P425/1
PURE MATHEMATICS
Paper 1
Nov./ Dec. 2017
3 hours

UGANDA NATIONAL EXAMINATIONS BOARD

Uganda Advanced Certificate of Education

PURE MATHEMATICS

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES:

Answer **all** *the* **eight** *questions in section* **A** *and any* **five** *from section* **B**.

Any additional question(s) answered will **not** *be marked.*

All *necessary working* **must** *be shown clearly.*

Begin each answer on a fresh sheet of paper.

Graph paper is provided.

Silent non programmable scientific calculators and mathematical tables with a list of formulae may be used.

SECTION A: (40 MARKS)

Answer **all** *questions in this section.*

1. The coefficients of the first three terms of the expansion of $\left(1+\frac{x}{2}\right)^n$ are in an Arithmetic Progression (AP). Find the value of n. *(05 marks)*

2. Solve the equation $3\tan^2\theta + 2\sec^2\theta = 2(5-3\tan\theta)$ for $0^\circ \leq \theta \leq 180^\circ$. *(05 marks)*

3. Differentiate $\left(\frac{1+2x}{1+x}\right)^2$ with respect to x. *(05 marks)*

4. Solve for x in the equation $4^{2x} - 4^{x+1} + 4 = 0$. *(05 marks)*

5. The vertices of a triangle are $P(4,3)$, $Q(6,4)$ and $R(5,8)$. Find angle RPQ using vectors. *(05 marks)*

6. Show that $\int_2^4 x \ln x \, dx = 14\ln 2 - 3$. *(05 marks)*

7. The equation of a curve is given by $y^2 - 6y + 20x + 49 = 0$.
   - (a) Show that the curve is a parabola. *(03 marks)*
   - (b) Find the coordinates of the vertex. *(02 marks)*

8. A container is in the form of an inverted right circular cone. Its height is 100 cm and base radius is 40 cm. The container is full of water and has a small hole at its vertex. Water is flowing through the hole at a rate of 10 $cm^3 s^{-1}$. Find the rate at which the water level in the container is falling when the height of water in the container is halved. *(05 marks)*

SECTION B: (60 MARKS)

Answer any **five** *questions from this section. All questions carry equal marks.*

9\. (a) Given that the complex number Z and its conjugate $\bar{Z}$ satisfy the equation $Z\bar{Z} - 2Z + 2\bar{Z} = 5 - 4i$, find the possible values of Z. *(06 marks)*

(b) Prove that if $\frac{Z-6i}{Z+8}$ is real, then the locus of the point representing the complex number Z is a straight line. *(06 marks)*

10\. A circle whose centre is in the first quadrant touches the x -and y-axes and the line $8x - 15y = 120$.
Find the:

(a) equation of the circle. *(10 marks)*

(b) point at which the circle touches the x-axis. *(02 marks)*

11\. A curve whose equation is $x^2y + y^2 - 3x = 3$ passes through points A (1,2) and B (-1,0). The tangent at A and the normal to the curve at B intersect at point C. Determine the:

(a) equation of the tangent. *(06 marks)*

(b) coordinates of C. *(06 marks)*

12\. (a) Express $\cos(\theta + 30^\circ) - \cos(\theta + 48^\circ)$ in the form $R\sin P\sin Q$, where R is a constant. Hence solve the equation.

$$\cos(\theta + 30^\circ) - \cos(\theta + 48^\circ) = 0.2$$ *(06 marks)*

(b) Prove that in any triangle ABC; $\frac{\sin(A-B)}{\sin(A+B)} = \frac{a^2 - b^2}{c^2}$. *(06 marks)*

13\. (a) Solve for x and y in the following simultaneous equations.

$$(x - 4y)^2 = 1$$
$$3x + 8y = 11$$ *(06 marks)*

(b) Find the set of values of x for which $4x^2 + 2x < -3x + 6$. *(06 marks)*

14. (a) The points A and B have position vectors $\boldsymbol{a}$ and $\boldsymbol{b}$. A point C with a position vector $\boldsymbol{c}$ lies on AB such that $\frac{AC}{AB} = \lambda$.

Show that $\boldsymbol{c} = (1 - \lambda)\,\boldsymbol{a} + \lambda\,\boldsymbol{b}$. *(04 marks)*

(b) The vector equations of two lines are

$$r_1 = 2i + j + \lambda\,(i + j + 2k) \quad \text{and}$$

$$r_2 = 2i + 2j + t\,k + \mu\,(i + 2j + k)$$

where $\boldsymbol{i}, \boldsymbol{j}$ and $\boldsymbol{k}$ are unit vectors and λ, μ and t are constants. Given that the two lines intersect, find

(i) the value of t.

(ii) the coordinates of the point of intersection. *(08 marks)*

15. (a) Sketch the curve $y = x^3 - 8$. *(08 marks)*

(b) The area enclosed by the curve in (a), the y-axis and the x-axis is rotated about the line $y = 0$ through 360°. Determine the volume of the solid generated. *(04 marks)*

16. (a) Solve the differential equation

$$\frac{dy}{dx} = (xy)^{1/2} \ln x, \text{ given that } y = 1 \text{ when } x = 1.$$

Hence find the value of y when $x = 4$. *(12 marks)*