FURTHERMATHS 2
0775

CAMEROON GENERAL CERTIFICATE OF EDUCATION BOARD

General Certificate of Education Examination

JUNE 2018 **ADVANCED LEVEL**

Subject Title	Further Mathematics
Paper No.	2
Subject Code No.	0775

THREE HOURS

INSTRUCTIONS TO CANDIDATES

Answer ALL 10 questions.

For your guidance, the approximate mark allocation for parts of each question is indicated.

Mathematical formulae and tables published by the Board, and noiseless non-programmable electronic calculators are allowed.

In calculations, you are advised to show all the steps in your working, giving your answer at each stage.

REPLACEMENT

1. Given the differential equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 4e^{-x}$$

Find

i) the complementary function in the form $y = f(x)$. **(3 marks)**

ii) the particular integral in the form $y = g(x)$ **(3 marks)**

iii) the solution of the differential equation for which $y = 0$, $\frac{dy}{dx} = 1$ when $x = 0$. **(4 marks)**

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2. The position vectors of four non-coplanar points A, B, C and D relative to the origin, O, are **a**,**b**,**c** and **d** respectively, where

$$\mathbf{a} = 2\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$$
$$\mathbf{b} = \mathbf{i} + 6\mathbf{j} - \mathbf{k}$$
$$\mathbf{c} = 5\mathbf{i} + \mathbf{k}$$
$$\mathbf{d} = 3\mathbf{j} - 2\mathbf{k}$$

Find

(i) The vector equation of the plane containing A, B and C. **(5 marks)**

(ii) the volume of the parallelpiped with vertices A, B, C and D. **(4 marks)**

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3. (i) Given that two roots of the complex equation

$$z^5 = 1$$

are $z = \cos\frac{2\pi}{5} + i\sin\frac{2\pi}{5}$ and $z = \cos\frac{4\pi}{5} + i\sin\frac{4\pi}{5}$

Find the other three roots in the form $z = \cos\theta + i\sin\theta$, $-\pi < \theta \le \pi$ **(4 marks)**

(ii) Given also that $w = \cos\frac{2\pi}{5} + i\sin\frac{2\pi}{5}$,

and that the structure $(\{1, w, w^2, w^3, w^4\}, \times)$

forms a group, find the identity element and the inverse of each element. **(5 marks)**

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4. Using the definition of $\sec hx, x \ge 0$ interms of e^x, prove that

$$\sec h^{-1}x = \ln\left(\frac{1+\sqrt{1-x^2}}{x}\right), \quad 0 < x \le 1$$

Hence, or otherwise, show that

(i) $\frac{d}{dx}\left(\sec h^{-1}x\right) = \frac{-1}{x\sqrt{1-x^2}}$ **(3 marks)**

(ii) $\int_{\frac{1}{2}}^{1} \sec h^{-1}x\,dx = \frac{\pi}{3} - \frac{1}{2}\ln\left(2+\sqrt{3}\right)$ **(6 marks)**

5. Given the matrix M, where

$$M = \begin{pmatrix} -1 & 1 & 1 \\ 3 & h & -2 \\ h & 1 & 1 \end{pmatrix}$$

(i) Find the real values of h for which M is non-invertible. (3 marks)

(ii) For $h = 0$, find the image of the line

$\mathbf{r} = 2\mathbf{i} - 3\mathbf{j} - 5\mathbf{k} + \lambda(\mathbf{i} - \mathbf{j} + \mathbf{k})$, $\lambda \in \mathbb{R}$

under transformation M. (3 marks)

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6. (i) Find, d, the greatest common divisor of 81 and 21. (2 marks)

(ii) Hence, find the values of a and b, $a, b \in \mathbb{Z}$, such that

$81a + 21b = d$. (3 marks)

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7. (i) Show that the curve with polar coordinates (r, θ) where

$$r^2 = \frac{12}{3 + \sin^2 \theta}$$

is an ellipse, E, in the (x, y) plane, where

$E : 3x^2 + 4y^2 = 12$ (3 marks)

(ii) Find the equation of the normal to E at the point $\left(1, \frac{3}{2}\right)$. (3 marks)

(iii) This normal cuts the coordinate axes at A and B. Show that the area of triangle OAB is $\frac{1}{16}$. (4 marks)

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8. (a) Prove by contradiction that $\frac{1}{\sqrt{3}}$ is irrational. (6 marks)

(b) The function, f, is defined by

$$f(x) = \ln\left(\frac{1}{1 - x^2}\right), \; |x| < 1$$

Prove that the length of arc of the curve traced by f for $0 \le x \le \frac{1}{\sqrt{3}}$ is

$$\ln\left(2 + \sqrt{3}\right) - \frac{1}{\sqrt{3}}$$

(7 marks)

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9. A function, f, is defined by

$$f(x) = \frac{2x^2 + x + 1}{x+1}, x \neq -1$$

Given that $\lim_{x\to\infty}\left(\frac{f(x)}{x}\right) = a$,

find

(i) the value of a **(2 marks)**

(ii) $\lim_{x\to\infty}[f(x) - ax]$. **(2 marks)**

(iii) Hence, or otherwise, find the equation of the oblique asymptote to the curve $y = f(x)$. **(1 mark)**

(iv) Show that $f(x) \leq -7$ or $f(x) \geq 1$. **(3 marks)**

(v) Sketch the curve $y = f(x)$. **(3 marks)**

(vi) Find the point of symmetry of the curve $y = f(x)$. **(2 marks)**

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10. Given three sequences (u_n), (v_n) and (w_n) defined by

$$u_0 = 1,\ u_{n+1} = \frac{1}{3}(u_n + 2v_n),$$

$$v_0 = 12,\ v_{n+1} = \frac{1}{4}(u_n + 3v_n),$$

$$w_n = v_n - u_n.$$

(i) Show that (w_n) is a geometric sequence with positive terms. **(2 marks)**

(ii) Find the limit of the sequence w_n. **(2 marks)**

(iii) Show that u_n is increasing. **(2 marks)**

(iv) Show that v_n is dereasing. **(2 marks)**

(v) Hence show that $u_0 \leq u_n \leq v_n \leq v_0$. **(2 marks)**

(vi) Show that the sequences u_n and v_n each converges to l. **(2 marks)**

Another sequence (t_n) is defined by $t_n = 3u_n + 8v_n$

(vii) Show that t_n is a constant sequence, and hence find l. **(4 marks)**