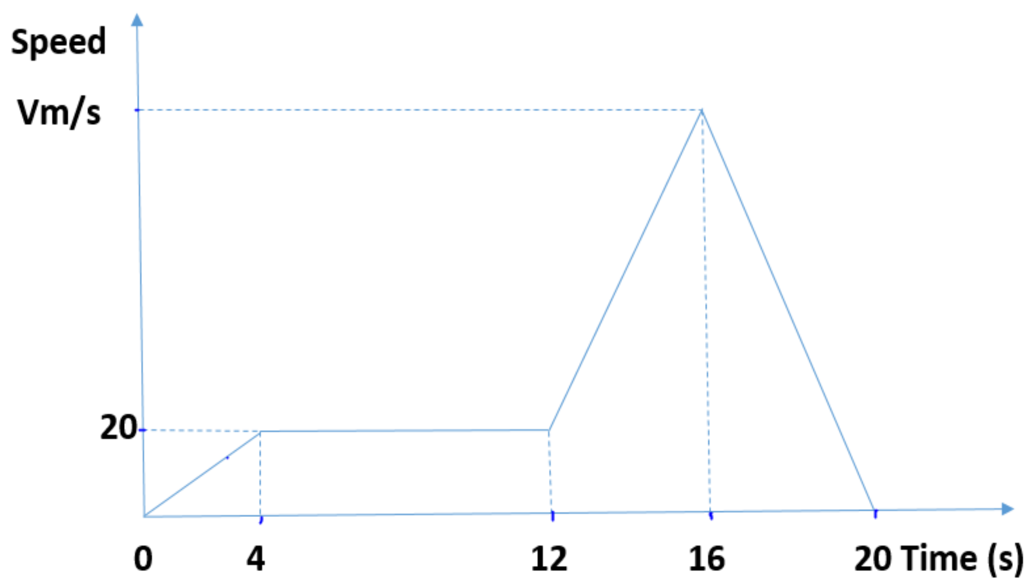
Hints; to find the coordinates of the turning point, just go to the graph and pick the point at which the graph is turning, that point is giving us the coordinates.

$$\therefore \textbf{Coordinates} = (\mathbf{0}, -\mathbf{1})$$

END OF LESSON 120

LESSON 121 SPECIMEN QUESTION 23 P1 2016 (page 1)

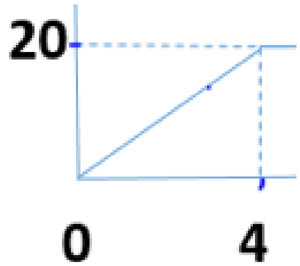
The speed-time graph below shows how the a Lady drove from her home to the night club,



a. **Find his acceleration during the first 4 seconds**
b. **If her deceleration was $10m/s^2$, what was the maximum speed, v, attained?**
c. **Calculate the distance that she covered between 12^{th} and 20^{th} seconds.**

SOLUTION (a PAGE 2)

Hints; the line pointing upwards is representing speed or velocity, the horizontal line is for time, in 4 seconds the velocity attained was 20m/s, she started with velocity 0m/s.



The formula for acceleration is as follows;

$$acceleration = \frac{final\ velocity - initial\ verocity}{time}$$

$$a = \frac{20-0}{4} \quad \Rightarrow \quad a = \frac{20}{4} \qquad \therefore a = 5m/s^2$$

SOLUTION b PAGE 3

Hints; deceleration means the reduction in velocity, in this case **V** is the highest velocity, the lowest velocity is 0m/s, calculate v using the formula for deceleration. She is moving from **V** downwards to the point where we have **20** seconds, at that point velocity is 0m/s

$$deceleration = \frac{final\ velocity - initial\ verocity}{time}$$

In this case

$$deceleration\ 10\mathrm{m}/s^2$$

$$final\ velocity = 0m/s$$

$$initial\ verocity = Vm/s$$

Deceleration happened between 16 and 20 seconds

$$time = 20 - 16 = 4\ seconds$$

SOLUTION b cont. PAGE 4

Substitute into the formula

$$10 = \frac{0 - v}{4}$$

Cross multiply

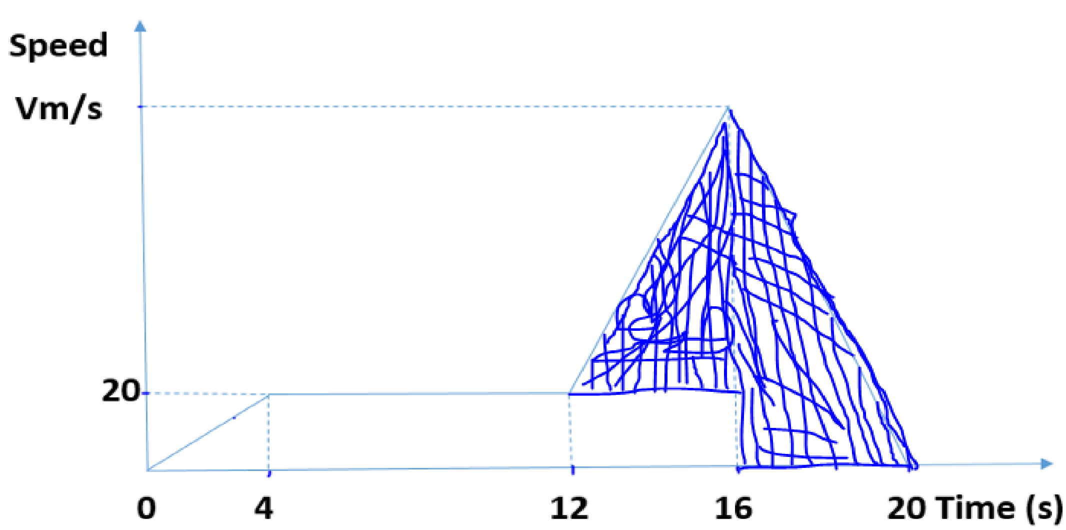
$$40 = 0 - v \quad \Rightarrow \quad -v = \quad 40$$

Now that this is speed, ignore the negative

$$\therefore v = \quad 40m/s$$

SOLUTION c PAGE 5

The distance is the un-dotted line above 12 and the one from upwards going to 20. These two lines are bounding two triangles; just calculate the shaded areas of the triangles and that will be the distance.



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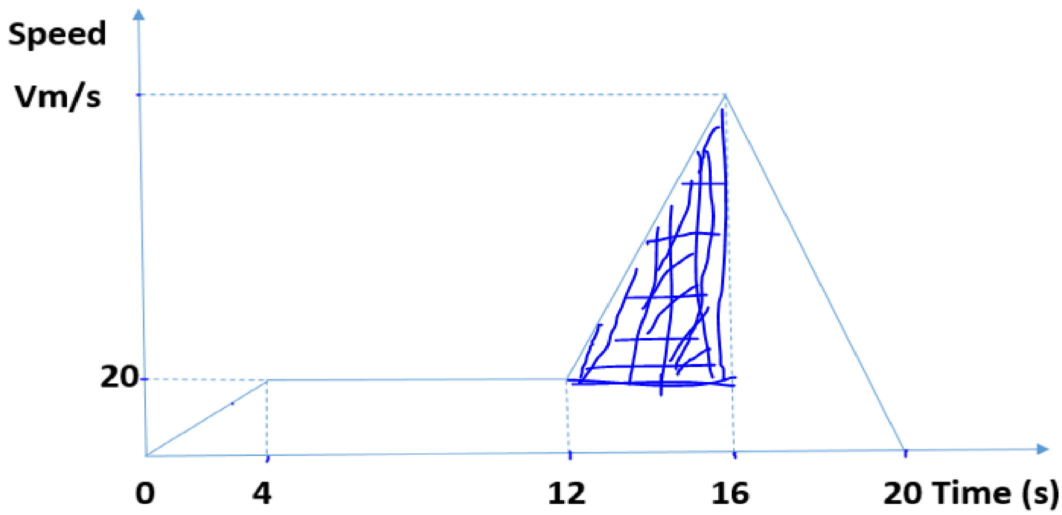
SOLUTION c cont. PAGE 6

Show triangles one by one and calculate area one by one also



Area of this triangle will be found by using

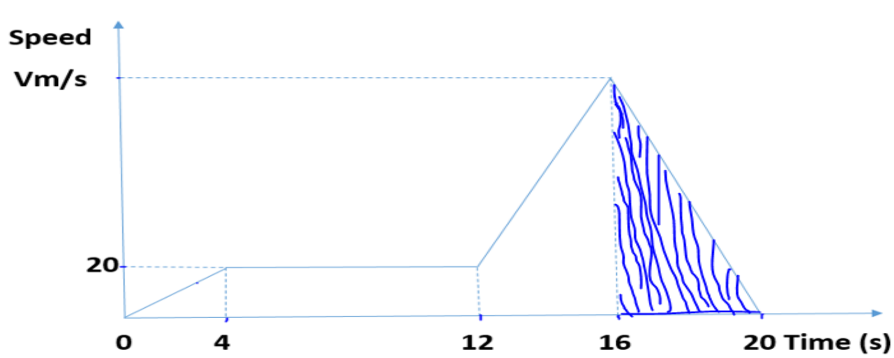
$$base = 16 - 12 = 4$$

$$Height = 40 - 20 = 20$$

$$A = \frac{1}{2} \times base \times height$$

$$A = \frac{1}{2} \times 4 \times 20 \quad \Rightarrow \quad A = 2 \times 20 \quad \boxed{\therefore A = 40}$$

SOLUTION c cont. PAGE 7



Area of this triangle will be found by using

$$base = 20 - 16 = 4$$

$$Height = 40$$

$$A = \frac{1}{2} \times base \times height$$

$$A = \frac{1}{2} \times 4 \times 40 \quad \Rightarrow \quad A = 2 \times 40 \quad \therefore A = 80$$

$$distance = Area + Area$$

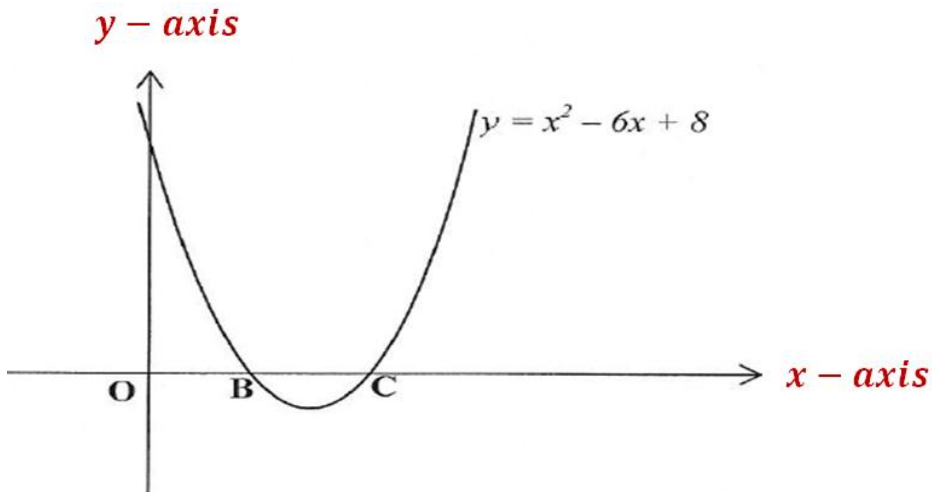
$$distance = 40 + 80$$

$$\therefore distance = 120m$$

END OF LESSON 121

LESSON 122 QUESTION 22 P1 OF 2017 PAGE 1

The diagram below shows a sketch of the graph of $y = x^2 - 6x + 8$ cutting the $x - axis$ at B and C.



a. Find the coordinates of B and C
b. Find the coordinates of the turning point of the graph

SOLUTION a PAGE 2

At point **B** and **C** the values of y are zero **(0)** because the graph is cutting the $x - axis$. This means that $y = x^2 - 6x + 8$ can be equated to **zero** because $y = 0$.

$$\therefore x^2 - 6x + 8 = 0$$

This is a quadratic equation just like

$$ax^2 + bx + c = 0$$

The coefficient of x^2 is a, the coefficient of x is **b** and the constant is **c**. Referring to the quadratic equation given

$$a = 1,\ b = -6 \text{ and } c = 8$$

Use the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

SOLUTION a cont. PAGE 3

Substitute into the formula

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(8)}}{2(1)}$$

Multiply first

$$x = \frac{6 \pm \sqrt{36 - 32}}{2}$$

Subtract in the root and find the square root

$$x = \frac{6 \pm \sqrt{4}}{2} \quad \text{and} \quad x = \frac{6 \pm 2}{2}$$

Add and subtract

$x = \frac{8}{2}$ and $x = \frac{4}{2}$

Divide to get the values

$\therefore x = 4$ and $\therefore x = 2$

SOLUTION a cont. PAGE 4

At Point B $x = 2$ and at point C $x = 4$. The value of x will be less at B because point B is closer to zero on the $x - axis$ than point C. Remember $y = 0$ at B and C in the $x - axis$.

Coordinates will be $B(x, y)$ and $C(x, y)$

$\therefore Ans = B(2, 0)$ and $\therefore Ans = C(4, 0)$

SOLUTION b PAGE 5

At the turning point, gradient of $y = x^2 - 6x + 8$ is zero, so use calculus differentiation to find the gradient function, equate to zero and find the value of x.

$$\frac{dy}{dx} = 2x^{2-1} - 6x^{1-1} + 0$$

Subtract the powers

$$\frac{dy}{dx} = 2x^1 - 6x^0$$

Ignore power 0 and any number to power zero is 1

$\frac{dy}{dx} = 2x - 6(1)$ $\therefore \frac{dy}{dx} = 2x - 6$

Continuation on the next page

SOLUTION b cont. PAGE 6

Equate the gradient function to zero

$$2x - 6 = 0$$

Collect like terms and subtract

$$2x = 0 + 6 \text{ and } 2x = 6$$

Divide both sides by 2

$$\frac{2x}{2} = \frac{6}{2} \qquad \therefore x = 3$$

You have found x now substitute for into $y = x^2 - 6x + 8$ to find y

$$y = (3)^2 - 6(3) + 8 \Rightarrow y = 9 - 18 + 8 \qquad \therefore y = -1$$

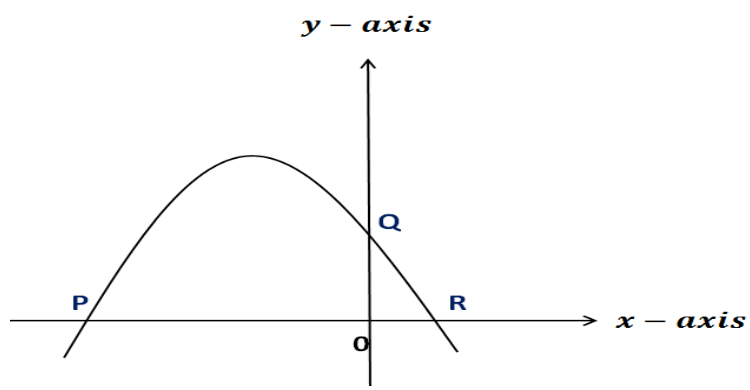
The turning point will be $Ans = P(3, -1)$

END OF LESSON 122

LESSON 123 QUESTION 22 P1 OF 2016 PAGE 1

a. Solve the equation $x^2 = 3x$

b. The diagram below shows a sketch of the graph of $y = 3 - 2x - x^2$, passing through P, Q and R.



Find the

i. Coordinates of the turning point of the graph

ii. Equation of the axis symmetry of the graph

SOLUTION a PAGE 2

$$x^2 = 3x$$

Cross $3x$ to the left and equate to zero

$$x^2 - 3x = 0$$

Factorise x because it's a common factor

$$x(x-3) = 0$$

Divide both sides by $(x-3)$

$$\frac{x(x-3)}{(x-3)} = \frac{0}{(x-3)}$$

$$\therefore x = 0$$

Divide both sides by x and collect like terms

$\frac{x(x-3)}{x} = \frac{0}{x}$ and $x - 3 = 0$

$$\therefore x = 3$$

SOLUTION b i PAGE 3

At the turning point, gradient of $y = 3 - 2x - x^2$ is zero, so use calculus differentiation to find the gradient function, equate to zero and find the value of x.

$$\frac{dy}{dx} = 0 - 2x^{1-1} - 2x^{2-1} \quad \Rightarrow \quad \frac{dy}{dx} = -2x^0 - 2x^1$$

Ignore power 1 and any number to power zero is 1

$$\frac{dy}{dx} = -2(1) - 2x$$

Equate the gradient function to zero and collect like terms

$-2 - 2x = 0$ and $-2x = 0 + 2 \;\Rightarrow\; -2x = 2$

Divide both sides by -2

$$\frac{-2x}{-2} = \frac{2}{-2} \qquad \therefore x = -1$$

SOLUTION b cont. i PAGE 4

To find the value of y at the turning point, substitute for x into $y = 3 - 2x - x^2$.

$$y = 3 - 2(-1) - (-1)^2 \Rightarrow y = 3 + 2 - 1 \quad \therefore y = 4$$

The turning point will be (x, y)

$$\therefore P(-1, 4)$$

SOLUTION b ii PAGE 5

At point **P** and **R** the values of y are zero **(0)** because the graph is cutting the $x - axis$. This means that $y = 3 - 2x - x^2$ can be equated to **zero** because $y = 0$.

$$\therefore 3 - 2x - x^2 = 0$$

This is a quadratic equation just like

$$ax^2 + bx + c = 0$$

The coefficient of x^2 is **a**, the coefficient of x is **b** and the constant is **c**. Referring to the quadratic equation given

$$a = -1,\ b = -2 \text{ and } c = 3$$

Use the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Continuation on the next page

SOLUTION b ii cont. PAGE 6

Substitute into the formula

$$x = \frac{-(-2)\pm\sqrt{(-2)^2-4(-1)(3)}}{2(-1)}$$

Multiply first

$$x = \frac{2\pm\sqrt{4+12}}{-2}$$

Add in the root and find the square root

$$x = \frac{2\pm\sqrt{16}}{-2} \quad \text{and} \quad x = \frac{2\pm 4}{-2}$$

Add and subtract

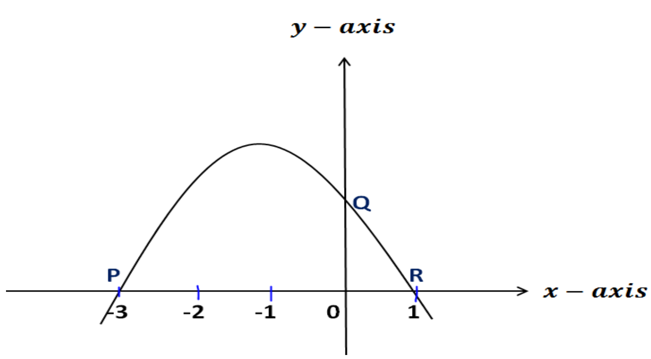
$$x = \frac{6}{-2} \quad \text{and} \quad x = \frac{-2}{-2}$$

Divide to get the values

$$\therefore x = -3 \quad \text{and} \quad \therefore x = 1$$

SOLUTION a cont. PAGE 7

At Point R $x = 1$ and at point P $x = -3$. The value of x will be less at P because point P is further to zero on the $x - axis$ than point R. Put these values of x on the graph,



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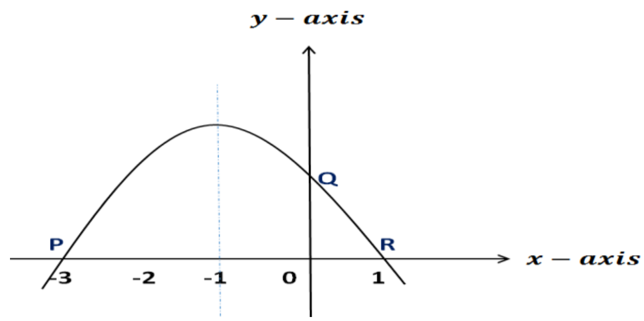
SOLUTION a cont. PAGE 8

The axis of symmetry should be a line dividing the graph into two equal halves. That line will pass through the middle most number between -3 and 1 on the $x - axis$



This shows that the line of symmetry will pass through a point where $x = -1$ and that will be the equation of symmetry

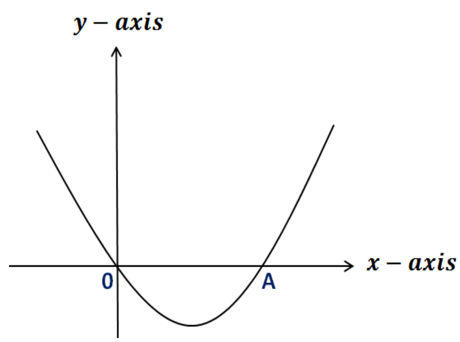
$$\therefore x = -1$$

END OF LESSON 123

LESSON 124 QUESTION 22 P1 OF 2017 PAGE 1

a. If $y = (1 - 2x)(1 + x) - 2$, find the values of x for which $y = -2$.

b. The sketch shown below represents a section of the curve $y = x(x - 2)$



i. Find the coordinates of the points where the curve cuts the $x = axis$

ii. What is the minimum value of the function?

SOLUTION a PAGE 2

$y = (1 - 2x)(1 + x) - 2$, for which $y = -2$.

Equate the function to $= -2$

$$(1 - 2x)(1 + x) - 2 = -2$$

Collect like terms and add

$(1 - 2x)(1 + x) = -2 + 2$ and $(1 - 2x)(1 + x) = 0$

Divide both sides by $(1 - 2x)$

$$\frac{(1-2x)(1+x)}{(1-2x)} = \frac{0}{(1-2x)} \Rightarrow 1 + x = 0$$

Collect like terms and evaluate

$$x = 0 - 1 \qquad \therefore x = -1$$

SOLUTION a cont. PAGE 3

$$(1 - 2x)(1 + x) = 0$$

Divide both sides by $(1 + x)$

$$\frac{(1-2x)(1+x)}{(1+x)} = \frac{0}{(1+x)} \Rightarrow 1 - 2x = 0$$

Collect like terms and evaluate

$-2x = 0 - 1$ and $-2x = -1$

Divide both sides by -2

$$\frac{-2x}{-2} = \frac{-1}{-2} \qquad \therefore x = \frac{1}{2}$$

SOLUTION b i PAGE 4

At point 0 and A the values of y are zero (0) because the graph is cutting the $x - axis$. This means that $y = x(x-2)$ can be equated to zero because $y = 0$.

$$x(x-2) = 0$$

Divide both sides by x

$$\frac{x(x-2)}{x} = \frac{0}{x} \quad \Rightarrow \quad x - 2 = 0$$

Collect like terms

$$x = 0 + 2 \qquad \therefore x = 2$$

Divide both sides by $(x-2)$

$$\frac{x(x-2)}{x-2} = \frac{0}{x-2} \quad \Rightarrow \therefore x = 0$$

Coordinates will be

$\therefore Ans = O(0,0)$ **and** $\therefore Ans = A(2,0)$

SOLUTION b ii PAGE 5

They are asking you to find the minimum value of y. this will be found at the turning point. First find x at the turning point then find y.

$$y = x(x-2)$$

Open the brackets

$$y = x^2 - 2x$$

Differentiate

$$\frac{dy}{dx} = 2x^{2-1} - 2x^{1-1} \quad \Rightarrow \quad \frac{dy}{dx} = 2x^{1} - 2x^{0}$$

Ignore the power 1, any number to power zero is 1

$$\therefore \frac{dy}{dx} = 2x - 2$$

SOLUTION b ii cont. PAGE 6

At the turning point, gradient is zero so equate the gradient function to zero.

$$2x - 2 = 0$$

Collect like terms

$$2x = 0 + 2 \quad \Rightarrow \quad 2x = 2$$

Divide both sides by 2

$$\frac{2x}{2} = \frac{2}{2} \qquad \therefore x = 1$$

SOLUTION b ii cont. PAGE 7

Substitute for x in $y = x(x-2)$

$$y = 1(1-2)$$

Subtract in brackets

$$y = 1(-1)$$

Multiply

$$\therefore y = -1$$

END OF LESSON 124

LESSON 125 VARIATIONS QUESTIONS P1

a. If z varies as directly as $2m + 1$ and $z = 6$ when $m = 1$. Find;

i. the equation of variation connecting z, m and the constant of variation k

ii. k (the constant of variation)

iii. The value of z when $m = 3$

iv. The value of m when $z = 11$

b. $\int (4x^4 + 4x)dx$

SOLUTION a i PAGE 2

This being direct variation means there will be no fraction and the equation of variation will be;

$$z = k(2m + 1)$$

From here connect to page 3

SOLUTION a ii PAGE 3

Use the equation on page 2 to find k

$$z = k(2m + 1)$$

Substitute

$$6 = k(2(1) + 1)$$

Multiply

$$6 = k(2 + 1)$$

Add and multiply

$$6 = k(3) \quad \Rightarrow \quad 3k = 6$$

Divide both sides by 3

$$\frac{3k}{3} = \frac{6}{3} \qquad \therefore k = 2$$

SOLUTION a iii PAGE 4

Use the equation on page 2 to find z when $m = 3$

$$z = k(2m + 1)$$

Substitute into the equation

$$z = 2(2(3) + 1)$$

Multiply and add

$$z = 2(6 + 1) \quad \Rightarrow \quad z = 2(7) \qquad \therefore z = 14$$

SOLUTION a iv PAGE 5

Use the equation on page 2 to find m when $z = 11$

$$z = k(2m + 1)$$

Substitute into the formula

$$11 = 2(2m + 1)$$

Open brackets

$$11 = 4m + 2$$

Collect like terms and subtract

$$4m = 11 - 2 \quad \Rightarrow \quad 4m = 9$$

Divide both sides by 4

$$\frac{4m}{4} = \frac{9}{4} \qquad \therefore m = 2\frac{1}{4}$$

SOLUTION b PAGE 6

$$\int (4x^4 + 4x)dx$$

When integrating you add 1 to the power and use the power as denominator and add a constant c at last

$$\frac{4x^{4+1}}{4+1} + \frac{4x^{1+1}}{1+1} + c$$

Add the powers and denominators

$$\frac{4x^5}{5} + \frac{4x^2}{2} + c$$

Divide 4 and 2 and rewrite the expression

$$Ans = \frac{4}{5}x^5 + 2x^2 + c$$

END OF LESSON 125

LESSON 126 MATRICES QUESTIONS PG 1

1. Given that $A = \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$ and $A^{-1} = k\begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix}$
   i. Find the value of k.
   ii. Find the matrix X, where $2A + X = \begin{pmatrix} 5 & -2 \\ 0 & 4 \end{pmatrix}$
   iii. Find the matrix Y, where $Y + A = \begin{pmatrix} 9 & 7 \\ 4 & 8 \end{pmatrix}$
2. Given that $y = 4x^3 - 6x^2 + 3x$ find $\frac{dy}{dx}$

SOLUTION 1 i PAGE 2

You have to know that A^{-1} means inverse of matrix A

This can be equated to formula for inverse is as follows

$$A^{-1} = k\begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix} = \frac{1}{Determinant} \times Adjoind$$

The k is the coefficient of the adjoind, so equate

$$k = \frac{1}{Determinant}$$

Find determinant of $A = \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$

SOLUTION 1 i cont. PAGE 3

$$Determinant = product[(major) - (minor)]diagonal$$

Numbers 3 and 2 are in major while 1 and -1 are in minor diagonal

$$Determinant = (3 \times 2) - (1 \times -1)$$

Multiply numbers

$$Determinant = 6 - (-1)$$

Multiply signs

$$Determinant = 6 + 1$$

$$\therefore Determinant = 7$$

$$\therefore k = \frac{1}{7}$$

SOLUTION 1 ii PAGE 4

You already have the formula

$$2A + X = \begin{pmatrix} 5 & -2 \\ 0 & 4 \end{pmatrix}$$

Substitute for A

$$2\begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix} + X = \begin{pmatrix} 5 & -2 \\ 0 & 4 \end{pmatrix}$$

Multiply 2 to matrix A on the left

$$\begin{pmatrix} 6 & 2 \\ -2 & 4 \end{pmatrix} + X = \begin{pmatrix} 5 & -2 \\ 0 & 4 \end{pmatrix}$$

Collect like terms

$$X = \begin{pmatrix} 5 & -2 \\ 0 & 4 \end{pmatrix} - \begin{pmatrix} 6 & 2 \\ -2 & 4 \end{pmatrix}$$

SOLUTION 1 ii cont. PAGE 5

Subtract in correspondence

$$X = \begin{pmatrix} 5-6 & -2-2 \\ 0--2 & 4-4 \end{pmatrix}$$

Subtract

$$X = \begin{pmatrix} -1 & -4 \\ 0+2 & 0 \end{pmatrix}$$

$$\therefore X = \begin{pmatrix} -1 & -4 \\ 2 & 0 \end{pmatrix}$$

SOLUTION 1 iii PAGE 6

You already have the formula

$$Y + A = \begin{pmatrix} 9 & 7 \\ 4 & 8 \end{pmatrix}$$

Substitute for A

$$Y + \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} 9 & 7 \\ 4 & 8 \end{pmatrix}$$

Collect like terms

$$Y = \begin{pmatrix} 9 & 7 \\ 4 & 8 \end{pmatrix} - \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$$

Subtract in correspondence

$$Y = \begin{pmatrix} 9-3 & 7-1 \\ 4--1 & 8-2 \end{pmatrix} \therefore Y = \begin{pmatrix} 6 & 6 \\ 5 & 6 \end{pmatrix}$$

From here connect to page 7

SOLUTION 2 PAGE 7

$$y = 4x^3 - 6x^2 + 3x \quad \text{find } \frac{dy}{dx}$$

They are asking to find $\frac{dy}{dx}$ meaning differentiating y with respect to x. When differentiating you multiply the power of x to the coefficient of x and subtract 1 from the power of x.

$$\frac{dy}{dx} = 3 \times 4x^{3-1} - 2 \times 6x^{2-1} + 1 \times 3x^{1-1}$$

Multiplying and subtracting

$$\frac{dy}{dx} = 12x^2 - 12x^1 + 3x^0$$

Do not write power 1 and any number to power 0 is equal to 1

$$\frac{dy}{dx} = 12x^2 - 12x^1 + 3(1)$$

$$\therefore \frac{dy}{dx} = 12x^2 - 12x + 3$$

END OF LESSON 126

LESSON 127 INTERATION PAGE 1

Evaluate each of the following

a. $\int_2^4 78x\,dx$

b. $\int_1^2 (3x^2 + 4x + 2)dx$

c. $\int_0^5 (3x^3 - 1)dx$

SOLUTION a PAGE 2

To integrate x^n add 1 to the power n and make the power $n+1$ as a denominator of term. The symbol for integration is $\int x^n dx$. This means that $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

$$\int_2^4 78x dx$$

First integrate $78x$ using the power 1 and use square brackets

$$\left[\frac{78x^{1+1}}{1+1}\right]_2^4$$

Add on power and denominator

$$\left[\frac{78x^2}{2}\right]_2^4$$

Divide 2 and 78 and add constant c

$$[39x^2 + c]_2^4$$

SOLUTION a cont. PAGE 3

Subtract the lower limit 2 from upper limit 4 by substitution

$$[39(4)^2 + c] - [39(2)^2 + c]$$

Multiply on powered terms

$$[39(16) + c] - [39(4) + c]$$

Multiply inside and remove brackets

$$624 + c - 156 - c$$

Collect like terms

$$624 - 156 + c - c$$

Subtract and add

$$\therefore Ans = 468$$

SOLUTION b PAGE 4

First integrate $3x^2 + 4x + 2$ but introduce x to constant 2

$$\left[\frac{3x^{2+1}}{2+1} + \frac{4x^{1+1}}{1+1} + 2x\right]_1^2$$

Add on powers and denominators

$$\left[\frac{3x^3}{3} + \frac{4x^2}{2} + 2x\right]_1^2$$

Divide where possible and add c as a constant

$$[x^3 + 2x^2 + 2x + c]_1^2$$

SOLUTION b cont. PAGE 5

Subtract the lower limit 1 from upper limit 2 by substitution

$$[2^3 + 2(2)^2 + 2(2) + c] - [(1)^3 + 2(1)^2 + 2(1) + c]$$

Multiply on powered terms

$$[8 + 2(4) + 2(2) + c] - [1 + 2(1) + 2(1) + c]$$

Multiply inside brackets

$$[8 + 8 + 4 + c] - [1 + 2 + 2 + c]$$

Add in brackets

$$[20 + c] - [5 + c]$$

Open brackets

$$20 + c - 5 - c$$

Collect like terms

$$20 - 5 + c - c$$

Add and subtract

$$\therefore Ans = 15$$

SOLUTION c PAGE 6

First integrate $3x^3 - 1$ but introduce x to constant 1

$$\left[\frac{3x^{3+1}}{3+1} - 1x\right]_0^5$$

Add on power and denominator and add constant c

$$\left[\frac{3x^4}{4} - x + c\right]_0^5$$

Subtract the lower limit 0 from upper limit 5 by substitution

$$\left[\frac{3(5)^4}{4} - 5 + c\right] - \left[\frac{3(0)^4}{4} - 0 + c\right]$$

Multiply on powered terms

$$\left[\frac{3(625)}{4} - 5 + c\right] - \left[\frac{3(0)}{4} + c\right]$$

SOLUTION c cont. PAGE 7

Multiply in numerators

$$\left[\frac{1875}{4} - 5 + c\right] - \left[\frac{0}{4} + c\right]$$

Divide where possible and open brackets

$$\frac{1875}{4} - 5 + c - 0 - c$$

Ignore zero and collect like terms

$$\frac{1875}{4} - 5 + c - c$$

Subtract constants

$$\frac{1875}{4} - \frac{5}{1}$$

SOLUTION c cont. PAGE 8

Find the common denominator

$$\frac{\qquad}{4}$$

Divide 4 into 4 and multiply the answer to 1875

$$\frac{1(1875)-}{4}$$

Divide 1 into 4 and multiply the answer to 5

$$\frac{1(1875)-4(5)}{4}$$

Open brackets

$$\frac{1875-20}{4}$$

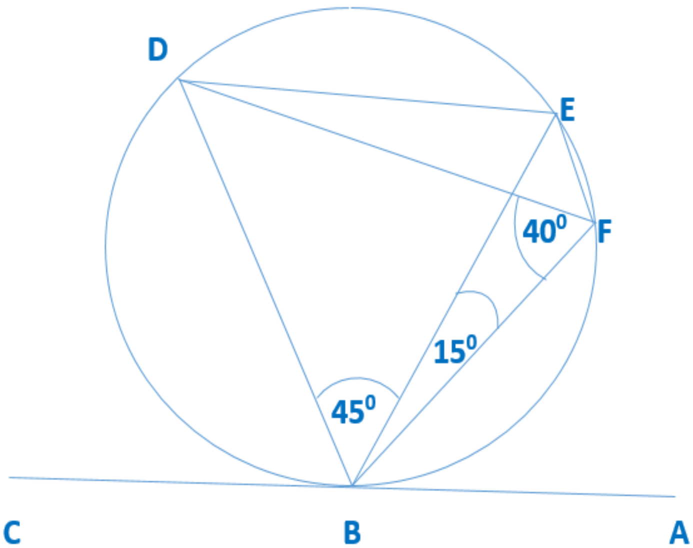
Subtract and divide to mixed fraction

$$\frac{1855}{4} \qquad \therefore Ans = 463\frac{3}{4}$$

END OF LESSON 127

LESSON 128 SPECIMEN QUESTION 15 P1 2016 PAGE 1

1. In the diagram below, ABC is a tangent to the circle BDEF at B, $D\hat{F}B = 40^0$, E$\hat{B}$F$= 15^0$ and $D\hat{B}E = 45^0$



Find

a. $B\hat{D}F$
b. $D\hat{E}B$
c. $D\hat{E}F$
d. $A\hat{B}E$

SOLUTION 1 a PAGE 2

To find $B\hat{D}F$ come up with the formula, BFD is a triangle and has interior angles 45°, 15°, and 40°, the sum of interior angles of a given triangle is 180^0, so

$$B\hat{D}F + B\hat{F}D + D\hat{B}F = 180^\circ$$

$$B\hat{D}F + 40^\circ + 60^\circ = 180^\circ$$

$$B\hat{D}F + 100^\circ = 180^\circ$$

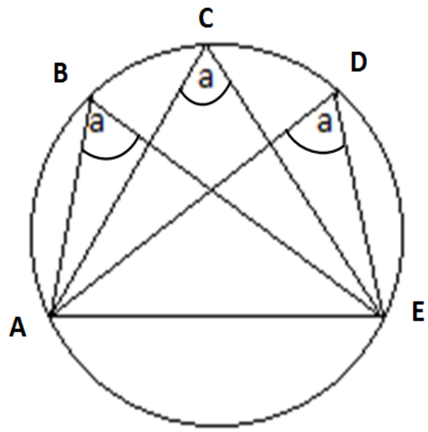
$$B\hat{D}F = 180^\circ - 100^\circ$$

$$\therefore B\hat{D}F = 80^\circ$$

SOLUTION 1 b PAGE 3

Two angles $D\hat{E}B$ and $D\hat{F}B$ are angles on the circumference subtended on the same chord DB hence they are equal.

This is a theorem on circle theorem; you can refer to lesson 97 page 1. This was the second theorem



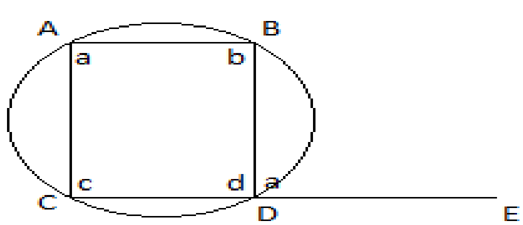
$$D\hat{E}B = D\hat{F}B$$

$$\therefore D\hat{E}B = 40^{\circ}$$

SOLUTION 1 c PAGE 4

$D\hat{E}F$ is opposite to $D\hat{B}F$ in a cyclic quadrilateral, opposite angle of that nature add up to 180^0 so we write the formula.

The opposite angles of a cyclic-quadrilateral are supplementary. The exterior angle of a cyclic-quadrilateral is equal to the opposite interior angle. You can refer to lesson 97 page 4.



$a + d = 180$

$b + c = 180$

$\angle CAB = \angle BDE$

SOLUTION 1 c cont. PAGE 5

In the question, $D\hat{E}F$ is opposite to $D\hat{B}F$

$$D\hat{E}F + D\hat{B}F = 180^{\circ}$$

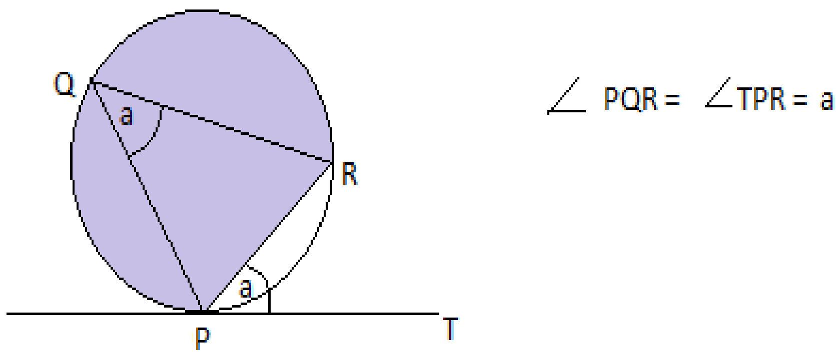
$$D\hat{E}F + 60^{\circ} = 180^{\circ}$$

$$D\hat{E}F = 180^{\circ} - 60^{\circ}$$

$$\therefore D\hat{E}F = 120^{0}$$

SOLUTION 1 d PAGE 6

Alternate Segment theorem: The angle between a chord and a tangent at the point of contact is equal to any angle in the alternate Segment. Refer to lesson 97 theorem 6 page 5.



SOLUTION 1 d cont. PAGE 7

Angle FBA outside is opposite to angle EDB inside the circle hence they are equal. So find $E\hat{D}B$

$$E\hat{D}B + D\hat{B}E + B\hat{E}D = 180^{\circ}$$

$$E\hat{D}B + 45^{\circ} + 40^{\circ} = 180^{\circ}$$

$$E\hat{D}B + 85^{\circ} = 180^{\circ}$$

$$E\hat{D}B = 180^{\circ} - 85^{\circ}$$

$$E\hat{D}B = 180^{\circ} - 85^{\circ}$$

$$\therefore E\hat{D}B = 95^{\circ}$$

$$E\hat{D}B = A\hat{B}E$$

$$A\hat{B}E = 95^{\circ}$$

END OF LESSON 128

LESSON 129 TREE DIAGRAMS (page 1)

Musonda bought 3 oranges and 2 apples, which she put in a bag. Later on she picked one fruit at random from the bag and ate it, after some time she picked another fruit at random and ate it.

i. **Construct a tree diagram to represent this information.**
ii. **Hence or otherwise find the probability that two fruits picked were of different types.**
iii. **Also find the probability that the two fruits picked are of the same type.**

SOLUTIONS PAGE 2

First of all we have to know the probability of picking each fruit from the bag. The total number of fruits is $3 + 2 = 5$ the probability of picking a particular fruit is found by the total number of that type of fruits divide by the total number of all the fruits in the bag. Therefore;

$$p(orange) = \frac{total\ number\ of\ oranges}{total\ number\ of\ fruits}$$

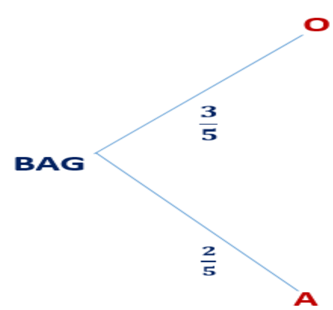
$$p(orange) = \frac{3}{3+2} = \frac{3}{5}$$

$$p(apple) = \frac{total\ number\ of\ apples}{total\ number\ of\ fruits}$$

$$p(apple) = \frac{2}{3+2} = \frac{2}{5}$$

SOLUTIONS i PAGE 3

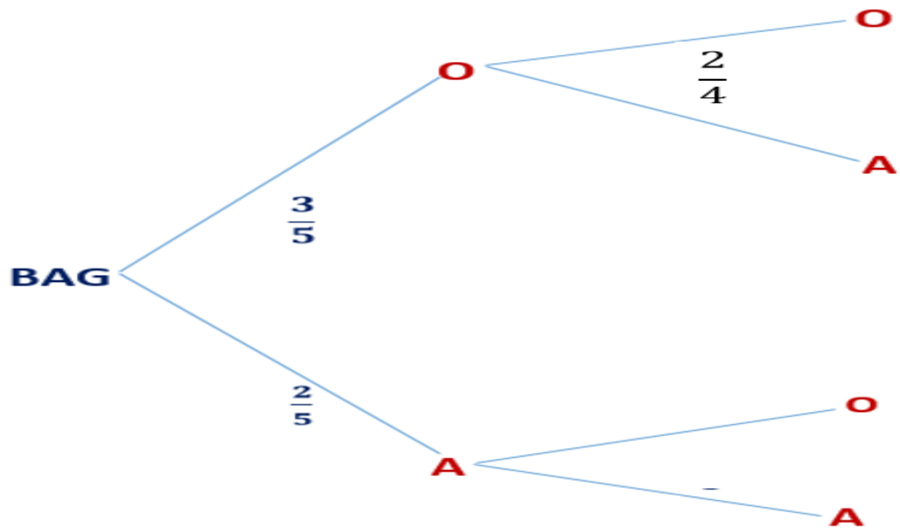
In the bag, there are two types of fruits, so when picking at random, you can either pick an apple or orange hence the tree diagram will have two branches. We can denote oranges by capital O and apples by capital A. The tree diagram will be as follows before picking is done. After the first picking, the tree diagram will be as follows;



This means that, one fruit has been picked at random and we are not sure of which type of fruit has been picked.

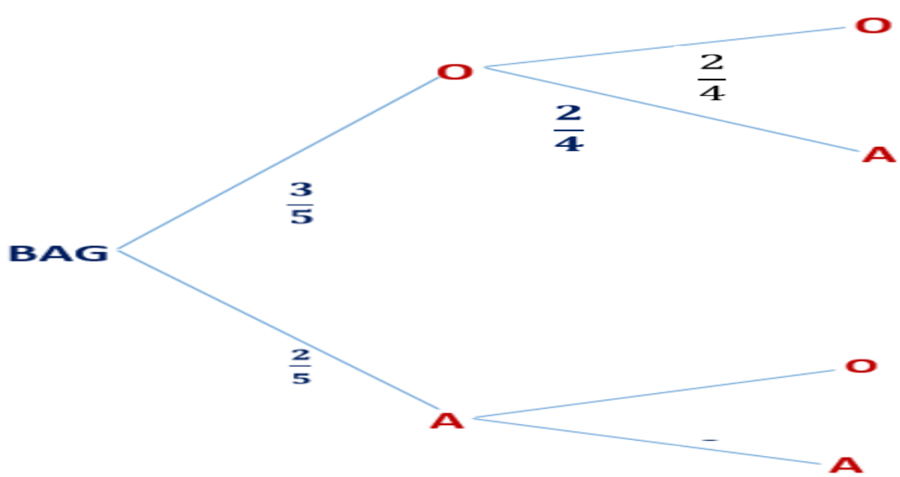
SOLUTIONS i cont. PAGE 4

When another fruit is picked, then another branch will be drawn as a continuation of the first tree diagram. If another orange has to be picked from that O to the second O then you can only pick from the remaining 2 oranges and the total number of oranges has also reduced to 4 because the first orange has been eaten so we have is $\frac{2}{4}$ as the probability of picking the second orange. So we have;



SOLUTIONS i cont. PAGE 5

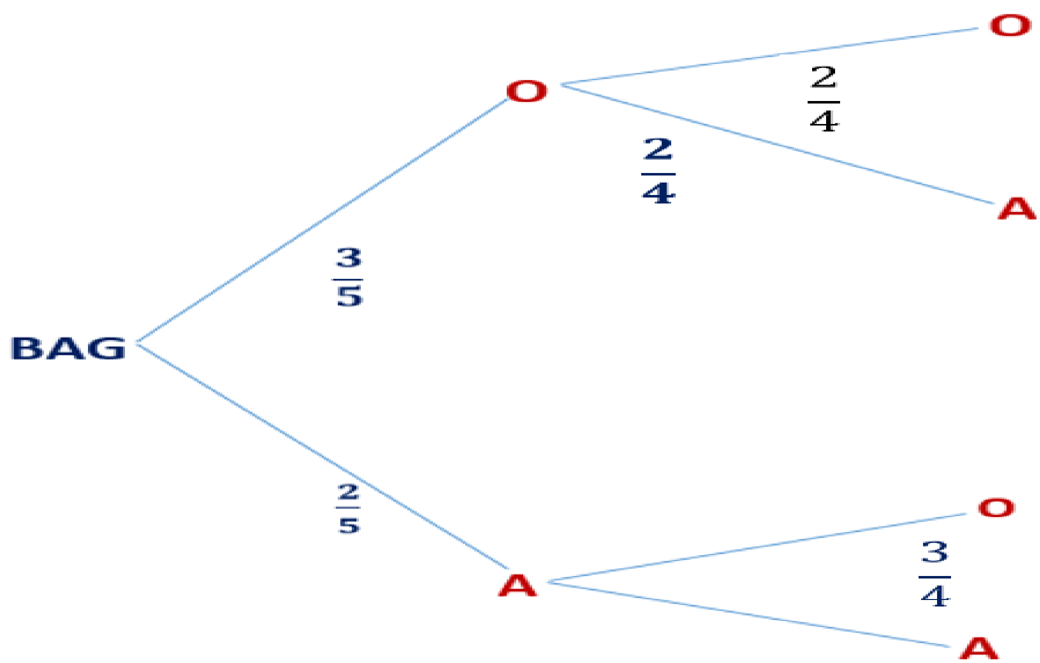
If the second fruit picked after picking an orange is an apple it means we are moving from O to A remmember that's the first apple, so we still have 2 apples but the total number of fruits has already reduced by 1. Hence the probability of picking an apple as the second fruit after picking an orange is $\frac{2}{4}$ so we have;



Continuation on the next page

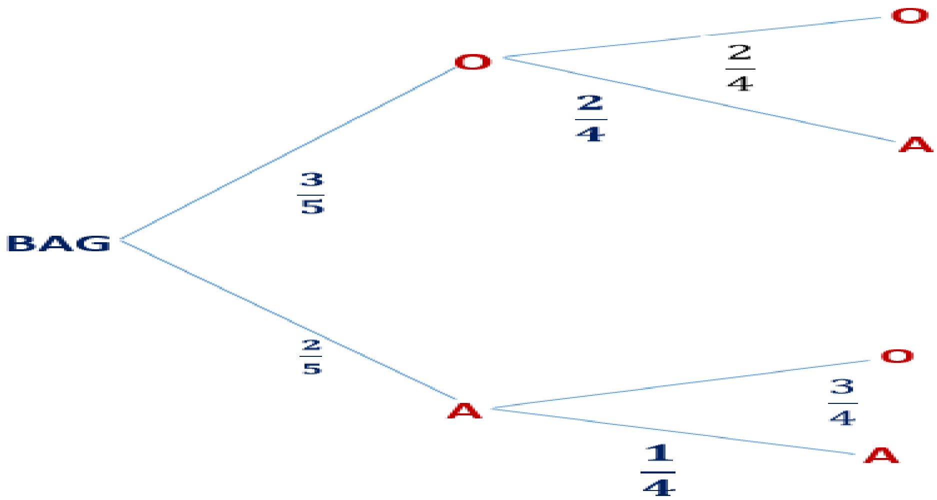
SOLUTIONS i cont. PAGE 6

Let us consider the branch from the bag to the Apple. If the first fruit picked is an apple, then to pick an orange as the second fruit means, we still have 3 oranges but 1 apple has been eaten already so the probability of picking an orange after picking an apple will be $\frac{3}{4}$ and we have



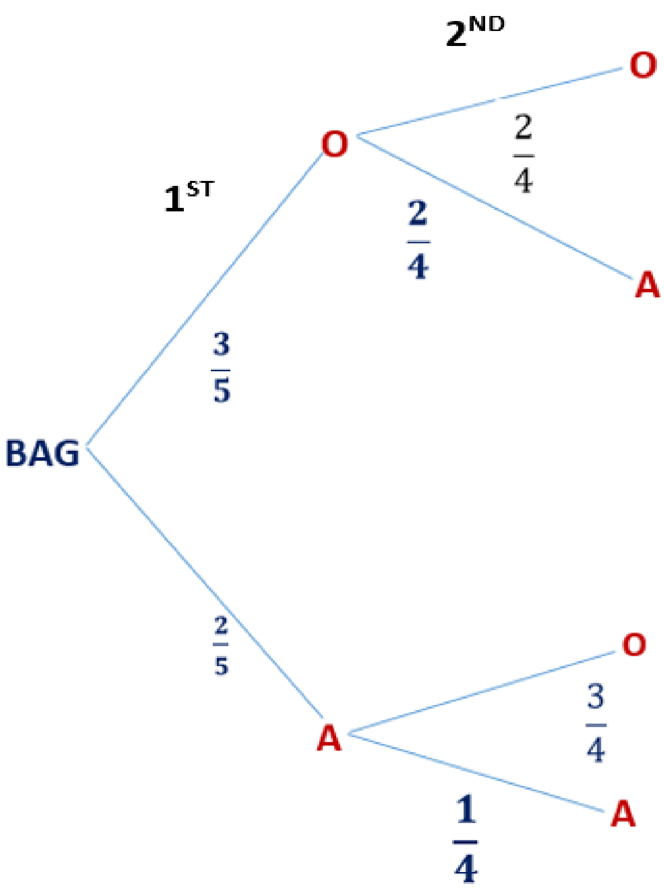
SOLUTIONS i cont. PAGE 7

If the second fruit picked was an apple after picking an apple as the first fruit, then there will be a reduction in the number of apples, such that she only remained with 1 more apple and the probability of picking that apple will be $\frac{1}{4}$ so we have;



SOLUTIONS i cont. PAGE 8

The following is the complete tree diagram, results and probabilities



Results	probabilities
OO	$P(OO) = \frac{3}{5} \times \frac{2}{4} = \frac{6}{20}$
OA	$P(OA) = \frac{3}{5} \times \frac{2}{4} = \frac{6}{20}$
AO	$P(AO) = \frac{2}{5} \times \frac{3}{4} = \frac{6}{20}$
AA	$P(AA) = \frac{2}{5} \times \frac{1}{4} = \frac{2}{20}$

SOLUTIONS ii page 9

The probability of two different types will come from $p(OA)$ and $P(AO)$

$$P(OA) = \frac{6}{20}$$

$$P(AO) = \frac{6}{20}$$

$$P(OA) + P(AO) = \frac{6}{20} + \frac{6}{20}$$

$$P(OA) + P(AO) = \frac{6+6}{20}$$

$$P(OA) + P(AO) = \frac{12}{20} \qquad \therefore Ans = \frac{3}{5}$$

SOLUTIONS iii page 10

The probability of two fruits of the same type will come from $p(OO)$ and $P(AA)$

$$P(OO) = \frac{6}{20}$$

$$P(AA) = \frac{2}{20}$$

$$P(OO) + P(AA) = \frac{6}{20} + \frac{2}{20}$$

$$P(OO) + P(AA) = \frac{6+2}{20} = \frac{8}{20} \qquad \therefore Ans = \frac{2}{5}$$

END OF LESSON 129

LESSON 130 TREE DIAGRAMS PAGE 1

1. A box contains 5 red cards and 7 blue cards. Two cards are drawn in succession at random without being replaced.
   a. Draw a tree diagram to show all the possible outcomes and corresponding probabilities
   b. Use your tree diagram to find the probability of getting;
      i. Two red cards
      ii. At least one red card

SOLUTIONS a PAGE 2

The following are the probabilities of drawing the first card according to colour.

$$p(red) = \frac{total\ number\ of red\ cards}{total\ number\ of\ cards\ in\ the\ box}$$

$$p(red) = \frac{5}{7+5}$$

$$\therefore p(red) = \frac{5}{12}$$

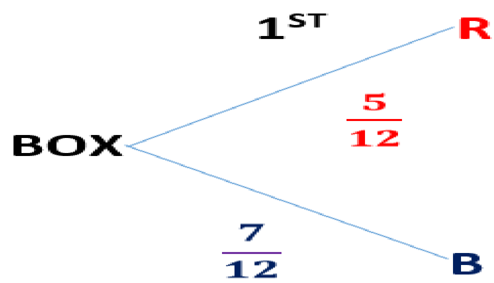
$$p(blue) = \frac{total\ number\ of blue\ cards}{total\ number\ of\ cards\ in\ the\ box}$$

$$p(blue) = \frac{7}{7+5}$$

$$\therefore p(blue) = \frac{7}{12}$$

SOLUTIONS a cont. PAGE 3

The probabilities of drawing the first card will be presented on the tree diagram as follows;

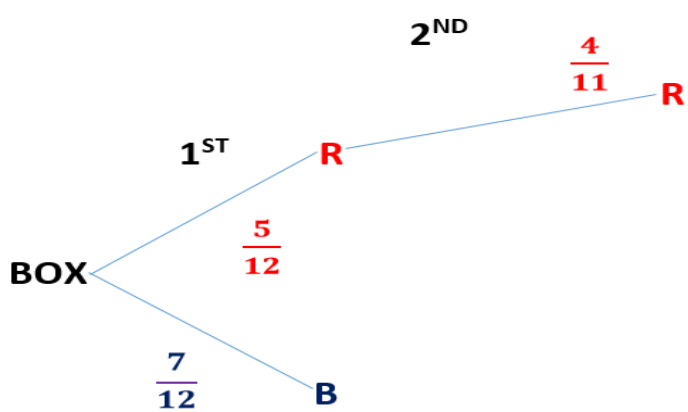


This means that, if you have to draw a red card, then you have to draw from the 5 red cards but the total number of cards in the box is 12 so its $\frac{5}{12}$

If you have to draw a blue card, then you have to draw from the 7 blue cards but the total number of cards in the box is 12 so its $\frac{7}{12}$.

SOLUTIONS a cont. PAGE 4

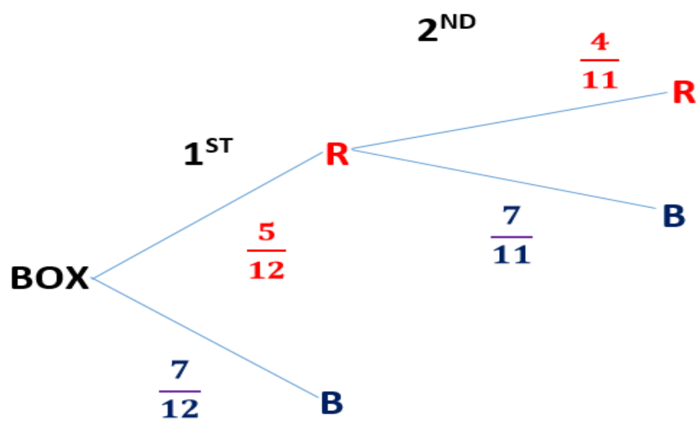
Drawing the second card, if the second card will be red after drawing a red card at first, then you have the following tree diagram



The probability becomes $\frac{4}{11}$ because 1 red card was drawn and not replaced from the box,

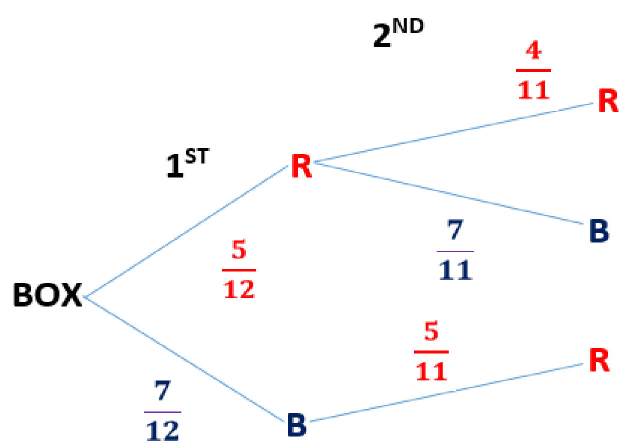
SOLUTIONS a cont. PAGE 5

If the second card drawn after drawing a red card is blue, then the number of blue cards will still be 7 but the total number of cards has already reduced to 11 so the probability will be $\frac{7}{11}$ and the tree diagram will be as follows;



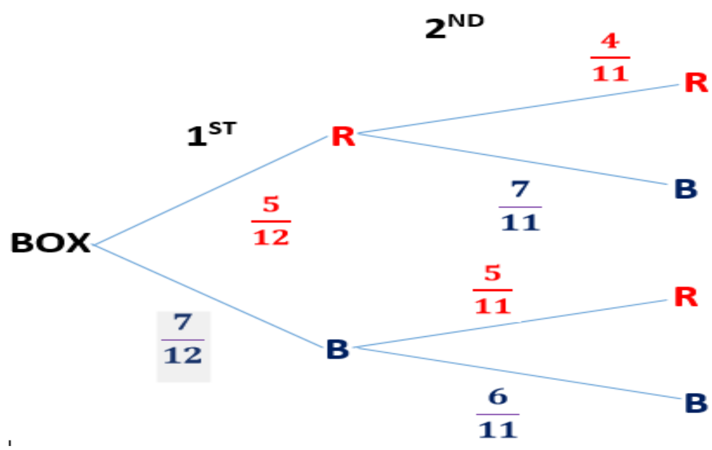
SOLUTIONS a cont. PAGE 6

If the first card drawn was Blue and the second card becomes red, then the probability of picking the second card colour red will be $\frac{5}{11}$ this means that you are drawing a red card from 5 red cards, but the total number of cards has already reduced to 11 because there was a blue card drawn. Tree diagram below

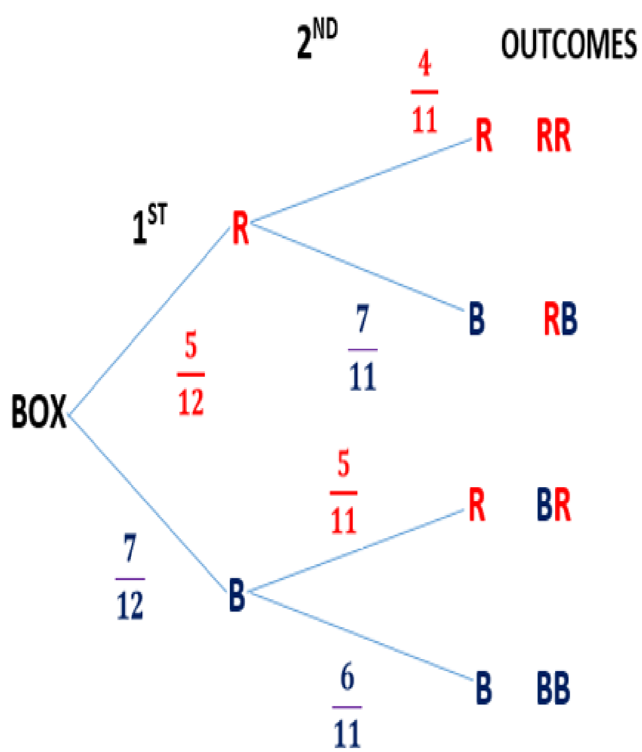


SOLUTIONS a cont. PAGE 7

If after drawing the first blue card the second car drawn is also blue, the probability of drawing a second blue card will be $\frac{5}{11}$ and the tree diagram will be as follows;



SOLUTIONS a cont. PAGE 8



PROBABILITIES

$$P(RR) = \frac{5}{\cancel{12}_{3}} \times \frac{\cancel{4}^{1}}{11} = \frac{5}{33}$$

$$P(RB) = \frac{5}{12} \times \frac{7}{11} = \frac{35}{132}$$

$$P(BR) = \frac{7}{12} \times \frac{5}{11} = \frac{35}{132}$$

$$P(BB) = \frac{7}{\cancel{12}_{2}} \times \frac{\cancel{6}^{1}}{11} = \frac{7}{22}$$

From here connect to page 9

SOLUTIONS b i PAGE 9

There is only one scenario where we have two red cards

$$P(RR) = \frac{5}{12} \times \frac{4}{11} = \frac{20}{132} = \frac{5}{33}$$

SOLUTIONS (b ii)

At least one read is found in $P(RR), P(RB) and\ P(BR)$ to find that we just add them.

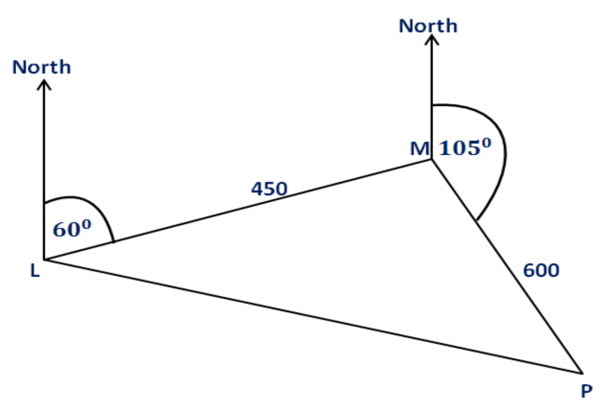
$$P(RR) + P(RB) + P(BR) = \frac{5}{33} + \frac{35}{132} + \frac{35}{132}$$

$$\frac{20+35+35}{132} == \frac{90}{132} = \frac{15}{22}$$

END OF LESSON 130

LESSON 131 QUESTION 7 P2 2016 PAGE 1

a. Three towns Luzi (L), Muzi (M) and Puzi (P) are spaced as shown in the diagram. M is on a bearing of 060° from L and P is on a bearing of 105° from M.



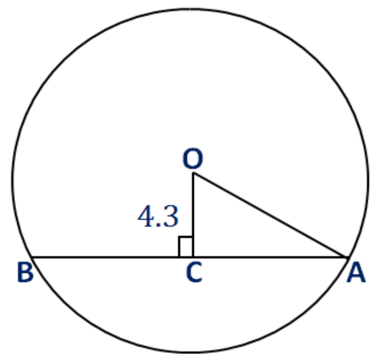
Given that M is 450km from L and 600km from P, calculate

i. The distance between L and P

ii. The area of triangle LMP

iii. Angle MPL

b. In the figure below, O is the centre of the circle, the chord AB is perpendicular to OC and $OC = 4.3cm$.



Given that the chord AB is 38.2cm, calculate the radius of the circle.

SOLUTIONS a i page 2

To find the distance between L and P, you have to use the angle LMP. Angle North(LP) and angle North(ML) are between two parallel lines (Norths) hence they are allied angles. Allied angles add up to 180°.

$$\text{North}(\text{LM}) + \text{North}(\text{ML}) = 180^{\circ}$$

$$60^{\circ} + \text{North}(\text{ML}) = 180^{\circ}$$

$$60^{\circ} + x = 180^{\circ}$$

$$x = 180^{\circ} - 60^{\circ}$$

$$\therefore x = 120^{\circ}$$

SOLUTIONS a i cont. PAGE 3

Now find angle LMP angle North(ML) plus angle North(MP) plus angle LMP gives 360°

$$North(ML) + North(MP) + LMP = 360^{\circ}$$

$$120^{\circ} + 105^{\circ} + LMP = 360^{\circ}$$

$$225^{\circ} + LMP = 360^{\circ}$$

$$LMP = 360^{\circ} - 225^{\circ}$$

$$\therefore LMP = 135^{\circ}$$

SOLUTIONS a i cont. PAGE 4

Then find LP using the cosine-rule, this states in formula as follows;

$$a^2 = b^2 + c^2 - 2abcos\theta$$

$$a = side\ opposite\ to\ the\ angle\ LMP$$

$$b\ and\ c = \ sides\ adjacent\ to\ the\ angle\ LMP$$

$$\theta = \ angle\ LMP$$

$$a^2 = 450^2 + 600^2 - 2(450 \times 600)cos\,135^{\circ}$$

Multiply where needed

$$a^2 = 202500 + 360000 - 2(270000)cos\,135^{\circ}$$

Press everything into the calculator the way they appear

$$a^2 = 944,337.6618$$

Find square-roots on both sides

$$\sqrt{a^2} = \sqrt{944,337.6618}$$

$$a = 971.7703751$$

$$\therefore LP = 971.77km$$

Revise more on lesson 34 page 88 of the maths pamphlet

SOLUTIONS a ii PAGE 5

Then find area of LMP using the sine- formula, this states in formula as follows;

$$Area = \frac{1}{2} \times a \times b \times sin\theta$$

$$a \text{ and } b = \text{ sides adjacent to the angle } LMP$$

$$\theta = \text{ angle } LMP$$

$$Area = \frac{1}{2} \times 450 \times 600 \times \sin 135^{\circ}$$

Divide 2 and 600

$$Area = 1 \times 450 \times 300 \times \sin 135^{\circ}$$

Press everything into the calculator the way they appear

$$Area = 95,459.41546$$

$$\therefore A = 95,459.42km^2$$

Revise more on lesson 36 page 97 of the maths pamphlet

SOLUTIONS a iii PAGE 6

To find angle MPL use the sine-rule, this states in formula as follows;

$$\frac{a}{\sin(a)} = \frac{b}{\sin(b)}$$

$$(a) = \text{angle } MPL$$

$$a = a \text{ side opposite to angle } MPL$$

$$(b) = \text{any angle inside the triangle}$$

$$a = a \text{ side opposite to angle } (b)$$

$$\text{Let } (b) = \text{angle } LMP = 135^{\circ} \text{ and Let } b = LP = 971.77$$

SOLUTIONS a iii cont. PAGE 7

Substitute into the formula

$$\frac{450}{\sin(a)} = \frac{971.77}{\sin 135^{\circ}}$$

Cross multiply

$$971.77sin(a) = 450sin\,135^{\circ}$$

Divide both sides by 971.77

$$\frac{971.77sin(a)}{971.77} = \frac{450sin\,135^{\circ}}{971.77} \Rightarrow \quad sin(a) = \frac{450sin\,135^{\circ}}{971.77}$$

Press $(450sin\,135^{\circ})\div 971.77$ into the calculator

$$sin(a) = 0.327441731$$

Press shift or 2ndf sin 0.327441731=

$$(a) = 19.11357237$$

$$\therefore \mathrm{M\widehat{P}L} = 19.11^{\circ}$$

Revise more on lesson 35 page 92 of the maths pamphlet

SOLUTIONS b PAGE 8

First find the length of CA by dividing AB by 2

$$CA = \frac{AB}{2} = \frac{38.2}{2}$$

$$\therefore CA = 19.1cm$$

Use Pythagoras theorem to find AO the radius

$$AO^2 = OC^2 + AC^2$$

Substitute into the formula

$$AO^2 = 4.3^2 + 19.1^2$$

Evaluate on squares

$$AO^2 = 18.49 + 364.81$$

Add on the right

$$AO^2 = 383.3$$

Find the square root on both sides

$$\sqrt{AO^2} = \sqrt{383.3}$$

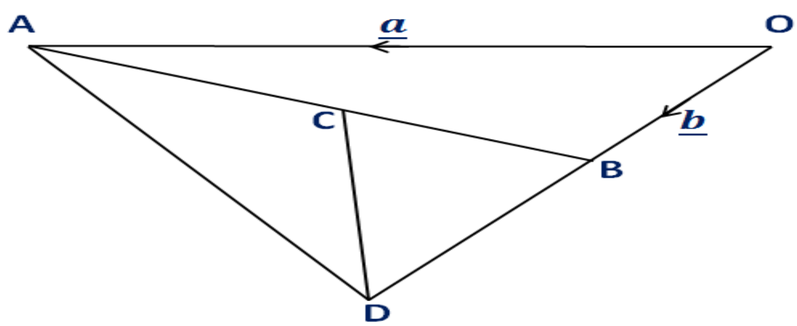
$$\therefore AO = 19.58cm$$

Revise more on lesson 23 page 61 of the maths pamphlet

END OF LESSON 131

LESSON 132 QUESTION 6 P2 GCE 2016 PAGE 1

a. In the diagram below, $\overrightarrow{OA} = \underline{a}$ and $\overrightarrow{OB} = \underline{b}$. C on AB is such that $AC:CB = 2:1$ and B on OD is such that $OB:OD = 1:2$.



Express as simple as possible in terms of $\underline{a}$ and/$\underline{b}$

i. $\overrightarrow{AB}$

ii. $\overrightarrow{AD}$

iii. $\overrightarrow{CD}$

b. Tickets for a variety show held at Mulonga Secondary School were sold at $\text{K}50.00$ each. Half of the money collected at the show was donated to charity. If the charity received $\text{K}2,500.00$, how many tickets were sold?

c. If 72 ladies were invited to a kitchen party. The ratio of those who took Fanta to those who did not was $13:5$

respectively. Given that 18 others came in uninvited and 8 of these took Fanta. Find the new ratio of those who took Fanta to those who did not.

SOLUTIONS a i PAGE 2

To find $\overrightarrow{AB}$ means moving from point A to B by using a different route not direct from A to B. make sure that the route you are using has vectors up to B.

$$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$$

Substitute including a negative on $\underline{a}$ as you move against the arrow

$$\overrightarrow{AB} = -\underline{a} + \underline{b}$$

Always start with a positive vector

$$\therefore \overrightarrow{AB} = \underline{b} - \underline{a}$$

SOLUTIONS a ii PAGE 3

To find $\overrightarrow{AD}$ means moving from point A to D by using a different route not direct from A to D. make sure that the route you are using has vectors up to D.

$$\overrightarrow{AD} = \overrightarrow{AO} + \overrightarrow{OD}$$

Substitute considering the ratio on $\overrightarrow{OD}$ and vector $\underline{b}$

$$\overrightarrow{AD} = -\underline{a} + (OB + OD)\underline{b}$$

Substitute in the ratios

$$\overrightarrow{AD} = -\underline{a} + (1 + 2)\underline{b} \quad \Rightarrow \quad \overrightarrow{AD} = -\underline{a} + (3)\underline{b}$$

$$\therefore \overrightarrow{AD} = 3\underline{b} - \underline{a}$$

SOLUTIONS a iii PAGE 4

To find $\overrightarrow{CD}$ means moving from point C to D by using a different route not direct from C to D. Make sure that the route you are using has vectors up to D.

$$\overrightarrow{CD} = \overrightarrow{CA} + \overrightarrow{AD}$$

Vector $\overrightarrow{CA}$ is on vector $\overrightarrow{AB}$ and its moving against the arrow hence include a negative, also consider the ratios and vector $\overrightarrow{AB}$.

$$\overrightarrow{CD} = -\left(\frac{AC}{AC + CB}\right)\overrightarrow{AB} + 3\underline{b} - \underline{a}$$

Substitute other values

$$\overrightarrow{CD} = -\left(\frac{2}{2+1}\right)(\underline{b} - \underline{a}) + 3\underline{b} - \underline{a}$$

Add in the denominator

SOLUTIONS a iii cont. PAGE 5

$$\overrightarrow{CD} = -\frac{2}{3}(\underline{b} - \underline{a}) + 3\underline{b} - \underline{a}$$

Open brackets

$$\overrightarrow{CD} = -\frac{2}{3}\underline{b} + \frac{2}{3}\underline{a} + 3\underline{b} - \underline{a}$$

Collect like terms and introduce denominators

$$\overrightarrow{CD} = \left(\frac{2\underline{a}}{3} - \frac{\underline{a}}{1}\right) + \left(\frac{3\underline{b}}{1} - \frac{2\underline{b}}{3}\right)$$

SOLUTIONS a iii cont. PAGE 6

Find the common denominator in each bracket

$$\overrightarrow{CD} = \left(\frac{\qquad}{3}\right) + \left(\frac{\qquad}{3}\right)$$

Divide denominators and multiply to numerators

$$\overrightarrow{CD} = \left(\frac{2\underline{a} - 3\underline{a}}{3}\right) + \left(\frac{9\underline{b} - 2\underline{b}}{3}\right)$$

Subtract in the numerator

$$\overrightarrow{CD} = -\frac{\underline{a}}{3} + \frac{7\underline{b}}{3}$$

$$\therefore \overrightarrow{CD} = 2\frac{1}{3}\underline{b} \quad -\frac{1}{3}\underline{a}$$

Revise more on lesson 10 and 45 in the maths pamphlet

SOLUTIONS b PAGE 7

Each ticket is $\mathrm{K}50$, let the number of tickets sold be x. The total amount of money realized will be $2500 \times 2 = \mathrm{K}5000$

$$50 \times x = 5000$$

Multiply and divide both sides by 50

$$\frac{50x}{50} = \frac{5000}{50}$$

$$\therefore x = 100$$

From here connect to page 8

SOLUTIONS c PAGE 8

First find the number of those who took Fanta from the 72. The number 13 in the ratio is for those who took Fanta.

$$\frac{13}{13+5} \times 72 = \frac{13}{18} \times 72 = 13 \times 4 = 52$$

This means that 52 people took Fanta from the invited; however 8 of the 18 uninvited also took Fanta. This makes $52 + 8 = 60$ of those that took Fanta. Those that did not take Fanta from the invited are $72 - 52 = 20$. However there were 10 from the uninvited that did not take Fanta so $20 + 10 = 30$. Come up with the ratio;

$$60: 30$$

Divide both sides by 30

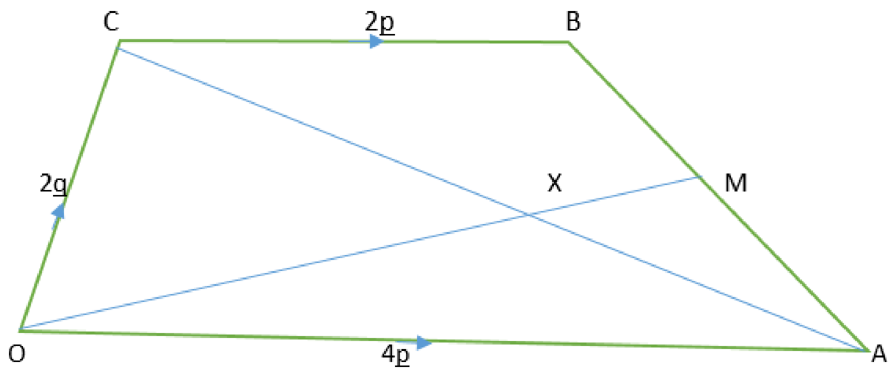
$$\frac{60}{30} : \frac{30}{30}$$

$$\therefore Ans = 2: 1$$

END OF LESSON 132

LESSON 133 QUESTION 5 P2 1983 PAGE 1

The diagram below shows a trapezium OABC. M is the midpoint of AB, OM and CA meet at X. $\overrightarrow{OA}$ = 4$\underline{p}$, $\overrightarrow{OC}$ =2$\underline{q}$ and $\overrightarrow{CB}$ = 2$\underline{p}$.



i. Express as simple as possible, in terms of p and/or q the vectors;

a. $\overrightarrow{CA}$

b. $\overrightarrow{BA}$

c. $\overrightarrow{OM}$

ii. Given that CX = h$\overrightarrow{CA}$, express CX in terms of $\underline{p}$, $\underline{q}$ and h.

iii. Hence show that $OX = 4h\underline{p} + 2(1-h)\underline{q}$

SOLUTIONS i a PAGE 2

To find $\overrightarrow{CA}$ means moving from point C to A by using a different route not direct from C to A. make sure that the route you are using has vectors up to A.

$$\overrightarrow{CA} = \overrightarrow{CO} + \overrightarrow{OA}$$

Substitute; put a negative on $\overrightarrow{CO}$ because it's against the arrow

$$\overrightarrow{CA} = -2\underline{q} + 4\underline{p}$$

Start with a positive term and factorise

$$\therefore \overrightarrow{CA} = 2\left(2\underline{p} - \underline{q}\right)$$

SOLUTIONS i b PAGE 3

To find $\overrightarrow{BA}$ means moving from point B to A by using a different route not direct from B to A. make sure that the route you are using has vectors up to A.

$$\overrightarrow{BA} = \overrightarrow{BC} + \overrightarrow{CA}$$

Substitute; put a negative on $\overrightarrow{BC}$ because it's against the arrow

$$\overrightarrow{BA} = -2\underline{p} + 2\left(2\underline{p} - \underline{q}\right)$$

Open brackets

$$\overrightarrow{BA} = -2\underline{p} + 4\underline{p} - 2\underline{q}$$

$$\overrightarrow{BA} = 2\underline{p} - 2\underline{q} \qquad \therefore \overrightarrow{BA} = 2\left(\underline{p} - \underline{q}\right)$$

SOLUTIONS i c PAGE 4

To find $\overrightarrow{OM}$ means moving from point O to M by using a different route not direct from O to M. make sure that the route you are using has vectors up to M.

$$\overrightarrow{OM} = \overrightarrow{OA} + \overrightarrow{AM}$$

Substitute; put a negative on $\overrightarrow{AM}$ because it's against the $\overrightarrow{BA}$

$$\overrightarrow{OM} = 4\underline{p} + \frac{1}{2}\left(-\overrightarrow{BA}\right)$$

$$\overrightarrow{OM} = 4\underline{p} + -\frac{1}{2}\left(\overrightarrow{BA}\right)$$

$$\overrightarrow{OM} = 4\underline{p} - \frac{1}{2}\left(2\underline{p} - 2\underline{q}\right)$$

$$\overrightarrow{OM} = 4\underline{p} - \underline{p} + \underline{q}$$

$$\therefore \overrightarrow{OM} = 3\underline{p} + \underline{q}$$

SOLUTIONS ii PAGE 5

If CX = h$\overrightarrow{CA}$ then just multiply h to $\overrightarrow{CA}$

$$\mathrm{CX} = \mathrm{h}\left(4\underline{p} - 2\underline{q}\right)$$

Open the brackets

$$\mathrm{CX} = \mathrm{h}\left(4\underline{p} - 2\underline{q}\right)$$

$$\therefore \mathrm{CX} = 4\mathrm{h}\underline{p} - 2\mathrm{h}\underline{q}$$

From here connect to page 6

SOLUTIONS iii PAGE 6

To find $\overrightarrow{OX}$ means moving from point O to X by using a different route not direct from O to X. make sure that the route you are using has vectors up to X.

$$\overrightarrow{OX} = \overrightarrow{OC} + \overrightarrow{CX}$$

Substitute in the vectors

$$\overrightarrow{OX} = 2\underline{q} + 4\mathrm{h}\underline{p} - 2\mathrm{h}\underline{q}$$

Collect vector like terms

$$\overrightarrow{OX} = 4\mathrm{h}\underline{p} + 2\underline{q} - 2\mathrm{h}\underline{q}$$

Factorise on vector like terms

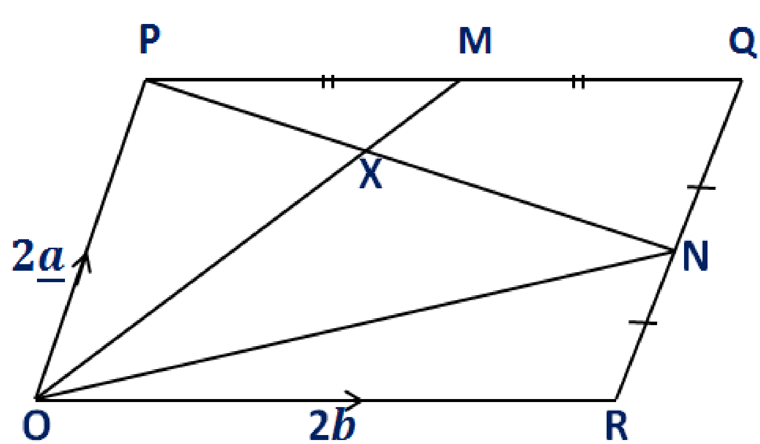
$$\therefore \overrightarrow{OX} = 4\mathrm{h}\underline{p} + 2(1-\mathrm{h})\underline{q}$$ (Hence shown)

Revise more on lesson 10 and 45 in this pamphlet

END OF LESSON 133

LESSON 134 QUESTION 9 P2 1989 PAGE 1

In the diagram OPQR is a parallelogram, M is the mid-point of PQ and N is the mid-point of QR. $\overrightarrow{OP} = 2\underline{a}$ and $\overrightarrow{OR} = 2\underline{b}$



i. Express in terms of $\underline{a}$ and/or $\underline{b}$ the vectors

a. $\overrightarrow{PM}$

b. $\overrightarrow{OM}$

c. $\overrightarrow{ON}$

d. $\overrightarrow{PN}$

ii. OM and PN meet at X, given that $\overrightarrow{PX} = h\overrightarrow{PN}$, express $\overrightarrow{PX}$ in terms of $\underline{a}$, $\underline{b}$ and h.

iii. Show that $\overrightarrow{OX} = (2-h)\underline{a} + 2h\underline{b}$

SOLUTIONS i a PAGE 2

This being a parallelogram means that $PQ = OR$ then I is also said that M is the mid-point of PQ.

$$\overrightarrow{PM} = \frac{1}{2}\overrightarrow{OR}$$

Substitute for the vector

$$\overrightarrow{PM} = \frac{1}{2}(2\underline{b})$$

Multiply

$$\therefore \overrightarrow{PM} = \underline{b}$$

SOLUTIONS i b PAGE 3

To find $\overrightarrow{OM}$ means moving from point O to M by using a different route not direct from O to M. make sure that the route you are using has vectors up to M.

$$\overrightarrow{OM} = \overrightarrow{OP} + \overrightarrow{PM}$$

Substitute in the vectors

$$\therefore \overrightarrow{OM} = 2\underline{a} + \underline{b}$$

SOLUTIONS i c PAGE 4

To find $\overrightarrow{ON}$ means moving from point O to N by using a different route not direct from O to N. make sure that the route you are using has vectors up to N. Point N is half of RQ and OP.

$$\overrightarrow{ON} = \overrightarrow{OR} + \overrightarrow{RN}$$

Substitute in the vectors

$$\overrightarrow{ON} = 2\underline{b} + \frac{1}{2}\overrightarrow{OP}$$

Substitute in the vectors

$$\overrightarrow{ON} = 2\underline{b} + \frac{1}{2}(2\underline{a})$$

Open brackets

$$\therefore \overrightarrow{ON} = 2\underline{b} + \underline{a}$$

SOLUTIONS i d PAGE 5

To find $\overrightarrow{PN}$ means moving from point O to N by using a different route not direct from O to N. make sure that the route you are using has vectors up to N. Point N is half of RQ and OP but $\overrightarrow{QN}$ is against the arrow on $\overrightarrow{OP}$.

$$\overrightarrow{PN} = \overrightarrow{PQ} + \overrightarrow{QN}$$

Substitute in the vectors

$$\overrightarrow{PN} = 2\underline{b} + \frac{1}{2}(-\overrightarrow{OP})$$

$$\overrightarrow{PN} = 2\underline{b} + -\frac{1}{2}(\overrightarrow{OP})$$

$$\overrightarrow{PN} = 2\underline{b} - \frac{1}{2}(2\underline{a})$$

$$\therefore \overrightarrow{PN} = 2\underline{b} - \underline{a}$$

SOLUTIONS ii PAGE 6

Given that $\overrightarrow{PX} = h\overrightarrow{PN}$ so just multiply h to $\overrightarrow{PN}$

$$\overrightarrow{PX} = h\overrightarrow{PN}$$

Substitute for $\overrightarrow{PN}$

$$\overrightarrow{PX} = h\overrightarrow{PN} = h(2\underline{b} - \underline{a})$$

Open the brackets

$$\therefore \overrightarrow{PX} = 2h\underline{b} - h\underline{a}$$

SOLUTIONS iii PAGE 7

To find $\overrightarrow{OX}$ means moving from point O to X by using a different route not direct from O to X. make sure that the route you are using has vectors up to X.

$$\overrightarrow{OX} = \overrightarrow{OP} + \overrightarrow{PX}$$

Substitute in the vectors

$$\overrightarrow{OX} = 2\underline{a} + 2h\underline{b} - h\underline{a}$$

Collect like terms in terms of vectors

$\overrightarrow{OX} = (2 - h)\underline{a} + 2h\underline{b}$ (Hence shown)

Revise more on lesson 10 and 45 in this pamphlet

END OF LESSON 134

LESSON 135 SPECIMEN QUESTION 16 P1 2016 PAGE 1

a. Given that $f(x) = 2x - 3$ and $g(x) = \frac{3x+1}{x+2}$, $x \neq -2$, find

a. $f^{-1}(x)$

b. $f^{-1}(11)$

c. $gf(x)$

b. Evaluate $\int_{-2}^{-6}(5x^4 - 3x^2 + 8\,)dx$

Copy the question above, scroll down for answers and listen to possible audios for each answer page

SOLUTION 1 a PAGE 2

First let $f(x) = y$ then write y where there is $f(x)$

$$y = 2x - 3$$

Make x the subject of the formula

$$y + 3 = 2x \;\Rightarrow\; 2x = y + 3$$

Divide both sides by 2

$$\frac{2x}{2} = \frac{y+3}{2} \qquad \therefore x = \frac{y+3}{2}$$

Write $f^{-1}(x)$ where there is x, and x where there is y

$$\therefore f^{-1}(x) = \frac{x+3}{2}$$

SOLUTION 1 b PAGE 3

To find $f^{-1}(11)$ where there is x in $f^{-1}(x)$ they have written 11, write 11 where there is x in $f^{-1}(x) = \frac{x+3}{2}$;

$$f^{-1}(11) = \frac{11+3}{2}$$

Add in the numerator

$$f^{-1}(11) = \frac{14}{2}$$

Divide 2 and 14

$$\therefore f^{-1}(11) = 7$$

SOLUTION 1 c PAGE 4

$gf(x)$ means substituting $f(x)$ for x in $g(x)$ so we have;

$$gf(x) = \frac{3f(x)+1}{f(x)+2}$$

But we know that $f(x) = 2x - 3$ so we substitute

$$gf(x) = \frac{3(2x-3)+1}{2x-3+2}$$

$$gf(x) = \frac{6x-9+1}{2x-1}$$

$$\therefore gf(x) = \frac{6x-8}{2x-1}$$

SOLUTION 2 PAGE 5

First integrate $5x^4 - 3x^2 + 8$ but introduce x to constant 8

$$\left[\frac{5x^{4+1}}{4+1} - \frac{3x^{2+1}}{2+1} + 8x\right]_{-2}^{-6}$$

Add on power and denominator and add constant c

$$\left[\frac{5x^5}{5} - \frac{3x^3}{3} + 8x + c\right]_{-2}^{-6}$$

Divide where possible

$$\left[x^5 - x^3 + 8x + c\right]_{-2}^{-6}$$

SOLUTION 2 cont. PAGE 6

Subtract the lower limit -2 from upper limit -6 by substitution

$$\left[(-6)^5 - (-6)^3 + 8(-6) + c\right] - \left[(-2)^5 - (-2)^3 + 8(-2) + c\right]$$

Multiply on powered terms

$$[-7776 - (-216) - 48 + c] - [-32 - (-8) - 16 + c]$$

Opening brackets

$$-7776 + 216 - 48 + c + 32 - 8 + 16 - c$$

Add collected like terms

$$-7608 + c + 40 - c$$

Collect like terms, add and subtract

$$-7944 + 40 + c - c \qquad \therefore Ans = -7,568$$

END OF LESSON 135

LESSON 136 QUESTION 12 PAPER 2 2016 (page 1)

a. Answer this part of the question on a sheet of graph paper
The values of x and y are connected by the equation;

$$y = x(x-2)(x+2)$$

Some corresponding values of x and y are given in the table below;

x	-3	-2	-1	0	1	2	3
y	-15	0	3	0	-3	0	k

i. Calculate the value of k

ii. Using a scale of $2cm$ to 1 unit on the $x-axis$ for $-3 \leq x \leq 3$ and 2cm to represent 5 units on the $y-axis$ for $-20 \leq y \leq 20$ draw the graph of $y = x(x-2)(x+2)$

iii. Use your graph to solve the equations

(a) $x(x-2)(x+2)=0$

(b) $x(x-2)(x+2)=x+2$

b. Express $\frac{2}{2x-1}-\frac{1}{3x+1}$ as a single fraction in its simplest form

OLUTIONS (i page 2)

To find k, we have to looks at the table of values and see the value of x corresponding to k in the box above k. The number in that box is $\mathbf{3}$ meaning that $x=3$ so substitute for x in $y=x(x-2)(x+2)$ where there is x write $\mathbf{3}$ in order to get y because y is the k at that point.

$$y=3(3-2)(3+2)$$

$$k=3(1)(5) \quad \therefore k=15$$

The table of values will be as follows;

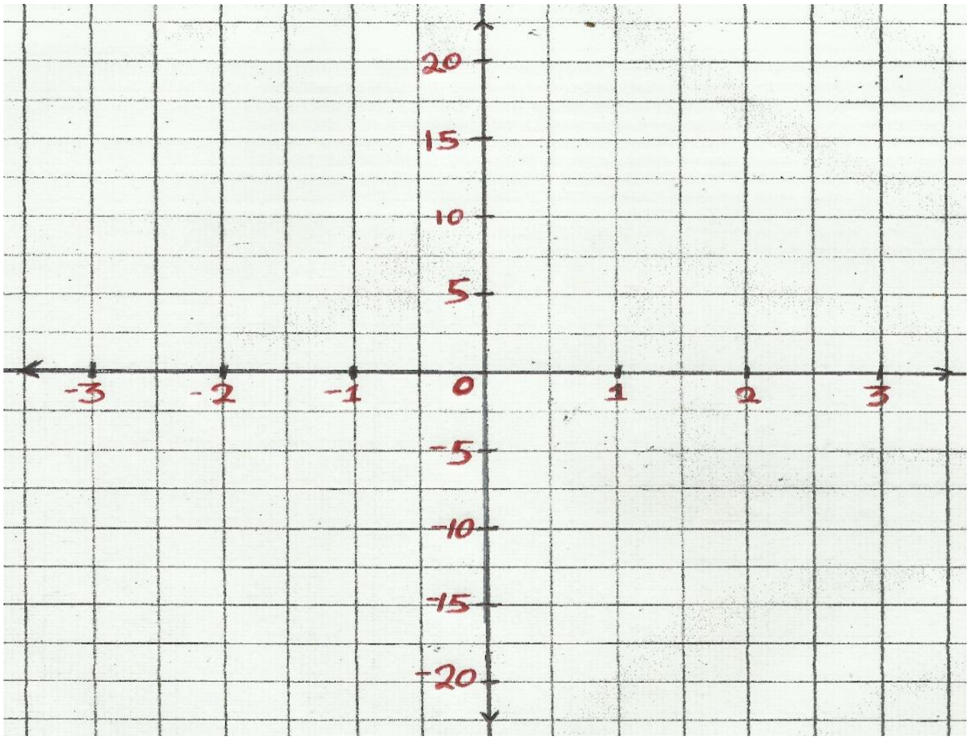
x	-3	-2	-1	0	1	2	3
y	-15	0	3	0	-3	0	15

The following will be the points as a point is (x,y)

$$(-3,-15),(-2,0),(-1,3),(0,0),(1,-3),(2,0),(3,15)$$

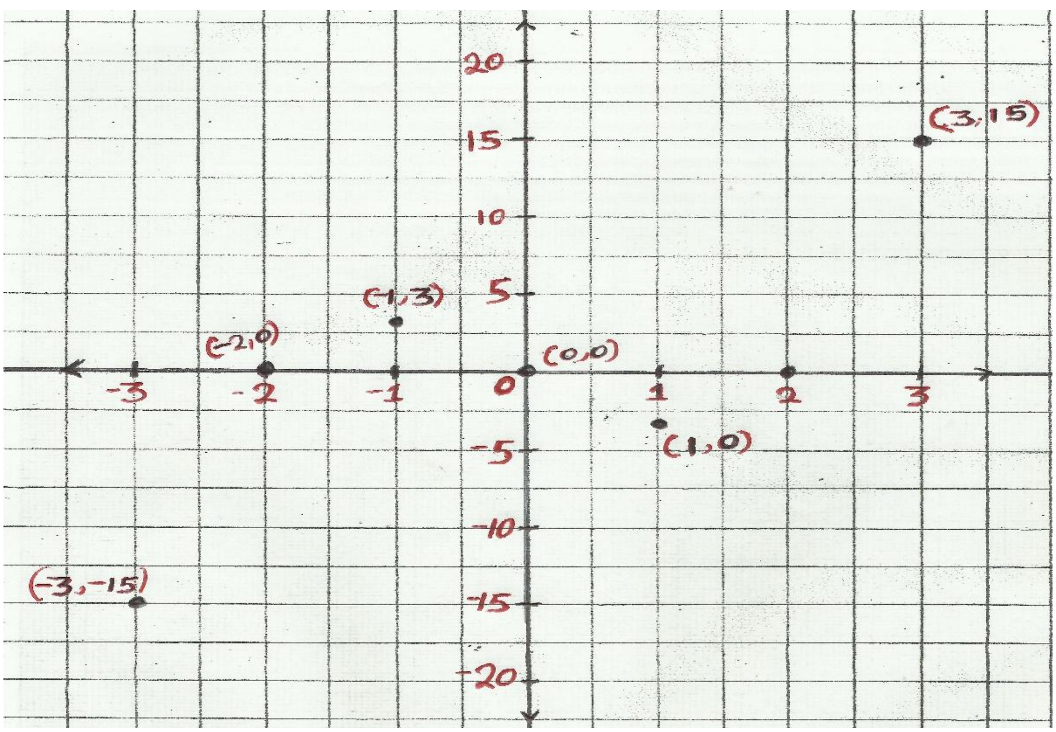
SOLUTIONS ii page 3

Plot these points one by one on the graph and connect them, the next graph shows how $x-axis$ and $y-axis$ will be scaled



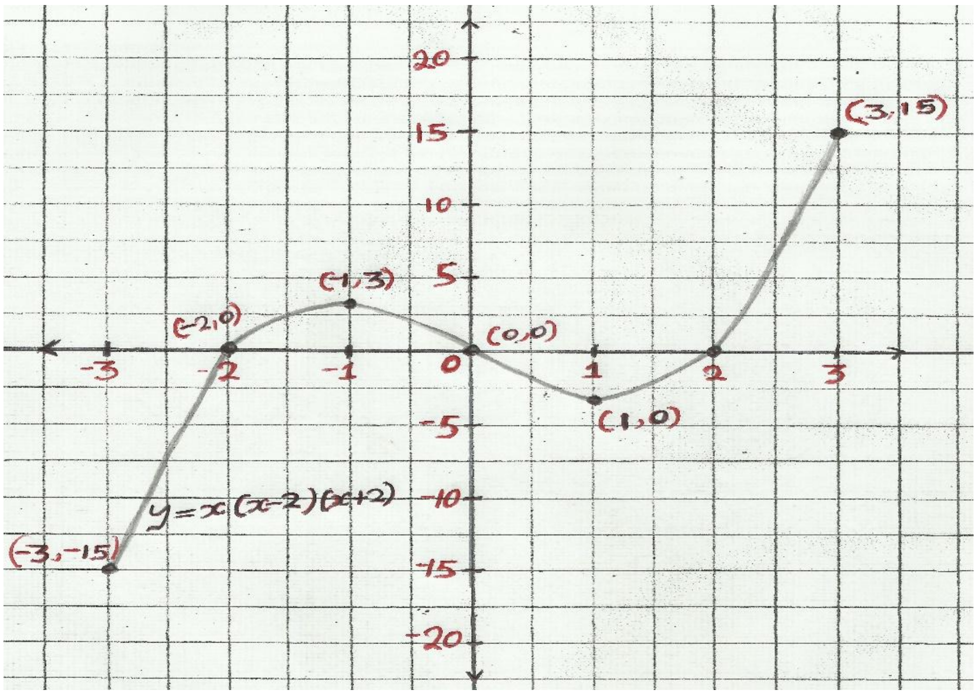
SOLUTIONS ii cont. page 4

The dotted and labeled points on the graph will appear as follows;



SOLUTIONS ii cont. page 5

The fully connected and labeled graph will now be as follows;



SOLUTIONS iii a page 6

To find the solution to iii a we have to remember that was written as $y = x(x-2)(x+2)$ but now it is written as follows

$$x(x-2)(x+2) = 0$$

This means that we have to find the value of x at the point where $y = 0$, this will be found at the points where the graph cuts the $x - axis$, because in the $x - axis\ y = 0$, there are three points where the graph cuts the $x - axis$. These points are $(-2, 0)$, $(0, 0)$ and $(2, 0)$. In these points

$$\therefore x = -2, 0\ and\ 2$$

From here connect to page 7

SOLUTIONS iii b page 7

To solve the equation $x(x-2)(x+2) = x+2$ plot the graph of $y = x+2$ then use the points where the graph of $x(x-2)(x+2)$ meets graph of $y = x+2$ first of all we **-2** and **2** as values of x from the table of values and substitute into $y = x+2$ so that we have two coordinates that we will connect to draw the line of $y = x+2$.

When $x = -2$

$$y = -2 + 2 = 0$$

$$\therefore \textit{The point is } (-2, 0)$$

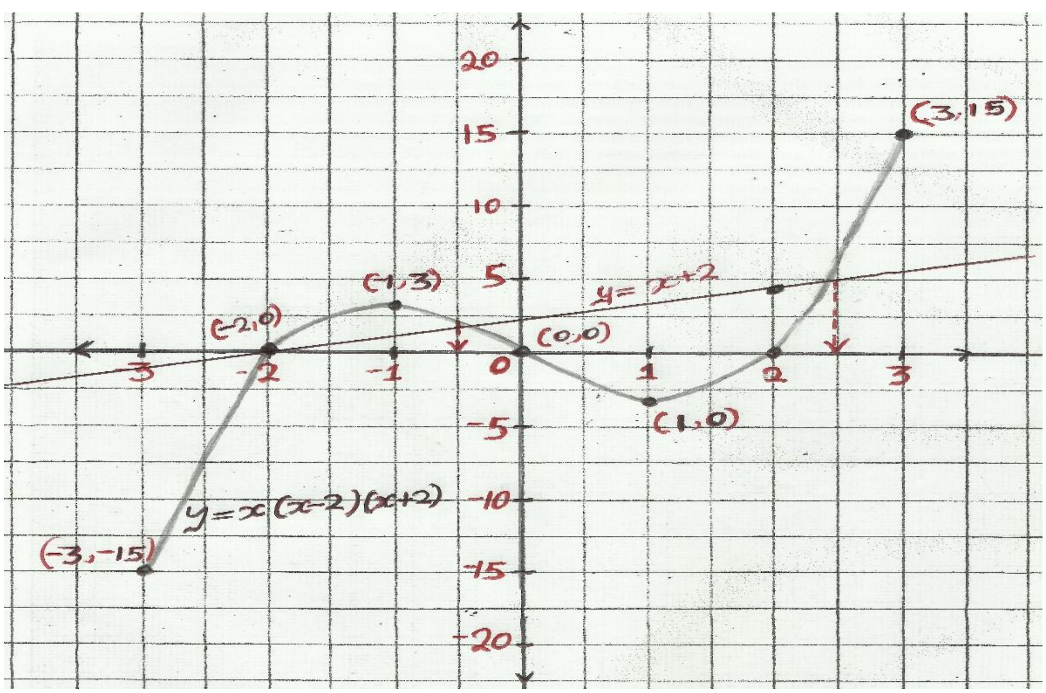
When $x = 2$

$$y = 2 + 2 = 4$$

$\therefore$The point is $(2, 4)$

SOLUTIONS iii b cont. page 8

We draw the line on the same graph as follows connecting $(-2, 0)$ and $(2, 4)$



The two graphs meet at $x = -2, x = -0.5$ and $x = 2.5$

SOLUTIONS iii b page 9

To express $\frac{2}{2x-1} - \frac{1}{3x+1}$ as a single fraction we first find the common denominator.

$$\frac{}{(2x-1)(3x+1)}$$

Divide and multiply

$$\frac{2(3x+1) - 1(2x-1)}{(2x-1)(3x+1)}$$

Opening brackets

$$\frac{6x+2-2x+1}{(2x-1)(3x+1)}$$

Collecting like terms

$$\frac{6x-2x+2+1}{(2x-1)(3x+1)}$$

$$\therefore Ans = \frac{4x+3}{(2x-1)(3x+1)}$$

END OF LESSON 136

LESSON 137 SPECIMEN QUESTION 19 P1 2016 PAGE 1

a. **Mambwe Company sold its bonds to interested members of the public and paid them at an interest rate of 15% per annum. How much did Mr. Hamakwebo receive if he bought a bond worth $K200\,000$?**

b. **If the equation of a straight line K is $y + 7 = 3x$, find the equation of the line passing through $(2, 8)$ and its Perpendicular to the line K.**

SOLUTION a PAGE 2

Hints; **Bonds are shares, in this case Mr. Hamakwebo bought shares so that he can be receiving interest pay annum (year), the percentage rate was 15% of the amount he paid for his shares. So they are asking you to calculate the interest he was receiving**

$$\mathit{Formula};\ \boldsymbol{interest\,rate} = \mathbf{15\%} \times \boldsymbol{total\,amount\,paid}$$

Know that $\mathbf{15\%} = \frac{\mathbf{15}}{\mathbf{100}}$ **and substitute**

$$\boldsymbol{Interest\,rate} = \frac{\mathbf{15}}{\mathbf{100}} \times \mathbf{200\,000}$$

Divide 100 and 200 000

$$\boldsymbol{Interest\,rate} = \mathbf{15} \times \mathbf{2000}$$

$$\therefore \boldsymbol{Received\ interest} = \boldsymbol{K}\mathbf{30\,000}$$

SOLUTION b PAGE 3

When two lines are parallel, then they are in the same direction and have the same gradient (m). So find the gradient using the equation of line K.

$$y + 7 = 3x$$

Make y the subject of the formula

$$y = 3x - 7$$

Coefficient of x is the gradient when y is the subject of the formula

$$\therefore m = 3$$

Continuation on the next page

SOLUTION b PAGE 4

If the two lines are perpendicular, then the product of their gradients will be – 1.

This means that

$$Gradient \times Gradient = -1$$

Substitute the gradient found on page 3

$$Gradient \times 3 = -1$$

Multiply on the left

$$3Gradient = -1$$

Divide both sides by 3

$$\frac{3Gradient}{3} = -\frac{1}{3} \qquad \therefore Gradient = -\frac{1}{3}$$

SOLUTION b cont. PAGE 5

The general equation of a straight line is $y = mx + c$ where m is the gradient and c is the constant. Substitute for m in $y = mx + c$.

$$y = -\frac{1}{3}x + c$$

Substitute, in $(2, 8)$ the value of $x = 2$ and $y = 8$

$$8 = -\frac{1}{3}(2) + c$$

Multiply and collect like terms

$$8 = -\frac{2}{3} + c \qquad \Rightarrow c = 8 + \frac{2}{3}$$

Find common denominator

SOLUTION b cont. PAGE 6

$$c = \frac{\qquad}{3}$$

Divide and multiply

$$c = \frac{3(8)+2}{3}$$

Multiply

$$c = \frac{24+2}{3}$$

Add

$$c = \frac{26}{3}$$

Divide

$$c = 8\frac{2}{3}$$

Substitute into the general equation

$$\therefore y = -\frac{1}{3}x + 8\frac{2}{3}$$

QUESTIONS FOR YOUR PRACTICE PAGE 7

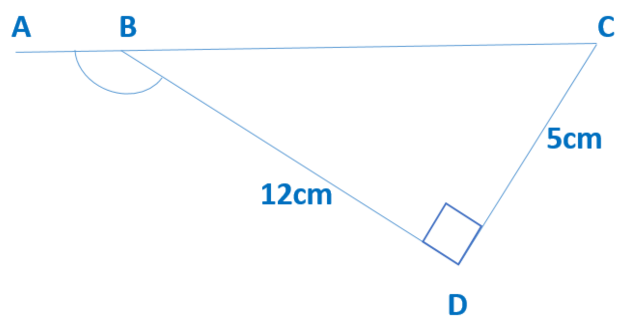
a. A photo shop sold its bolds to the general public and paid them interest of 12% per annum. If Mr. Bwalya bought a bond worth K36 000, calculate how much he received after a decade.

b. Give that the equation of the straight line B is $2y - 4x = 6$. If there is another line M perpendicular to line B and its passing through the point $(2, 9)$. Find the equation of the line M

END OF LESSON 137

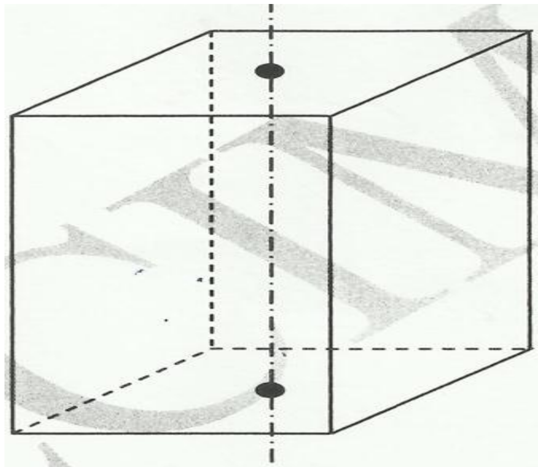
LESSON 138 SPECIMEN QUESTION 17 P1 2016 PAGE 1

a. In the diagram below, ABC is a straight line, $BD = 12cm$, $CD = 5cm$ and $B\hat{D}C = 90^\circ$



Express as a fraction, the value of sin $A\hat{B}D$

b. The diagram below is a cuboid with a square base



Write down the order of rotational symmetry of the cuboid along the given axis.

SOLUTION a PAGE 2

First of all, calculate the length of BC using Pythagoras theorem. BC is the hypotenuse so using Pythagoras we have BD and DC as adjacent sides, the formula will be;

$$BC^2 = BD^2 + DC^2$$

Substituting we have;

$$BC^2 = 12^2 + 5^2$$

$$BC^2 = (12)(12) + (5)(5)$$

$$BC^2 = 144 + 25$$

$$BC^2 = 169$$

$$\sqrt{BC^2} = \sqrt{169}$$

$$\therefore BC = 13cm$$

SOLUTION a cont. PAGE 3

Identify the sides, when you stand at $C\hat{B}D$ facing inside the triangle BCD your face will be opposite to $DC = 5cm$, this means that DC is the opposite. The longest side of the right angled triangle is the hypotenuse so $BC = 13cm$ is the hypotenuse. Substitute into the sine trig-ratio.

$$SinC\hat{B}D = \frac{opposite}{hypotenuse}$$

$$\therefore SinC\hat{B}D = \frac{5}{13}$$

What has been found is $SinC\hat{B}D$ to find $C\hat{B}D$ divide both sides by Sin.

$$\frac{SinC\hat{B}D}{sin} = sin^{-1}\left(\frac{5}{13}\right)$$

$$\therefore C\hat{B}D = sin^{-1}\left(\frac{5}{13}\right)$$

Continuation on the next page

SOLUTION a cont. PAGE 4

In the diagram $A\hat{B}D$ and $C\hat{B}D$ are on the same straight line, meaning they are adding up to 180°. To find $A\hat{B}D$ subtract $C\hat{B}D$ from 180°

$$A\hat{B}D = 180^{\circ} - C\hat{B}D$$

Substitute for $C\hat{B}D$ and introduce denominator 1 on 180°

$$A\hat{B}D = 180^{\circ} - sin^{-1}\left(\frac{5}{13}\right)$$

Now find sin $A\hat{B}D$

$$SinA\hat{B}D = Sin\left[180^{\circ} - sin^{-1}\left(\frac{5}{13}\right)\right]$$

Open brackets

$$SinA\hat{B}D = Sin180^{\circ} - Sin \times sin^{-1}\left(\frac{5}{13}\right)$$

Trig ratios $Sin \times sin^{-1}$ will cancel

$$SinA\hat{B}D = Sin180^{\circ} - \frac{5}{13}$$

Introduce a denominator on $Sin180^{\circ}$

$$\frac{Sin180^{\circ}}{1} - \frac{5}{13}$$

Find the common denominator

$$\frac{\qquad}{13}$$

Divide denominators and multiply to numerators

$$\frac{13Sin180^{\circ} - 5}{13}$$

Using special angles $Sin180^{\circ} = 0$

$$\frac{13(0) - 5}{13}$$

$$\therefore sinA\hat{B}D = -\frac{5}{13}$$

SOLUTION a ALTENATIVE METHOD 5

The angles $C\hat{B}D$ and $A\hat{B}D$ are on the same straight line.

When you find $SinC\hat{B}D = \frac{5}{13}$

Just put a negative to find $sinA\hat{B}D = -\frac{5}{13}$

SOLUTION b PAGE 6

When determining the order of rotational symmetry, consider the base on which the shape can possible stand. In this case the base is a square, we know that a square has 4 equal sides, since equal sides are 4 it means the order of rotational symmetry 4.

$$\therefore Ans = 4 \; order \; of \; rotational \; symmetry$$

If the base was not a square but a rectangle, we know that a rectangle has two pairs of equal sides so the order of rotational symmetry would have been 2.

END OF LESSON 138

LESSON 139 QUESTION 7 SECTION B P2 2015 PAGE 1

a. The variables x and y are connected by the equation; $y = x^2 - 2x + 1$. Some of the corresponding values of x and y correct to 1 decimal place are given in the table below.

x	-1.5	-1	-0.5	0	0.5	1	1.5	2	2.5	3
y	p	4	2.3	1	0.3	0	0.3	1	2.3	4

i. Calculate the value of p

ii. Using a scale of $2cm$ to represent 1 unit on both axes, draw the graph of $y = x^2 - 2x + 1$. For $-2 \leq x \leq 3$ and $0 \leq y \leq 10$.

iii. Calculate an estimate of the gradient of the curve at the point $(0, 1)$.

iv. Showing your method clearly, use your graph to solve $x^2 - 2x + 1 = 1.5$.

b. Given that Favor and Loveness uses $\text{K}46.90$ to buy $\$7$,

i. Calculate the cost of buying $\$1$

ii. How much Kwacha will they require to by $\$3$

SOLUTION a i PAGE 2

Look at the table of values, the value of x corresponding to p is -1.5 and p corresponds to y. Substitute these into the equation $y = x^2 - 2x + 1$

$$p = (-1.5)^2 - 2(-1.5) + 1$$

Expand and multiply

$$p = (-1.5)(-1.5) - 2(-1.5) + 1$$

Multiply observe signs

$$p = 2.25 + 3 + 1$$

Add

$$\therefore p = 6.25 = 6.3$$

x	-1.5	-1	-0.5	0	0.5	1	1.5	2	2.5	3
y	6.3	4	2.3	1	0.3	0	0.3	1	2.3	4

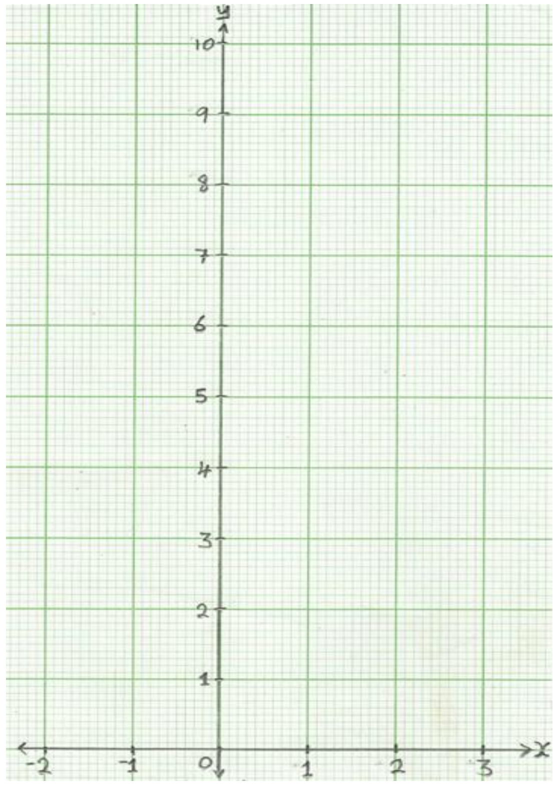
List the coordinates for corresponding (x, y) from the table

$(-1.5, 6.3)(-1, 4)(-0.5, 2.3)(0, 1)(0.5, 0.3)(1, 0)(1.5, 0.3)$
$(2, 1)(2.5, 2.3)(3, 4)$

From here connect to page 3

SOLUTION a ii PAGE 3

Scale of $2cm$ to represent 1 unit means after every $2cm$ you have 1 unit. The range $-2 \leq x \leq 3$ means in the $x - axis$ numbers will range from -2 to 3 and $0 \leq y \leq 10$ means numbers will range from 0 to 10.



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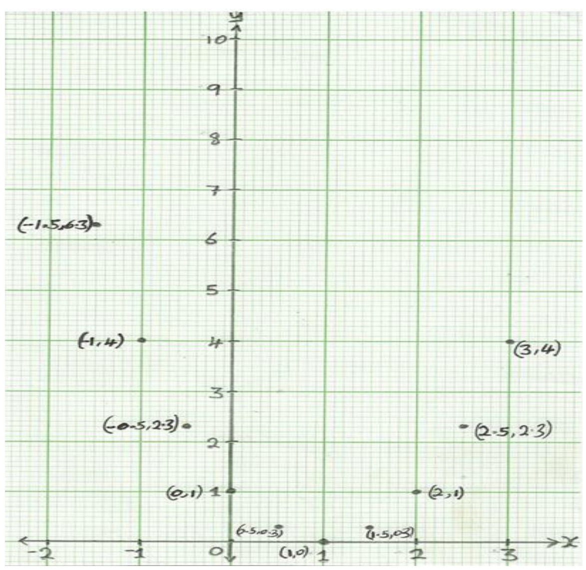
SOLUTION a ii cont. PAGE 4

Plot the points and label them clearly



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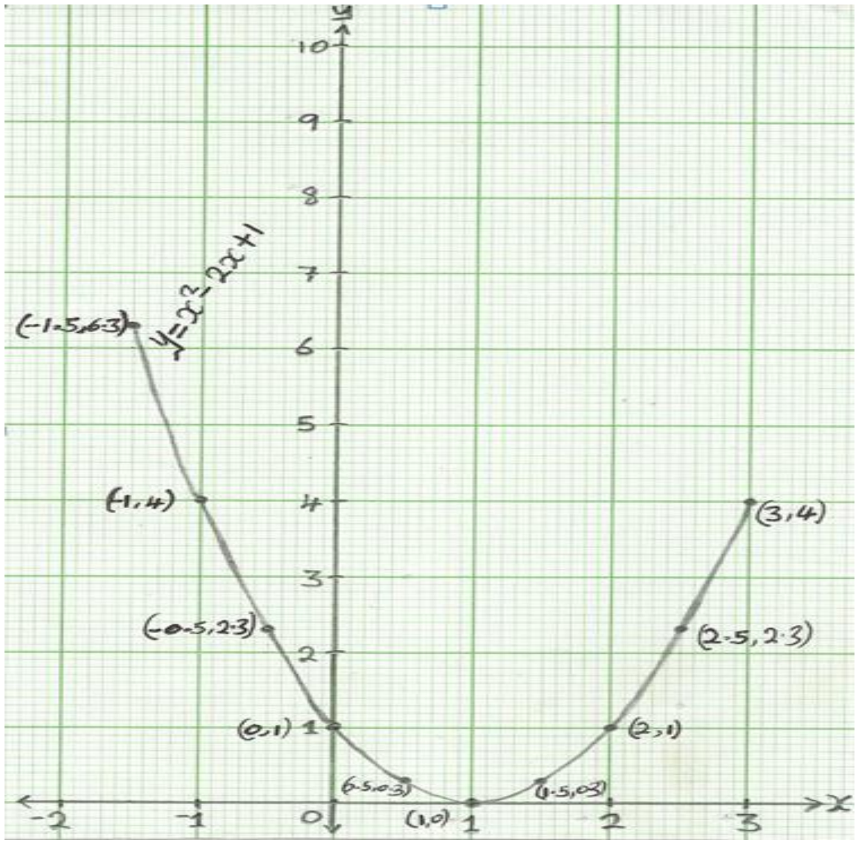
SOLUTION a ii cont. PAGE 5

Draw the graph by connecting the points using a free hand



From here connect to page 6

SOLUTION iii PAGE 6

To find the estimate of gradient, you can use calculus by differentiating y with respect to x then substituting for x.

$$y = x^2 - 2x + 1$$

Multiply the power to coefficient and subtract 1 from power but the constant 1 becomes 0.

$$\frac{dy}{dx} = 2x^{2-1} - 2x^{1-1} + 0$$

Subtract on powers

$$\frac{dy}{dx} = 2x^1 - 2x^0$$

Ignore power 1 and any number to power 0 is 0

$$\frac{dy}{dx} = 2x - 2(1) \quad \therefore \frac{dy}{dx} = 2x - 2$$

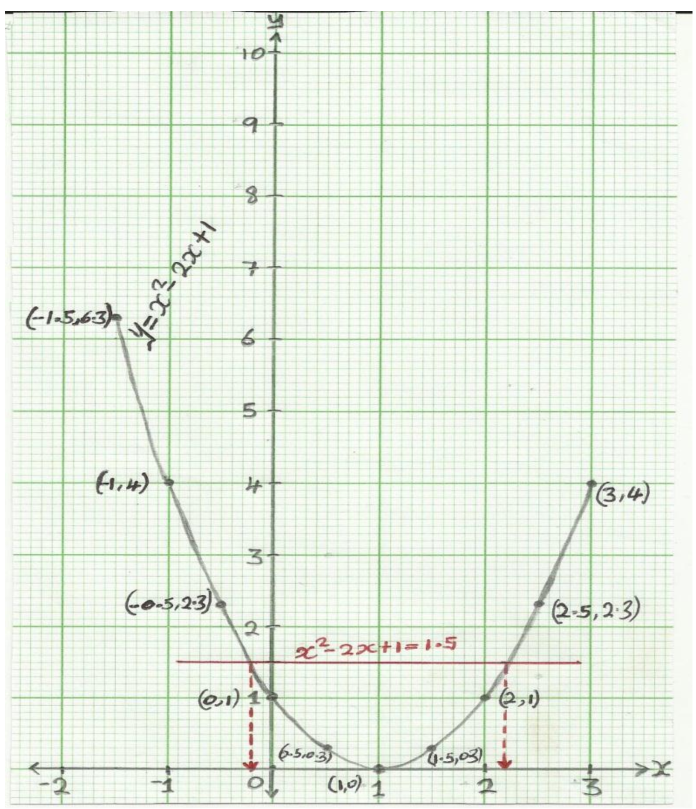
Use the point $(0, 1)$ substitute for $x = 0$

$$\frac{dy}{dx} = 2(0) - 2 \therefore \frac{dy}{dx} = -2$$

From here connect to page 7

SOLUTION a iv PAGE 7

To solve $x^2 - 2x + 1 = 1.5$ it means you have to draw a straight line along the line where $y = 1.5$. Then pick the values of x where the straight line cuts the graph.



From here connect to page 8

SOLUTION b i PAGE 8

Use proportion and ratio

$$7 = 46.90$$

$$1 = x$$

Cross multiply

$$7x = 46.90$$

Divide both sides by 7

$$\frac{7x}{7} = \frac{46.90}{7} \therefore x = 6.7$$

$$\therefore 1\$ = \text{K}6.7$$

SOLUTION b ii

Use proportion and ratio

$$1 = 6.7$$

$$3 = x$$

Cross multiply

$$x = \text{K}20.1$$

$$\therefore 3\$ = \text{K}20.1$$

End of lesson 139

LESSON 140 SERIES AND SEQUENCES PAGE 1

Arithmetic progressions

A sequence is a set of numbers following a certain pattern. A sequence becomes an arithmetic progression (AP) when it has a common difference denoted by d. The first number in a sequence is denoted by a.

Examples;

1. In the sequence $2, 4, 6, 8, 10, 12, 14, 16, \dots \dots$ The common difference is 2 therefore the sequence is an AP
2. In the sequence $1, 3, 6, 10, 15, 21$ there is no common difference hence the sequence is not an AP.

From here connect to page 2

QUESTIONS PAGE 2

d. For the sequence $2, 4, 6, 8, 10, 12, 14, 16$ find the 15th term.

e. For the sequence $14, 17, 20, 23, \dots\dots\dots$ Find the,

   iii. 18th term

   iv. Sum of the first 30 terms.

f. Find the sum 24 terms of $1, 5, 9, 13, 17 \ \dots \dots$

g. The fifth term of an AP is 10 and the tenth term is 20, find the values of a and d, hence find the 20th term.

Copy the question above, scroll down for answers and listen to possible audios for each answer page

SOLUTION a PAGE 3

The formula for finding a given term is $t_n = a + (n - 1)d$ we want to find term number 15 so we have $n = 15$ then we have to know that in $2, 4, 6, 8, 10, 12, 14, 16 \dots \dots \dots$

$a = 2$ (The first term)

$d = 4 - 2 = 2$ (Second - first) term

We substitute into the formula

$$t_{15} = 2 + (15 - 1)2$$
$$t_{15} = 2 + (14)2$$
$$t_{15} = 2 + 28$$
$$\therefore t_{15} = 30$$

From here connect to page 4

SOLUTIONS b i PAGE 4

The formula for finding a given term is $t_n = a + (n-1)d$ we want to find term number 18 so we have $n = 18$ then we have to know that in $14, 17, 20, 23, \ldots..$

$$a = 14 \text{ (The first term)}$$

$$d = 17 - 14 = 3 \text{ (Second - first) term}$$

We substitute into the formula

$$t_{18} = 14 + (18 - 1)3$$
$$t_{18} = 14 + (17)3$$
$$t_{18} = 14 + 51$$
$$\therefore t_{18} = 65$$

From here connect to page 4

SOLUTIONS b ii PAGE 5

Sum of the first 30 terms formula $S_n = \frac{n}{2}[2a + (n-1)d]$ so we just substitute

$$S_{30} = \frac{30}{2}[2(14) + (30 - 1)3]$$

Multiply and subtract inside brackets

$$S_{30} = 15[28 + (29)3]$$

Multiply inside brackets

$$S_{30} = 15[28 + 87]$$

Add inside brackets

$$S_{30} = 15[115]$$

Multiply to find the final answer

$$S_{30} = 1725$$

From here connect to page 6

SOLUTIONS c PAGE 6

Sum of 24 terms for $1, 5, 9, 13, 17 \ldots\ldots$

The formula for sum is $S_n = \frac{n}{2}[2a + (n-1)d]$

$n = 24$ (the number of terms)

$a = 1$ (The first term)

$d = 5 - 1 = 4$ (Second- first=Common difference)

$$S_{24} = \frac{24}{2}[2(1) + (24-1)4]$$

$$S_{24} = 12[2 + (23)4]$$

$$S_{24} = 12[2 + 92]$$

$$S_{24} = 12[94]$$

$$\therefore S_{24} = 1128$$

From here connect to page 7

SOLUTIONS d PAGE 7

The formula for term of an AP is $t_n = a + (n-1)d$

Fifth term is 10

Meaning term number 5 is 10, so $n = 5,$ then term number 5 is 10. So we have;

$$T_5 = a + (5-1)d = 10$$

$$a + (4)d = 10$$

$$a + 4d = 10 \ldots \ldots \ldots \ldots\ i$$

The tenth term is 20

Meaning term number 10 is 20, so $n = 10,$ then term number 10 is 20. So we have;

$$T_{10} = a + (10-1)d = 20$$

$$T_{10} = a + (9)d = 20$$

$$a + 9d = 20 \dots \dots \dots \dots \dots \; ii$$

Continuation on the next page

SOLUTIONS d cont. PAGE 8

Solve equation i and ii simultaneously

$$a + 4d = 10 \dots \dots . . i$$
$$a + 9d = 20 \dots \dots . . ii$$

Use equation i to make a the subject of the formula we have;

$$\therefore a = 10 - 4d$$

Substitute for a in equation ii.

$$10 - 4d + 9d = 20$$

Add and collect like terms $-4d + 9d = 5d$ you have

$$5d = 20 - 10 \quad \Rightarrow \qquad 5d = 10$$

Divide both sides by 5

$$\frac{5d}{5} = \frac{10}{5} \qquad \therefore d = 2$$

From here connect to page 9

SOLUTIONS d cont. PAGE 9

To find a substitute for d in $a = 10 - 4d$

The value of $d = 2$, so substitute for d

$$a = 10 - 4(2)$$

Multiply

$$a = 10 - 8$$

$$\therefore a = 2$$

Find the 20th term

$$t_{20} = 2 + (20 - 1)2$$

$$t_{20} = 2 + (19)2$$

$$t_{20} = 2 + 38$$

$$\therefore t_{20} = 40$$

End of lesson 140

LESSON 141 SERIES AND SEQUENCES (PAGE 1)

b. For the sequence $2, 5, 8, 11, 14 \ldots \ldots$ find

i. An expression for the n^{th} term

ii. The 9^{th} term using an expression in (i)

iii. The 100^{th} term using an expression in (i)

iv. An expression for the sum of the first n terms

v. The sum of the first 40 terms using an expression in (i)

Copy the question above, scroll down for answers and listen to possible audios following each answer page

SOLUTION (a. i. page 2)

To find the expression for the n^{th} term in this sequence, use the formula $T_n = a + (n - 1)d$ in this formula T_n stands for n^{th} term a stands for first term, n stands of number of terms and d stands for common difference. In $2, 5, 8, 11, 14$

$$a = 2$$
$$d = 5 - 2 = 3$$

Substitute into the formula

$$T_n = 2 + (n - 1)3$$

Open brackets

$$T_n = 2 + 3n - 3$$

Collect like terms

$$T_n = 3n + 2 - 3$$

$$\therefore T_n = 3n - 1$$

From here connect to page 3

SOLUTION (a. ii page 3)

To find the 9^{th} term in $2, 5, 8, 11, 14$ … … just substitute for n in the expression for the n^{th} on page 2.

$$n^{th} = 3n - 1$$

Substitute into the formula

$$9^{th} = 3(9) - 1$$

Multiply first

$$9^{th} = 27 - 1$$

Subtract

$$\therefore 9^{th} = 26$$

From here connect to page 4

SOLUTION (iii. page 4)

To find the 100^{th} term use the expression $T_n = 3n - 1$ in this case $n = 100$.

Substitute into the expression on page 2

$$T_{100} = 3(100) - 1$$

Multiply first

$$T_{100} = 300 - 1$$

Subtract

$$\therefore T_{100} = 299$$

From here connect to page 5

SOLUTION (iv. page 5)

To find the expression for the sum of the first n terms in this sequence, use the formula $S_n = \frac{n}{2}[2a + (n-1)d]$ in this formula S_n stands for sum of the first n terms, a stands for first term, n stands of number of terms and d stands for common difference. In $2, 5, 8, 11, 14$

$$a = 2 \quad \text{and} \quad d = 5 - 2 = 3$$

Substitute into the formula

$$S_n = \frac{n}{2}[2(2) + (n-1)3]$$

Continuation on the next page

SOLUTION (iv. Cont. page 6)

Multiply inside brackets

$$S_n = \frac{n}{2}[4 + 3n - 3]$$

Collect like terms

$$S_n = \frac{n}{2}[4 - 3 + 3n]$$

Subtract inside

$$S_n = \frac{n}{2}[1 + 3n]$$

Open brackets

$$\therefore S_n = \frac{n}{2} + \frac{3n^2}{2}$$

From here connect to page 7

SOLUTION (iv. page 7)

$$S_n = \frac{n}{2} + \frac{3n^2}{2}$$

We already have all the numbers so we just substitute

$$S_{40} = \frac{40}{2} + \frac{3(40)^2}{2}$$

Multiply first

$$S_{40} = \frac{40}{2} + \frac{3(1600)}{2}$$

$$S_{40} = \frac{40}{2} + \frac{4800}{2}$$

$$\frac{40+4800}{2} = \frac{4840}{2}$$

$$\therefore S_{40} = 2420$$

QUESTION FOR YOUR PRACTICE (page 8)

b. For the sequence $1, 5, 9, 13, \ldots \ldots \ldots \ldots$ find

v. The 7th term

vi. An expression for the nth term

vii. The 20th term using your expression in (ii)

viii. The sum of the first 40 terms

PRACTICE MAKES PERFECT, TIME IS NOW

END OF LESSON 141

LESSON 142 GEOMETRIC PROGRESSIONS (PAGE 1)

When we have $1, 2, 4, 8, 16, 32, \ldots \ldots \ldots ..$ the first term is 1 and the remaining terms are formed by multiplying each preceding term by 2 (common ratio). The formed set of these numbers forms a geometric progression (GP). The first term is denoted by a while the common ratio is denoted by r.

From here connect to page 2

QUESTIONS PAGE 2

For each of the following sequences, say whether it is a geometric progression or not.

e. $10, 11, 12, 13, \ldots$
f. $3, 6, 12, 24, \ldots$
g. $-100, 50, -25, 12.5$
h. $\frac{9}{100}, \frac{3}{10}, 1, \frac{10}{3}$

2. For each of the following GPs find the 6th term
   c. $100, 50, 25, 12.5$
   d. $3, 6, 12, 24, \ldots$

SOLUTION (1 a page 3)

The sequence $10, 11, 12, 13, \ldots$ can only be a GP if it has a common ration r that is a number which each term is being multiplied by in order to get the next term, to get r we have to divide the second term by the first term or the third term by the second, the answer must be the same.

Test for common ratio

Second divide by first

$$r = 11 \div 10 = 1.1$$

Third divide by second

$$r = 12 \div 11 = 1.090909091$$

Fourth divide by third

$$r = 13 \div 12 = 1.0833333$$

There is no common ratio hence $10, 11, 12, 13,$ is not a GP

From here connect to page 4

SOLUTION (1 b page 4)

$$3, 6, 12, 24, \ldots$$

Test for common ratio

Second divide by first

$$r = 6 \div 3 = 2$$

Third divide by second

$$r = 12 \div 6 = 2$$

Fourth divide by third

$$r = 24 \div 12 = 2$$

The common ratio is 2 hence $3, 6, 12, 24, \ldots,$ is a GP

From here connect to page 5

SOLUTION (1 c page 5)

$$-100, 50, -25, 12.5$$

Test for common ratio

Second divide by first

$$r = 50 \div -100 = -0.5$$

Third divide by second

$$r = -25 \div 50 = -0.5$$

Fourth divide by third

$$r = 12.5 \div -25 = -0.5$$

Common ratio -0.5, hence $-100, 50, -25, 12.5$ is a GP

From here connect to page 6

SOLUTION (1 d page 6)

$$\frac{9}{100}, \frac{3}{10}, 1, \frac{10}{3}$$

Test for common ratio

Second divide by first

$$r = \frac{3}{10} \div \frac{9}{100} = \frac{3}{10} \times \frac{100}{9} = \frac{10}{3}$$

Third divide by second

$$r = 1 \div \frac{3}{10} = 1 \times \frac{10}{3} = \frac{10}{3}$$

Fourth divide by third

$$r = \frac{10}{3} \div 1 = \frac{10}{3} \times \frac{1}{1} = \frac{10}{3}$$

There is a common ration of $\frac{10}{3}$ hence $\frac{9}{100}, \frac{3}{10}, 1, \frac{10}{3}$ is a GP

From here connect to page 7

SOLUTION (2 a page 7)

To find the 6th term for $100, 50, 25, 12.5$ we use the formula

$$T_n = ar^{n-1}$$

Where $n = 6$, $a = 100$ and $r = 0.5$

$$T_6 = (100)(0.5)^{6-1}$$

$$T_6 = (100)(0.5)^5$$

$$T_6 = (100)(0.5 \times 0.5 \times 0.5 \times 0.5 \times 0.5)$$

$$T_6 = (100)(0.03125) \therefore T_{10} = 3.125$$

From here connect to page 8

SOLUTION (2 b page 8)

For $3, 6, 12, 24, \ldots$

$$T_n = ar^{n-1}$$

$$T_6 = (3)(2)^{6-1}$$

$$T_6 = (3)(2)^5$$

$$T_6 = 3 \times 32$$

$$\therefore T_6 = 96$$

End of lesson 142

LESSON 143 SPECIMEN P2 QUESTIONS 5 2016 (PAGE 1)

c. The 3rd and 4th terms of a geometric progression are 4 and 8 respectively. Find
   iv. The common ratio, first term and second term
   v. The sum of the first 10 terms
   vi. The sum infinity of this geometric progression

d. Simplify $\frac{13k}{20a^2} \times \frac{5}{39k^2}$

SOLUTION (a i page 2)

To find the common ratio we divide 4 into 8 because they are coming one after the other;

$$\therefore r = 8 \div 4 = 2$$

Now that 4 is the third term, then we obtain the second term by dividing r into the third term.

$$Second\ term = 4 \div r = 4 \div 2 = 2$$

$$\therefore second\ term = 2$$

To get the first term we divide r into the second term

$$\therefore First\ term = 2 \div r = 2 \div 2 = 1$$

From here connect to page 3

SOLUTION (a ii page 3)

Sum of the first 10 terms, we use the following formula;

$$S_n = \frac{a(1-r^n)}{(1-r)}$$

$$n = 10, r = 2\ and\ a = 1$$

$$S_{10} = \frac{1(1-2^{10})}{(1-2)}$$

$$S_{10} = \frac{1(1-1024)}{-1}$$

$$S_{10} = \frac{1(-1023)}{-1} \Rightarrow S_{10} = \frac{-1023}{-1}$$

$$\therefore S_{10} = 1023$$

From here connect to page 4

SOLUTION (a iii page 4)

To find the sum of infinity (∞) terms, it means we do not know the number of terms because they are too many to count, so we first find the value of $S_n = \frac{a(1-r^n)}{(1-r)}$ such that we do not know the real value of n as follows;

$$S_n = \frac{1(1-2^n)}{(1-2)}$$

subtruating in the denominator

$$S_\infty = \frac{1(1-2^\infty)}{-1}$$

Dividing 1 outside brackets by denominator -1

$$S_\infty = -1(1-2^\infty)$$

opening brackes by multiplying by -1 outside

$$S_\infty = 2^\infty - 1$$

This shows that, when n becomes too large, then the value of $S_\infty = 2^\infty - 1$ becomes too large or an infinite number too

$$\therefore S_\infty = 2^\infty - 1 = \infty$$

SOLUTION (b page 5)

$$\frac{13k}{20a^2} \times \frac{5}{39k^2}$$

Divide 13k and $39k^2$ by 13k, $20a^2$ and 5 by 5

$$\frac{1}{4a^2} \times \frac{1}{3k}$$

multiply numerator by numerator

$$\frac{1}{3k\times4a^2}$$

multiply denominator by denominator

$$\therefore Ans = \frac{1}{12ka^2}$$

End of lesson 143

LESSON 144 QUETSION 5 P2 2016 PAGE 1

a. Simplify $\frac{x-1}{x^2-1}$

b. The first three terms of a geometric progression are $x+1$, $x-3$ and $x-1$. Find;

i. The value of x

ii. The first term

iii. The sum to infinite

SOLUTION 5 a PAGE 2

The expression $\frac{x-1}{x^2-1}$ is a difference of two squares. The 1 in the denominator can also be raised to power 2.

$$\frac{x-1}{x^2-1^2}$$

Factorise in the denominator.

$$\frac{x-1}{(x-1)(x+1)}$$

Divide the common factors

$$\therefore Ans = \frac{1}{(x+1)}$$

From here connect to page 3

EXPLAINATION PAGE 3

When we have $1, 2, 4, 8, 16, 32, 64, 128 \ldots \ldots \ldots ..$ the first term is 1 and the remaining terms are formed by multiplying each preceding term by 2 (common ratio). The formed set of these numbers forms a geometric progression (GP). The first term is denoted by a while the common ratio is denoted by r.

$$common\ ratio = \frac{second\ term}{first\ term}$$

Substitute the terms

$$r = \frac{2}{1} \qquad \therefore r = 2$$

$$preceding \times r = next\ term$$

Substitute the terms

$$1 \times 2 = 2$$

$$2 \times 2 = 4$$

$$4 \times 2 = 8$$

$$8 \times 2 = 16$$

$$16 \times 2 = 32$$

$$32 \times 2 = 64$$

$$64 \times 2 = 128$$

From here connect to page 4

SOLUTION 5 b i PAGE 4

In GPs you get the next term by multiplying a common ratio r to the preceding term using the formula.

$$T = ar$$

$a = preceding\ term$ **and** $T = term$. **To find the second term, multiply the first term by** r

$$First\ term \times r = second\ term$$

Substitute the terms

$$(x+1)r = x-3$$

Open brackets to find equation i

$$xr + r = x - 3 \ \ldots\ldots..i$$

To find the third term, multiply the second term by r

$$second\ term \times r = third\ term$$

Substitute the terms

$$(x-3)r = x-1$$

Open brackets to form equation ii

$$xr - 3r = x - 1 \ldots\ldots\ldots\ldots\ldots..ii$$

Solve equation i and ii simultaneously

Continuation on the next page

SOLUTION 5 b i cont. PAGE 5

$$xr + r = x - 3 \ldots\ldots\ldots.i$$
$$xr - 3r = x - 1 \ldots\ldots\ldots ii$$

Subtract ii from i

$$-\begin{vmatrix} xr & +r = & x-3 \\ xr & -3r = & x-1 \end{vmatrix}$$

The subtraction is based on corresponding values

$$(xr - xr) + (r - -3r) = (x - x) + (-3 - -1)$$

Work out the signs

$$(xr - xr) + (r + 3r) = (x - x) + (-3 + 1)$$

Add and subtract

$$0 + 4r = 0 - 3 + 1$$

$$4r = -2$$

Divide both sides by 4

$$\frac{4r}{4} = \frac{-2}{4} \qquad \therefore\ r = -\frac{1}{2}$$

Continuation on the next page

SOLUTION 5 b i cont. PAGE 7

Use one of the equations to find x

$$xr + r = x - 3$$

Substitute for r

$$x\left(-\frac{1}{2}\right) + \left(-\frac{1}{2}\right) = x - 3$$

Open brackets

$$-\frac{x}{2} - \frac{1}{2} = x - 3$$

Introduce 2 to get rid of the fractions

$$\left(-\frac{x}{2} - \frac{1}{2} = x - 3\right)2$$

Continuation on the next page

SOLUTION 5 b i cont. PAGE 8

Multiply throughout by 2 to remove the fractions

$$\left(-\frac{x}{2}\times 2\right)+\left(-\frac{1}{2}\times 2\right)=(x\times 2)-(3\times 2)$$

Multiply

$$-x-1=2x-6$$

Collect like terms

$$-x-2x=-6+1$$

Subtract and add

$$-3x=-5$$

Divide both sides by -3

$$\frac{-3x}{-3}=\frac{-5}{-3} \qquad \therefore x=1\frac{2}{3}$$

SOLUTION 5 b ii PAGE 9

The first term is $x+1$ substituting for x

$$T_1=\frac{5}{3}+1$$

Find the common denominator

$$\frac{\qquad}{3}$$

Divide and multiply

$$\frac{1(5)+3(1)}{3}$$

Multiply

$$\frac{5+3}{3}=\frac{8}{3}$$

$$\therefore Ans=2\frac{2}{3}$$

SOLUTION 5 b iii PAGE 10

The formula for sum of terms is

$$S_n = \frac{a(1-r^n)}{(1-r)}$$

The sum we want it's the sum to infinite or when the power if too large.

$$S_\infty = \frac{a(1-r^\infty)}{(1-r)}$$

Substitute for a and r

$$S_\infty = \frac{\frac{8}{3}\left(1--\frac{1}{2}^\infty\right)}{1--\frac{1}{2}}$$

SOLUTION 5 b iii cont. PAGE 11

Evaluate in the denominator

$$S_\infty = \frac{\frac{8}{3}\left(1--\frac{1}{2}^\infty\right)}{1+\frac{1}{2}} = \frac{\frac{8}{3}\left(1+\frac{1}{2}^\infty\right)}{\frac{2+1}{2}} = \frac{\frac{8}{3}\left(1+\frac{1}{2}^\infty\right)}{\frac{3}{2}}$$

This can be written as

$$S_\infty = \frac{8}{3}\left(1+\frac{1}{2}^\infty\right) \div \frac{3}{2}$$

Change the sign and fraction

$$S_\infty = \frac{8}{3}\left(1+\frac{1}{2}^\infty\right) \times \frac{2}{3}$$

Divide common factors

$$S_\infty = \frac{8}{3}\left(1+\frac{1}{2}^\infty\right) \times \frac{2}{3} \quad \Rightarrow \quad S_\infty = \frac{16}{9}\left(1+\frac{1}{2}^\infty\right)$$

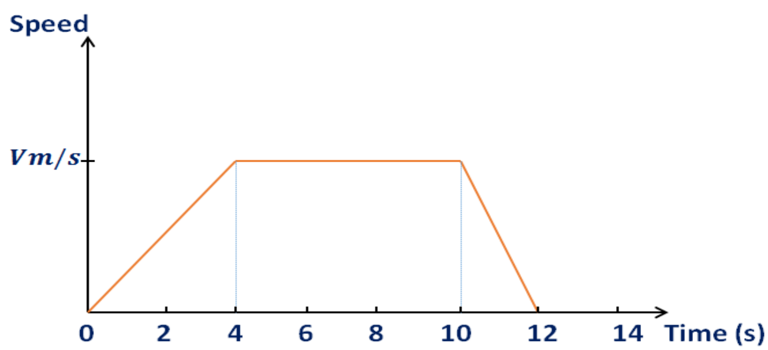
Multiply

$$\therefore S_\infty = 1\frac{7}{9}\left(1+\frac{1}{2}^{\infty}\right)$$

End of lesson 144

LESSON 145 TRAVEL GRAPHS PAGE 1

The diagram shows the speed of a car which starts from rest and increases its speed at a constant rate for 4 seconds and then travels at a constant speed of vm/s for further 6 seconds before it comes to halt in 2 seconds



Given that the car travelled 80m in the first 4 seconds calculate

a. Maximum speed (v) this car had reached

b. Acceleration of the car in the first 4 seconds

c. Retardation in the last 2 seconds

d. Total distance travelled in the period of 12 seconds

SOLUTION a PAGE 2

The formula for speed is $Speed = \frac{Distance}{time}$ where $time = 4s$ and $Distance = 80m$ so we substitute

$$Speed = \frac{Distance}{time}$$

Substituting we have

$$Speed = \frac{80}{4}$$

Dividing by 4 we have

$$\therefore Speed = 20m/s$$

SOLUTION b PAGE 3

The formula for acceleration is $a = \frac{v-u}{t}$ where a is $acceleration$, v is $final\ velocity$ or $speed$ and u is $the\ initial\ verocity$ or $speed$. In this case v is the speed we calculated in a then u is the speed it was at when the car was not moving or at the start of the graph, that will be zero and t is $4s$ so we substitute and subtract

$$a = \frac{v-u}{t} \quad \Rightarrow \quad a = \frac{20-0}{4} \quad \Rightarrow \quad a = \frac{20}{4}$$

Dividing by 4 we have

$$\therefore a = 5m/s^2$$

SOLUTION c PAGE 4

Retardation is the opposite of acceleration because the speed is reducing from 20 to 0 the formula will still be $a = \frac{v-u}{t}$ in this case speed is reducing from $20m/s$ to $0m/s$ so $u = 20m/s$ and $v = 0m/s$ so we substitute

$$a = \frac{v-u}{t} \quad \Rightarrow \quad a = \frac{0-20}{2} \quad \Rightarrow \quad a = \frac{-20}{2}$$

Dividing by 2 we have

$$\therefore a = -10m/s^2$$

SOLUTION d PAGE 5

The graph forms a trapezium, to calculate the total distance covered, you just find area of a trapezium using the following formula.

$$A = \frac{1}{2}(a + b)h$$

In this case $a = base$, $b = upper\ base$ and $h = height$

Substitute into the formula

$$D = \frac{1}{2}(12 + 6)20$$

Add in brackets

$$D = \frac{1}{2}(18)20$$

Divide 2 into 20

$$D = (18)10$$

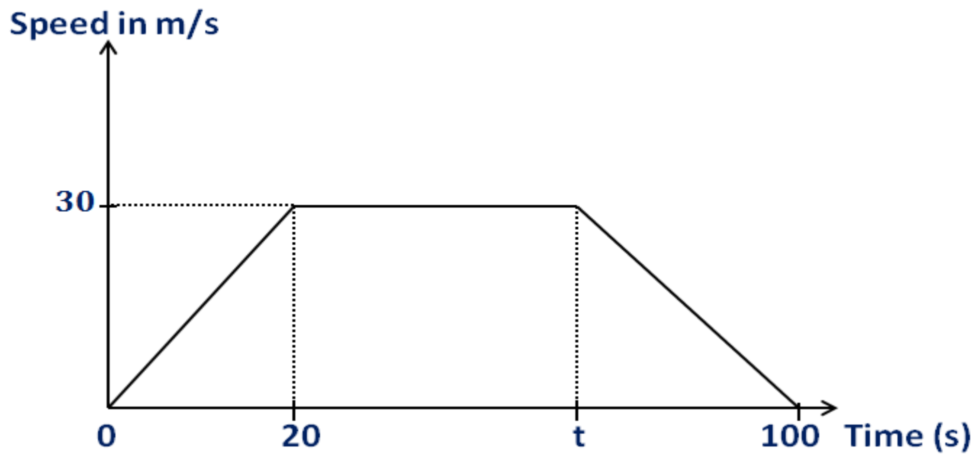
Multiply 18 and 10

$$\therefore D = 180m$$

End of lesson 145

LESSON 146 TRAVEL GRAPHS QN 23 P1 2016 PAGE 1

The diagram below is a speed-time graph of a car which starts from rest and accelerates uniformly for 20 seconds till it reaches a speed of 30m/s. it then moves at a constant speed for some more time before it starts decelerating. It comes to rest after 100 seconds.

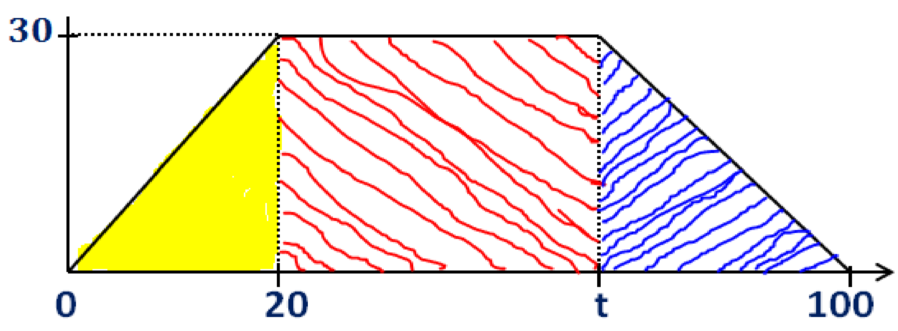


Given that the total distance travelled is 2 400m, calculate;

d. The value of t
e. The retardation in the last part of the journey
f. Speed of the car at the ninety fifth second

SOLUTION a PAGE 2

You have to know that total distance in a travel graph is found by calculating the area of the space enclosed by the graph and the time line. This graph has, three parts a triangle in yellow, rectangle in red and triangle in blue.



SOLUTION a cont. PAGE 3

Formula and finding area of triangle yellow

$$A_{yellow} = \frac{1}{2} \times base \times height$$

$base = 20$ and $height = 30$

Substituting into the formula and multiplying

$$A_{yellow} = \frac{1}{2} \times 20 \times 30 \quad \Rightarrow \quad A_{yellow} = \frac{600}{2}$$

Dividing 2 into 600

$$\therefore A_{yellow} = 300$$

SOLUTION a cont. PAGE 4

Formula and finding area of rectangle red

$$A_{red} = length \times breadth$$

$length = t - 20$ and $breadth = height = 30$

Substituting into the formula

$$A_{red} = (t - 20) \times 30$$

Multiplying t and 20 by 30

$$\therefore A_{red} = 30t - 600$$

Formula and finding area of triangle blue

$$A_{blue} = \frac{1}{2} \times base \times height$$

$base = 100 - t$ and $height = 30$

Substituting and multiplying $\frac{1}{2}$ and 30

$$A_{blue} = \frac{1}{2} \times (100 - t) \times 30 \quad \Rightarrow \quad A_{blue} = (100 - t) \times 15$$

Multiplying $(100 - t)$ and 15

$$\therefore A_{blue} = 1500 - 15t$$

To find t we have to use the formula for total area

$$A_{yellow} + A_{red} + A_{blue} = A_{total}$$

Area total is equal to total distance so we substitute

$$300 + 30t - 600 + 1500 - 15t = 2400$$

Collecting like terms and changing signs to the right

$$30t - 15t = 2400 - 300 + 600 - 1500$$

Subtracting and adding on both sides

$$15t = 1200$$

Dividing both sides by 15

$$\frac{15t}{15} = \frac{1200}{15}$$

$$\therefore t = 80\ \ seconds$$

Retardation is moving from high speed to stopping the car

$$a = \frac{final\ speed - initial\ speed}{time\ taken}$$

The final speed is zero because it goes to rest, the initial speed is 30m/s, the time taken is $100 - t$ or $100 - 80 = 20$

Substitution we have

$$a = \frac{0-30}{20} \Rightarrow a = \frac{-30}{20}$$

Dividing we have

$$\therefore a = -1.5m/s^2$$

To find speed of the car at the ninety fifth second we consider that retardation is taking place in the blue triangle. At ninety fifth second it has moved 15 seconds into the blue triangle so the base becomes 15. Let's find the distance in form of area

$$A = \frac{1}{2} \times base \times height \Rightarrow A = \frac{1}{2} \times 15 \times 30 \Rightarrow A = 15 \times 15$$

$$\therefore A = 225$$

We now find the speed

$$Speed = \frac{distance}{time}$$

Substituting we have

$$Speed = \frac{225}{15}$$

$$\therefore Speed = 15m/s$$

End of lesson 146

LESSON 147 STATISTICS PAGE 1

Answer the whole of this question on a sheet of graph paper. The table below shows the masses of 100 babies at birth, recorded at a hospital.

Mass (xkg)	1.5<x≤2.0	2.0<x≤2.5	2.5<x≤3.0	3.0<x≤3.5	3.5<x≤4.0	4.0<x≤4.5	4.5<x≤5.0
№. of babies	3	12	20	24	25	14	2

a. Copy and complete the cumulative frequency table below;

Mass (xkg)	≤1.5	≤2.0	≤2.5	≤3.0	≤3.5	≤4.0	≤4.5	≤5.0
№ of	0	3	15					100

babies								

b. Using a horizontal scale of 2cm to represent 0.5kg for masses from 1.5kg to 5.0kg and a vertical scale of 2cm to represent 10 babies, draw a smooth cumulative frequency curve.

c. Showing your method clearly, use you graph to estimate.

i. The median mass

ii. The interquartile range

iii. The 40^{th} percentile

d. How many babies weighed more than 4.3kg?

SOLUTION a PAGE 2

✓ To find the cumulative number of babies corresponding to **≤3.0** add 15 and 20.

Mass (xkg)	≤1.5	≤2.0	≤2.5	≤3.0	≤3.5	≤4.0	≤4.5	≤5.0
№ of babies	0	3	15	35				100

✓ To find the cumulative number of babies corresponding to **≤3.5** add 35 and 24.

Mass (xkg)	≤1.5	≤2.0	≤2.5	≤3.0	≤3.5	≤4.0	≤4.5	≤5.0
№ of babies	0	3	15	35	59			100

SOLUTION a cont. PAGE 3

✓ To find the cumulative number of babies corresponding to **≤4.0** add 59 and 25.

Mass (xkg)	≤1.5	≤2.0	≤2.5	≤3.0	≤3.5	≤4.0	≤4.5	≤5.0

№ of babies	0	3	15	35	59	84		100

- ✓ To find the cumulative number of babies corresponding to ≤4.5 add 84 and 14.

Mass (xkg)	≤1.5	≤2.0	≤2.5	≤3.0	≤3.5	≤4.0	≤4.5	≤5.0
№ of babies	0	3	15	35	59	84	98	100

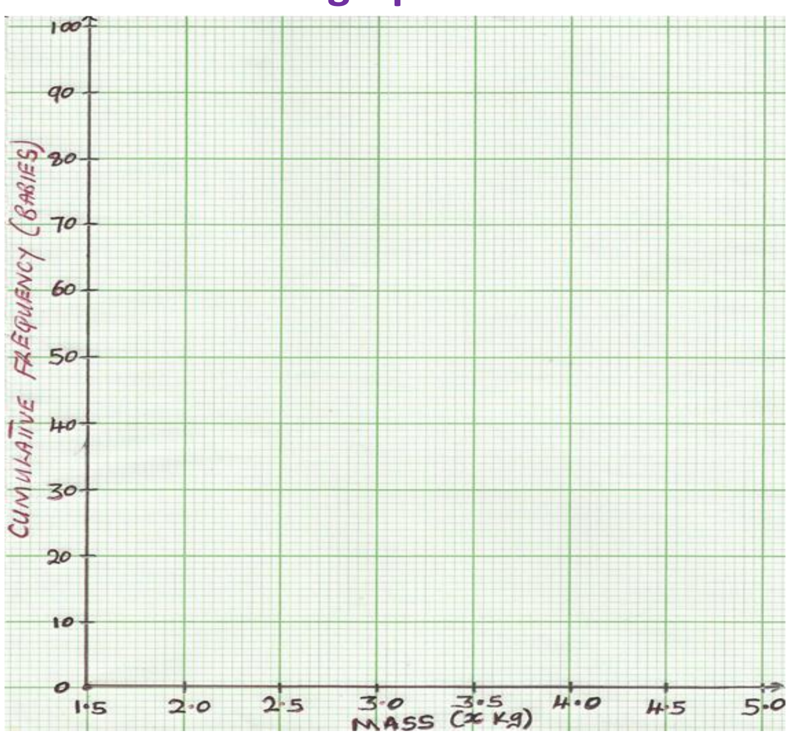
Pick points from the complete cumulative frequency table

Mass (xkg)	≤1.5	≤2.0	≤2.5	≤3.0	≤3.5	≤4.0	≤4.5	≤5.0
№ of babies	0	3	15	35	59	84	98	100

$$(1.5, 0), (2.0, 3), (2.5, 15), (3.0, 35),$$
$$(3.5, 59)(4.0, 84)(4.5, 98)(5.0, 100)$$

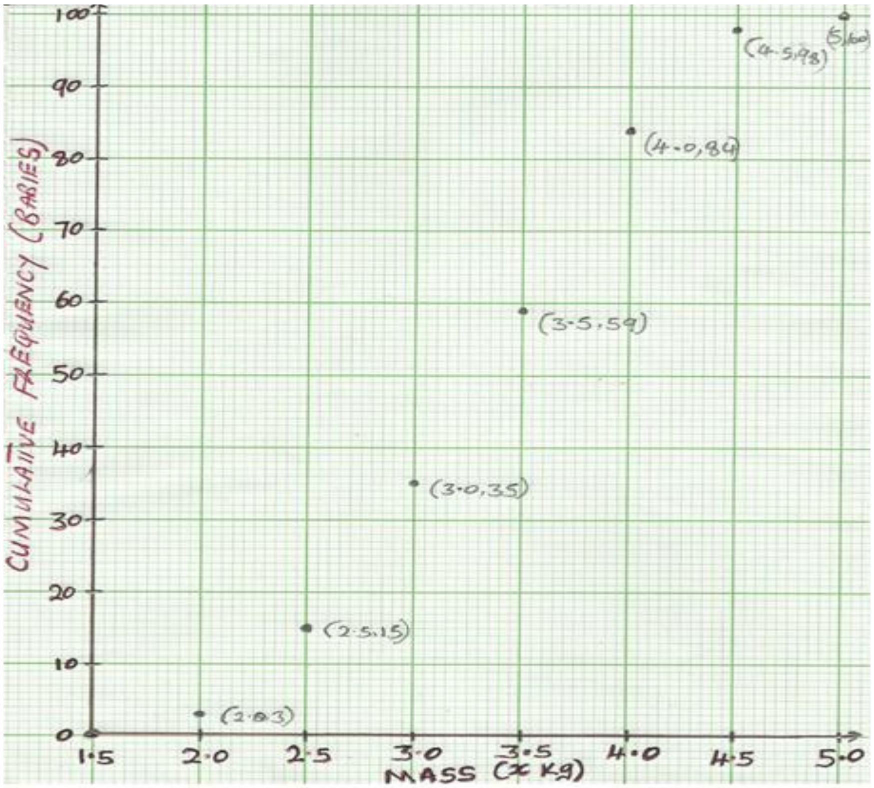
SOLUTION b PAGE 4

Scale the graph as follows



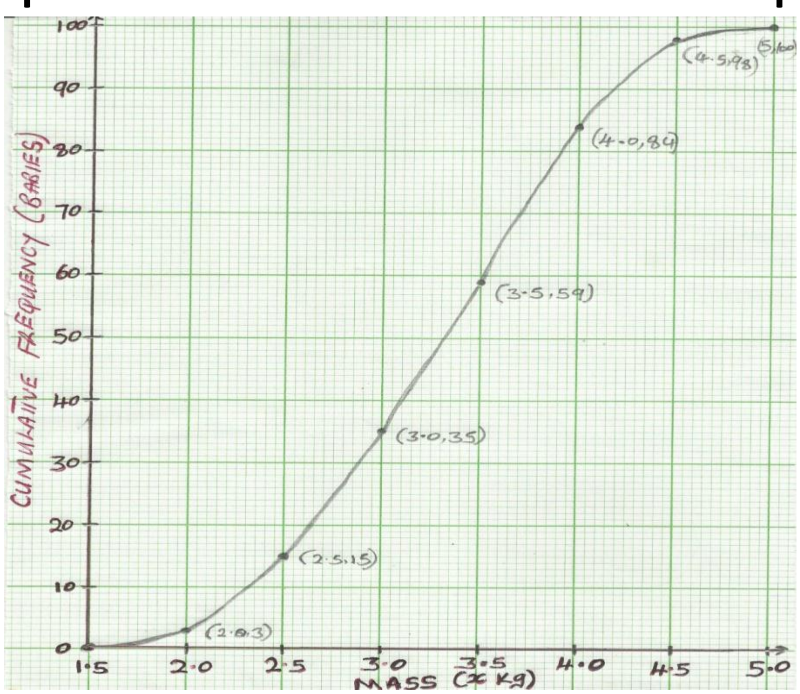
SOLUTION b cont. PAGE 5

Plot the points as follows;



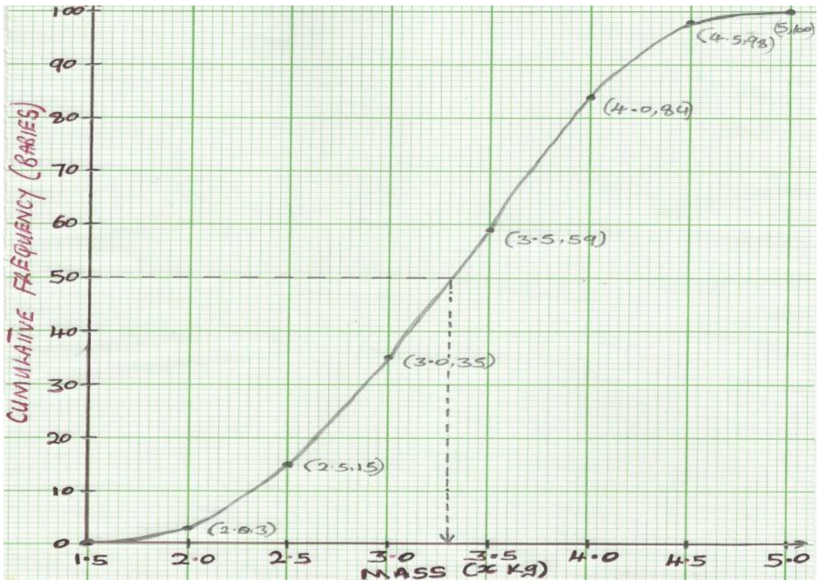
SOLUTION b cont. PAGE 6

Use a pencil and a free hand to connect the points



SOLUTION c i PAGE 7

To find the median mass you consider half of the total cumulative frequency. The total is 100 so $\frac{1}{2} \times 100 = 50$. Draw a dotted line from 50 to the curve, when it reaches the curve drop it down and pick the value on the mass line.



$\therefore \boldsymbol{Median = 3.3kg}$

SOLUTION c ii PAGE 9

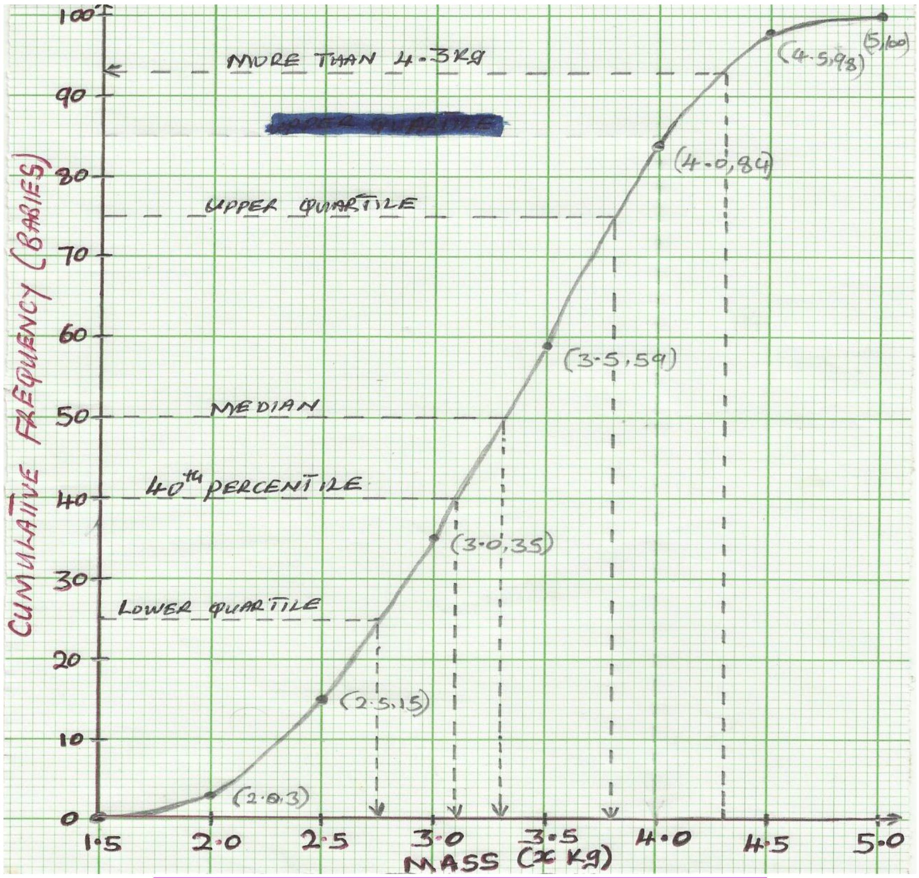
To find the interquartile range, you first find the lower quartile and upper quartile then subtract the lower from the upper.

To find the lower quartile mass you consider quarter of the total cumulative frequency. The total is 100 so $\frac{1}{4} \times 100 = 25$. Draw a dotted line from 25 to the curve, when it reaches the curve drop it down and pick the value on the mass line.

To find the upper quartile mass. Draw a dotted line from 75 to the curve, when it reaches the curve drop it down and pick the value on the mass line.

SOLUTION c ii cont. PAGE 10

Plot lower and upper quartile



$$\therefore \text{ lower quartile} = 2.75kg$$

$$\therefore \text{ Upper quartile} = 3.8kg$$

Continuation on the next page

SOLUTION c ii cont. PAGE 11

$$Interquartile\ range = (upper - lower)quartile$$

Substitute the values

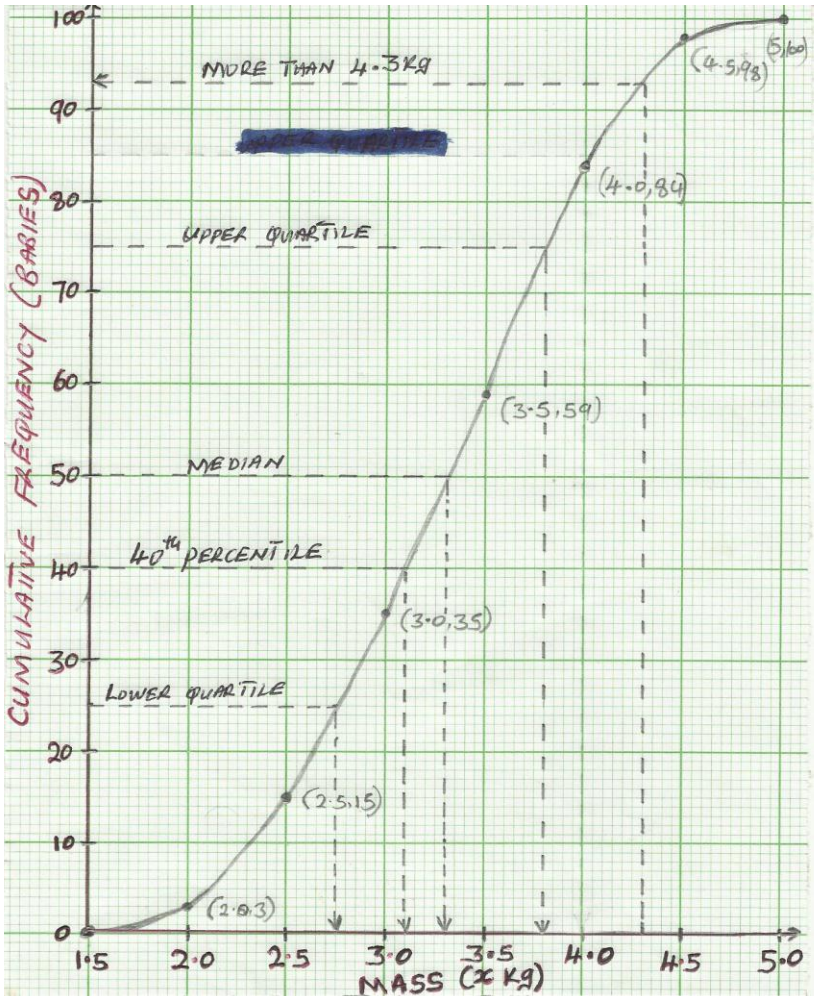
$$Interquartile = 3.8 - 2.75$$

Subtract

$$\therefore Interquartile\ range = 1.05kg$$

SOLUTION c iii PAGE 12

To find 40th percentile draw a dotted line from 40% of the total cumulative frequency, drop it when it reaches the curve and pick the value from the mass line.

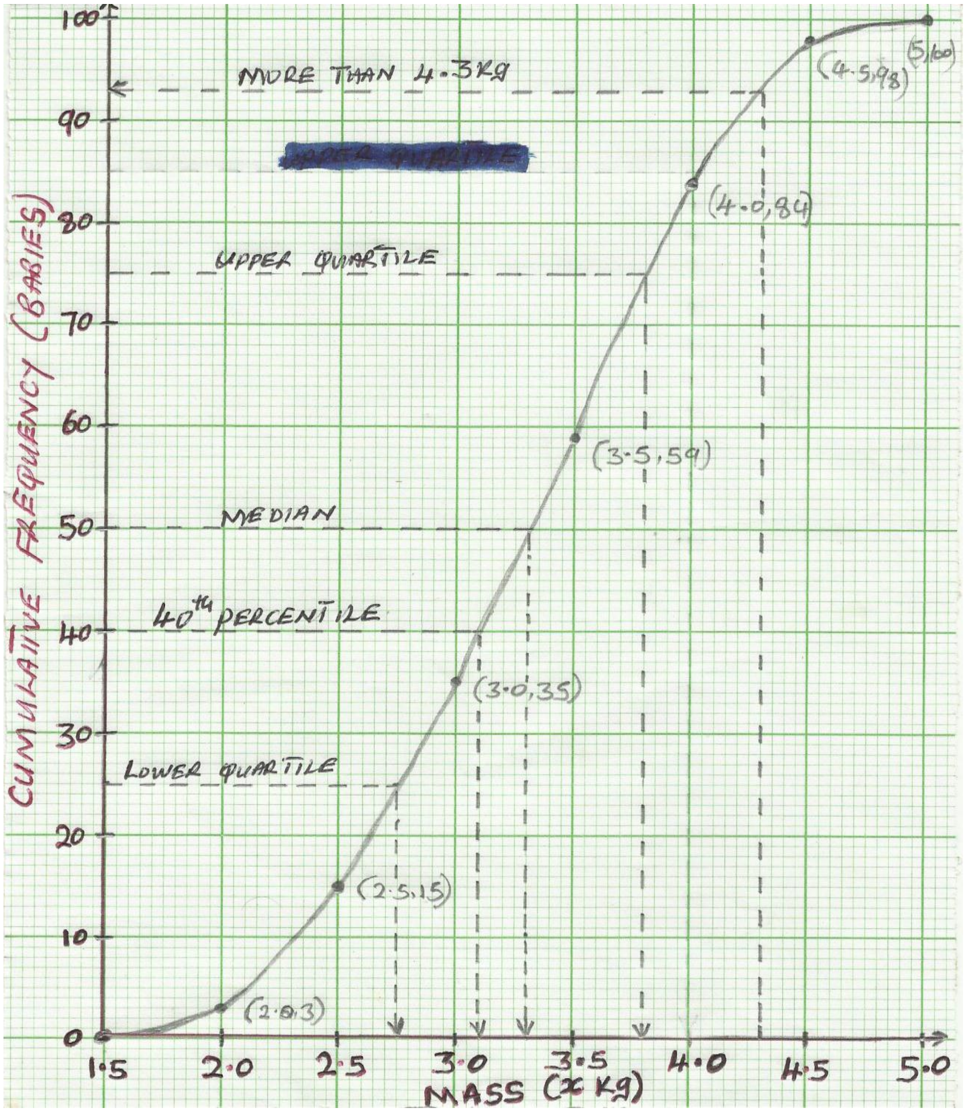


$$\therefore 40^{th} percentile = 3.1kg$$

From here connect to page 13

SOLUTION d PAGE 13

To find the number of babies who weighed more than 4.3kg, draw a dotted line from 4.3kg to the curve, then bend to the left and pick the value on the cumulative frequency line to be subtracted from the total frequency.



The dotted line reaches 93, to find the number of babies weighing more than 4.3kg. Subtract 93 from 100

$$\boldsymbol{more\ than\ 4.3 = 100 - 93}$$

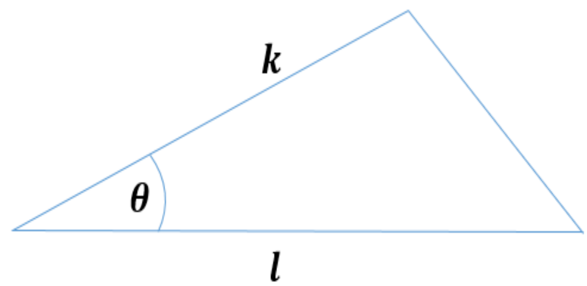
$$\boldsymbol{\therefore more\ than\ 4.3 = 7\ babies}$$

End of lesson 147

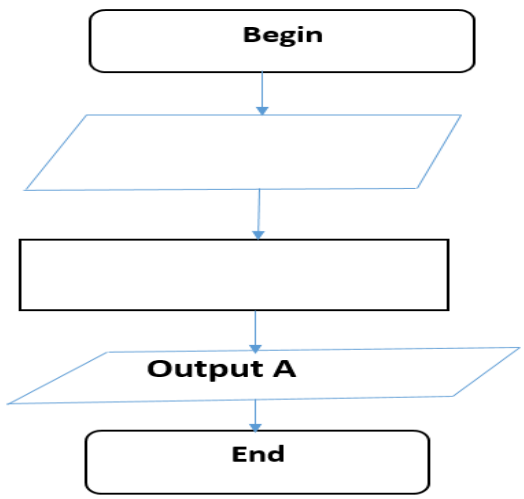
LERSSON 148 SPECIMEN QUESTION 20 P1 2016 (page 1)

a. Given that, $y = 2x^3 - \frac{3}{x^2}$, find $\frac{dy}{dx}$.

b. The diagram below is an incomplete program flow chart to calculate the area, A, of the triangle below



Complete the follow chart below by writing appropriate statements in the blank symbols.



Copy the question above, scroll down for answers and listen to possible audios following each answer page

SOLUTIONS (a page2)

Hints; **for the two terms in $y = 2x^3 - \frac{3}{x^2}$, the first term does not require any changes, but the second term has to be changed using indices**

$$\frac{3}{x^2} = 3 \times \frac{1}{x^2}$$

We can now express $\frac{1}{x^2}$ with a negative index we have

$$3 \times \frac{1}{x^2} = 3 \times x^{-2} = 3x^{-2}$$

So the equation becomes $y = 2x^3 - 3x^{-2}$

Hints; $\frac{dy}{dx}$ is coming from calculus, use the formula $\frac{dy}{dx} = nx^{n-1}$ n is power of x for $2x^3$ $n = 3$, for $3x^{-2}$ $n = -2$ remember the power n drops and multiplies the coefficient of x, the power is then subtracted by 1.

For $y = 2x^3 - 3x^{-2}$

(Observe colours to see where numbers are coming from)

$$\frac{dy}{dx} = (3)2x^{3-1} - (-2)3x^{-2-1}$$

Working out coefficients we have

$$\frac{dy}{dx} = 6x^{3-1} + 6x^{-2-1}$$

Working out powers we have $3 - 1 = 2$ and $-2 - 1 = -3$

$$\therefore \frac{dy}{dx} = 6x^2 + 6x^{-3}$$

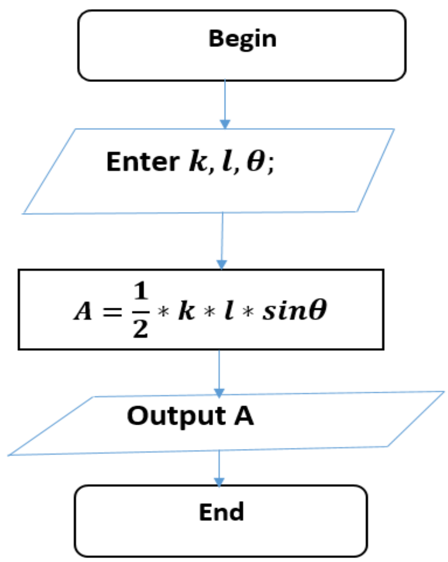
We have to express $6x^{-3}$ with a positive index, the negative on -3 becomes a numerator 1 of x^3

$$6x^{-3} = 6 \times x^{-3} = 6 \times \frac{1}{x^3} = \frac{6}{x^3}$$

$$\therefore \frac{dy}{dx} = 6x^2 + \frac{6}{x^3}$$

SOLUTIONS (b page 3)

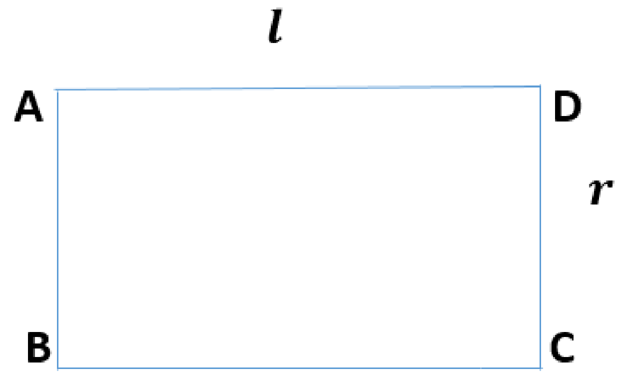
Hints; **determine the formula to use for finding the area of a triangle. You have two sides $\boldsymbol{k}$ and $\boldsymbol{l}$ adjacent (attached) to the angle $\boldsymbol{\theta}$. Use trigonometry formula $\boldsymbol{A = \frac{1}{2} \times a \times b \times sin\theta}$ for this formula, $\boldsymbol{a}$ and $\boldsymbol{b}$ are the sides adjacent to angle $\boldsymbol{\theta}$. So take a=k and b=l, so when the computer program begin, you have to feed it with the values of $\boldsymbol{k}$, $\boldsymbol{l}$ and $\boldsymbol{\theta}$ as indicated in the follow chart. Enter the formula $\boldsymbol{A = \frac{1}{2} * k * l * sin\theta.}$ In programming use * in place of $\times$. So the answer will be;**



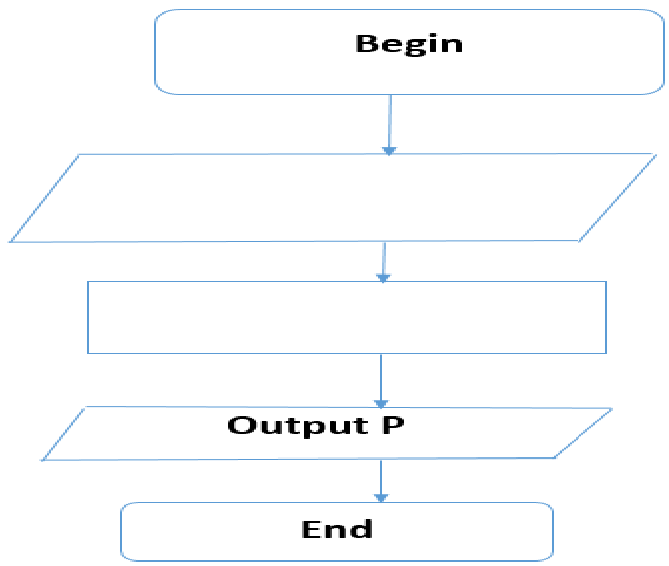
SIMILAR QUESTION FOR YOUR PRACTICE PAGE 5

a. **Find** $\int (3x^4 - 4x + 5)dx$

b. **The diagram below is an incomplete program flow chart to calculate the perimeter (P) of a rectangle ABCD where $CD = r, AD = l$**



Complete the flow chart below



END OF LESSON 148

LESSON 149 INTRODUCTION TO PERCENTAGES PAGE 1

A percentage is a fraction with 100 as its denominator, it is always calculated out of 100.

$N\%$ of K will be found as N over 100 time K.

$$N\%\ of\ K = \frac{N}{100} \times K$$

The percentage of a fraction is that fraction times 100, expressing $\frac{N}{K}$ as a percentage means.

$$\frac{N}{K} \times 100$$

Questions on the next page

QUESTIONS PAGE 2

1. Express $\frac{1}{4}$ as a percentage
2. A car was bought at K20 000 and later sold at K25 000. Calculate the percentage of the profit
3. In grade 12 class of 50 pupils, 30 are girls. Express the number of boys as a percentage of the total number of pupils in class.
4. A street vendor is paid a salary of K1200 per month plus $1\frac{1}{2}\%$ commission on sales. In one month he sold goods worth K3800. What were his total earnings for that month?
5. The price of shirt was K55 after an increase of 10%. What was the price of the shirt before the increment?
6. Express 65% as a fraction in its simplest form?
7. Bupe is a street vendor who sales hand bags. One day he sold a bag at K42 and made a loss of 30%. Calculate the cost price of the bag
8. After an increment of 15%, the population of a new town became 230 000. How many people were there before the increase?

SOLUTION 1 PAGE 3

To express a fraction as percentage, just multiply the fraction by hundred.

$$\frac{1}{4}\text{ as a percentage} = \frac{1}{4} \times 100$$

Divide 4 and 100

$$= 1 \times 25 \quad \therefore Ans = 25\%$$

SOLUTION 2 PAGE 4

First calculate the profit.

$$Profit = sales - cost$$

$$Profit = 25\,000 - 20\,000$$

$$\therefore Profit = K5\,000$$

$$Percentage = \frac{profit}{cost} \times 100$$

$$Percentage = \frac{5000}{20\,000} \times 100$$

$$Percentage = \frac{5}{20} \times 100$$

$$Percentage = 5 \times 5$$

$$\therefore profit\ Percentage = 25\%$$

SOLUTION 3 PAGE 5

First find the number of boys

$$boys = total\ number\ in\ class - girls$$

$$boys = 50 - 30$$

$$\therefore \boldsymbol{boys = 20}$$

To find the percentage of boys use

$$percentage\ of\ boys = \frac{boys}{total\ number\ in\ class} \times 100$$

$$percentage\ of\ boys = \frac{20}{50} \times 100$$

$$percentage\ of\ boys = 20 \times 2$$

$$\therefore percentage\ of\ boys = 40\%$$

SOLUTION 4 PAGE 6

$$total\ earnings = salary + commission$$

First calculate the commission

$$commission = 1\frac{1}{2}\% \times sales$$

$$commission = 1\frac{1}{2}\% \times 3800$$

We first express $1\frac{1}{2}\%$ as a simple fraction

$$1\frac{1}{2}\% = \frac{1\frac{1}{2}}{100} = \frac{\frac{3}{2}}{100} = \frac{3}{2} \div 100 = \frac{3}{2} \times \frac{1}{100} = \frac{3}{200}$$

$$commission = \frac{3}{200} \times 3800$$

$$commission = 3 \times 19$$

$$\therefore commission = K57$$

$$total\ earnings = 1200 + 57$$

$$\therefore total\ earnings = K1257$$

SOLUTION 5 PAGE 7

$$Price\ before = 55 - increment$$

First fine 10% (increment) of the price.

$$10\% \times 55 = \frac{10}{100} \times 55 = \frac{1}{10} \times 55 = \frac{1}{2} \times 11 = \frac{11}{2} = 5.5$$

$$\therefore increment = K5.5$$

$$Price\ before = 55 - increment$$

$$Price\ before = 55 - 5.5$$

$$\therefore Price\ before = 49.5$$

SOLUTION 6 PAGE 8

Know that $65\% = \frac{65}{100}$, Write in simplest form.

Divide by 5

$$\frac{65}{100} = \frac{13}{20}$$

$$\therefore 65\% = \frac{13}{20}$$

SOLUTION 7 PAGE 9

First find 30% of the price he sold at

$$30\%\ of\ 42 = \frac{30}{100} \times 42$$

Divide 10 into 30 and 100

$$\frac{3}{10} \times 42$$

Divide 2 into 10 and 42

$$\frac{3}{5} \times 21$$

Multiply and divide

$$\frac{63}{5} = K12.6$$

$$\therefore \text{Loss} = K12.6$$

To find the cost add 12.5 to 42

$$Cost = 42 + 12.6$$

$$\therefore Cost = 54.6$$

SOLUTION 8 PAGE 10

First find 15% of 230 000 then subtract from current population.

$$\frac{15}{100} \times 230\,000$$

Divide 100 and 230 000

$$15 \times 2300$$

Multiply

$$\therefore 15\%\ of\ 230\,000 = 34\,500$$

Now find the old population by subtraction

$$old\ population = 230\,000 - 34\,500$$

$$\therefore old\ population = 195\,500$$

End of lesson 149

LESSON 150QUETSION 6 P2 2016 PAGE 1

The equation of a curve is $y = x^3 - \frac{3}{2}x^2$. Find

i. The value of $\frac{dy}{dx} = 6$

ii. The equation of the normal where $x = 2$,

iii. The coordinates of the stationary point.

SOLUTION i PAGE 2

$$y = x^3 - \frac{3}{2}x^2$$

Multiply the power to x and subtract 1 from the power

$$\frac{dy}{dx} = 3x^{3-1} - \frac{3}{2} \times 2x^{2-1}$$

Subtract and divide on the fraction

$$\frac{dy}{dx} = 3x^2 - 3x^1$$

Ignore the power 1 and equate $\frac{dy}{dx} = 6$

$$3x^2 - 3x = 6$$

Divide throughout by 3

$$\frac{3x^2}{3} - \frac{3x}{3} = \frac{6}{3} \quad \Rightarrow \quad x^2 - x = 2$$

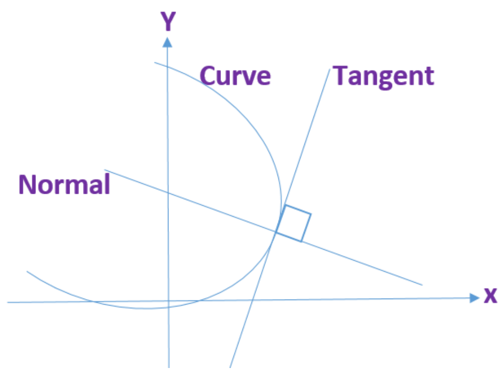
Factorise x

$$x(x-1) = 2$$

$\therefore x = 2$ and $\therefore x = -1$

SOLUTION ii PAGE 3

Explanation (page 2)



- The line touching the curve at one point is the tangent
- The line pointing into the curve is the normal line
- The tangent/curve and normal are perpendicular at the touching point, meaning they cut each other at 90°
- The product of gradient for tangent and gradient for normal is -1.

$$Gradient(tangent \times normal) = -1$$

From page 2 formula for gradient is

$$\frac{dy}{dx} = 3x^2 - 3x$$

Find gradient of the curve at $x = 2$ by substitution

$$\frac{dy}{dx} = 3(2)^2 - 3(2)$$

Multiply and subtract

$$\frac{dy}{dx} = 3(4) - 6 \quad \Rightarrow \quad \frac{dy}{dx} = 12 - 6$$

$$\therefore\ gradient\ of\ the\ curve = 6$$

Gradient of the curve multiplied by gradient of the normal gives negative 1

$$6 \times x = -1$$

Then you have

$$6x = -1$$

Divide both sides by 6

$$\frac{6x}{6} = \frac{-1}{6}$$

$$\therefore gradient\ of\ the\ normal = -\frac{1}{6}$$

Find the value of y from $y = x^3 - \frac{3}{2}x^2$ substitute $x = 2$

$$y = (2)^3 - \frac{3}{2}(2)^2$$

Multiply

$$y = 8 - \frac{3}{2}(4)$$

Divide 2 and 4

$$y = 8 - 3(2)$$

Multiply 3 and 2

$$y = 8 - 6$$

Subtract

$$\therefore y = 2$$

The general equation of a straight line (normal line) is $y = mx + c$ where m is the gradient and c constant. Find the value of c substituting $y = 2, x = 2$ and $m = -\frac{1}{6}$.

$$2 = -\frac{1}{6}(2) + c$$

Divide 2 and 6

$$2 = -\frac{1}{3} + c$$

Collect like terms

$$2 + \frac{1}{3} = c$$

Add on the left

$$\frac{6+1}{3} = c$$

$$\therefore c = \frac{7}{3}$$

Substitute into the equation

$$\therefore y = -\frac{1}{6}x + 2\frac{1}{3}$$

SOLUTION iii PAGE 4

At the stationary point $\frac{dy}{dx} = 0$ substitute

$$3x^2 - 3x = 0$$

Cross $-3x$ to the right

$$3x^2 = 3x$$

Divide both sides by $3x$

$$\frac{3x^2}{3x} = \frac{3x}{3x}$$

$$\therefore x = 1$$

Now you have to substitute for $x = 1$ in $y = x^3 - \frac{3}{2}x^2$

$$y = (1)^3 - \frac{3}{2}(1)^2$$

Multiply on powers

$$y = 1 - \frac{3}{2} \quad \Rightarrow y = \frac{2-3}{2} \quad \Rightarrow \quad \therefore y = -\frac{1}{2}$$

The stationary point will be (x, y)

$$\therefore Ans = \left(1, -\frac{1}{2}\right)$$

End of lesson 150