

P425/1  
PURE MATHEMATICS  
Paper 1  
Nov./ Dec. 2017  
3 hours



## UGANDA NATIONAL EXAMINATIONS BOARD

Uganda Advanced Certificate of Education

PURE MATHEMATICS

Paper 1

3 hours

### INSTRUCTIONS TO CANDIDATES:

*Answer all the eight questions in section A and any five from section B.*

*Any additional question(s) answered will not be marked.*

*All necessary working must be shown clearly.*

*Begin each answer on a fresh sheet of paper.*

*Graph paper is provided.*

*Silent non programmable scientific calculators and mathematical tables with a list of formulae may be used.*

**SECTION A: (40 MARKS)**

*Answer all questions in this section.*

1. The coefficients of the first three terms of the expansion of  $\left(1 + \frac{x}{2}\right)^n$  are in an Arithmetic Progression (AP). Find the value of  $n$ . (05 marks)
2. Solve the equation  $3\tan^2\theta + 2\sec^2\theta = 2(5-3\tan\theta)$  for  $0^\circ \leq \theta \leq 180^\circ$ . (05 marks)
3. Differentiate  $\left(\frac{1+2x}{1+x}\right)^2$  with respect to  $x$ . (05 marks)
4. Solve for  $x$  in the equation  $4^{2x} - 4^{x+1} + 4 = 0$ . (05 marks)
5. The vertices of a triangle are  $P(4,3)$ ,  $Q(6,4)$  and  $R(5,8)$ . Find angle  $RPQ$  using vectors. (05 marks)
6. Show that  $\int_2^4 x \ln x \, dx = 14\ln 2 - 3$ . (05 marks)
7. The equation of a curve is given by  $y^2 - 6y + 20x + 49 = 0$ .
  - (a) Show that the curve is a parabola. (03 marks)
  - (b) Find the coordinates of the vertex. (02 marks)
8. A container is in the form of an inverted right circular cone. Its height is 100 cm and base radius is 40 cm. The container is full of water and has a small hole at its vertex. Water is flowing through the hole at a rate of  $10 \text{ cm}^3 \text{ s}^{-1}$ . Find the rate at which the water level in the container is falling when the height of water in the container is halved. (05 marks)

**SECTION B: (60 MARKS)**

Answer any five questions from this section. All questions carry equal marks.

9. (a) Given that the complex number  $Z$  and its conjugate  $\bar{Z}$  satisfy the equation  $Z\bar{Z} - 2Z + 2\bar{Z} = 5 - 4i$ , find the possible values of  $Z$ . (06 marks)
- (b) Prove that if  $\frac{Z-6i}{Z+8}$  is real, then the locus of the point representing the complex number  $Z$  is a straight line. (06 marks)
10. A circle whose centre is in the first quadrant touches the  $x$ - and  $y$ -axes and the line  $8x - 15y = 120$ . Find the:
- (a) equation of the circle. (10 marks)
- (b) point at which the circle touches the  $x$ -axis. (02 marks)
11. A curve whose equation is  $x^2y + y^2 - 3x = 3$  passes through points  $A(1,2)$  and  $B(-1,0)$ . The tangent at  $A$  and the normal to the curve at  $B$  intersect at point  $C$ . Determine the:
- (a) equation of the tangent. (06 marks)
- (b) coordinates of  $C$ . (06 marks)
12. (a) Express  $\cos(\theta + 30^\circ) - \cos(\theta + 48^\circ)$  in the form  $R\sin P\sin Q$ , where  $R$  is a constant. Hence solve the equation.  
 $\cos(\theta + 30^\circ) - \cos(\theta + 48^\circ) = 0.2$  (06 marks)
- (b) Prove that in any triangle  $ABC$ ,  $\frac{\sin(A-B)}{\sin(A+B)} = \frac{a^2 - b^2}{c^2}$ . (06 marks)
13. (a) Solve for  $x$  and  $y$  in the following simultaneous equations.  
 $(x - 4y)^2 = 1$   
 $3x + 8y = 11$  (06 marks)
- (b) Find the set of values of  $x$  for which  $4x^2 + 2x < -3x + 6$ . (06 marks)

14. (a) The points  $A$  and  $B$  have position vectors  $a$  and  $b$ . A point  $C$  with position vector  $c$  lies on  $AB$  such that  $\frac{AC}{AB} = \lambda$ .  
Show that  $c = (1 - \lambda)a + \lambda b$ . (04 marks)

- (b) The vector equations of two lines are  
 $r_1 = 2i + j + \lambda(i + j + 2k)$  and  
 $r_2 = 2i + 2j + t k + \mu(i + 2j + k)$

where  $i, j$  and  $k$  are unit vectors and  $\lambda, \mu$  and  $t$  are constants.  
Given that the two lines intersect, find

- (i) the value of  $t$ .  
(ii) the coordinates of the point of intersection. (08 marks)

15. (a) Sketch the curve  $y = x^3 - 8$ . (08 marks)

- (b) The area enclosed by the curve in (a), the  $y$ -axis and the  $x$ -axis is rotated about the line  $y = 0$  through  $360^\circ$ . Determine the volume of the solid generated. (04 marks)

16. (a) Solve the differential equation

$$\frac{dy}{dx} = (xy)^{1/2} \ln x, \text{ given that } y = 1 \text{ when } x = 1.$$

Hence find the value of  $y$  when  $x = 4$ . (12 marks)