This summary notes or let's say quick notes on sequences & series is destined to help students of the class of form 5 get some of the key points on what sequences & series are all about/

As earlier said, this is just a summary lesson note and we will be touhing jus the key points.

What are sequences and series?

A sequence

A sequence is a set of terms that progresses or regresses in a defined order. For example 1, 2, 3 …

A series

A series is formed when the terms of a sequence are added. For example 1+3+5+7.

Arithmetic progression (AP)

Qn arithmethic progression is a series or sequence which progresses or regresses with a common or constant difference between the terms denoted by *d*. For example, 1, 3, 5, 7… Here, the common difference d is 3 -1 or 5- 3 = 2 = *d*.

The terms of an AP

If the first term of an AP is a and the common difference is d, then the terms of the AP are, a, a+d, a + 2d, a + 3d … a + (n – 1)d.

Therefore, the number of terms of the AP is given by:

$$T_n = a + (n - 1)d$$

Example

Consider the series 2, +4, +6, +8 …., find the fifth term of this series.

Solution

$T_n = a + (n - 1)d$

The first term a = 2, d = 4 -2 or d = 6 – 4

$\Rightarrow$ d = 2

From $T_n = a + (n - 1)d$

$\therefore T_5 = 2 + (5 - 1)2 = 2 + 4(2)$

$\Leftrightarrow T_5 = 10$

Sum of the terms of an AP

The sum of the first n term of an AP whose first term is *a* and the common difference is *d* is given by;

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

Example:

Find the sum of the first 10 terms of the sequence 3, 10, 17 …

Solution:

The first term $a = 3$

The common difference $d = 10 - 3 = 17 - 10 \ = 7$

$S_{10} = \frac{10}{2}[2(3) + (10 - 1)7]$

$S_{10} = 5[6 + 63]$ = 345

The arithmetic mean AM

This is the average of a set of numbers which is calculated by dividing the sum of all terms by the number of terms.

Example:

Find the AM of 3, 4, 5, 6, 7

Solution:

The number of terms is 5,

$AM = \frac{3+4+5+6+7}{5} = \frac{25}{5} = 5$

If *a*, *b*, *c* are three consecutive terms of an AP, then *b* is called the arithmetic mean of *a* and *c*.

That is;

$$d = b - c = c - b$$

$$2b = a + c$$

$$\Rightarrow b = \frac{q + c}{2}$$

Geometric progression GP

A geometric progression is a series or sequence which progresses or regresses with a constant multiplier called the common ratio and is denoted by *r*. e.g 3, 6, 12, 24 … each term is obtaioned by multiplying the previous term by 2.

The common ratio r is therefore calculated as;

$$r = \frac{6}{3} = \frac{12}{6}\ \frac{24}{12} = 2$$

Terms of a geometric progression

If the first term of a GP is ***a*** and the common ratio is ***r***, then the terms of the GP are: ***a, ar,*** ar^2, ar^3, …ar^{n-1}

Therefore, the nth term of the GP is given by;

$$T_n = ar^{n-1}$$

Example:

Find the (th term of the series 16+8+4+2

Solution:

$$a = 16, r = \frac{8}{16} \ or \ \frac{4}{8} \ or \ \frac{2}{4} = \frac{1}{2}, n = 5$$

From $T_n = ar^{n-1}$

$$T_5 = (16)\left(\frac{1}{2}\right)^{5-1} = 1$$

Sum of the terms of a GP

The sum of the first ***n*** terms of a GP whose first term is ***a*** and common ratio ***r*** is given by;

$$S_n = \frac{a(1 - r^n)}{1 - r}, \text{Where } r < 1$$

OR

$$S_n = \frac{a(r^n - 1)}{r - 1}, \text{Where } r > 1$$

Example:

Find the sum of the first 8 terms of the sequence 2, 6, 18 …

Solution:

$$a = 2, \qquad r = \frac{6}{2} \ or \ \frac{18}{6} \Rightarrow r = 3. \ i.e \ r > 1$$

$$\therefore S_n = \frac{a(r^n - 1)}{r - 1}$$

$$\Rightarrow S_8 = \frac{2(3^8 - 1)}{3 - 1} = 6560$$

If ***x, y, z*** are the three consecutive terms of a Geometric progression, then ***y*** is the geometric mean of ***y*** and ***z***.

I.e $r = \frac{y}{x}$ or $\frac{z}{y} \Rightarrow \frac{y}{x} = \frac{z}{y} \Rightarrow y = \sqrt{xz}$

The Geometric Mean (G.M) of a series containing n observations is the nth root of the product of the values.

Consider, if x_1, x_2 .... X_n are the observation, then the G.M is defined as:

$$G.M = \sqrt[n]{x_1 x_2 \dots x_n}$$

The sum to infinity of a GP

If the number of terms in a GP is not finite, then the GP is called infinite GP.

The sum to infinity of a GP whose first term is ***a*** and common ratio ***r*** is given by ;

$$S_\infty = \frac{a}{1-r} \ where \ |a| < 1$$

The sum of the terms of a sequence

The ***n***th term of a sequence which is not necessarily an AP or GP is given by

$$T_n = S_n - S_{n-1}$$