FURTHER MATHEMATICS PAPER 2
0775

CAMEROON GENERAL CERTIFICATE OF EDUCATION BOARD

General Certificate of Education Examination

JUNE 2019 **ADVANCED LEVEL**

Subject Title	**Further Mathematics**
Paper No.	**2**
Subject Code No.	**0775**

THREE HOURS

Answer ALL 10 questions.

For your guidance, the approximate mark allocation for parts of each question is indicated.

Mathematical formulae and tables published by the Board, and noiseless non-programmable electronic calculators are allowed.

In calculations, you are advised to show all the steps in your working, giving your answer at each stage.

Turn Over

1. Find the complementary function of the differential equation

$$2\frac{d^2y}{dx^2} - \frac{dy}{dx} - 3y = 5e^{-x},$$

in the form $y = f(x)$. **(3 marks)**

Hence find the particular integral and the general solution in the form $y = f(x)$. **(4 marks)**

2. (a) Express $f(x)$ in partial fractions where

$$f(x) = \frac{2x^3 + x + 2}{(x^2+1)(x+1)(x-2)}, \quad x \neq -1, 2.$$ **(4 marks)**

Hence, or otherwise, show that

$$\int_0^1 f(x)\,dx = -\tfrac{1}{12}\left[13\ln 2 + \pi\right]$$ **(4marks)**

3. (a) Solve the equation

$$\tanh^{-1}\left(\frac{x-2}{x+1}\right) = \ln 2.$$ **(4 marks)**

(b) Show that the set $\{1, 2, 4, 8\}$ under $\times_{15}$, multiplication $\bmod 15$, forms a group. **(4 marks)**

4. (a) Given that the matrix M is defined by

$$M = \begin{pmatrix} 3 & 1 \\ 0 & 2 \end{pmatrix}$$

Prove by induction that

$$M^n = \begin{pmatrix} 3^n & 3^n - 2^n \\ 0 & 2^n \end{pmatrix}, \text{ for all } n \geq 1.$$ (4 marks)

(b) A curve is given by the parametric equations

$$x = t^2, \quad y = t\left(1 - \tfrac{1}{3}t^2\right), \quad 0 \leq t \leq \sqrt{3}.$$

Show that the length of the curve is $2\sqrt{3}$. **(3 marks)**

5. Show that the curve with polar coordinates (r, θ) where

$$r = \frac{4}{-3 + 3\sin\theta}, \theta \neq n\pi + (-1)^n \tfrac{\pi}{2}, n \in \mathbb{Z}.$$

is a parabola, P, in the (x, y) plane. **(2 marks)**

Show that the point $(2, -\frac{5}{6})$ lies on P and find the equation of the tangent to P at this point.

(4 marks)

6. (a) By the use of the Chinese Remainder Theorem, or otherwise, solve the system of congruences

$$x \equiv 3 \pmod 4$$
$$x \equiv 4 \pmod 7$$ **(5 marks)**

(b) A complex number z is defined by $z = \frac{1}{2}(\cos\theta + i\sin\theta)$, such that

$$z^n = \frac{1}{2^n}(\cos n\theta + i\sin n\theta).$$

Using De Moivre's theorem, or otherwise, show that

i) $\displaystyle\sum_{r=0}^{\infty} \frac{1}{4^r}\sin 2r\theta$ is a convergent geometric progression. **(1 mark)**

ii) $$\sum_{r=0}^{\infty} \frac{1}{4^r}\sin 2r\theta = \frac{4\sin 2\theta}{17 - 16\cos 2\theta}.$$ **(4 marks)**

7. A transformation, f, on a complex plane is defined by

$$z' = 2z + 3 - 4i \; .$$

(i) Find the image of the point $z = 2 - i$. **(1 mark)**

(ii) Determinethe invariant point of f in the form $a + ib$, $a, b \in \mathbb{R}$. . **(2 marks)**

(iii) Show that f is a similarity transformation (similitude), stating its radius. **(2 marks)**

(iv) Give the geometrical interpretation of f **(1 mark)**

8. Given two vectors

$$\mathbf{a} = \alpha\mathbf{i} - \mathbf{j} - 4\mathbf{k} \quad \text{and} \quad \mathbf{b} = 3\mathbf{i} + 2\mathbf{j} + (1 + 2\beta)\mathbf{k}, \; \alpha, \beta \in \mathbb{Z},$$

$$\mathbf{a} \times \mathbf{b} = 3\mathbf{i} - 21\mathbf{j} + 6\mathbf{k},$$

(i) Calculate the values of the real constants α and β. **(3 marks)**

(ii) By using the values of α and β, state the vectors $\mathbf{a}$ and $\mathbf{b}$. **(1 mark)**

(iii) Show that $\mathbf{a}$ and $\mathbf{b}$ are linearly independent. **(2 marks)**

(iv) Find the Cartesian equation of the plane containing $\mathbf{a}$ and $\mathbf{b}$. **(2 marks)**

9. A function, f, is defined by

$$f(x) = \frac{1}{(1 + e^x)^2} \; .$$

(i) Find the domain of f. **(1 mark)**

(ii) Find the intercept(s) of the curve $y = f(x)$. **(2 marks)**

(iii) Find

$$\lim_{x \to -\infty} f(x) \quad \text{and} \quad \lim_{x \to +\infty} f(x),$$

and state the asymptotes of the curve $y = f(x)$. **(3 marks)**

(iv) Determine $f'(x)$ and $f''(x)$. **(4 marks)**

(v) Prove that there are no turning points. **(2 marks)**

(vi) Prove, also, that $\left(-\ln 2, \frac{4}{9}\right)$ is the only point of inflexion. **(2 marks)**

(vii) Obtain the intervals on which f is concave up and intervals on which f is concave down.

(2 marks)

(viii) Obtain a variation table for f. **(2 marks)**

(ix) Sketch the curve, $y = f(x)$. **(2 marks)**

10. Two sequences, (u_n) and (v_n), for $n \in \mathbb{N}$ are defined as follows;

$$\begin{cases} u_0 = 3 \\ u_{n+1} = \frac{1}{2}(u_n + v_n) \end{cases} \text{and} \begin{cases} v_0 = 4 \\ v_{n+1} = \frac{1}{2}(u_{n+1} + v_n) \end{cases}$$

(i) Calculate u_1, v_1, u_2 and v_2. **(5 marks)**

(ii) Another sequence (w_n) is defined by

$$w_n = v_n - u_n, \ \forall n \in \mathbb{N}.$$

(iii) Show that (w_n) is a convergent geometric sequence. **(2 marks)**

(iv) Express w_n as a function of n and obtain its limit. **(4 marks)**

(v) Study the sense of variation (monotony) of (u_n) and (v_n).
What can you deduce? **(4 marks)**

(vi) Consider another sequence, t_n, defined by

$$t_n = \frac{u_n + 2v_n}{3}, \ \forall n \in \mathbb{N}$$

(vii) Show that (t_n) is a constant sequence. **(2 marks)**

(viii) Hence, obtain the limits of the sequences (u_n) and (v_n). **(3 marks)**

END