P425/1
PURE MATHEMATICS
PAPER 1
Nov. / Dec. 2016
3 hours

UGANDA NATIONAL EXAMINATIONS BOARD

Uganda Advanced Certificate of Education

PURE MATHEMATICS

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES:

Answer **all** *the* **eight** *questions in section* **A** *and* **five** *questions from section* **B**.

Any additional question(s) answered will **not** *be marked.*

All *working* **must** *be shown clearly.*

Begin each answer on a fresh sheet of paper.

Graph paper is provided.

Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

SECTION A: (40 MARKS)

Answer **all** *the questions in this section.*

1. Without using mathematical tables or a calculator, find the value of

$$\frac{\left(\sqrt{5}+2\right)^2-\left(\sqrt{5}-2\right)^2}{8\sqrt{5}}$$

(*05 marks*)

2. Find the angle between the lines $2x-y=3$ and $11x+2y=13$. (*05 marks*)

3. Evaluate $\displaystyle\int_{1/2}^{1} 10x\sqrt{\left(1-x^2\right)}\,dx$

(*05 marks*)

4. Solve the equation $\frac{dy}{dx}=1+y^2$ given that $y=1$ when $x=0$. (*05 marks*)

5. Given that $2x^2+7x-4$, x^2+3x-4 and $7x^2+ax-8$ have a common factor, find the:

(a) factors of $2x^2+7x-4$ and x^2+3x-4,

(b) value of a in $7x^2+ax-8$.

(*05 marks*)

6. Solve the equation $\sin 2\theta+\cos 2\theta\cos 4\theta = \cos 4\theta\cos 6\theta$ for $0\leq\theta\leq\frac{\pi}{4}$.

(*05 marks*)

7. Using small changes, show that $(244)^{1/5}=3\frac{1}{405}$. (*05 marks*)

8. Three points A (2, -1, 0), B (-2, 5, -4) and C are on a straight line such that $3AB=2AC$. Find the coordinates of C.

(*05 marks*)

SECTION B: (60 MARKS)

Answer any five questions from this section. All questions carry equal marks.

9. (a) If $Z_1 = \frac{2i}{1+3i}$ and $Z_2 = \frac{3+2i}{5}$, find $|Z_1 - Z_2|$. *(06 marks)*

(b) Given the complex number $Z = x + iy$;

(i) find $\frac{Z+i}{Z+2}$.

(ii) show that the locus of $\frac{Z+i}{Z+2}$ is a straight line when its imaginary part is zero. State the gradient of the line.

(06 marks)

10. (a) Solve the equation $\cos 2x = 4\cos^2 x - 2\sin^2 x$ for $0 \leq x \leq 180^o$.

(06 marks)

(b) Show that if $\sin(x+\alpha) = P\sin(x-\alpha)$ then

$$\tan x = \left(\frac{P+1}{P-1}\right)\tan\alpha.$$

Hence solve the equation $\sin(x+20^o) = 2\sin(x-20^o)$ for $0^o \leq x \leq 180^o$.

(06 marks)

11. Given that $x = \frac{t^2}{1+t^3}$ and $y = \frac{t^3}{1+t^3}$, find $\frac{d^2y}{dx^2}$. *(12 marks)*

12. (a) Line A is the intersection of two planes whose equations are

$$3x - y + Z = 2 \text{ and } x + 5y + 2Z = 6.$$

Find the cartesian equation of the line. *(05 marks)*

(b) Given that line B is perpendicular to the plane $3x - y + Z = 2$ and passes through the point $C\,(1, 1, 0)$, find the:

(i) cartesian equation of line B.

(ii) angle between line B and line A in (a) above. *(07 marks)*

13. (a) Find $\int \frac{1+\sqrt{x}}{2\sqrt{x}}\,dx$. *(03 marks)*

(b) The gradient of the tangent at any point on a curve is $x - \frac{2y}{x}$. The curve passes through the point (2, 4). Find the equation of the curve. *(09 marks)*

14. (a) The points $P(at_1^2, 2at_1)$ and $Q(at_2^2, 2at_2)$ are on the parabola $y^2 = 4ax$.

OP is perpendicular to OQ, where O is the origin. Show that $t_1t_2 + 4 = 0$.

(*04 marks*)

(b) The normal to the rectangular hyperbola $xy = 8$ at a point (4, 2) meets the asymptotes at M and N. Find the length of MN. (*08 marks*)

15. (a) Prove by induction

$$1.3 + 2.4 + \ldots + n(n+2) = \frac{1}{6}n(n+1)(2n+7)$$

for all integral values of n.

(*06 marks*)

(b) A man deposits Shs150,000 at the beginning of every year in a micro-finance bank with the understanding that at the end of seven years he is paid back his money with 5% per annum compound interest. How much does he receive? (*06 marks*)

16. (a) If $x^2 + 3y^2 = k$, where k is a constant, find $\frac{dy}{dx}$ at the point (1, 2).

(*04 marks*)

(b) A rectangular field of area 7200m^2 is to be fenced using a wire mesh. On one side of the field, is a straight river. This side of the field is not to be fenced. Find the dimensions of the field that will minimize the amount of wire mesh to be used. (*08 marks*)