

P425/1  
PURE MATHEMATICS  
PAPER 1  
Nov. / Dec. 2016  
3 hours



**UGANDA NATIONAL EXAMINATIONS BOARD**

**Uganda Advanced Certificate of Education**

**PURE MATHEMATICS**

**Paper 1**

**3 hours**

**INSTRUCTIONS TO CANDIDATES:**

*Answer all the eight questions in section A and five questions from section B.*

*Any additional question(s) answered will not be marked.*

*All working must be shown clearly.*

*Begin each answer on a fresh sheet of paper.*

*Graph paper is provided.*

*Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.*

**SECTION A: ( 40 MARKS)**

*Answer all the questions in this section.*

1. Without using mathematical tables or a calculator, find the value of

$$\frac{(\sqrt{5}+2)^2 - (\sqrt{5}-2)^2}{8\sqrt{5}}$$

(05 marks)

2. Find the angle between the lines  $2x - y = 3$  and  $11x + 2y = 13$ .

(05 marks)

3. Evaluate  $\int_{1/2}^1 10x\sqrt{1-x^2} dx$

(05 marks)

4. Solve the equation  $\frac{dy}{dx} = 1 + y^2$  given that  $y = 1$  when  $x = 0$ . (05 marks)

5. Given that  $2x^2 + 7x - 4$ ,  $x^2 + 3x - 4$  and  $7x^2 + ax - 8$  have a common factor, find the:

- (a) factors of  $2x^2 + 7x - 4$  and  $x^2 + 3x - 4$ ,  
(b) value of  $a$  in  $7x^2 + ax - 8$ .

(05 marks)

6. Solve the equation  $\sin 2\theta + \cos 2\theta \cos 4\theta = \cos 4\theta \cos 6\theta$  for  $0 \leq \theta \leq \frac{\pi}{4}$ . (05 marks)

7. Using small changes, show that  $(244)^{1/5} = 3\frac{1}{405}$ . (05 marks)

8. Three points  $A(2, -1, 0)$ ,  $B(-2, 5, -4)$  and  $C$  are on a straight line such that  $3AB = 2AC$ . Find the coordinates of  $C$ .

(05 marks)

**SECTION B: (60 MARKS)***Answer any five questions from this section. All questions carry equal marks.*

9. (a) If  $Z_1 = \frac{2i}{1+3i}$  and  $Z_2 = \frac{3+2i}{5}$ , find  $|Z_1 - Z_2|$ . (06 marks)

(b) Given the complex number  $Z = x + iy$ ;

(i) find  $\frac{Z+i}{Z+2}$ .

(ii) show that the locus of  $\frac{Z+i}{Z+2}$  is a straight line when its imaginary part is zero. State the gradient of the line.

*(06 marks)*

10. (a) Solve the equation  $\cos 2x = 4 \cos^2 x - 2 \sin^2 x$  for  $0 \leq x \leq 180^\circ$ . (06 marks)

(b) Show that if  $\sin(x + \alpha) = P \sin(x - \alpha)$  then

$$\tan x = \left( \frac{P+1}{P-1} \right) \tan \alpha.$$

Hence solve the equation  $\sin(x + 20^\circ) = 2 \sin(x - 20^\circ)$  for  $0^\circ \leq x \leq 180^\circ$ .

*(06 marks)*

11. Given that  $x = \frac{t^2}{1+i^3}$  and  $y = \frac{t^3}{1+i^3}$ , find  $\frac{d^2y}{dx^2}$ . (12 marks)

12. (a) Line  $A$  is the intersection of two planes whose equations are

$$3x - y + Z = 2 \text{ and } x + 5y + 2Z = 6.$$

Find the cartesian equation of the line.

*(05 marks)*

(b) Given that line  $B$  is perpendicular to the plane  $3x - y + Z = 2$  and passes through the point  $C(1, 1, 0)$ , find the:

(i) cartesian equation of line  $B$ .

(ii) angle between line  $B$  and line  $A$  in (a) above.

*(07 marks)*

13. (a) Find  $\int \frac{1+\sqrt{x}}{2\sqrt{x}} dx$ . (03 marks)

(b) The gradient of the tangent at any point on a curve is  $x - \frac{2y}{x}$ . The

curve passes through the point  $(2, 4)$ . Find the equation of the

curve.

*(09 marks)*

14. (a) The points  $P(at_1^2, 2at_1)$  and  $Q(at_2^2, 2at_2)$  are on the parabola  $y^2 = 4ax$ .  
 $OP$  is perpendicular to  $OQ$ , where  $O$  is the origin. Show that  $t_1t_2 + 4 = 0$ .  
 (04 marks)
- (b) The normal to the rectangular hyperbola  $xy = 8$  at a point  $(4, 2)$  meets the asymptotes at  $M$  and  $N$ . Find the length of  $MN$ .  
 (08 marks)
15. (a) Prove by induction  
 $1 \cdot 3 + 2 \cdot 4 + \dots + n(n+2) = \frac{1}{6}n(n+1)(2n+7)$   
 for all integral values of  $n$ .  
 (06 marks)
- (b) A man deposits Shs150,000 at the beginning of every year in a micro-finance bank with the understanding that at the end of seven years he is paid back his money with 5% per annum compound interest. How much does he receive?  
 (06 marks)
16. (a) If  $x^2 + 3y^2 = k$ , where  $k$  is a constant, find  $\frac{dy}{dx}$  at the point  $(1, 2)$ .  
 (04 marks)
- (b) A rectangular field of area  $7200\text{m}^2$  is to be fenced using a wire mesh. On one side of the field, is a straight river. This side of the field is not to be fenced. Find the dimensions of the field that will minimize the amount of wire mesh to be used.  
 (08 marks)