Cambridge IGCSE

Cambridge International Examinations
Cambridge International General Certificate of Secondary Education

CANDIDATE NAME	
CENTRE NUMBER	
CANDIDATE NUMBER	

CAMBRIDGE INTERNATIONAL MATHEMATICS **0607/61**

Paper 6 (Extended) **May/June 2018**

1 hour 30 minutes

Candidates answer on the Question Paper.

Additional Materials: Graphics Calculator

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
Do not use staples, paper clips, glue or correction fluid.
You may use an HB pencil for any diagrams or graphs.
DO **NOT** WRITE IN ANY BARCODES.

Answer both parts **A** (Questions 1 to 4) and **B** (Questions 5 to 9).
You must show all the relevant working to gain full marks for correct methods, including sketches.
In this paper you will also be assessed on your ability to provide full reasons and communicate your mathematics clearly and precisely.
At the end of the examination, fasten all your work securely together.
The total number of marks for this paper is 40.

DC (LK/SG) 153611/3

CAMBRIDGE International Examinations

[Turn over

Answer **both** parts **A** and **B**.

A INVESTIGATION (QUESTIONS 1 to 4)

LARGEST PRODUCTS (20 marks)

You are advised to spend no more than 45 minutes on this part.

This investigation looks at finding the largest product when two or more positive integers have a given sum.

For the positive integers 2 and 5

- the sum $2 + 5$ is 7
- the product 2×5 is 10.

1 (a) (i) Complete the table for all the **different** pairs of positive integers that have a sum of 8.

Integers		Sum	Product
1		8	
2		8	
3	5	8	
4		8	16

Write down the calculation that gives the largest product.

.....

(ii) Find the largest product of two positive integers that have a sum of 20.

.....

Write down the calculation that gives this product.

.....

(b) (i) Complete this table for all the **different** pairs of positive integers that have a sum of 9.
Note that 2 and 7 is the same pair as 7 and 2.

Integers		Sum	Product
1		9	
		9	
		9	
		9	

Write down the calculation that gives the largest product.

.....

(ii) Find the largest product for two positive integers that have a sum of 21.

.....

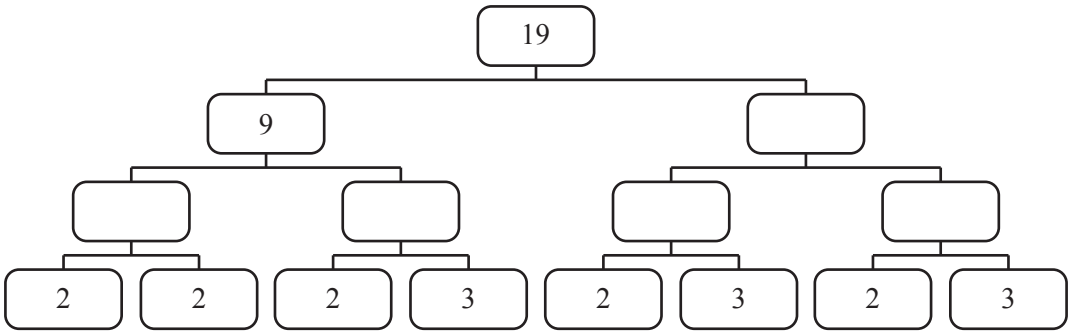
Write down the two positive integers that give this product.

.....

2 Tom wants to find the set of positive integers that have a sum of 19 and give the largest product.
He uses the ideas from **question 1** and the following method.

- Write 19 in the top box of the diagram.
- Write the two positive integers that give the largest product in the two boxes on the next row.
- Complete the remaining boxes in the same way.

(a) Complete the diagram.



The largest product using Tom's method is

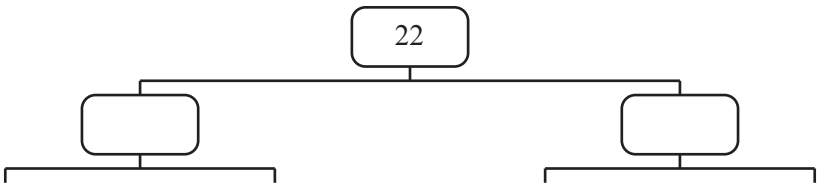
$$2 \times 2 \times 2 \times 3 \times 2 \times 3 \times 2 \times 3 = 2^5 \times 3^3 = 864.$$

(b) Explain why Tom does not replace the 2s and the 3s to make another row in the diagram.

.....

.....

(c) Complete the diagram and find the largest product that Tom's method gives for positive integers that have a sum of 22.



.....

3 In **any** set of positive integers, the replacement of a set of three 2s with a set of two 3s will not change the sum.

(a) Explain why this is correct.

.....

(b) Explain why this replacement will **always** increase the product of a set of positive integers.

.....

.....

(c) Here is Tom's result from **question 2(a)**.

$$2 \times 2 \times 2 \times 3 \times 2 \times 3 \times 2 \times 3 = 2^5 \times 3^3 = 864$$

(i) Use the replacement to calculate the largest product of a set of positive integers that have a sum of 19.

.....

(ii) Write your answer to **part (i)** as a product of powers of 2 and 3.

.....

4 **(a)** The sum of a set of positive integers is N.
After replacing each set of three 2s in Tom's method with two 3s, the largest product will be $2^x \times 3^y$.

(i) Find, in terms of x and y, a formula for the sum N.

.....

(ii) Explain why x can only be 0, 1 or 2.

.....

.....

(b) Use **part (a)** to find the largest product of a set of positive integers that have a sum of

(i) 60,

.....

(ii) 62.

.....

(c) 6 377 292 is the largest product of a set of positive integers that have a sum of N.

Find the value of N.

.....

B MODELLING (QUESTIONS 5 to 9)

COUNTING PRIME NUMBERS (20 marks)

You are advised to spend no more than 45 minutes on this part.

Prime numbers have only two factors: 1 and the number itself.
In this task you will use models for the number of primes in a given range.

The function P(x) gives the number of primes that are less than x.

For example, P(10) = 4 because there are exactly four primes that are less than 10.
These four primes are 2, 3, 5 and 7.

5 **(a)** Show that P(20) = 8.

(b) P(40) = 12. Find P(50).

.....

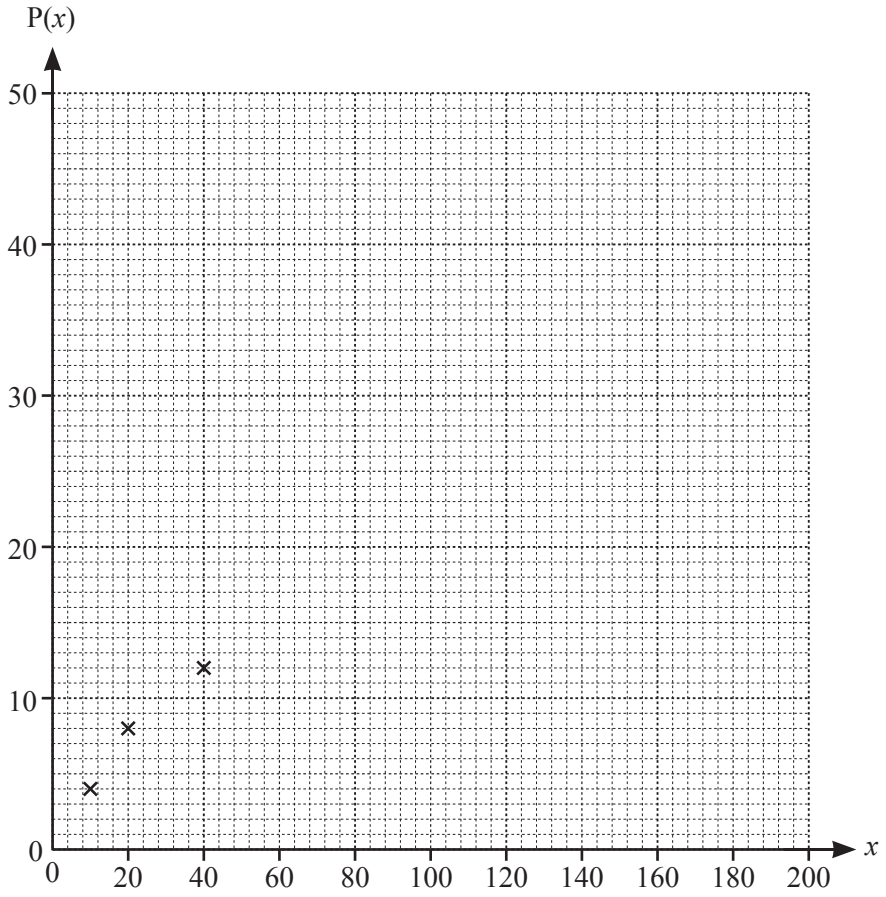
(c) P(100) = 25. Find P(90).

.....

6 Use your answers to **question 5** to complete the table.

x	10	20	40	50	90	100	140	200
P(x)	4	8	12			25	34	46

On the grid plot this information. The first three points have been plotted for you.



7 A model, L(x), for the number of primes less than x, is a straight line that passes through the points (40, 12) and (140, 34).

(a) Find the equation of the line.

.....

(b) Use your model to estimate the number of primes that are less than 1000.

.....

8 A quadratic model, Q(x), for the number of primes that are less than x, is

$$Q(x) = kx - 0.001x^2 .$$

(a) The model estimates that there are 34 primes that are less than 140.

Find the value of k, giving your answer correct to 1 decimal place.

.....

(b) The model is quite accurate for the number of primes when $x < 200$.

Why is the model unsuitable for the number of primes when $x > 200$?

.....

.....

9 A very good model for the number of primes less than x, where x is less than 1000, is

$$\mathrm{N}(x) = \frac{x}{(2\log_{10} x)}.$$

(a) Use the model to find how many primes there are that are

(i) less than 400,

.....

(ii) less than 800.

.....

(b) Use your answers to **part (a)** to complete the table.

	$0 \leqslant x < 200$	$200 \leqslant x < 400$	$400 \leqslant x < 600$	$600 \leqslant x < 800$
Number of primes	44		31	

Question 9(c) and 9(d) are printed on the next page.

(c) The model is still a good model for $x \geqslant 1000$.

(i) A number is chosen at random from the numbers 1 to 10 000.

Use the model to estimate the probability that it is prime.

.....

(ii) A number is chosen at random from numbers in the range 1 to x.

Find, in its simplest form, a model for the probability that it is prime.

.....

(d) Use the model $\mathrm{N}(x) = \dfrac{x}{(2\log_{10} x)}$ to estimate the 100th prime.

.....