# Nov. / Dec. 2016 

$2 \frac{2}{3}$ hours.


## INSTRUCTIONS TO CANDIDATES:

Answer all the eight questions in section A and only four questions in section B. Any additional question(s) will not be marked.

Each question in section A carries 5 marks while each question in section $\mathbf{B}$ carries 15 marks.

All working must be shown clearly.
Begin each answer on a fresh sheet of paper.
Where necessary, take acceleration due to gravity $g=9.8 \mathrm{~ms}^{-2}$.
Graph paper is provided.
Silent non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

SECTION A: (40 MARKS) Answer all the questions in this section.

1. Given that $(x+1)$ and $(x-2)$ are factors of the polynomial $a x^{3}-3 x^{2}-b x+2$, find the values of $a$ and $b$.
(05 marks)
2. The table below shows the oral interview rank $(X)$ and written interview rank $(Y)$ for 12 candidates.

| Candidate | $A$ | $B$ | $C$ | $D$ | $E$ | $F$ | $G$ | $H$ | $I$ | $J$ | $K$ | $L$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Oral interview <br> Rank ( $X$ | 8 | 10 | 9 | 4 | 12 | 5 | 11 | 7 | 3 | 6 | 1 | 2 |
| Written Interview <br> Rank (Y) | 11 | 12 | 9 | 7 | 10 | 6 | 8 | 5 | 2 | 4 | 1 | 3 |

Calculate Spearman's rank correlation coefficient and comment on your result.
(05 marks)
3. The sum to infinity of a Geometric Progression (GP) is $\frac{25}{4}$ and the first term is 5 . Find the
(a) common ratio of the GP.
(03 marks)
(b) sum of the first ten terms of the GP.
(02 marks)
4. The table below shows the number of crates of soda sold by a certain shop in 2010.

| MONTH | JAN | FEB | MAR | APR | MAY | JUN | JUL | AUG | SEPT | OCT | NOV | DEC |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| NUMBER OF <br> CRATES | 175 | 783 | 351 | 228 | 378 | 297 | 823 | 338 | 230 | 391 | 410 | 742 |

Calculate the four - month moving averages for the data.
(05 marks)
5. Determine the coordinates of the stationary point of the curve

$$
y=\frac{1}{4} x^{2}-2 x-5
$$

(05 marks)
6. Two independent events $A$ and $B$ are such that $P(A)=\frac{1}{4}$ and $P(B)=\frac{3}{5}$.

Find $P(A \cup B)$.
(05 marks)
7. Given the matrices $A=\left(\begin{array}{cc}3 & 5 \\ -2 & 4\end{array}\right)$ and $B=\left(\begin{array}{cc}8 & -3 \\ -4 & 7\end{array}\right)$, find
(a) matrix $C$ such that $3 A-2 C+B=I$, where $I$ is a $2 \times 2$ identity matrix.
(b) the determinant of $\boldsymbol{C}$.
8. A car of mass 2000 kg ascends an incline of $\sin ^{-1}\left(\frac{1}{10}\right)$ to the horizontal. The resistance force to the motion of the car is 1000 N . The power of the car engine is $59,200 \mathrm{~W}$. Calculate the maximum speed of the car. ( 05 marks)

## SECTION B: (60 MARKS)

Answer only four questions from this section.
9. The table below shows a frequency distribution of marks scored by 55 students in a test.

| Marks | $10-$ | $20-$ | $30-$ | $40-$ | $50-$ | $60-$ | $70-$ | $80-\leq 90$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Number of students | 2 | 6 | 12 | 15 | 10 | 6 | 3 | 1 |

(a) Draw a histogram for the data and use it to estimate the modal mark.
(b) Calculate the
(i) mean mark.
(ii) standard deviation.
(10 marks)
10. Chemical $A$ is converted into another chemical by a chemical reaction. The rate at which chemical $A$ is being converted is directly proportional to the amount present at any time. Initially 100 g of chemical $A$ was present. After 5 minutes, 90 g of $A$ is present.
(a) Form a differential equation for the chemical reaction.
(03 marks)
(b) By solving the differential equation formed in (a), determine the
(i) amount of chemical $A$ present after 20 minutes.
(ii) time taken for the amount of chemical $A$ to be reduced to 20 g .
(12 marks)
11. The table below shows the prices in US dollars and weights of the five components of an engine, in 1998 and 2005.

| COMPONENT | $A$ | $B$ | $C$ | $D$ | $E$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| PRICE(\$) 1998 | 35 | 70 | 43 | 180 | 480 |
| PRICE (\$) 2005 | 60 | 135 | 105 | 290 | 800 |
| WEIGHT | 6 | 5 | 3 | 2 | 1 |

Taking 1998 as the base year,
(a) Calculate for 2005 the :
(i) simple aggregate price index.
(03 marks)
(ii) price relative of each component. (03 marks)
(iii) weighted aggregate price index.
(06 marks)
(b) Estimate the cost of an engine in 1998 given that its cost in 2005 was 1600 US dollars.
12. (a) Solve the equation $1+\cos \theta=2 \sin ^{2} \theta$ for values of $\theta$ between $0^{\circ}$ and $360^{\circ}$.
(09 marks)
(b) By eliminating $\theta$ from the equations $x=a \sec \theta$ and $y=b+C \cos \theta$, show that $x(y-b)=C a$.
(06 marks)
13. A random variable $X$ has a probability density function $f(x)$, defined by

$$
f(x)=\left\{\begin{array}{cl}
k x(x+2), & 0 \leq x \leq 2 \\
0, & \text { Oiherwise }
\end{array}\right.
$$

where $k$ is a constant.
Determine the
(a) value of $k$.
(b) $P(1 \leq X \leq 1.5)$
(c) Expectation, $E(X)$.
(d) Variance, $\operatorname{Var}(X)$.
14. A car initially at rest accelerated uniformly to a speed of $20 \mathrm{~ms}^{-1}$ in 16 seconds. The car then travelled at the attained speed for 2 minutes. The car then accelerated uniformly at $2.5 \mathrm{~ms}^{-2}$ for 8 seconds. It finally decelerated uniformly at $2.5 \mathrm{~ms}^{-2}$ to rest.
(a) Find the
(i) greatest speed attained by the car.
(ii) total time taken by the car to come to rest.
(b) Sketch the velocity- time graph for the motion of the car.
(c) Use your graph to find the total distance travelled by the car. (05 marks)

