ZIMBABWE SCHOOL EXAMINATIONS COUNCIL

General Certificate of Education Advanced Level

MECHANICAL MATHEMATICS 6021/1

PAPER 1

SPECIMEN PAPER 3 hours

Additional materials:
Answer paper
Graph paper
List of Formulae
Electronic calculator

TIME 3 hours

INSTRUCTIONS TO CANDIDATES

Write your name, Centre number and candidate number in the spaces provided on the answer paper/answer booklet.

Answer all questions

If a numerical answer cannot be given exactly, and the accuracy required is not specified in the question, then in the case of an angle it should be given to the nearest degree, and in other cases it should be given correct to 2 significant figures.

If a numerical value for g is necessary, take $g = 9.81\ ms^{-2}$.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 120.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

This question paper consists of 8 printed pages.

Answer all questions

1 The trajectory of a projectile's motion is described by the equation $y = 2x - 0.01x^2$ where x is the horizontal displacement and y is the vertical displacement from the point of projection.

Find the

(i) angle of projection, [1]

(ii) initial velocity of projection. [2]

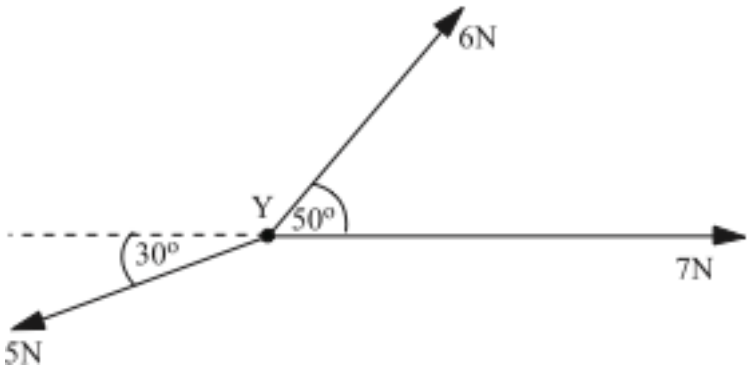
2 Model trolleys P and Q of masses 0.5 kg and 0.8 kg respectively are held in contact on a smooth horizontal surface. A compressed spring inside one of the trolleys is released and they then move apart. The speed of P is 4.5 ms^{-1}.

Calculate the speed of Q. [3]

3 A tractor of mass 5.5 tonnes moves with a constant speed of 4 ms^{-1} up a slope inclined 30° to the horizontal.

Given that the engine of the tractor is working at a constant rate of 160 kW, find the resistance to the motion. [4]

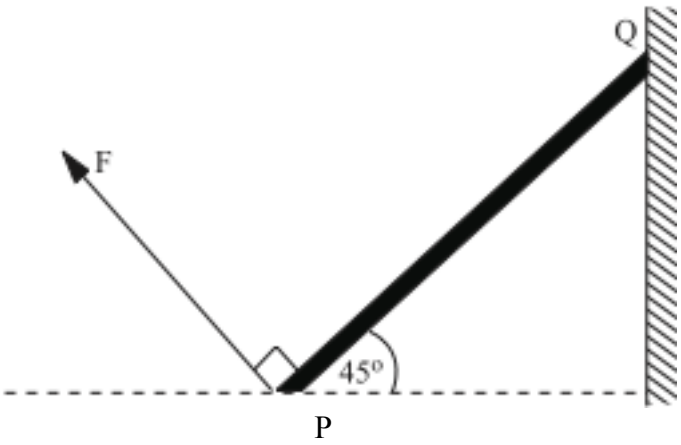
4



The diagram shows three coplanar forces acting at point Y. The 7 N force is horizontal, the 6 N force is inclined to it at 50° above the horizontal and the 5 N force acts at an angle of 30° below the horizontal.

Find the magnitude and direction of the resultant of the three forces. [5]

5



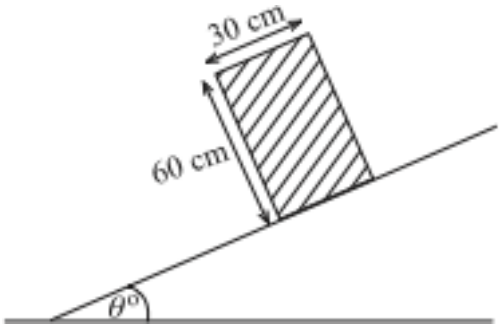
The diagram shows a straight uniform rod PQ of length $5a$ and weight $2W$ hanging in equilibrium. The end Q of the rod rests on a rough vertical wall. A force, F, whose line of action is perpendicular to the length of the rod, is applied at point P to maintain equilibrium.

Find the

(i) magnitude of F in terms of W, [2]

(ii) coefficient of friction between the rod and the wall. [5]

6



The diagram shows a 5 kg rectangular block of height 60 cm and width 30 cm, resting on a plane inclined at an angle, θ, to the horizontal. The coefficient of friction between the block and the plane is 0.2.

(a) Find the value of the angle θ at which the block begins to slide as the plane is tilted gradually. [4]

(b) If the block is prevented from sliding, and the angle θ is slowly increased, find the value of θ at which the block topples over. [4]

7 A particle starts from rest and accelerates at 2 ms^{-2} for 3 seconds. It then maintains the attained velocity for 4 seconds and then decelerates at 5 ms^{-2} for 2 seconds.

(a) Sketch a velocity-time graph for the motion of the particle for the 9 seconds. [4]

(b) Calculate the

(i) total distance travelled by the particle in the 9 seconds, [2]

(ii) displacement of the particle at the end of the 9 seconds. [2]

8 Two identical small trays each of mass 0.2 kg, are connected by a light inextensible string which passes over a fixed smooth pulley, as shown in **Fig. 8.1**. The trays remain balanced.

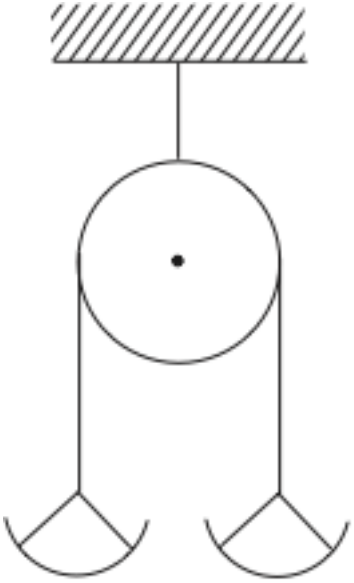


Fig. 8.1

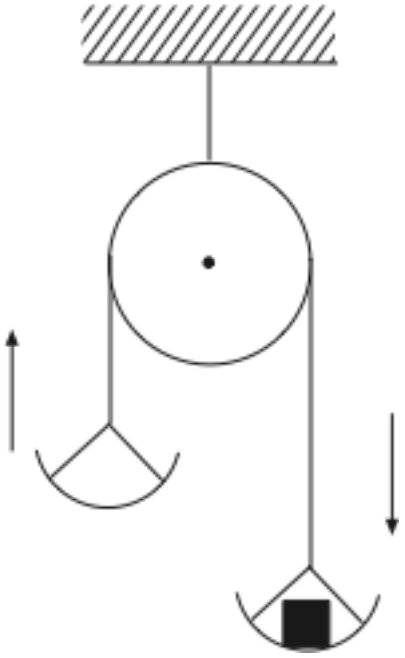


Fig. 8.2

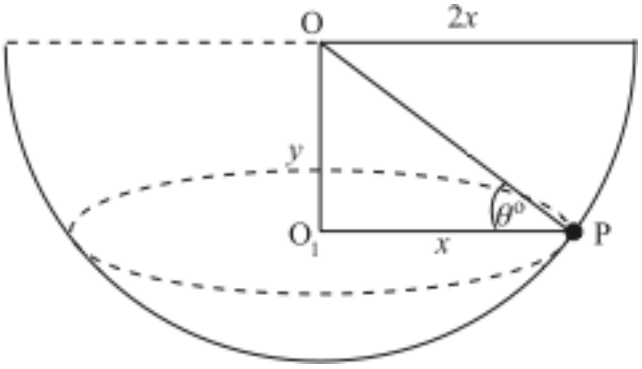
A mass of 80 grammes is then placed on one of the trays, which begins to move downwards, see **Fig 8.2**.

Calculate the

(i) acceleration of the trays and the tension in the string, [5]

(ii) force exerted on the 80 gramme mass by the tray when in motion. [3]

9



The diagram shows a particle P moving with a constant speed of 10 ms^{-1} in a horizontal circle of radius x metres on the inside of a fixed smooth spherical shell of internal radius $2x$ metres. The centre, O_1 of rotation of particle P is y metres vertically below O the centre of the sphere. The line OP makes an angle of θ with the radius line PO_1.

(i) Show that $y = x\sqrt{3}$. [2]

(ii) Find the exact value of $\tan\theta$. [2]

(iii) Hence, or otherwise, show that $xg = 100\sqrt{3}$. [4]

10 A helicopter is flying horizontally at a steady speed of 50 ms^{-1}. A parachutist of mass 90 kg falls from the helicopter and falls freely for 80 m before the parachute opens.

(i) Show that the vertical component of the velocity is approximately 40 ms^{-1} when the parachutist has fallen 80 m. [2]

(ii) Determine the magnitude and direction of the resultant velocity of the parachutist at the 80 m point. [3]

(iii) State the assumption made in the calculations in **(i)** and **(ii)**. [1]

(iv) The parachutist lands with a vertical velocity of 30 ms^{-1} taking 25 s to reach the ground.

Calculate the average retarding force on the parachutist during the landing. [2]

11 A train is moving along a straight track with uniform acceleration. The train passes through a level crossing **P** with a speed of 22 ms^{-1} and another level crossing **R**, 1 500 m from **P**, at 28 ms^{-1}.

(a) Calculate the

(i) time taken by the train to move from **P** to **R**, [3]

(ii) acceleration of the train. [2]

(b) **Q** is a sign post between **P** and **R** such that **PQ** : **QR** = 2 : 3.

Calculate the

(i) distance **PQ**, [2]

(ii) the speed of the train as it passes sign post **Q.** [3]

12 A train starts from rest at station P and accelerates uniformly until it reaches a speed of 30 ms^{-1}. It maintains this speed for 120 seconds until brakes are applied and brought to rest with uniform retardation at station Q. The magnitude of the retardation is three times that of the acceleration. The total time taken for the whole journey is 480 seconds.

(a) Sketch the (v,t) graph for the train's journey. [3]

(b) Find the

(i) time taken when accelerating, [4]

(ii) total distance travelled by the train, [2]

(iii) acceleration. [2]

13

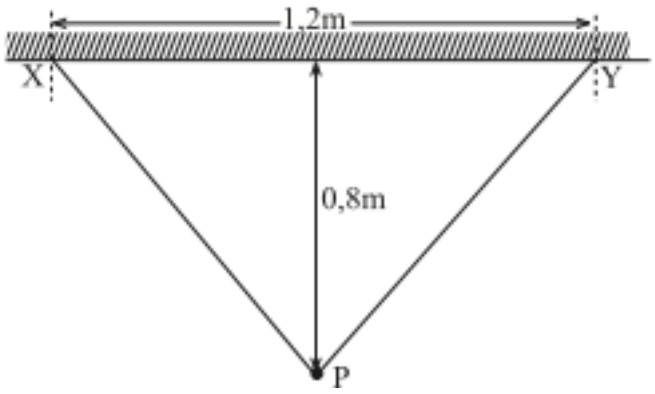


Fig. 11.1

Two light elastic strings each of natural length l metres and modulus of elasticity 24 N, are fastened to a particle P, of mass 0.4 kg and the other ends are attached to fixed points X and Y on a horizontal ceiling where XY = 1.2 m. The particle is then released from rest at ceiling level between X and Y, falls vertically and comes instantaneously to rest at a point 0.8 m below the level of the ceiling as shown in **Fig. 11.1**.

(a) Show that the extension of each string, x, is given by $x = 1 - l$. [3]

(b) Using the principle of conservation of energy or otherwise, determine the value of l, to 2 decimal places. [9]

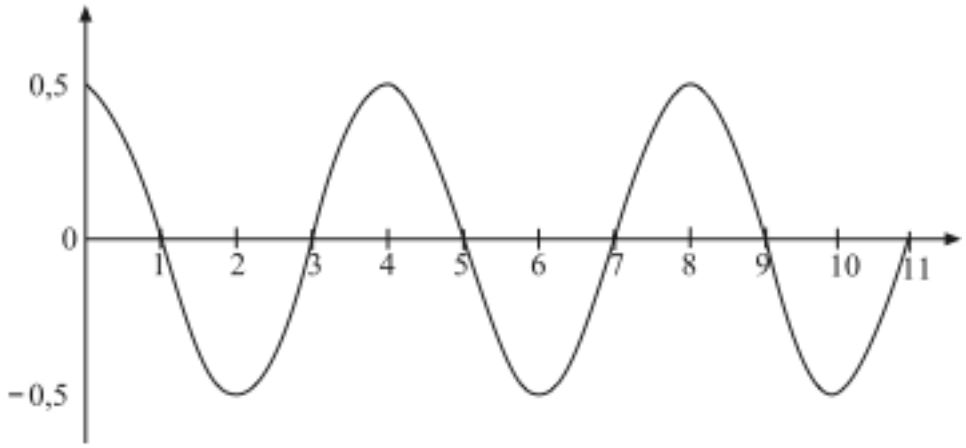
14 A car of mass 1 500 kg, travelling on a horizontal surface, has its engine working at a constant rate of 25 kW against a resisting force of 40 v N, where v is the speed of the car in ms^{-1}.

(i) Find the maximum speed of the car. [3]

(ii) 1. Show that $\frac{dv}{dt} = \frac{2\,500-4v^2}{150\,v}$. [3]

2. Hence find the time taken for the car to increase its speed from 10 ms^{-1} to 20 ms^{-1}. [6]

15



The diagram shows a displacement-time graph for the centre of a loudspeaker cone executing simple harmonic motion.

(a) Determine, from the graph, the

(i) amplitude, [1]

(ii) period, [1]

(iii) acceleration at maximum displacement. [1]

(b) Use your answers in **(a)** to calculate the

(i) maximum speed of the centre of the cone, [2]

(ii) acceleration of the centre of the cone at maximum displacement. [2]

(c) Initially, the centre of the cone is at rest.

Find the

(i) distance moved by the centre of cone until its speed is $\frac{2}{5}$ of the maximum value, [3]

(ii) time taken by the centre of the cone to move the distance in **c(i)**. [3]