



UGANDA NATIONAL EXAMINATIONS BOARD

Uganda Advanced Certificate of Education

PURE MATHEMATICS

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES:

Answer all the eight questions in section A and any five from section B.

Any additional question(s) answered will not be marked.

All necessary working must be shown clearly.

Begin each answer on a fresh sheet of paper.

Squared paper is provided.

Silent non programmable scientific calculators and mathematical tables with a list of formulae may be used.

SECTION A: (40 MARKS)

Answer all questions in this section.

In triangle ABC , $a = 7\text{cm}$, $b = 4\text{cm}$ and $c = 5\text{cm}$. Find the value of:

(a) $\cos A$

(b) $\sin A$

(05 marks)

Determine the angle between the line $\frac{x+4}{8} = \frac{y-2}{2} = \frac{z+1}{-4}$ and the plane $4x + 3y - 3z + 1 = 0$

(05 marks)

Find $\int x^2 e^x dx$.

(05 marks)

Express the function $f(x) = x^2 + 12x + 32$, in the form $a(x+b)^2 + c$.
Hence find the minimum value of the function $f(x)$.

(05 marks)

A point P moves such that its distances from two points $A(-2,0)$ and $B(8,6)$ are in the ratio $AP : PB = 3 : 2$. Show that the locus of P is a circle.

(05 marks)

Determine the equation of the tangent to the curve $y^3 + y^2 - x^4 = 1$ at the point $(1,1)$.

(05 marks)

Show that $2\log 4 + \frac{1}{2}\log 25 - \log 20 = 2\log 2$.

(05 marks)

The region bounded by the curve $y = x^2 - 2x$ and the x -axis from $x = 0$ to $x = 2$, is rotated about the x -axis. Calculate the volume of the solid formed.

(05 marks)

SECTION B: (60 MARKS)

Answer any five questions from this section. All questions carry equal marks.

9. The position vectors of the vertices of a triangle are \mathbf{o} , \mathbf{r} and \mathbf{s} , where O is the origin. Show that its area (A) is given by

$$4A^2 = |\mathbf{r}|^2 |\mathbf{s}|^2 - (\mathbf{r} \cdot \mathbf{s})^2 \quad (06 \text{ marks})$$

Hence, find the area of a triangle when $\mathbf{r} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ and $\mathbf{s} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$. (06 marks)

10. Express $5 + 12i$ in polar form.

Hence, evaluate $\sqrt[3]{(5+12i)}$, giving your answers in the form $a + ib$ where a and b are corrected to two decimal places. (12 marks)

11. (a) Differentiate $\frac{x^3}{\sqrt{(1-2x^2)}}$ with respect to x . (06 marks)

- (b) The period, T of a swing of a simple pendulum of length, l is given by the equation

$$T^2 = \frac{4\pi^2 l}{g}$$

where g is the acceleration due to gravity.

An error of 2% is made in measuring the length, l . Determine the resulting percentage error in the period, T .

(06 marks)

12. (a) Show that $\tan 4\theta = \frac{4t(1-t^2)}{t^4 - 6t^2 + 1}$, where $t = \tan \theta$. (06 marks)

- (b) Solve the equation

$$\sin x + \sin 5x = \sin 2x + \sin 4x \quad \text{for } 0^\circ < x < 90^\circ. \quad (06 \text{ marks})$$

13. (a) The first three terms of a Geometric Progression (G.P) are 4, 8, and 16. Determine the sum of the first ten terms of the G.P. (04 marks)

- (b) An Arithmetic Progression (A.P) has a common difference of 3. A Geometric Progression (G.P) has a common ratio of 2. A sequence is formed by subtracting the terms of the A.P from the corresponding terms of the G.P. The third term of the sequence is 4. The sixth term of the sequence is 79. Find the first term of the

(i) A.P.

(ii) G.P.

(08 marks)

14. Evaluate:

(a) $\int_0^{\frac{\pi}{2}} \sin 5x \cos 3x \, dx.$

(06 marks)

(b) $\int_0^{\frac{\sqrt{3}}{2}} \frac{dx}{9+4x^2}$

(06 marks)

5. The line $y = mx + c$ is a tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$

(a) Obtain an expression for c in terms of a , b , and m . (06 marks)

(b) Calculate the gradients of the tangents to the ellipse through the point $(\sqrt{a^2 + b^2}, 0)$. (06 marks)

6. The rate at which the temperature of a body falls is proportional to the difference between the temperature of the body and that of its surrounding. The temperature of the body is initially 60°C . After 15 minutes the temperature of the body is 50°C . The temperature of the surrounding is 10°C .

(a) Form a differential equation for the temperature of the body. (09 marks)

(b) Determine the time it takes for the temperature of the body to reach 30°C . (03 marks)