

P425/1
RE MATHEMATICS
Paper 1
Nov. / Dec. 2013
3 hours



UGANDA NATIONAL EXAMINATIONS BOARD

Uganda Advanced Certificate of Education

PURE MATHEMATICS

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES:

Answer all the eight questions in Section A and any five from Section B.

*Any additional question(s) answered will **not** be marked.*

*All necessary working **must** be clearly shown.*

Begin each answer on a fresh sheet of paper.

Graph paper is provided.

Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

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Turn Over

SECTION A: (40 MARKS)

Answer all questions in this section.

1. Solve $\log_x 5 + 4 \log_5 x = 4$. (05 marks)
2. In a Geometric Progression (G.P.), the difference between the fifth and the second term is 156. The difference between the seventh and the fourth term is 1404. Find the possible values of the common ratio. (05 marks)
3. Given that $r = 3 \cos \theta$ is an equation of a circle, find its Cartesian form. (05 marks)
4. The position vector of point A is $2i + 3j + k$, of B is $5j + 4k$ and of C is $i + 2j + 12k$. Show that ABC is a triangle. (05 marks)
5. Solve $5 \cos^2 3\theta = 3(1 + \sin 3\theta)$ for $0^\circ \leq \theta \leq 90^\circ$. (05 marks)
6. If $y = (x - 0.5)e^{2x}$, find $\frac{dy}{dx}$.
Hence determine $\int_0^1 x e^{2x} dx$. (05 marks)
7. The region bounded by the curve $y = \cos x$, the y -axis and the x -axis from $x = 0$ to $x = \frac{\pi}{2}$ is rotated about the x -axis. Find the volume of the solid formed. (05 marks)
8. Solve $(1 - x^2)\frac{dy}{dx} - xy^2 = 0$, given that $y = 1$ when $x = 0$. (05 marks)

SECTION B: (60 MARKS)

Answer any five questions from this section. All questions carry equal marks.

9. (a) The complex number $Z = \sqrt{3} + i$. \bar{Z} is the conjugate of Z .
 - (i) Express Z in the modulus argument form.
 - (ii) On the same Argand diagram plot \bar{Z} and $2\bar{Z} + 3i$. (05 marks)(b) What are the greatest and least values of $|Z|$ if $|Z - 4| \leq 3$?

10. Given the equation $x^3 + x - 10 = 0$,

(a) show that $x = 2$ is a root of the equation. (03 marks)

(b) deduce the values of $\alpha + \beta$ and $\alpha\beta$ where α and β are the other roots of the equation.

Hence form a quadratic equation whose roots are α^2 and β^2 .

(09 marks)

11. (a) Find the point of intersection of the lines $\frac{x-5}{4} = \frac{y-7}{4} = \frac{z+3}{-5}$
and $\frac{x-8}{7} = \frac{y-4}{1} = \frac{z-5}{3}$. (06 marks)

(b) The equations of a line and a plane are $\frac{x-2}{1} = \frac{y-2}{2} = \frac{z-3}{2}$
and $2x + y + 4z = 9$ respectively. P is a point on the line where $x = 3$. N is the foot of the perpendicular from point P to the plane. Find the coordinates of N . (06 marks)

12. (a) Find the equation of the tangent to the hyperbola whose points are of the parametric form $(2t, 2/t)$. (05 marks)

(b) (i) Find the equations of the tangents in (a), which are parallel to $y + 4x = 0$. (04 marks)

(ii) Determine the distance between the tangents in (i). (03 marks)

13. A curve has the equation $y = \frac{2}{1+x^2}$.

(a) Determine the nature of the turning point on the curve. (07 marks)

(b) Find the equation of the asymptote. Hence sketch the curve. (05 marks)

14. (a) Prove that $\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$.

Hence show that $\frac{1 - \tan 15^\circ}{1 + \tan 15^\circ} = \frac{1}{\sqrt{3}}$. (06 marks)

(b) Given that $\cos A = \frac{3}{5}$ and $\cos B = \frac{12}{13}$ where A and B are acute, find the value of

(i) $\tan(A + B)$.

(ii) $\operatorname{cosec}(A + B)$. (06 marks)

15. Resolve $y = \frac{x^3 + 5x^2 - 6x + 6}{(x-1)^2(x^2 + 2)}$ into partial fractions.

Hence find $\int y dx$ and $\frac{dy}{dx}$. (12 marks)

16. The differential equation $\frac{dp}{dt} = kp(c - p)$ shows the rate at which information flows in a student population c . p represents the number who have heard the information in t days and k is a constant.

(a) Solve the differential equation. (06 marks)

(b) A school has a population of 1000 students. Initially, 20 students had heard the information. A day later, 50 students had heard the information. How many students heard the information by the tenth day? (06 marks)