P425/2
APPLIED MATHEMATICS
PAPER 2
Nov. / Dec. 2016
3 hours

UGANDA NATIONAL EXAMINATIONS BOARD

Uganda Advanced Certificate of Education

APPLIED MATHEMATICS

Paper 2

3 hours

INSTRUCTIONS TO CANDIDATES:

Answer **all** *the* **eight** *questions in section* **A** *and* **five** *questions from section* **B.**

Any additional question(s) answered will **not** *be marked.*

All *working* **must** *be shown clearly.*

Begin each answer on a fresh sheet of paper.

Graph paper is provided.

Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

In numerical work, take g to be 9.8 ms^{-2}

SECTION A: (40 MARKS)

Answer **all** *the questions in this section.*

1. A ball is projected vertically upwards and it returns to its point of projection 3 seconds later. Find the :

   (a) speed with which the ball was projected.

   (b) greatest height reached. *(05 marks)*

2. The table below shows the values of two variables P and Q.

P	14	15	25	20	15	7
Q	30	25	20	18	15	22

   Calculate the rank correlation coefficient between the two variables. *(05 marks)*

3. Use the trapezium rule with 4 sub-intervals to estimate

$$\int_0^{\frac{\pi}{2}} \cos x \, dx$$

   correct to **three** decimal places. *(05 marks)*

4. A body of mass 4 kg is moving with an initial velocity of 5 ms^{-1} on a plane. The kinetic energy of the body is reduced by 16 Joules in a distance of 40 m. Find the deceleration of the body. *(05 marks)*

5. A continuous random variable X has a cumulative distribution function:

$$F(x) = \begin{cases} 0, & x \le 0, \\ \lambda x^3, & 0 \le x \le 4, \\ 1, & x \ge 4. \end{cases}$$

   Find the

   (a) value of the constant λ. *(02 marks)*

   (b) probability density function, $f(x)$. *(03 marks)*

6. The table below shows the values of $f(x)$ for given values of x.

x	0.4	0.6	0.8
$f(x)$	-0.9613	-0.5108	-0.2231

Use linear interpolation to determine $f^{-1}(-0.4308)$ correct to 2 decimal places. *(05 marks)*

7. A particle of mass 2 kg rests in limiting equilibrium on a rough plane inclined at 30° to the horizontal. Find the value of the coefficient of friction. *(05 marks)*

8. A bag contains 5 Pepsi Cola and 4 Mirinda bottle tops. Three bottle tops are picked at random from the bag one after the other without replacement. Find the probability that the bottle tops picked are of the same type. *(05 marks)*

SECTION B: (60 MARKS)

Answer any **five** *questions from this section. All questions carry equal marks.*

9. The data below shows the length in centimetres of different calendars produced by a printing press. A cumulative frequency distribution was formed.

Length (cm)	<20	<30	<35	<40	<50	<60
Cumulative frequency	4	20	32	42	48	50

(a) Construct a frequency distribution table. *(02 marks)*

(b) Draw a histogram and use it to estimate the modal length. *(06 marks)*

(c) Find the mean length of the calendars. *(04 marks)*

10. Five forces of magnitudes 3N, 4N, 4N, 3N and 5N act along the lines AB, BC, CD, DA and AC respectively, of a square $ABCD$ of side 1m. The direction of the forces is given by the order of the letters. Taking AB and AD as reference axes; find the

(a) magnitude and direction of the resultant force. *(08 marks)*

(b) point where the line of action of the resultant force cuts the side AB. *(04 marks)*

11. Given the equation $x^3-6x^2+9x+2=0$;

(a) show that the equation has a root between −1 and 0. *(03 marks)*

(b) (i) show that the Newton Raphson formula for approximating the root of the equation is given by

$$x_{n+1}=\frac{2}{3}\left[\frac{x_n^3-3x_n^2-1}{x_n^2-4x_n+3}\right].$$

(04 marks)

(ii) use the formula in b(i) above, with an initial approximation of $x_0=-0.5$, to find the root of the given equation correct to two decimal places. *(05 marks)*

12. A newspaper vendor buys 12 copies of a sports magazine every week. The probability distribution for the number of copies sold in a week is given in the table below.

Number of copies sold	9	10	11	12
Probability	0.2	0.35	0.30	0.15

(a) Estimate the

(i) expected number of copies that she sells in a week.

(ii) variance of the number of copies sold in a week.

(05 marks)

(b) The vendor buys the magazine at Shs1,200 and sells it at Shs1,500. Any copies not sold are destroyed. Construct a probability distribution table for the vendor's weekly profit from the sales.
Hence calculate her mean weekly profit. *(07 marks)*

13. A particle starts from rest at a point (2, 0, 0) and moves such that its acceleration at any time $t>0$ is given by

$$a=[16\cos 4t\,i+8\sin 2t\,j+(\sin t-2\sin 2t)\,k]\ \text{ms}^{-2}$$

Find the:

(a) speed when $t=\frac{\pi}{4}$. *(06 marks)*

(b) distance from the origin when $t=\frac{\pi}{4}$. *(06 marks)*

14. The numbers x and y are approximated by X and Y with errors Δx and Δy respectively.

(a) Show that the maximum relative error in xy is given by

$$\left|\frac{\Delta x}{X}\right| + \left|\frac{\Delta y}{Y}\right|.$$

(*05 marks*)

(b) If $x = 4.95$ and $y = 2.013$ are each rounded off to the given number of decimal places, calculate the

(i) percentage error in xy,

(ii) limits within which xy is expected to lie. Give your answer to three decimal places. (*07 marks*)

15. The drying time of a newly manufactured paint is normally distributed with mean 110.5 minutes and standard deviation 12 minutes.

(a) Find the probability that the paint dries between 104 and 109 minutes. (*06 marks*)

(b) If a random sample of 20 tins of the paint was taken, find the probability that the mean drying time of the sample is between 108 and 112 minutes. (*06 marks*)

16. A particle of mass 2 kg moving with Simple Harmonic Motion (SHM) along the x-axis, is attracted towards the origin O by a force of $32x$ Newtons. Initially the particle is at rest at $x = 20$. Find the

(a) amplitude and period of the oscillation. (*05 marks*)

(b) velocity of the particle at any time, $t > 0$. (*05 marks*)

(c) speed when $t = \frac{\pi}{4}s$. (*02 marks*)