ZIMBABWE SCHOOL EXAMINATIONS COUNCIL

General Certificate of Education Advanced Level

ADDITIONAL MATHEMATICS 6002/1

PAPER 1

SPECIMEN PAPER 3 hours

Additional materials:
Answer paper
Graph paper
List of Formula
Scientific calculator

TIME 3 hours

INSTRUCTIONS TO CANDIDATES

Write your name, Centre number and candidate number in the spaces provided on the answer paper/answer booklet.

Answer **all** questions.

If a numerical answer cannot be given exactly, and the accuracy required is not specified in the question, then in the case of an angle it should be given to the nearest degree, and in other cases it should be given correct to 2 significant figures.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 120.

The use of a scientific calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

This question paper consists of 6 printed pages and 2 blank pages.

1 A machine is programmed to cut lengths of string from a uniform string coming up continuously from a factory belt. The lengths are all measured in metres, and are as follows:

$$\frac{21^3}{100};\quad \frac{22^3}{100};\quad \frac{23^3}{100};\quad \ldots,\quad \frac{120^3}{100}.$$

Calculate the total length of the string cut, giving your answer in kilometres. [4]

2 The points A, B and C have position vectors

$\boldsymbol{a} = i - 3j,\quad \boldsymbol{b} = 2i - j + k$ and $\boldsymbol{c} = 10j + 10k$ respectively.

(i) Calculate the product $\boldsymbol{a}.(\boldsymbol{b} \times \boldsymbol{c})$. [4]

(ii) Describe fully the geometrical meaning of this product. [2]

3 A matrix, M, is given by

$$\mathrm{M} = \begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}.$$

(i) Find M^3. [3]

(ii) Hence or otherwise find M^6. [1]

(iii) Describe fully, the geometrical transformation represented by the matrix M^3. [3]

4 The function $f(x)$ is given by $f(x) = 4x^2 - 4x - 3$.

(i) Express $f(x)$ in the form $(ax+b)^2 + c$, where a, b and c are integers. [2]

(ii) Using an appropriate change of variable, or otherwise, show that

$$\int_3^4 \frac{1}{\sqrt{f(x)}}\,dx = \frac{1}{2}\ln\left(\frac{7+\sqrt{45}}{5+\sqrt{21}}\right).$$ [6]

5 **(i)** The integral I_n is given by

$$I_n = \int \sin^n 5x\,dx.$$

Use integration by parts, or otherwise, to show that

$$I_n = -\frac{1}{5n}\cos 5x \sin^{n-1} 5x + \left(\frac{n-1}{n}\right) I_{n-2}.$$ [6]

(ii) Hence evaluate

$$\int_0^{\frac{\pi}{6}} \sin^4 5x\,dx.$$ [3]

6 A curve, C, has a function $y = k\sqrt{x},$ where k is a positive constant and $2 \le x \le 4$.

(i) Find $\frac{dy}{dx}$, giving your answer in terms of x and k. [2]

(ii) The curve, C, is rotated completely about the x-axis.

Show that the area of the surface generated, S_k, is

$$S_k = \frac{\pi k}{6}\left\{\left(16+k^2\right)^{\frac{3}{2}} - \left(8+k^2\right)^{\frac{3}{2}}\right\}.$$ [6]

(iii) Hence evaluate $S_2 - S_1$ giving your answer correct to 3 decimal places. [2]

7 It is given that a matrix

$$A = \begin{pmatrix} 1 & 2 & -4 \\ 2 & 0 & -1 \\ -2 & 3 & k \end{pmatrix} \text{ has a determinant 3.}$$

(a) **(i)** Find the value of k.

(ii) Using this value of k, find adj.A, the adjoint matrix of A.

(iii) Hence or otherwise, write down A^{-1}.

[7]

(b) It is given that when the point (a; b; c) is transformed by the matrix A, its image is $(-8; -9; 3)$.

Find the values of a, b and c. [3]

8 A set $G = \{0, 1, 2, 3, 4\}$, where the elements are defined from the turning of a minute hand of a clock as follows:

0 = a turn through 0 minutes or multiples of 60 minutes.

1 = a turn through 12 minutes

2 = a turn through 24 minutes

3 = a turn through 36 minutes

4 = a turn through 48 minutes

Defining $a * b$ as " a turn of the element a followed by a turn of the element b".

(i) Construct the multiplication table for all the elements of the set G. [2]

(ii) Show that the set G is a group. [5]

(iii) Find the order of each of the elements. [2]

(iv) Establish whether the set G forms an abelian group. Give a reason for your answer. [2]

9 A curve, C, is given parametrically as

$y = \cosh bt$ and $x = \sinh at$ where a and b are constants and t is the parameter.

(a) Find $\frac{dy}{dx}$ in terms of a, b, and t. [2]

(b) In the case where $b = 4$ and $a = 2$, find the cartesian equation of curve C in the form $y = f(x)$ and show that $\frac{dy}{dx} = 4x$. [4]

(c) Hence show that the length, L, of curve C between the points where $x = 1$ and $x = 2$ is

$$L = \sqrt{65} - \frac{1}{2}\sqrt{17} + \frac{1}{8}\ln\left\{\frac{8+\sqrt{65}}{4+\sqrt{17}}\right\}$$ [6]

$\Bigg[$ You may make use of the standard integration given as

$$\int \sqrt{a^2+x^2}\,dx = \frac{x}{2}\sqrt{a^2+x^2} + \frac{a^2}{2}\sinh^{-1}\frac{x}{a}\Bigg]$$

10 The function $f(x)$ is given by $f(x) = \frac{16}{(x+1)^3} + 3x$.

(i) Write down the expression for $f'(x)$ and $f''(x)$. [2]

(ii) Hence find the coordinates of the turning point(s) for the curve $y = f(x)$, stating their nature. [6]

(iii) Write down the equations of the asymptote(s) of the curve $y = f(x)$ and hence sketch the curve showing any intercepts with the axes. [5]

11 **(a)** Solve the differential equation

$$\frac{dy}{dx} + 10y(10x-3)^{-1} = (2x)^{-1}, \text{ given that } y = 0 \text{ when } x = 1.$$ [6]

(b) A curve, S, has its differential equation

$$\frac{d^2y}{dx^2} - 2k\frac{dy}{dx} + k^2y = 0, \text{ where } k \text{ is a constant.}$$

(i) Find the general solution of the differential equation.

(ii) Hence find the exact particular solution of the differential equation, given that the curve, S, passes through the points (0; 4), $(\ln 2; 0)$ and $\left(\ln\frac{1}{2};\ 32\right)$.

[8]

12 A line, l_1, has an equation $\boldsymbol{r} = \boldsymbol{i} - \boldsymbol{j} - 3\boldsymbol{k} + \lambda(\boldsymbol{i} + 4\boldsymbol{j} - \boldsymbol{k})$, and the line, l_2, passes through the points A(0; 6; 6) and $\mathrm{B}(8; -6; -34)$.

(i) Find the vector equation of l_2 and show that the lines l_1 and l_2 are perpendicular to each other. [4]

(ii) Find the point of intersection of l_1 and l_2. [4]

(iii) 1. Find the cartesian equation of the plane, π, which contains both l_1 and l_2.

2. Calculate the distance of the point $\mathrm{P}(1; -7; 1)$ from the plane π. [8]

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