# ZIMBABWE SCHOOL EXAMINATIONS COUNCIL General Certificate of Education Ordinary Level 

## ADDITIONAL MATHEMATICS

Additional materials:
Answer paper
Graph papers
Formula Booklet
Electronic calculator

TIME
2 hours 30 minutes

INSTRUCTIONS TO CANDIDATES
Write your Name, Centre number and Candidate number on the answer sheet or answer booklet provided.
TIME 2 hour 30 minutes

Answer all questions in Section $\mathbf{A}$ and any four questions from Section B.
Write your answers on the spaces provided below each question or part question.
All working must be shown clearly.
Electronic calculators may be used.

## INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question. If the degree of accuracy is not specified in the question and if the answer is not exact, the answer should be given to three significant figures. Answers in degrees should be given to one decimal place.

This question paper consists of 6 printed pages and 2 blank pages.

## 2

## Section A (52 marks)

Answer all questions in this section.
1 It is given that $x-2$ is a factor of the polynomial $\mathrm{f}(x)=x^{3}+2 x^{2}-5 x+k$, where k is a constant.
(a) Find the numerical value of k .
(b) Hence, factorise $\mathrm{f}(x)$ completely.

2 Find the gradient of the straight line through the points $(-5,-2)$ and $(7,6)$.

3 Solve the pair of simultaneous equations.

$$
\begin{align*}
& 2 x+y=1 \\
& 4 x^{2}-y^{2}=13 \tag{5}
\end{align*}
$$

4 Write down, in ascending powers of $x$, the first 3 terms of the expansion of $(2-x)^{5}$.
(a) Functions $f$ and g are defined by

$$
\begin{array}{ll}
f: & x \rightarrow \frac{x+1}{4}, x \in \mathrm{R} \text { and } \\
\mathrm{g}: & x \rightarrow 2 x-1, x \in \mathrm{R} .
\end{array}
$$

Find an expression for
(i) $\quad f \mathrm{~g}(x)$
(ii) $\quad f^{-1}(x)$
(b) Evaluate $\mathrm{g} f^{-1}(2)$

6 A particle starts from a point O and moves in a straight line so that its displacement, $S$ metres from O, in $t$ seconds after leaving O , is given by $S=t(t-6)^{2}$.
(a) Obtain an expression for the velocity of the particle in terms of $t$.

6 (b) (i) Hence, determine the value of $t$ when the particle first comes to instantaneous rest.
(ii) Find the acceleration of the particle at the instant in $\mathbf{b}(\mathbf{i})$.

7 The fourth term of a geometric progression is 6 and the seventh term is -48 .

Calculate the
(a) common ratio and the first term,
(b) sum of the first eleven terms.

8 (a) An arithmetic progression has 14 terms. The sum of the odd terms (i.e. $1^{\text {st }}, 3^{\text {rd }}, 5^{\text {th }}$, etc.) is 40 and the sum of the even terms (i.e. $2^{\text {nd }}, 4^{\text {th }}, 6^{\text {th }}$, etc.) is 161 . Find the first term and the common ratio.
(b) Prove the following identity

$$
\begin{equation*}
\cot ^{2} \theta-\cos ^{2} \theta \equiv \cot ^{2} \theta \cos ^{2} \theta \tag{2}
\end{equation*}
$$

9 (a) Differentiate with respect to $x$

$$
\begin{equation*}
\text { (i) } x \sin x \tag{2}
\end{equation*}
$$

(ii) $e^{2-x}$
(iii) $\tan x$
(b) Find the equation of the tangent to the curve
$y^{2}-8 x-2 y+13=0$, at the point $(2 ; 3)$.

## Section B (48 marks)

Answer any four questions in this section.

## Each question carries 12 marks

10 (a) (i) Find the area of the region enclosed by the curve $y=\frac{12}{x}$, the $x$-axis and the lines $x=1$ and $x=3$.
(ii) The area of the region enclosed by the curve $y=\frac{12}{x}$, the axes and the lines $x=2$ and $x=a$, where $a>2$, is 3,6 square units.

Find the value of $a$.
(b) Fig. 10.1 shows the curve $y=4 \sqrt{x}$, and the line OA, where $0(0,0)$ and $A(4,8)$.


Fig.10.1
Find the volume, in terms of $\pi$, when the shaded region is rotated through $360^{\circ}$ about the $x$-axis.

11 (a) (i) Show that the curve $y=x^{3}-2 x^{2}+x$ has a stationary point at $x=1$.
(ii) Hence, determine the nature of the stationary point.
(b) The parametric equations of a curve are

$$
\begin{equation*}
x=t-e^{-2 t}, \quad y=t+e^{-2 t} \tag{3}
\end{equation*}
$$

(i) Find $\frac{d y}{d x}$, in terms of $t$.
(ii) Hence, find the exact value of $t$, for which the gradient of the curve is zero.

12 (a) Solve the equation $3 x^{3}+7 x^{2}-22 x-8=0$, giving answers in exact form.
(b) Given that, for all values of $x, 2 x^{3}+3 x^{2}-14 x-5$ is identical to $(\mathrm{A} x+\mathrm{B})(x+3)(x+1)+\mathrm{C}$.

Evaluate A, B and C.

13 (a) The function $g(x)=\frac{h}{x}+k$ is such that

$$
g(-1)=1 \frac{1}{2} \text { and } g(2)=9
$$

(i) State the value of $x$ for which $g(x)$ is undefined.
(ii) Find the value of $h$ and the value of $k$,
(iii) Evaluate $g^{-1}(8)$.

## 6

13 (b) Given that

$$
f: x \rightarrow 3 x+5 \text { and } g: x \rightarrow \frac{8}{x}, \text { for } x \neq 0
$$

express in simplest form
(i) $\quad f \mathrm{~g}(x)$,
(ii) $\quad \mathrm{g}^{2}(x)$.

14 A rectangular cake box is open at the top and is made of thin cardboard. The volume of the box is $500 \mathrm{~cm}^{3}$. The base of the box is a square with sides of length $x \mathrm{~cm}$.
(a) Show that area, $\mathrm{A} \mathrm{cm}^{2}$, of cardboard used to make such an open box, is given by the formula

$$
\begin{equation*}
\mathrm{A}=x^{2}+\frac{2000}{x} \tag{4}
\end{equation*}
$$

(b) (i) Given that $x$ varies, find the value of $x$ for which $\frac{d A}{d x}=0$.
(ii) Find the height of the box when $x$ has this value in (i).
(c) The radius of a circular disc is increasing at a constant rate of $0,05 \mathrm{~cm} / \mathrm{sec}$.

Find the rate at which the area is increasing when the radius is 20 cm .

15 (a) (i) Express $3 \operatorname{Sin} \theta-\operatorname{Cos} \theta$ in the form $\mathrm{R} \operatorname{Sin}(\theta-\alpha)$ for $0<\alpha<90^{\circ}$.
(ii) Hence or otherwise solve the equation

$$
\begin{equation*}
3 \operatorname{Sin} \theta-\operatorname{Cos} \theta=2 \text { for which } 0^{\circ}<\theta<360^{\circ} . \tag{4}
\end{equation*}
$$

(b) By first expanding $\operatorname{Cos}(2 \mathbf{A}+2 \mathbf{A})$,

Show that $\operatorname{Cos} 4 \mathbf{A}=1-8 \operatorname{Sin}^{2} \mathbf{A}+8 \operatorname{Sin}^{4} \mathbf{A}$.

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