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## ADVANCED LEVEL GCE

## CORRECTIONS JUNE 2005

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I. (a) An equation is homogeneolls if all the terms in the equation have the same base units or dimensions.
(b) $\mathrm{C}^{2} \mu_{0 \varepsilon_{0}}=1$, from $F=\frac{\mathrm{Q}^{2}}{4 \pi \varepsilon \varepsilon_{0} r^{2}} \Rightarrow \varepsilon_{0}=\frac{\mathrm{Q}^{2}}{4 \pi \mathrm{Fr}^{2}} \Rightarrow\left[\varepsilon_{0}\right]=\frac{Q^{2}}{\left[F I \mathcal{F}_{2}\right.}=\mathrm{kg}^{-1} \mathrm{~m}^{-3} \mathrm{~s}^{4} \mathrm{~A}^{2}$

From $F=\frac{\mu_{0} I^{2}}{2 \pi r} \Rightarrow\left[\mu_{0}\right]=\frac{[r][F]}{[1]^{2}[1]} \Rightarrow\left[\mu_{0}\right]=\mathrm{kgms}^{-2} \mathrm{~A}^{-2}$
Units of LHS $=\left(\mathrm{ms}^{-1}\right)^{2}\left(\mathrm{kgms}^{-2} \mathrm{~A}^{-2}\right)\left(\mathrm{kg}^{-1} \mathrm{~m}^{-3} \mathrm{~s}^{4} \mathrm{~A}^{2}\right)=1$. Hence the equation is homogeneous.
2.(i) For a simple pendulum, $T=2 \pi \sqrt{\frac{1}{g}} \Rightarrow T^{2}=(2 \pi)^{2} \frac{1}{g}$, but $f=\frac{1}{T} \Rightarrow f^{2}=\left(\frac{g}{4 T^{2}}\right) \cdot \frac{1}{l}$. Thus a graph of $f^{2}$ against $\frac{1}{l}$ is straight line passing through the origin with slope $-\frac{g}{4 \pi^{2}}$
From the graph, slope $=\frac{1.5-0.5}{6-2}=0.25 \mathrm{~ms}^{-2} \Rightarrow \frac{g}{4 \pi^{2}}=0.25 \Rightarrow \mathrm{~g}=0.25 \times 4 \pi^{2}=9.87 \mathrm{~ms}^{-2}$
(ii) We know that $\mathrm{f}^{2}=\left(\frac{\mathrm{g}}{4 \pi^{2}}\right) \cdot \frac{1}{\mathrm{l}} \Rightarrow \mathrm{l}=\left(\frac{\mathrm{g}}{4 \pi^{2}}\right) \cdot \frac{1}{\mathrm{f}^{2}}=\frac{987}{4 \pi^{2}\left(20^{0}\right)}=6.25 \times 10^{-4}=0.625 \mathrm{~mm}$
3.By the law of conservation of linear momentum,

- Horizontally, $m_{A} v_{A}=m_{A} v_{A} \cos 60+m_{B} m_{B} \cos \theta$, but $m_{A}=m_{B}=m \Longrightarrow 5.5=2.5 \cos 60+v_{B} \cos \theta$

$$
\Rightarrow \mathrm{v}_{\mathrm{B}} \cos \theta=4.25-----\cdots--(1)
$$

- Vertically, $0=\mathrm{mv}_{\mathrm{A}} \sin 60-\mathrm{mvB} \sin \theta \Rightarrow \mathrm{vB}_{\mathrm{B}} \sin \theta=\frac{\sqrt{3}}{2}(2.5)$
$\Rightarrow v_{B} \sin \theta=1.25 \sqrt{3}$
Solving (1) and (2) simultaneously, gives $v_{B}=4.77 \mathrm{~ms}^{-1}$ and $\theta=\tan ^{-1}\left(\frac{125 \sqrt{3}}{4.25}\right)=27.0^{\circ}$
4.(a) $v_{p}=\frac{c}{n_{p}}=\frac{3.0 \times 10^{B}}{1.40}=2.14 \times 10^{8} \mathrm{~ms}^{-1}$ and $v_{Q}=\frac{c}{n_{Q}}=\frac{3.0 \times 10^{8}}{1.45}=2.07 \times 10^{8} \mathrm{~ms}^{-1}$

Sincev $\mathrm{V}_{\mathrm{P}}>\mathrm{v}_{\mathrm{Q}}$, the light will emerge first from P
(b) Optical path of $X, d_{x}=n_{p} t=1.40 \times 20 \times 10^{-6}=2.8 \times 10^{-5} \mathrm{~m}$

Optical path of $Y, d_{Y}=n_{Q} t=1.45 \times 20 \times 10^{-6}=2.9 \times 10^{-5} \mathrm{~m}$
Optical path difference, $\Delta x=d_{y}-d_{x}=1.0 \times 10^{-6}$
Phase difference, $\Delta \varphi=\frac{2 \pi \Delta x}{\lambda}=\frac{2 \pi \times 1.0 \times 10^{-6}}{450 \times 10^{-9}}=13.96 \mathrm{rad}$
5.


For the convex lens in position, the image is formed at L when the concave lens is now placed between the screen and the convex lens, it diverges the rays from the convex lens slightly so that they now converge at the point I $I^{\prime}$ Therefore the screen must be shifted through a distanced as indicated on the diagram
The image formed by the convex lens acts like a virtual object to the concave lens. Thus for the concave lens, $f=-30.0 \mathrm{~cm}, u=-15 \mathrm{~cm} \Rightarrow \frac{1}{f}=\frac{1}{v}+\frac{1}{u} \Rightarrow v=\frac{u f}{u-f^{\prime}}=\frac{(-30)(-15)}{-15--30} \Rightarrow v=30 \mathrm{~cm}$ Therefore the screen must be shifted a distanced $=30 \mathrm{~cm}-15 \mathrm{~cm}=15 \mathrm{~cm}$ backwards.
6.

(i) Applying KVL

$$
\begin{aligned}
& \text { Loop } 1, V_{2}+V_{1}-6=0 \\
& \Rightarrow 5 I_{3}+10 I_{1}-6=0 \Rightarrow 5 I_{3}-10 I_{1}=6--(1) \mathrm{In}
\end{aligned}
$$

loop 2,- $V_{3}+12-V_{2}=0$
$\Rightarrow-5 I_{2}+12-5 I_{3}=0 \Rightarrow I_{2}+I_{3}=2.4-$ - (2)
By KCL, $I_{1}+I_{2}-I_{3}=0--------(3)$
Solving the above equations simultaneously, gives

$$
I_{2}=I_{3}=1.2 \mathrm{~A}, I_{1}=0
$$

(ii) $V_{x y}=5 I_{3}=5 \times 1.2=6.0 \mathrm{~V}$
7.(i)

$F_{A P}=\frac{K Q_{A Q B}}{\left(d_{1}+d_{2}\right)_{2}}=\frac{9.0 \times 109 \times 4.0 \times 10^{-8} \times 2 \times 10^{-8}}{\left(d_{1}+d_{2}\right)_{2}}=\frac{7.2 \times 10^{-6}}{\left(d_{1}+d_{2}\right)_{2}}$
$F_{B P}=\frac{K Q B Q_{P}}{d_{2}^{2}}=\frac{9.0 \times 10^{9} \times 1.0 \times 10^{-}-8 \times 2 \times 10^{-8}}{d_{2}^{2}}=\frac{1.8 \times 10^{-5}}{d_{2}^{2}}$
The net force on $P$ due to $A$ and $B, F=F_{B P}-F_{A P}=\frac{18 \times 10^{-6}}{d_{2}^{2}}-\frac{7.2 \times 10^{-6}}{\left(d_{1}+\mathrm{d}_{2}\right)^{2}}$
If the distanees are known then the magnitude of F ean be gotten.
(ii) Since the charge $\operatorname{Pis}$ positive, $B$ is negative and $A$ is positive, if P is placed between $A$ and $B$, the force on $P$ due to $A$ is to the right and that due to $B$ is also to the right. Hence the net force cannot be zero,
8.(a) (i) For the Newton's laws of motion, see your textbooks.
(ii) Consider two bodies A and B with masses $\mathrm{m}_{\mathrm{A}}$ and $\mathrm{m}_{\mathrm{B}}$ respectively. If initially A and B bave speeds $u_{A}$ and $u_{B}$ respectively and collide such that after the collision, the have respective speeds $v_{A}$ and $v_{B}$, then By Newton's second law, the forces exerted on A and Bare

$$
F_{A}=\frac{m_{A}\left(v_{A}-v_{B}\right)}{\Delta t} \text { and } F_{B}=\frac{m_{A}\left(v_{B}-v_{B}\right)}{\Delta t}
$$

By Newton's third law, $F_{A}=-F_{B} \Rightarrow \frac{m_{A}\left(v_{A}-v_{B}\right)}{\Delta x}=-\frac{m_{A}\left(v_{B}-v_{B}\right)}{\Delta t} \Rightarrow m_{A} v_{A}-m_{A} u_{A}=-m_{B} v_{B}-m_{B} u_{B}$ $\Rightarrow m_{A} u_{A}+m_{B} u_{B}=m_{B} v_{B}+m_{A} v_{A}$, which is the principle of conservation of lincar momentum.
(b) See your textbooks
(c) A conservative force is one in which the total work done under its influence in a closed path is zero.

Examples are: gravitational force, electrostatic force A non- conservative force is one in which the total work done in a closed path under it influence is not zero, Examples are: friction, drag force, upthrust etc. d) (i) Kirchhoff's current law states that the total current entering a junction equal the total current leaving the junction. The law is essentially a law of conservation of charge because current is the rate of flow of charge with time, If current entering a junction equal current leaving the junction, it therefore implies that the quantity of charge arriving the junction per unit time equal the quantity of charge leaving the junction per unit time hence charge is conserved.

Kirchhoff's voltage law states that in a closed loop, the algebraic sum of the p.d drops across loads equal algebraic sum of emf. This law is the law of conservation of energy in that the total work done per unit charge in converting other forms of energy to electrical energy is equal to the work done per unit charge in converting electrical energy to other forms of energy. Hence from the work energy principle, energy is conserved.
(e) See you notebooks
(f)


$$
\begin{aligned}
& \mathrm{KCL}, l=l_{1}+I_{2}-\cdots---(1) \\
& \text { KVL, loop (1): } 12-8 I-6-6 I_{2}=0 \\
& \Rightarrow 4 I+3 l_{2}=3----- \text { (2) } \\
& \text { KVE, loop (2): } 6 I_{2}-12 I_{1}=0 \\
& \Rightarrow I_{2}=2 I_{1}-\cdots-\cdots \text {-(3) } \\
& \text { Solving, } I=0.5 \mathrm{~A}, I_{1}=0.167 \mathrm{~A}, I_{2}=0.33 \mathrm{~A} \\
& V_{5 \Omega}=V=I_{2} R=0.333 \times 6=2.0 \mathrm{~V} .
\end{aligned}
$$

9. 

Molecules of a particular gas are identical
$\checkmark$ Collision between the molecules and with the walls of the container is perfectly elastic.
$\checkmark$ The molecules exert no force on each other except during impact which may be assumed to be of negligible duration.
$\checkmark$ There are sufficiently large numbers of molecules for the laws of statistic to be meaningfully applied.
$\checkmark$ The sizes of molecules are negligible compare to their separation.


Consider a gas molecule enclosed in a cube of sides L. let each molecule of the gas have mass m. consider a single molecule with $x$ - component of speed, $\mathrm{u}_{4}$ moving towards the wall X the x component of the momentum of the molecules is $\mathrm{mu}_{1}$ towards the wall. The molecule will reverse its direction after colliding with the wall. Since the collision is perfectly elastic, its $x$ - component in the reverse direction will be $-m u_{j}$. The change in the $x$-component of the momentum is $m u_{l}-m u_{i}=2 \mathrm{mu}_{1}$. The molecule has travelled a distance 2 L (to and fro). The time for the molecule to move to and from the wall is is $2 \mathrm{~L} / \mathrm{u}_{1}$
The rate of change of momentum of the molecule due to the collision is $\frac{2 m}{2 L_{1}} \frac{\mu_{1}}{u_{1}}=\frac{m u_{1}^{2}}{L}$. By Newton's second law, the rate of clange of momentum is equal to the force exerted on the wall X
i.e Force on wall, $F=\frac{m u_{2}^{2}}{L}$

Therefore force per unit area, pressure $P=\frac{m u_{1}^{2} / L}{L^{2}}=\frac{m u^{2}}{L^{13}}$ (area side $x$ side $=L^{2}$ )
if there are N molecules in the container with x - components of velocity $\mathrm{u}_{1}, \mathrm{u}_{2}, \mathrm{u}_{3}, \ldots, \mathrm{u}_{\mathrm{N}}$, then the total pressure exerted on the wall will be given by

$$
\mathrm{P}=\frac{\mathrm{m}}{\mathrm{~L}^{3}}\left(\mathrm{u}_{\mathrm{I}}^{2}+\mathrm{u}_{2}^{2}+\cdots+\mathrm{u}_{\mathrm{N}}^{2}\right)
$$

Therefore, $\mathrm{P}=\frac{\mathrm{m}}{\mathrm{L}^{3}} N \overline{u^{2}}$, where $\overline{\mathrm{u}^{2}}$ is the mean square velocity in the x -direction.
The total mass of all the molecules is mN . Therefore the density of the gas is given by $\rho=\frac{\mathrm{m}}{\mathrm{L}} \frac{\mathrm{N}}{}{ }^{3}$. Thus $\mathrm{P}=\rho \overline{\mathrm{u}^{2}}-$ ${ }^{+}+$
If the $c$ is the resultant speed of a molecule whose $x-y$ - and $z$-components of velocity are $u, \omega$ and $\vartheta$ respectively, then $c^{2}=u^{2}+\omega^{2}+\vartheta_{2} \Rightarrow \overline{c^{2}}=\overline{u^{2}+\omega^{2}+\vartheta^{2}} \Rightarrow \overline{c^{2}}=\overline{u^{2}}+\overline{\omega^{2}}+\overline{\vartheta^{2}}$
Since there are large number of molecules and are moving randomly, $\overline{\mathrm{u}^{2}}=\overline{\omega^{2}}=\overline{9^{2}} \Rightarrow \overline{\mathrm{u}^{2}}=\frac{1}{3} \overline{\mathrm{c}^{2}}$.
Therefore, $(+)$ becomes $P=\frac{1}{3} \rho \bar{c}^{2}$
(b) (i) $\mathrm{P}=\frac{1}{3} \mathrm{pc}^{2} \Rightarrow$ slope of graph $=\frac{1}{3} \overline{\mathrm{c}}^{2}$. From the graph, slope $=\frac{(1: 3-0) 10^{5}}{1.5-0}=8.87 \times 10^{4} \mathrm{~m}^{2} \mathrm{~s}^{-2}$
$\Rightarrow \frac{1}{3} \bar{c}^{2}=8.87 \times 10^{4} \Rightarrow \overline{c^{2}}=3 \times 8.87 \times 10^{4} \Rightarrow \mathrm{c}_{\mathrm{rms}}=\sqrt{3 \times 8.87 \times 10^{4}}=510 \mathrm{~ms}^{-1}$
(ii) $T<300 \mathrm{~K}$ Because slope of graph at the temperature Tis less than the slope at the temperature 300 K Or by calculations,
Slope at temperature T is given by $\frac{(1.0-0) \times 105}{1.85-0}=5.4 \times 10^{4} \mathrm{~m}^{2} \mathrm{~s}^{-2}$

$$
\Rightarrow \mathrm{c}_{\mathrm{rms}}=\sqrt{3 \times 5.4 \times 10^{4}}=\quad 402.7 \mathrm{~ms}^{-1}
$$

Now, $\frac{c_{1}}{c_{2}}=\sqrt{\frac{r_{1}}{T_{2}}} \Rightarrow \mathrm{~T}_{2}=\left(\frac{c_{1}}{c_{2}}\right)^{2} \mathrm{~T}_{1} \Rightarrow \mathrm{~T}_{2}=\left(\frac{402.7}{510}\right)^{2} \times 300=187.0 \mathrm{~K}$. Thus $\mathrm{T}=187.0 \mathrm{~K}<300 \mathrm{~K}$.
(i)

(d) See June 2003
(e)

f) (i) Maximum energy= area between the curve and the extension axis Area of one square $0.5 \times 1000$ 500 J . Approximate number of squares under the graph $=16.5$

Total area $=16.5 \times 500 \mathrm{~J}=8250 \mathrm{~J}$. therefore maximum energy $=8250 \mathrm{~J}$
(ii) Young's modulus, $=\frac{\text { stress }}{\text { strain }}=\frac{F / A}{e / l}=\frac{F}{A} \times \frac{l}{e} \Rightarrow e=\frac{l}{E A} \cdot F \Rightarrow$ slope $=\frac{l}{E A} \Rightarrow \frac{l}{\text { slopexA }}$, where the area of the material is calculated as follows $A=\pi\left(\frac{d}{2}\right)^{2,}$ where d is the diameter.

$D=\frac{\lambda}{d}--\frac{\lambda}{d}=\frac{2 \lambda}{d}=\frac{2 \times 5.5 \times 10^{-7}}{0.15 \times 10^{-3}}=7.33 \times 10^{-3} \mathrm{rad}$
(d) (i) (ii) See June 2001
(e) (i)
$\checkmark$ The effect electrostatics force can be shielded while that of gravitational force cannot be shielded
$\checkmark$ Electrostatic force is attractive or repulsive while gravitational force is only attractive.
$\checkmark$ When elementary particles are considered, the electrostatic force is stronger than the gravitational force.
(ii)
$\checkmark$ Both of them obey the inverse square law
$\checkmark$ Both are action at a distance forces
$\checkmark$ Both field strength equal their negative potential gradient
(f) $U=\frac{G M m}{(R+h)}$
(i) On the surface of the earth, the potential of the earth is given by $V=\frac{-G M}{R}$, while at a height $h$ above the surfice, the potential is given by $V=\frac{-G M}{(R+h)}$. The potential energy gained in lifting a mass $m$ to a height $h$ above the surface of the earth is given by
$U=m \Delta V=m\left(\frac{-G M}{R+h}--\frac{G M}{R}\right)=G M m\left(\frac{1}{R}-\frac{1}{R+h}\right)=\frac{G M m h}{R(R+h)}=\frac{G M m h}{R^{2}\left(1+\frac{h}{R}\right)}$. Ash<< $R, \frac{h}{R} \approx 0$, and one bas the following relation $U=\frac{G M m h}{R^{2}}$, but $g=\frac{G M}{R^{2}} \Rightarrow U=g m h=m g h$ hence.
(ii) If the space craft is to leave the earth completely, the initial kinetic energy should be equal to the potential energy needed to take the space craft to infinity.
i.e,K.e $=0-\frac{G M m}{-K}=\frac{G M m}{-}, \operatorname{but} g R^{2}=G M \Rightarrow K . e=\frac{g R_{2} m}{-K}=g R m$
$\Rightarrow K . e=100 \times 9.8 \times 6.4^{6}=6.3 \times 10^{9]}$
(iii) If the energy is less, the space craft will move in an elliptical path. If the energy is is more, the space craft will escape from the earth, and the path will describe a hyperbola

