MATHEMATICS

Paper 1521/42
Paper 42 (Extended)

Key messages

To achieve well in this paper, candidates need to be familiar with all aspects of the extended syllabus.

The recall and application of formulae and mathematical facts to apply in varying situations is required as well as the ability to interpret situations mathematically and problem solve with unstructured questions.

Candidates must learn to hold accurate values in their calculators when possible and not to approximate during the working of a question. If they need to approximate, then they should use at least four significant figures.

General comments

Candidates scored across the full mark range for this paper and many demonstrated a good understanding of key areas of the syllabus.

Candidates generally performed well on the more routine questions but had more difficulty when questions required interpretation of a topic or involved an element of problem solving.

Many candidates gave answers to the required accuracy, but some rounded intermediate values in multi-stage calculations which led to inaccurate final answers. This was seen often on **Question 2** and **Question 8**.

Most candidates showed full working with their answers and thus ensured that method marks were considered where answers were incorrect.

The areas that caused most difficulty were problem-solving with bounds, vectors, setting up and simplifying algebraic fractions, conditional probability, using precise language in geometric reasoning, problem-solving with area and perimeter and reasoning with turning points.

The areas where candidates scored well were ratio, sine rule and cosine rule, mid-point of a line, drawing algebraic graphs, reverse percentage, finding an estimate of the mean and interpreting a cumulative frequency diagram.

Comments on specific questions

Question 1

(a) (i) Most candidates answered this correctly. The most common wrong answer was \$1.85, which came from $37 \div 20$.

(ii) Many candidates showed all of the stages and steps required to reach \$124. Most candidates scored a method mark for 6×8. Fewer scored the second method mark which required the whole calculation, $5 \times 3 + 24$, to be evident. Common errors included writing 48 or $15 + 24$ without the multiplications being explicitly shown, calculating $24 \times 3 + 5 \times 3$, or omitting to add on 37.

(iii) Many candidates answered this question correctly. Other candidates scored one mark for working out $\frac{800}{124}$ but then left a non-integer number of weeks, usually 6.45 weeks, as their final answer. Some candidates merely wrote 6.4 without evidence of a clear method and did not score any marks.

(b) Almost all candidates answered this question correctly. The occasional errors that were seen included arithmetic slips when adding 5 + 4 + 6 or finding the value of one part, 23, but not progressing further, or finding the compost used in one of the other two gardens.

(c) Several candidates answered this question correctly. However, a significant number of candidates scored only one mark because they worked out the upper bound of the difference as Upper Bound − Upper Bound $(355 - 245 = 110)$ rather than Upper Bound − Lower Bound $(355 - 235 = 120)$. Other errors included using bounds such 350.5 and 240.5 or 360 and 240. A further common misunderstanding was to first calculate the difference as $350 - 240 = 110$ and then apply the bounds.

Question 2

(a) Most candidates correctly used the sine rule to find angle *MCB*. The most common error was to lose accuracy by approximating during the working, and these candidates usually scored 1 or 2 marks depending on whether they showed full working. Other errors included rearranging their equation incorrectly, using sine of the lengths instead of the angles, or using $\sin C = \frac{15}{31}$.

(b) The majority of candidates used the cosine rule on triangle *AMB* and most then found *AM* to the required accuracy. A common error was to correctly write $15^2 + 18^2 - 2 \times 15 \times 18\cos 76$ but then evaluate this incorrectly as $(15^2 + 18^2 - 2 \times 15 \times 18)\cos 76$. Other errors seen including forgetting to square root or losing accuracy during the working of the question or using methods that assumed angle *AMC* = 90°. Some candidates used longer methods such as dropping a perpendicular from *M* to *AB* and using trigonometry and Pythagoras, or finding *BC* and using the cosine rule for triangle *MAC*.

(c) A number of candidates found the area of triangle *MAC* to the required accuracy using $\frac{1}{2}ab\sin C$. Most found and correctly summed the area of the two small triangles. Others were successful in finding the area of the large triangle, *MAC*, after first finding length *BC*. The most common errors were in loss of accuracy or only finding the area of one of the small triangles.

Question 3

(a) (i) This was very well answered with most candidates giving the correct coordinates. The occasional errors seen were usually from errors in arithmetic, forgetting to divide by 2, subtracting rather than adding, or giving the coordinates in reverse.

(ii) This was often well answered with the length of *AB* frequently given to the required accuracy. The most common errors came from substituting values into incorrectly learned formulae. These incorrect formulae often had errors in signs, or the squares were either omitted or in the wrong places. Other candidates had the correct formula but substituted incorrectly. Arithmetic errors were seen often with the negative signs.

(iii) There were many clear, accurate and mathematically well-presented solutions to this question. Successful candidates usually showed clearly the three key steps, namely, finding the gradient of *AB*, finding the gradient of the perpendicular and substituting (9, 4) into $y = mx + c$

$\frac{2}{3}$ such as 0.66. A common

$4 = \frac{2}{3} \times 9 + c$ but then incorrectly state $12 = 18 + c$ and hence $y = \frac{2}{3}x - 6$.

(b) Only a very small number of candidates completed this question correctly. A diagram was a good way to start this question and whilst many candidates attempted to draw a diagram it frequently did not have *P*, *Q* and *X* on a straight line, in that order. This error came from not combining the phrase '*PQ* is extended to point *X*' carefully with the ratio. Candidates often scored one mark for stating $\overrightarrow{PQ} = \boldsymbol{t} - \boldsymbol{s}$ or $\overrightarrow{OX} = \overrightarrow{OP} + \overrightarrow{PX}$ but a significant number of candidates did not incorporate an origin on their diagram and it was evident that they did not understand the phrase 'position vector'. The majority of candidates made no further progress because they dealt with the ratio *PX* : *QX* = 7:3 incorrectly, with statements such as $7PX = 3QX$ or $QX = \frac{3}{7}PQ$ or $QX = \frac{3}{10}PX$ commonly seen.

Question 4

(a) Almost all candidates correctly completed the table.

(b) There were some very good graphs. The curve shape was rarely distorted near the ends where $x = -0.2$ and $x = 0.2$ and only a few candidates let the curve slip out of the range or gave excessive vertical parts to the curve. The main errors were in the plots at $x = -0.2$ and 0.2. Only a few candidates joined the two branches of the curve across the *y*-axis.

(c) Given that there were many correct or almost correct graphs, some answers were out of range when quoted to only one decimal place. Some candidates did not use their graph to solve the equation and attempted an algebraic solution.

(d) (i) Most drew an accurate ruled line. The main errors were to misplot one of the points at (–1, 1.75) or (1,1.75). Some candidates plotted correctly but then missed these points when drawing the line. Where an incorrect line was drawn it was usually with a negative gradient through the origin. There were a number of omissions for this part.

(ii) A good graph and straight line usually gave a correct answer, 0.7 and –0.7 being the most common. Errors here were the result of inaccuracy with the line or the graph.

(iii) This was very well answered. The most common error here was in the first step where some had $3y = 5x$ resulting in $y = \frac{5x}{3}$ as the incorrect answer.

(iv) An algebraic attempt here was very rarely seen and the majority of candidates used one or both of their solutions from **part (d)(ii)** to get an answer. This was a misunderstanding of the question which did not ask them to use their graph but was asking them to show set up and solve algebraic equation linked to **part (d)(iii)**.

Question 5

(a) (i) This was well answered. A few candidates truncating the answer to 0.62 without showing a more accurate value. A few did $\frac{8}{5}$ or 8×5.

(ii) Many answered this well. Few candidates used the area of a trapezium, preferring to use the three separate areas. The most common error was to do 5×54. Other errors included the loss of $\frac{1}{2}$ when calculating one of the triangle areas.

(b) Candidates rarely used the trapezium formula for the area and instead used the three separate areas. The letter *v* was often used in the area calculations when setting up the equation but when a letter was not seen it resulted in an error. The most common error was to use $240 \div 45$.

Question 6

(a) (i) A majority of candidates were able to put the information into a compound interest formula using a decrease. A correct answer was then seen, but many did not round the answer to a whole number to represent the number of people. Some candidates attempted a percentage increase or used a simple interest approach. Some attempted a stepped 18 per cent decrease, year on year, and this was rarely successful.

(ii) Many candidates obtained the correct answer of 17. A large number of candidates were able to set up a correct first step, equating the compound decrease to 1000, and a number of candidates reached 16 or 16.2 by trials but then did not interpret this in the context of the question. Although using logarithms is not a topic on the syllabus, several candidates used this method successfully and were given full credit. A few did who struggled with **part (a)(i)** were unable to set up a correct compound decrease statement to begin with.

(b) Very few candidates scored full marks**.** Most candidates were successful in converting dollars to euros, with a few preferring to convert euros to dollars. The answers \$19.5 or \$19.53 were the most common, with few candidates correctly rounding to the nearest cent. The most common errors were to calculate incorrectly with 469×0.9046 or $538 \div 0.9046$.

(c) The majority of candidates were successful in recognising and calculating a reverse percentage. The most common error was to increase the sale price by 16 per cent.

Question 7

(a) (i) Most candidates were able to identify the midpoints and use the correct method to calculate an estimate of the mean. Some gave an answer to 2 significant figures only and others truncated the answer to 5.11 rather than rounding to 5.12. The most common errors were to use a midpoint of 1 rather than 0.5 for the first group or 6 rather than 6.5 for the fourth group. Most candidates showed clear working allowing method marks to be awarded if the final answer was incorrect. Only a small number of candidates used values other than the midpoints in their products, usually the group width, or divided by the number of groups rather than the total frequency.

(ii) Many candidates found the correct frequency densities and attempted to draw the correct histogram. Some heights were drawn inaccurately, for example a height of 84 in place of 83. It was common for candidates to draw the final bar with the incorrect width, continuing to the edge of the graph paper, rather than drawing the vertical line at $t = 10$.

(b) (i)(a) Most candidates found the median correctly. The most common incorrect answer was 30, the cumulative frequency required for the median.

(i)(b) Many candidates found the interquartile range correctly. Some misread one of the values, but usually wrote down the upper and lower quartiles so a mark and earned a mark for this. A small number gave either the upper or lower quartile as their answer. Some candidates identified frequencies of 15 for the lower quartile and 45 for the upper quartile then subtracted these to give 30 and then read the graph at that point, which gave the median as the answer rather than the interquartile range.

(i)(c) Many candidates gave the correct answer in this part, with a small number giving the answer 54 rather than 6.

(ii)(a) The most common response here was to complete the table with the cumulative frequencies rather than the frequencies. Some candidates who did use frequencies made errors with one or more of them.

(ii)(b) Candidates found this question very challenging. Some identified that there were 55 people who spent more than \$1 and that 12 of these spent more than \$6, even when they had completed the table using cumulative frequencies. The correct probability of $\frac{12}{55}$

$\frac{12}{55} \times \frac{11}{54}$, although some incorrectly multiplied this product by 2. Some candidates did not appreciate the conditional aspect of the question and used the total frequency of 60 as the denominator rather than 55.

Question 8

(a) (i) Few candidates gave a correct reason in this part that referred to both the radius and the tangent. Reasons often mentioned the tangent but were imprecise and were not given credit: answers such as '*the angle from the centre to the tangent is 90°*' or '*the tangent forms a right-angled triangle on the circle*' were common. Weaker candidates did not mention either the radius or the tangent and simply mentioned '*perpendicular*' or '*90°*'.

(ii) Many candidates gave the correct answer of 30°. The common incorrect answers were 60°, usually from identifying an angle other than *OAM*, 45° and 90°.

(b) Many candidates calculated *AM* correctly using trigonometry. Those who had given the answer 60° in the previous part often found the correct value here because their 60° was either angle *AOM* or angle *PAM*. Correct methods using either tangent or the sine rule were often seen here, although a small number of candidates used cosine in place of tangent. Very few candidates used a Pythagoras method. Some candidates rounded their answer incorrectly to 8.67 or gave 8.7 without a more accurate value and others truncated to 8.6 rather than rounding to 8.66 as required by the rubric.

(c) There were many possible approaches to find the shaded area and the most successful was to find the area of triangle *ABC*, subtract the area of the circle and then divide the answer by 3. Some long methods involved intermediate rounding and led to an answer just outside the acceptable range. Candidates who tried to use the small triangles and sector to find the area often made errors, for example subtracting the area of the sector from the area of triangle *APM* rather than from quadrilateral *OPAM*. Some candidates used their *AM* as the radius of the circle rather than the given value of 5 cm. Most candidates who attempted this part scored at least one method mark for correctly finding the area of the circle or of one small triangle.

(d) Candidates were more successful in finding the perimeter than the area, possibly because there were fewer parts to consider. Many identified that the arc length was one third of the circumference of the circle and added the lengths of the straight sides on to this, although some added three times their value for *AM*. As in the previous part, some candidates used their *AM* as the radius of the circle.

Question 9

(a) (i) Many candidates identified the angle correctly and the correct reason of vertically opposite angles or opposite angles was often stated. Some candidates were unable to state the reason clearly and answers such as '*vertical angles*' or '*opposite sides*' were seen. Common incorrect answers included corresponding angles, alternate angles, same segment or stating that the triangles were similar. A number of candidates gave the angle as *BXD* rather than *BXC*.

(ii) Many candidates identified the correct angle and stated the correct reason of angles in the same segment. Some stated angles on the same arc, which was acceptable, but others stated angles on the same chord, which is not acceptable. Reasons such as '*the butterfly rule*' are not acceptable and reasons such as '*angles touching the circumference*' are imprecise.

(iii) Most candidates correctly stated similar here. The most common incorrect answers were congruent, equal, opposite and parallel.

(iv) Many candidates used the ratio correctly to find *CX* correctly. Candidates who knew the triangles were similar sometimes set the ratio up incorrectly, starting with $\frac{4.6}{CX} = \frac{2.4}{1.6}$ which led to the answer 3.07, and some started with the correct ratio but rearranged it incorrectly. A common misconception was to find the difference between *AX* and *DX*, then add this to *BX* which led to *CX* = 5.4.

(v) Those candidates who had answered the previous part correctly often found the correct answer here. Common errors were to find $7.2\times\left(\frac{2.4}{1.6}\right)^2$ rather than $7.2\times\left(\frac{1.6}{2.4}\right)^2$ or to multiply by the scale factor rather than the scale factor squared. A small number of candidates used the square root of the scale factor. A few long methods were seen, using the sine formula to find angle *BXC* and then using this with the given sides from triangle *AXD* which led to the correct answer if sufficient accuracy was retained. A common misconception was to use $\frac{1}{2}$ base $\times$ height with the given values of *AX* and *DX*.

(b) Many candidates started by setting up the relationship $\frac{448}{189}=\left(\frac{h}{12}\right)^3$ and they were often able to manipulate this to reach the correct answer of 16. Some rounded values found in their working which led to an inaccurate final answer of 15.99. Common errors were to invert the ratio of the volumes or to start with $\left(\frac{448}{189}\right)^3=\frac{h}{12}$, $\frac{448}{189}=\left(\frac{h}{12}\right)^2$ or $\frac{448}{189}=\frac{h}{12}$.

Question 10

(a) Many candidates were successful in this part. As this was a show that question, it was important that candidates showed full working including all of the relevant products to arrive at the given answer. Errors occurred when some of these products were omitted

(b) There were many correct answers, almost all of which used a full volume calculation $5\times 12\times 18\times 3600$ for the volume of water in the channel in winter and then compared this to the summer volume. Some arrived at 750 per cent by not finding the difference in the winter and summer volumes.

The most common error when finding the percentage increase was to use the winter volume as the denominator in the percentage calculation rather than the summer volume.

(c) Candidates found this part challenging and there was some confusion about the increase in depth of 4 mm in the vertical cylinder storage tank with a number of candidates confusing this with an increase of 4 mm in the depth in the channel and using 3.4 cm as the cylindrical depth rather than 0.4.

Candidates were given credit for setting up a correct equation for the cylinder with the volume 518 400 using a depth of 4 mm. Incorrect attempts at unit conversion were ignored at this point and method marks were awarded. Many were able to manipulate their equations to find the radius but earlier errors in unit conversion often resulted in wrong answers.

The most common errors were to use a depth other than 4 mm when setting up the initial equation or to use an incorrect formula for the volume of the cylinder.

Question 11

(a) The majority of candidates recognised that the starting point would be to find the derivative of the equation of the curve. There were some errors in finding the derivative. A number went on unnecessarily to find and use the second derivative in this part.

The next step, equating the derivative to zero, was not always explicitly stated by candidates and if this was then followed by incorrect solutions or spurious working to solve the cubic, then this mark was not awarded. Candidates are advised to make a clear statement here.

A number were successful in finding the coordinates of the three turning points although some found the *x*- values but then overlooked the *y*- values.

Some having set up a correct equation with the first derivative were unable to solve it with the most efficient method of factorisation, and a number that attempted to use the quadratic formula made errors in one or more of the solutions.

(b) A small number of candidates were successful here. Almost all that made a reasonable attempt used the second derivative as the method. Credit was given for correctly finding the second derivative and then evaluating the value of this derivative for the three turning points. Some candidates having done this did not clearly explain how this value determined the nature of the turning points, which was essential to earn full credit.

There were a number of omissions in this part as well as many candidates that were not able to use a reasonable method.