P425/1
PURE MATHEMATICS
Paper 1
Nov. / Dec. 2015
3 hours

UGANDA NATIONAL EXAMINATIONS BOARD

Uganda Advanced Certificate of Education

PURE MATHEMATICS

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES:

Answer **all** *the* **eight** *questions in Section* **A** *and any* **five** *from Section* **B**.

Any additional question(s) answered will **not** *be marked.*

All necessary working **must** *be clearly shown.*

Begin each answer on a fresh sheet of paper.

Graph paper is provided.

Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

SECTION A: (40 MARKS)

Answer **all** *questions in this section.*

1. The first term of an Arithmetic Progression (A.P.) is equal to the first term of a Geometric Progression (G.P.) whose common ratio is 1/3 and sum to infinity is 9. If the common difference of the A.P is 2, find the sum of the first ten terms of the A.P. *(05 marks)*

2. Find the equation of a line through the point (5, 3) and perpendicular to the line $2x - y + 4 = 0$. *(05 marks)*

3. Solve for x in: $\log_a (x+3) + \dfrac{1}{\log_x a} = 2\log_a 2.$ *(05 marks)*

4. Given that $D(7,1,2)$, $E(3,-1,4)$ and $F(4,-2,5)$ are points on a plane, show that $\boldsymbol{ED}$ is perpendicular to $\boldsymbol{EF}$. *(05 marks)*

5. In a triangle ABC all the angles are acute. Angle $ABC = 50^{\circ}$, $a = 10$cm and $b = 9$ cm. Solve the triangle. *(05 marks)*

6. Differentiate $e^{-x^2} x^3 \sin x$ with respect to x. *(05 marks)*

7. The region enclosed by the curve $y = x^2$, the $x-$ axis and the line $x = 2$, is rotated through one revolution about the $x-$ axis. Find the volume of the solid generated. *(05 marks)*

8. Solve $\dfrac{dy}{dx} = e^{x+y}$ given that $y = 2$ when $x = 0$. *(05 marks)*

SECTION B: (60 MARKS)

Answer any **five** *questions from this section.* **All** *questions carry equal marks.*

9. (a) Given that $f(x) = (x-\alpha)^2 g(x)$, show that $f'(x)$ is divisible by $(x-\alpha)$. *(03 marks)*

   (b) A polynomial $P(x) = x^3 + 4ax^2 + bx + 3$ is divisible by $(x-1)^2$. Use the result in (a) above, to find the values of a and b. Hence solve the equation $P(x) = 0$. *(09 marks)*

10. Sketch on the same co-ordinate axes the graphs of the curve $y=2+x-x^2$ and the line $y=x+1$.
Hence determine the area of the region enclosed between the curve and the line. *(12 marks)*

11. (a) Solve $Z\overline{Z}-5iZ=5(9-7i)$, where $\overline{Z}$ is the complex conjugate of Z. *(06 marks)*

(b) (i) Find the Cartesian equation of the curve given as
$|Z+2-3i| = 2|Z-2+i|$.

(ii) Show that it represents a circle. Find the centre and radius of the circle. *(06 marks)*

12. (a) Simplify $\dfrac{\cos 3\theta+\cos 5\theta}{\sin 5\theta-\sin 3\theta}$. *(03 marks)*

(b) Show that $\cot 2\theta = \dfrac{1-\tan^2\theta}{2\tan\theta}$.

Hence solve the equation $\cot 2\theta = 4-\tan\theta$ for values of θ between 0° and 360°. *(09 marks)*

13. Express $\dfrac{1}{x^2(x-1)}$ as partial fractions.

Hence evaluate $\displaystyle\int_2^3 \frac{dx}{x^2(x-1)}$ correct to 3 decimal places. *(12 marks)*

14. (a) Show that the lines $\boldsymbol{a}=\begin{pmatrix}3\\-4\\2\end{pmatrix}+\alpha\begin{pmatrix}-1\\1\\2\end{pmatrix}$

and $\boldsymbol{b}=\begin{pmatrix}5\\0\\-2\end{pmatrix}+\beta\begin{pmatrix}-1\\-1\\2\end{pmatrix}$ intersect. *(06 marks)*

(b) Find the

(i) point of intersection, P of the two lines in (a).

(ii) cartesian equation of the plane which contains $\boldsymbol{a}$ and $\boldsymbol{b}$. *(06 marks)*

15. The tangents at the points $P(cp, c/p)$ and $Q((cq, c/q)$ on the rectangular hyperbola $xy = c^2$ intersect at R. Given that R lies on the curve $xy = \frac{c^2}{2}$, show that the locus of the mid-point of PQ is given by $xy = 2c^2$.

(12 marks)

16. The rate of increase of a population of certain birds is proportional to the number in the population present at that time. Initially, the number in the population was 32,000. After 70 years the population was 48,000.

Find the

(a) number of birds in the population after 82 years.

(b) time when the population doubles the initial number.

(12 marks)