P425/1
URE MATHEMATICS
Paper 1
Nov. / Dec. 2019
3 hours

UGANDA NATIONAL EXAMINATIONS BOARD

Uganda Advanced Certificate of Education

PURE MATHEMATICS

Paper 1

3 hours

NSTRUCTIONS TO CANDIDATES:

nswer **all** *the* **eight** *questions in section* **A** *and any* **five** *from section* **B**.

ny additional question(s) answered will **not** *be marked.*

ll *necessary working* **must** *be shown clearly.*

egin each answer on a fresh sheet of paper.

quared paper is provided.

ilent, non-programmable scientific calculators and mathematical tables with a ist of formulae may be used.

Turn Over

SECTION A: (40 MARKS)

Answer **all** *questions in this section.*

1. Show that the modulus of $\frac{(1-i)^6}{1+i} = 4\sqrt{2}$. *(05 marks)*

2. Solve $2\cos 2\theta - 5\cos\theta = 4$ for $0^\circ \leq \theta \leq 360^\circ$. *(05 marks)*

3. Using the substitution $u = \tan^{-1} x$, show that $\int_0^1 \frac{\tan^{-1} x}{1+x^2}\,dx = \frac{\pi^2}{32}$. *(05 marks)*

4. Given the plane $4x + 3y - 3z - 4 = 0$;
   (a) show that the point A (1,1,1) lies on the plane. *(02 marks)*
   (b) find the perpendicular distance from the plane to the point B (1,5,1). *(03 marks)*

5. Find the equation of the tangent to the curve $y = \frac{a^3}{x^2}$ at the point $P\left(\frac{a}{t},\ at^2\right)$. *(05 marks)*

6. Given that $\alpha + \beta = \frac{-1}{3}$ and $\alpha\beta = \frac{2}{3}$, form a quadratic equation whose roots are $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$. *(05 mark*

7. Find the area enclosed between the curve $y = 2x^2 - 4x$ and the x– axis. *(05 mark*

8. Given that $Q = \sqrt{80 - 0.1P}$ and $E = \frac{-dQ}{dP} \cdot \frac{P}{Q}$, find E when $P = 600$. *(05 marl*

SECTION B: (60 MARKS)

Answer any **five** *questions from this section.* **All** *questions carry equal marks.*

9. (a) Determine the perpendicular distance of the point (4, 6) from the line $2x + 4y - 3 = 0$. *(03 marks)*

(b) Show that the angle θ, between two lines with gradients λ_1 and λ_2 is given by $\theta = \tan^{-1}\left(\frac{\lambda_1 - \lambda_2}{1 + \lambda_1\lambda_2}\right)$.

Hence find the acute angle between the lines $x + y + 7 = 0$ and $\sqrt{3}x - y + 5 = 0$. *(09 marks)*

10. (a) Given that $26\left(1 - \frac{1}{26^2}\right)^{\frac{1}{2}} = a\sqrt{3}$, find the values of a. *(05 marks)*

(b) Solve the simultaneous equations:

$$2x = 3y = 4z,$$
$$x^2 - 9y^2 - 4z + 8 = 0.$$ *(07 marks)*

11. Express $7\cos 2\theta + 6\sin 2\theta$ in the form $R\cos(2\theta - \alpha)$, where R is a constant and α is an acute angle.
Hence solve $7\cos 2\theta + 6\sin 2\theta = 5$ for $0° \le \theta \le 180°$. *(12 marks)*

12. (a) Given that $y = ln\left\{e^x\left(\frac{x-2}{x+2}\right)^{\frac{3}{4}}\right\}$, show that $\frac{dy}{dx} = \frac{x^2 - 1}{x^2 - 4}$. *(05 marks)*

(b) Evaluate $\int_0^4 \frac{dx}{x^2\sqrt{(25 - x^2)}}$. *(07 marks)*

13. Four points have coordinates A (3,4,7), B (13,9,2), C (1,2,3) and D (10,k,6). The lines AB and CD intersect at P. Determine the;

(a) vector equations of lines AB and CD. *(06 marks)*

(b) value of k. *(04 marks)*

(c) coordinates of P. *(02 marks)*

14. Expand $\sqrt{\left(\frac{1+2x}{1-x}\right)}$ upto the term in x^2.

Hence find the value of $\sqrt{\left(\frac{1.04}{0.98}\right)}$ to **four** significant figures. (*12 marks*)

15. (a) Differentiate $y = 2x^2 + 3$ from first principles. (*04 marks*)

(b) A rectangular sheet is 50 cm long and 40 cm wide. A square of x cm by x cm is cut off from each corner. The remaining sheet is folded to form an open box. Find the maximum volume of the box. (*08 marks*)

16. (a) Find $\int \frac{\ln x}{x^2}\,dx$. (*04 marks*)

(b) Solve the differential equation

$\frac{dy}{dx} + y\cot x = x$, given that $y = 1$ when $x = \frac{\pi}{2}$. (*08 marks*)