P425/1
PURE
MATHEMATICS
Paper 1
Nov. / Dec. 2014
3 hours

UGANDA NATIONAL EXAMINATIONS BOARD

Uganda Advanced Certificate of Education

PURE MATHEMATICS

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES:

Answer **all** *the* **eight** *questions in section* **A** *and any* **five** *from section* **B.**

Any additional question(s) answered will **not** *be marked.*

All necessary working **must** *be shown clearly.*

Begin each answer on a fresh sheet of paper.

Graph paper is provided.

Silent non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

Turn Over

SECTION A (40 MARKS)

Answer **all** *questions in this section.*

1. Solve the simultaneous equations:

$$x - 2y - 2z = 0$$
$$2x + 3y + z = 1$$
$$3x - y - 3z = 3$$

(*05 marks*)

2. A focal chord PQ, to the parabola $y^2 = 4x$, has a gradient $m = 1$. Find the coordinates of the midpoint of PQ. (*05 marks*)

3. Given that $\cos 2A - \cos 2B = -p$ and $\sin 2A - \sin 2B = q$, prove that $\sec(A + B) = \frac{1}{q}\sqrt{p^2 + q^2}$. (*05 marks*)

4. Differentiate $\log_5\left(\frac{e^{\tan x}}{\sin^2 x}\right)$ with respect to x. (*05 marks*)

5. Find the equation of a line through $S(1, 0, 2)$ and $T(3, 2, 1)$ in the form $\boldsymbol{r} = \boldsymbol{a} + \lambda\boldsymbol{b}$.
Hence, deduce the Cartesian equation of the line. (*05 marks*)

6. Solve the equation $\sqrt{2x+3} - \sqrt{x+1} = \sqrt{x-2}$.
Verify your answer. (*05 marks*)

7. Find $\int x(1 - x^2)^{\frac{1}{2}}\, dx$. (*05 marks*)

8. A cylinder has radius r and height $8r$. The radius increases from 4cm to 4.1 cm. Find the approximate increase in the volume. (Use $\pi = 3.14$). (*05 marks*)

SECTION B (60 MARKS)

Answer any **five** *questions from this section.* **All** *questions carry equal marks.*

(a) Given that the complex number Z and its conjugate $\overline{Z}$ satisfy the equation $Z\overline{Z} + 2iZ = 12 + 6i$, find Z. *(07 marks)*

(b) One root of the equation $Z^3 - 3Z^2 - 9Z + 13 = 0$ is $2 + 3i$. Determine the other roots. *(05 marks)*

A circle is described by the equation $x^2 + y^2 - 4x - 8y + 16 = 0$. A line given by the equation $y = 2(x - 1)$ cuts the circle at points A and B. A point $P(x,y)$ moves such that its distance from the mid-point of AB is equal to its distance from the centre of the circle.

(a) Calculate the coordinates of A and B. *(05 marks)*

(b) Determine the centre and radius of the circle. *(03 marks)*

(c) Find the locus of P. *(04 marks)*

(a) Differentiate $y = \cot^{-1}(\ln x)$ with respect to x. *(06 marks)*

(b) Evaluate $\int_{\frac{\pi}{3}}^{\pi} x \sin x \, dx$. *(06 marks)*

(a) Find the Cartesian equation of the plane through the points whose position vectors are $2i + 2j + 3k$, $3i + j + 2k$ and $-2j + 4k$. *(06 marks)*

(b) Determine the angle between the plane in (a) and the line $\frac{x-2}{2} = \frac{y}{-4} = z - 5$. *(06 marks)*

(a) Find the first three terms of the expansion $(2 - x)^6$ and use it to find $(1.998)^6$ correct to **two** decimal places. *(07 marks)*.

(b) Expand $(1 - 3x + 2x^2)^5$ in ascending powers of x as far as the x^2 term. *(05 marks)*

14. (a) Find the equation of a normal to a curve whose parametric equatio[n] are $x = b\sec^2\theta$, $y = b\tan^2\theta$. (06 ma[rks])

(b) The area enclosed by the curve $x^2 + y^2 = a^2$, the y – axis and the lin[e] $y = -\frac{1}{2}a$ is rotated through 90^0 about the y – axis. Find the volume [of] the solid generated. (06 mar[ks])

15. Solve

(a) $4\sin^2\theta - 12\sin 2\theta + 35\cos^2\theta = 0$, for $0^0 \le \theta \le 90^0$. (06 mar[ks])

(b) $3\cos\theta - 2\sin\theta = 2$, for $0^0 \le \theta \le 360^0$. (06 mar[ks])

16. A substance loses mass at a rate which is proportional to the amount M present at time t.

(a) Form a differential equation connecting M, t and the constant of proportionality k. (02 mar[ks])

(b) If initially the mass of the substance is M_o, show that $M = M_o e^{-kt}$ (05 mar[ks])

(c) Given that half of the substance is lost in 1600 years, determine the number of years 15 g of the substance would take to reduce to 13.6 (05 mark[s])